

A Rule Based Biped Dynamic Walking

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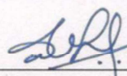
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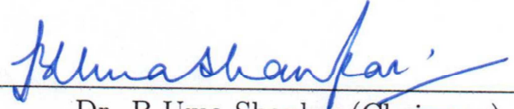
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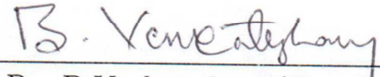
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To
The Almighty

Abstract

Dynamic walking approach has got its significance because of its energy efficiency in walking. Walking models are made using this approach which would consume energy as low as the energy required for human being walking. The basis of this dynamic walking is purely passive walking which takes no energy for walking.

For a simple compass model passive walking can be achieved only for particular initial conditions (angular positions and velocities) which are found by trial and error or from previous experience. Various ways are derived to make the model walk on a level ground by supplying external energy through some means i.e torques at hip joint and ankle joints which is called *active walking*. Two approaches are available for active walking, one is creating virtual slope and then by applying equivalent torques at ankle and hip as the functions of virtual slope; other approach is using torsional springs and dampers at hip as well as ankles such that the torques are given in terms of springs' stiffness coefficient and damping coefficient. The stability is analyzed based on ZMP position. When ZMP of the system falls within the foot support area then system is said to be stable.

Methods to make the walker start from stable condition are tried. Simple rules are derived to make the walker walk starting from stable configuration and to walk continuously. For this two concepts are followed. One is by ankle push-off concept. After the walker starts from standing still condition, the ankle push-off is given until the front leg crosses vertical. Another approach is to apply a constant angular velocity at the rear ankle until front leg crosses vertical. The analogy with a simple four-bar mechanism is used for simulation. Results are mathematically derived and compared with a dynamic simulation software results (ADAMS). Also different walking models are designed and tested in ADAMS to realize the practical walking scenario.

Simple rules are derived based on the physical behaviour of different models in ADAMS. Two such rules are made from the above discussed models and numerical simulations. Any walking model that is following these rules would walk continuously without stopping.

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Chapter 1

Introduction

1.1 Motivation

Legged robots are those which have human-like legs or animal-like legs. A robot having two legs is called *bipedal robot* or *biped* in short, and a robot with four legs as *quadruped* and that with six legs as *hexapod* etc.

Legged robots are effective in many cases than other robots. It is difficult for wheeled robots or tracked robots to perform some common activities like moving in rough terrains or for using staircases. Where as it is easy to do using bipeds. Bipedal robots will be better adapted to an environment that is usually destined for humans as its structure and shape imitates human beings.

Human beings uses very less energy to walk. So bipedal walking is also useful in reducing the amount of energy required for moving.

1.2 Understanding Human Walking

There has been a large influence of bipedal walking on the evolution of human beings which made them more efficient than any other animal. And that is the reason for there was interest to find how human beings walking is achieved, the kinematics, dynamics and energetics behind them. The early researches that were carried out in biomechanics enabled us some facts about human walking and the energy transfers that make human like walking possible. Realization of mechanical models that are inspired from the original human beings walking made us to develop equivalent methods like the muscles in humans act like actuators and shock absorbers during walking and running.

1.2.1 Terminology

Some terms are defined below which are generally used in robotic literature also used throughout this thesis.

Phase in which only one leg will be in contact with ground is called *Single Support Phase* (SSP). Phase where both legs will be in contact with the ground is called *Double Support Phase* (DSP).

While walking, the leg on which the walker is supported is called *stance leg* and the leg which is moving in the air is called *swing leg*.

The joint between two legs at the upper end is called *hip* and the joint between foot and leg are called *ankle*.

There are two events defined for one cycle of gait. The event *heel-strike* is the instance when a foot touches the ground. The event *toe-off* is the instance when foot starts leaving the ground.

1.2.2 Difference between walking and running:

It is observed that walking cycle starts when a contact is made on the ground with right heel followed by a left step then left heel comes in contact with ground followed by a right step. Here the walker maintains that at least one leg is in contact with ground through entire walking gait. SSP will be more than 80 percent in normal walking step. DSP will be less than 20 percent of total gait cycle duration. The duration of the DSP decreases with walking speed. When the DSP ceases to exist such that the phase is instantaneous then walking transitions into running [22].

While walking, the potential and kinetic energies of the human body are in opposition phases. The level of energy is maintained due to the transfer between the potential and kinetic energies. The potential and kinetic energies reach both a maximum and minimum value during the phases of double support. The potential energy reaches its maximum value when the mass center point goes over the grounded foot. When in a running gait, the potential and kinetic energies are synchronized and they reach their minimum and maximum values in the middle of the grounded phase and in the middle of the airborne phase. The muscles must therefore accomplish more mechanical work during running than during walking [22].

The design of bipedal robots and especially humanoid robots is naturally inspired from the functional mobilities of the human body. Nevertheless, the complex nature of the skeletal structure as well as the human muscular system cannot be reproduced in bipedal robots. The

number of internal mobilities is therefore limited to the essential and the actuating system must be simplified. A bipedal robot or a humanoid robot therefore has fewer DoF than a human body. The choice of the number of DoF for each articulation is very important.

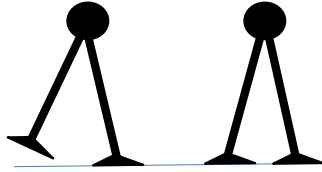


Figure 1.1: Single support and double support phases

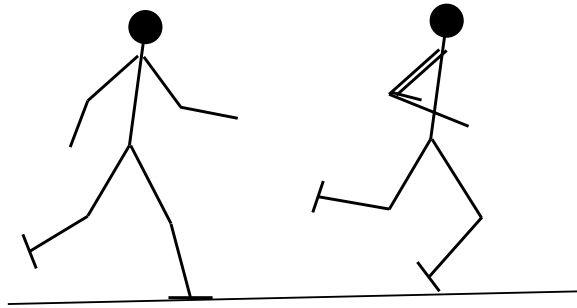


Figure 1.2: Human's walking and running

1.2.3 Static walking and dynamic walking:

Static walking maintains static equilibrium throughout its motion. It also imposes a speed limit, since cyclic accelerations must be limited in order to minimize inertial effects. Examples of static walkers are the Odex series [12]. Dynamic Walking is more like human walking. It exploits the natural dynamics of the motion for the movement of legs [2].

Chapter 2

Literature Survey

2.1 Walking Robots

Leonardo Da Vinci's machine 'Automatons' was the first ever known humanoid mechanism. In 19th century *steam man* (moved by steam-engine) was built by John Brainerd and the *Electric man* was built by Frank Reade Junior. At the beginning of the 20th century, the Westinghouse society made the Elektro humanoid [1].

Work of Kato and Vukobratovic was characterized by the design of relevant experimental systems. The first anthropomorphic robot, WABOT-1, was demonstrated in 1973 by I. Kato and his team at Waseda University, Japan [3]. Using a very simple control scheme it was able to realize a few slow steps in static equilibrium. The most recent robot The WABIAN-2R has 41 motorized joints. It is 1.53 m in height and weighs a total of 64.5 kg. The control is based on the ZMP feed forward drive (see Chapter on ZMP). Its average walking speed is 0.36 m/sec, with a period of 0.96 seconds per step.

The University of Lomonosov in Moscow and the University of Saint Petersburg built legged robots very early on robots [5, 6]. In USA, the first legged robot creations controlled by computers were created [7]. The hexapod CMU, built during the period 1980 - 1983, reached a maximum speed of 0.11 m/sec.

Following the early work of R.McGhee in the 1960s at University of Southern California, then in the 1970s at Ohio State University, M. Raibert started to study dynamically stable running at CMU (Carnegie Mellon University). In MIT (Massachusetts Institute of Technology) in the 1980s [8, 9] work was carried out on jumping robots. Miura and Shimoyama [10] developed the bipedal robot family called BIPER which was statically unstable but which had a dynamically stable walk. The analogy of an inverted pendulum was used to define its

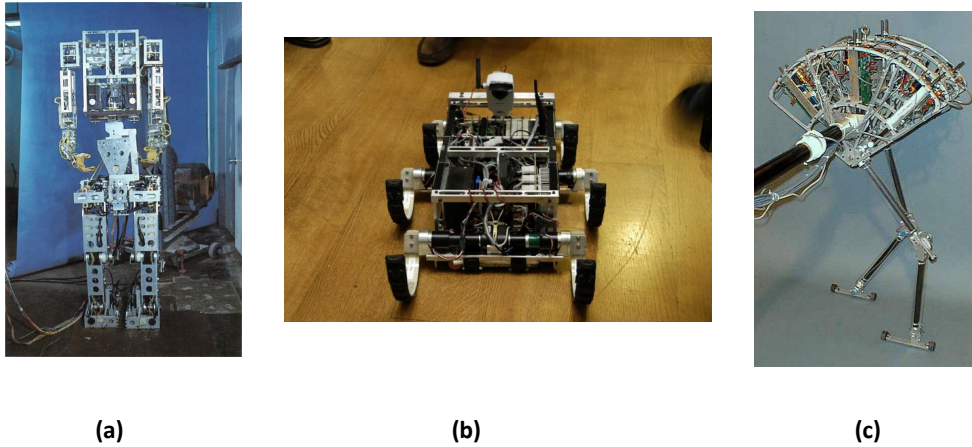


Figure 2.1: (a) WABOT1 (b) CMU Hexapod (c) MIT's Spring Flamingo [22]

gait. The idea of studying purely passive mechanical systems was pioneered by McGeer [13]. In his paper, McGeer introduced the concept of natural cyclic behavior, for a class of very simple systems, i.e. a plane compass on an inclined plane. Stable walking results from the balance between increase of the energy due to the slope and loss at the impacts.

With many extensions like adding trunk, feet and knees [13], semi-passive control, walking/running under actuated systems like the Rabbit [11] were made. The aim of the Rabbit project started in 1998 in France was to obtain walking and running gaits which were dynamically stable, based on a simple mechanical anthropomorphic structure with few DoF. The project resulted a five-bodied biped with four motors, two on the hip and two at the top of the thighs. The biped is maintained in the vertical position by a pivoting horizontal beam, and moves along a circular trajectory. This secure layout enabled them to perform successful walking and running trials for significant distances [11].

In Japan, the first humanoid robot, P2, was exhibited by Honda in 1996, followed by several more. The most advanced Humanoid Robots till now are said to be Honda's ASIMO (Advanced Step in Innovative Mobility), QRIO (Sony) and HRP (Kawada). The ASIMO robot is 1.4 m high, has 26 DoF and is moved by 26 electric motors. This robot uses a controlled trajectory approach to move its legs. In this approach the number of actuators and also the energy required for walking is high compared to human beings (for ASIMO it

is 20times) [1].

Very recently researchers at the Korean Institute of Science and Technology, created robot named HUBO which they claim is the smartest robot in the world among that were created up to now. This robot is linked to a computer via a high-speed wireless connection, the computer does all of the thinking for the robot [4].



(a)



(b)

Figure 2.2: (a) HONDA's ASIMO (b) KIST's HUBO [22]

2.2 Dynamic Walking Models

Industrial robotic manipulators are easily controllable, as the base is firmly attached to the floor and all joints possess powerful actuators. Control is a simple matter of accurate trajectory following. But for walking robots, the stability is always an issue as it is an inverted pendulum and the feet are not firmly attached to the floor and thus they may tip over.

McGeer [13] showed that a simple walking model consisting two legs hinged at a point (hip) can walk down a slope without any external actuation using gravity and dynamic nature of the swinging legs. He called this model as *Passive Dynamic Walking*.

Adding actuation to passive dynamic walkers result in highly efficient robotic walkers. Such walkers can be implemented at lower mass and use less energy because they walk

effectively with only a couple motors (one at hip and other at one of legs). This combination results greater efficiency in walking.

The passive dynamic walking approach is more promising in terms of efficiency and simplicity. Delft University made one more 2D autonomous biped based on passive dynamic walking called MIKE. It is a fully autonomous biped that can walk on a level floor with the same energy [24] efficiency as a human being. They used pneumatic actuation system for passive dynamic walking system to reduce the system weight for maximum efficiency.

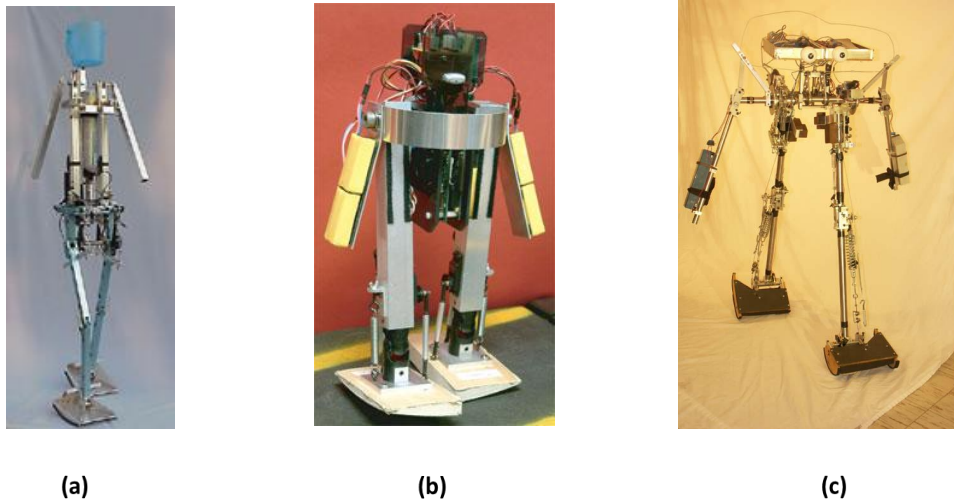


Figure 2.3: (a) Delft's Denise [29] (b) MIT's learning bipedal (c) Cornell's biped [14]

Researchers at Cornell University and MIT also have built robots that seem more close to the human gait. Cornell University's biped design aspects include the freely rotating hip joint with angle bisecting mechanism, freely rotating knee joints with latches, direct actuation of the ankles with a spring, release mechanism, reset motor, wide feet that are shaped to aid lateral stability and simple control algorithm [25].

MIT's learning biped is based on the simple ramp walkers. It has six internal degrees of freedom (two servo motors in each ankle and two passive hips), each arm is mechanically linked to the opposite leg and the body hangs passively. It uses reinforcement learning to acquire a control policy [14].

Researcher's at Delft University have built a biped robot based on passive dynamics called Denise. This biped stands $1.5m$ tall and is pneumatically powered. It has achieved a

Table 2.1: Comparison of costs of transport [14]

Model	Mech COT	Sp. COT	Walking speed(m/s)	Mass(Kg)
Cornell biped	0.055	0.2	0.4	13
Humans	0.05	0.19	1.25	60
Denise(Delft biped)	0.08	0.8	0.4	8
Honda ASIMO	1.6	3.2	-	52
Dribbel	-	1.01	0.22	9.2

walking speed of 4 m/sec. It has only one contact switch under each foot for sensors. When the right foot lands, the left knee is released and the hip muscles pull the leg forward. When the left foot touches the ground, the right leg is pulled.

2.3 Cost of Transport (COT)

There is one parameter by which one can compare the walking models in terms of the energy they consume for walking. The term is *Cost of transport* which is defined below. And Later we have compared the COT of some of the models that were explained above.

Energy efficiency in level-ground transport is quantified in terms of the dimensionless specific cost of transport, which is the amount of energy required to carry a unit weight a unit distance. In order to isolate effectiveness of mechanical design and controller from the actuator efficiency, we define two terms called *Specific energetic cost of transport*, which uses the total energy consumed by the system and *Mechanical COT* which considers only the positive mechanical work of the actuators.

$$COT = \frac{EnergyCost}{(BodyWeight)(DistanceTraveled)} \quad (2.1)$$

Passive dynamic walkers such as the Cornell efficient biped have the same specific cost of transport as humans, 0.20. Passive dynamic walkers have human-like gaits also. By comparison, Honda's biped ASIMO, which does not utilize the passive dynamics of its own limbs, has a specific cost of transport of 3.23. The Cornell's walker is almost as efficient as humans in walking [14].

Comparison of the costs of transport for different models is given in the Table. 2.1.

2.4 Objective of the Thesis

While examining the history above and the present state of the art, it is clear that robotic researchers are now facing a challenge. Very nice technological achievements are available, especially biped robots. However, the ability of these systems to walk truly autonomously on uneven and various terrains in a robust way, i. e., in daily life, remains to be demonstrated. This thesis focuses mainly on these areas. The thesis tries to find some simple and robust rules or logics in biped dynamic walking. We could identify in almost all of the models some problems in common even though some try to overcome them .

We want to design simplest walking model which walks on level ground. Almost all the existing bipedal walkers based on passive dynamic walking starts walking only through certain initial conditions and they have other problems like stopping or falling while walking when a disturbance comes. To overcome these problems we want to derive algorithms for a simplest walker. Our objectives of the work include,

- Deriving the walking algorithms for active walking level ground for a simple walking model such that
 - It starts walking from any configuration and any initial conditions
 - It starts walking again even if it stops in between

Chapter 3

Concept and Background Work

We start with a study of a simple inverted pendulum which forms the basis for the analysis of any walking model. Then we analyze a double inverted pendulum which is closely comparable with walking without knees. Compass gait model is the simplest model of a dynamic walker. We can have walking models with knees, with a torso on hip with joints at ankle or combination of any of these etc. But now we are considering only the simplest case of compass gait model and we will see different cases possible with this model. If we can make this model walk on a slope without any external energy or power supply, that is called *Passive dynamic walking*, if it walks with external power supply it is called *Active walking*.

3.1 Simple Inverted Pendulum

An inverted pendulum is a pendulum which has its mass above its pivot point. A normal pendulum is stable when hanging downwards, but an inverted pendulum is inherently unstable and must be actively balanced in order to remain upright either by applying a torque at the pivot point or by moving the pivot point horizontally. An inverted pendulum model is shown in Fig. 3.1.

The equation of motion is similar to that for a simple pendulum except that the sign of the angular position

$$\ddot{\theta} - \frac{g}{L} \sin\theta = 0, \quad (3.1)$$

where g is the acceleration due to gravity and L is the length of the pendulum. Thus, inverted pendulum will accelerate away from the vertical unstable equilibrium in the direction initially displaced and the acceleration is inversely proportional to the length. Tall pendulums fall more slowly than short ones.

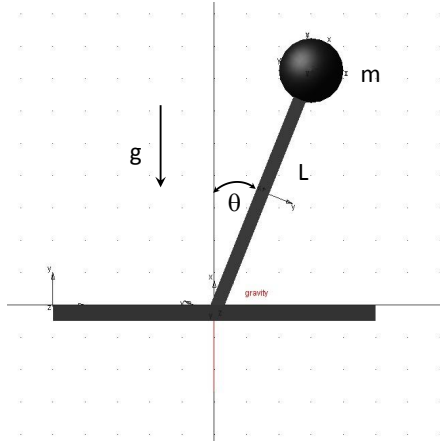


Figure 3.1: Simple inverted pendulum

3.2 Rimless Wagon Wheel

Most wheels that people are used to have a rim. This means that the wheel has a center hub and spokes that connect that center hub to a smooth outer rim. This rim is what is covered with a rubber tire on an automobile or what rests on the tracks when talking about a train. A rimless wheel shown in Fig. 3.2, on the other hand has the central hub and the spokes pressing outward from it, but the spokes are not connected to a rim because there is no rim. This means that when the wheel turns it rests solely on the spokes as it moves.

Rimless wheel is the simplest model of passive dynamic walking. This model has no hinges it has rigid legs and a center mass. This model provides most basic and essential features of walking as it has a discrete set of points of contacts with ground and at the end of each cycle there will be energy conversion because of the collision of leg with the ground.

Rimless wheels imitate are legs which are adept at moving over rough terrain but which lack the speed of wheels. By putting these things together rimless wheels can move quicker than legs on flat surfaces and they can navigate better on rough terrain.

The following motions are possible for a rimless wheel going down the slope.

- Rocking to a stop on two spokes
- Rolling down hill at a constant speed swinging up and balancing on one spoke (takes infinite time)
- Rolling down hill at ever increasing speeds (for large slopes only)
- Swinging up and balancing on one spoke (takes infinite time)

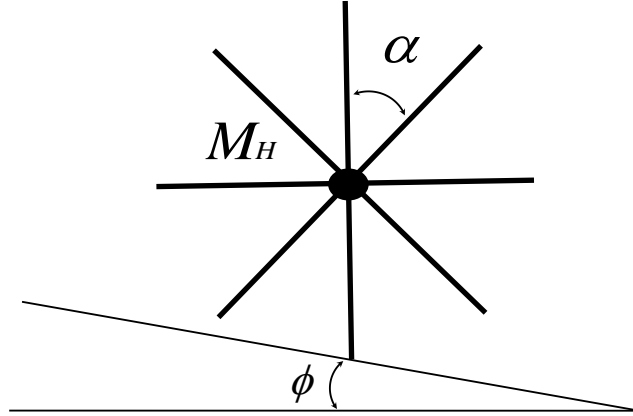


Figure 3.2: Rimless wheel model

For number of spokes 'n', $\alpha = \left(\frac{2\pi}{n}\right)$ and change in height of the center of mass of the wheel, $\Delta Z = 2L \sin \alpha \sin \left(\frac{\beta}{2}\right) = 2L \sin \alpha \sin \left(\frac{\pi}{n}\right)$

Therefore gain in potential energy of the wheel is given by

$$E_{GAIN} = 2MgL \sin \alpha \sin \left(\frac{\pi}{n}\right) \quad (3.2)$$

The relation between the pre and post impact velocities is given by

$$\dot{\theta}_n^+ = \mu \dot{\theta}_n^- \quad (3.3)$$

Kinetic energy loss

$$E_{LOSS} = \frac{1}{2}I_B(\dot{\theta}^-)^2 - \frac{1}{2}I_B(\dot{\theta}^+)^2 = (1 - \mu^2)\frac{1}{2}I_B(\dot{\theta})^2 \quad (3.4)$$

$$\mu = \frac{I_c + mL^2 \cos \left(\frac{2\pi}{n}\right)}{I_c + mL^2} \quad (3.5)$$

But we are neglecting the inertia of the spokes as we are considering the spokes as massless and so,

$$\mu = \cos \left(\frac{2\pi}{n}\right) \quad (3.6)$$

3.2.1 Equations of motion

L is the length of the spoke of the wheel.

$$\ddot{\theta} - \frac{g}{L} \sin \theta = 0 \quad (3.7)$$

If the wheel is given certain initial velocity, the stable rotation is achieved. For ground slope ϕ , EOM will be:

$$\ddot{\theta} - \frac{g}{L}(\theta + \phi) = 0 \quad (3.8)$$

We can calculate minimum angular velocity to be given to wheel for steady state to be achieved. Which is given by

$$\dot{\theta} > \frac{1}{\cos \alpha} \sqrt{\frac{g \sin \phi}{L \cos \alpha}} \quad (3.9)$$

The inequality can be solved for the value of ground slope for a particular number of spokes so that stable rolling is achieved.

3.3 Double Inverted Pendulum

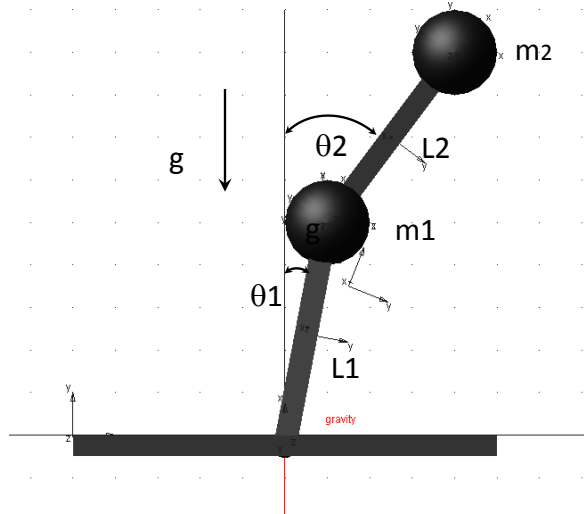


Figure 3.3: Double inverted pendulum

A double inverted pendulum is a vertical inversion of a normal double pendulum as shown in Fig. 3.3. Unlike a normal double pendulum a double inverted pendulum is inherently unstable. A way to make it remain upright under perturbation is to attach it to an oscillatory base. If the oscillation is strictly vertical, its frequency is high enough and its amplitude is large enough then double inverted pendulum can be stabilized. A double inverted pendulum is much more complicated than an inverted pendulum.

The above model shows a double inverted pendulum with one mass connected at the end of one link and connected with another link which has a second mass at the tip. The second

mass movement is not regular in this case. If the model has to be stable, a torque should be supplied at the joint which can be calculated from the equations of motions.

To derive its equations of motion, one of the possible ways is to use Lagrange equations:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \left(\frac{\partial L}{\partial \theta} \right) = Q \quad (3.10)$$

Where $L = T - P$, is is a Lagrangian, Q is a vector of generalized forces (or moments) acting in the direction of generalized coordinates and not accounted for in formulation of kinetic energy T and potential energy P . Kinetic and potential energies of the system are given by the sum of energies of its individual components

$$T = T_1 + T_2 \text{ and } P = P_1 + P_2.$$

Here '1' refers to first link and '2' to second link of the pendulum. Kinetic energies of both the links are given by,

$$T_1 = \frac{1}{2}m_2[(L_1\dot{\theta}_1\cos\theta_1)^2 + (L_1\dot{\theta}_1\sin\theta_1)^2] + \frac{1}{2}I_1\dot{\theta}_1^2 \quad (3.11)$$

$$T_2 = \frac{1}{2}m_2[(L_1\dot{\theta}_1\cos\theta_1 + L_2\dot{\theta}_2\cos\theta_2)^2 + (L_1\dot{\theta}_1\sin\theta_1 + L_2\dot{\theta}_2\sin\theta_2)^2] + \frac{1}{2}I_2\dot{\theta}_2^2 \quad (3.12)$$

and potential energies are given by the expressions,

$$P_1 = m_1gl_1\cos\theta_1, P_2 = m_1gl_1\cos\theta_1 + m_2gl_2\cos\theta_2 \quad (3.13)$$

Here, l_1 and l_2 are the distances of centers of masses of the links from their ends. Two equations of motions can be obtained from the above expressions.

3.4 Compass - Gait Bipedal Walker

The compass gait is a simple bipedal robot with two legs and a pin joint connecting them at the hip. It describes human locomotion (gait) very closely than any other models. It is shown in Fig. 3.4. The state of the robot is described by the position and velocities of the legs.

In the above problem, compass gait model is also considered as a double inverted pendulum with three masses. Two masses one each in the centers of the links called leg masses which are equal and third at the joint of two links called hip joint.

$$L = a + b \quad (3.14)$$

Equations of motion,

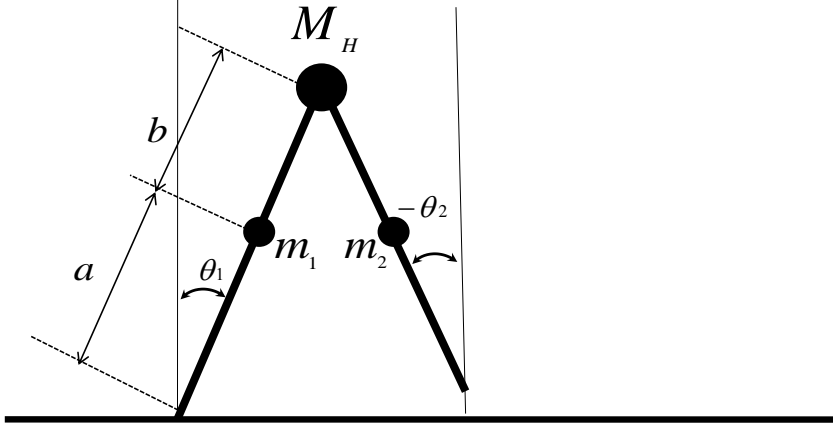


Figure 3.4: Compass gait model

Now the equations of motions of this system can be derived from Lagrange's method stated above. As the model is a two degree of freedom system we get two equations of motions which can be written in matrix form as shown below.

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = \tau \quad (3.15)$$

where $\theta = [\theta_1 \ \theta_2]^T$ is the angle vector of robot's configuration. The detailed matrices are given below.

$$M = \begin{bmatrix} ML^2 + mL^2 + ma^2 & -mbl \cos(\theta_1 - \theta_2) \\ -mbl \cos(\theta_1 - \theta_2) & mb^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & -mbl \sin(\theta_1 - \theta_2) \dot{\theta}_2 \\ -mbl \sin(\theta_1 - \theta_2) \dot{\theta}_1 & 0 \end{bmatrix}$$

and

$$g(\theta) = \begin{bmatrix} -(ML + mL + ma)g \sin \theta_1 \\ mbg \sin \theta_2 \end{bmatrix}$$

These two equations of motions can be solved for finding positions and velocities of the two links (legs). And the torques on the both the legs are given by

$$\tau = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 + u_2 \\ u_2 \end{bmatrix}$$

The stance leg rotates about a hinge and swing legs moves freely. In the gait any leg tip can touch ground. This case is considered as collision of mass with the ground. To find the post impact velocities of the legs and energy loss at collisions the impact is assumed to be purely inelastic. Thus we can find the post impact velocities given pre-impact velocities. These can be calculated from two conditions given below:

- Angular momentum of whole system is conserved about collision point. i.e. point where stance leg touching the ground
- Angular momentum of stance leg is conserved about hip joint.

From these two statements we can get the relation between pre impact and post impact angular velocities of the legs,

$$Q^+(\theta_1, \theta_2)(\dot{\theta})^+ = Q^-(\theta_1, \theta_2)(\dot{\theta})^- \quad (3.16)$$

where

$$Q^+(\theta_1, \theta_2) = \begin{bmatrix} -mab + 2maL \cos(\theta_1 - \theta_2) + M_H L^2 \cos(\theta_1 - \theta_2) & 0 \\ -maL \cos(\theta_1 - \theta_2) & -mab \end{bmatrix}$$

$$Q^-(\theta_1, \theta_2) = \begin{bmatrix} maL - mab \cos(\theta_1 - \theta_2) + M_H L^2 & 0 \\ maL & -mab \end{bmatrix}$$

and

$$(\dot{\theta})^+ = \begin{bmatrix} \dot{\theta}_1^+ \\ \dot{\theta}_2^+ \end{bmatrix}$$

$$(\dot{\theta})^- = \begin{bmatrix} \dot{\theta}_1^- \\ \dot{\theta}_2^- \end{bmatrix}$$

From the above relation we can get expressions for post impact angular velocities,

$$(\dot{\theta})^+ = [Q^+(\theta_1, \theta_2)]^{-1}[Q^-(\theta_1, \theta_2)](\dot{\theta})^- \quad (3.17)$$

This is the basis of a compass gait bipedal walker. Now we can use this model to analyze different cases in passive and active dynamic walking. But there are a few assumptions considered for the above model which we also going to consider in the models from now.

3.4.1 Assumptions:

- Foot scuffing is neglected

When the walker's swing legs move from one end to another end, there is a possibility that the leg touches the ground and because of this some energy is lost due to friction between leg and ground. This is called *Foot scuffing*. In our models we are neglecting this foot scuffing. In practical this can be achieved by making our model walk on surface little above the ground.

- All Joints are frictionless

We have joints at the connection of both links at springs dampers and some other places in the models we see below. We consider all the joints to be frictionless so that losses need not be considered for energy calculations.

- In the entire system motion we assume that the motion is planar i.e. the system moves only in XY plane of the coordinate system.

3.5 Passive Dynamic Walking Down the Slope

This term is coined by Mc.Geer in 1989. If we keep the compass gait model we studied in the first chapter on a down slope and is given some intimal speed, at some angle of ground slope it starts walking on its own as shown in Fig. 3.5. Here the energy lost dew to potential energy lost supplied the required kinetic energy to move the model and make it walk. And so there is no need for external energy supply. The robot must walk down a decline and the conversion of potential energy to kinetic energy drive the motion. Steady state is attained by the balance of energy losses in the foot collisions and friction either at the foot-ground interface or internal joints.

In the walker there is energy storage as kinetic energy (while moving) and potential energy (because of gravity). These energies are exchanged between each other. Potential energy is at its highest when the swing leg passes the stance leg whereas the kinetic energy is minimal at that point (and maximal right after foot impact). Kinetic energy is maximal after impact. It is not maximal directly after impact because the foot to ground impact

results in some kinetic energy being lost. It is minimal when the swing leg passes the stance leg (but not zero because there is some kinetic energy storage in the swing leg). It is not at the exact same point as when kinetic energy is minimal as would be expected. The reason for this is that the big drop in kinetic energy does not come from the energy exchange between potential and kinetic energy. To compensate for this loss of energy we need to inject energy into the system. For example, using motors. In this case, torques acting on both ankle and

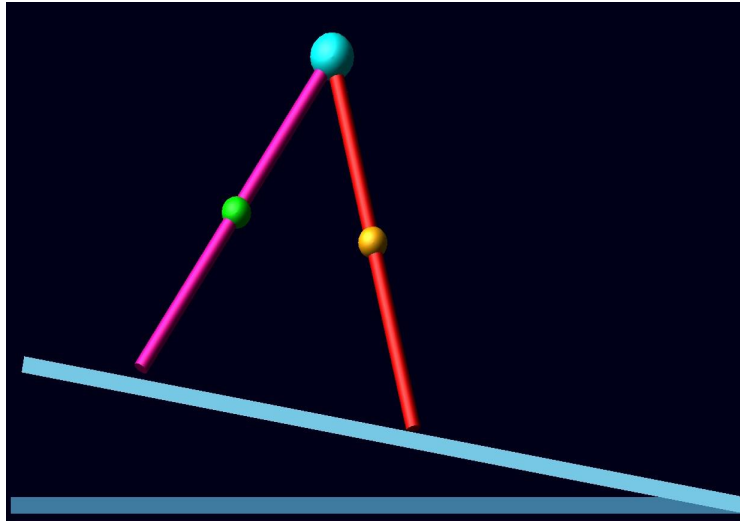


Figure 3.5: Passive dynamic walker model

hip will be zero. The energy required for moving the swing leg forward here is given by the gain in kinetic energy of the biped. But total energy of the system remains the same. For certain initial conditions we find whether the biped attains a stable gait or not with the help of a program. Plots for potential, kinetic and total energy were drawn for a step for comparison. COT in this case will be *zero* as it is not taking energy from external sources.

The above differential equations are solved for particular model and initial conditions. For one step position of the tip of both the legs and angular velocity change of tips of both the legs are plotted.

Typical model considered for simulation is $M_H=10\text{kg}$ $m=5\text{kg}$ $L=1\text{m}$ $a = b=0.5\text{m}$

Initial conditions for solving the differential equation are given as follows

Positions of the both legs, $\theta_1 = -0.2720$ and $\theta_2 = 0.2720$,

Angular velocities of both the legs, $\dot{\theta}_1 = 1.09$ and $\dot{\theta}_2 = 0.3$

We have solved the governing differential equations using the intimal conditions taken above and found the positions and angular velocities of the two legs for one step. We can also calculate potential, kinetic and total energies of the walker for one step at different

positions. The cycle is repeated for many times until a stable gait velocity is reached. If it cannot reach the stable gait then we try with different positions and angular velocities (initial conditions).

We make some assumptions while solving this problem, that is foot scuffing. Which is already explained. There may be possibilities of achieving *bifurcation* (two different gaits for one set of initial conditions). But we are avoiding the case of bifurcation here for simplicity.

All the plots were shown below. From the above plots we can see, total energy of the system always remains same in one step. At the end of each step some energy is lost because of the inelastic collision of swing leg with the ground. At that moment leg is interchanged and the swing leg becomes the new stance leg.

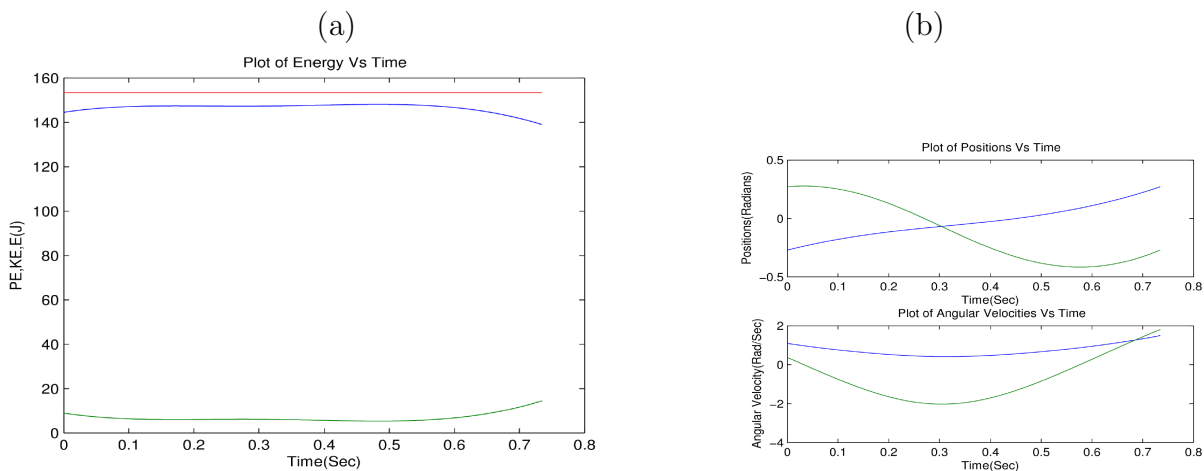


Figure 3.6: (a) Energy changes in one step (b) Positions and angular velocities of swing leg and stance leg

3.6 Level Ground - Virtual Slope Approach

In this case biped is on level ground and so it cannot walk by taking the help of gravity as in the case of sloped ground. But a virtual slope can be created by providing enough amount of torque to the biped to create the required slope. If we want to create a virtual slope then the equations of motions will change accordingly. We will find the torque required for the bipedal to attain stable gait and also find the kinetic, potential and total energy change for a step. COT is also calculated.

Here ϕ is the virtual slope created. Torques are calculated from the equations

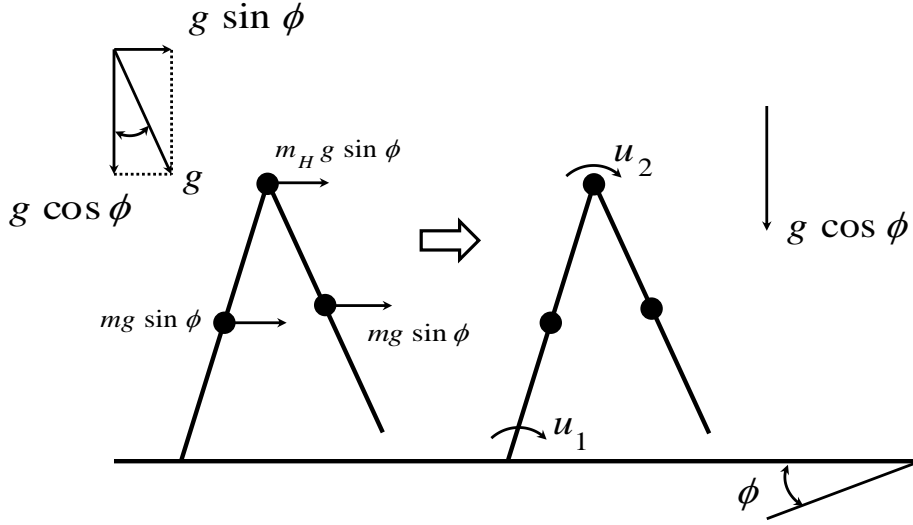


Figure 3.7: Simple walker model with virtual slope [14]

Simulations are done for the same values as in the above case and virtual slope value is taken as 4° .

3.7 Level Ground- Springs and Dampers Approach

Above problem can be solved by providing the torques required for the biped to take steps by means of one spring and one damper each at hip and the ankles of each leg as shown in Fig. 3.8. Ankle springs will be actuated when the leg is acting as stance leg. Critical damping is applied.

EOM is same as in the case of virtual slope approach. But values of the torques are computed from springs' stiffness and dampers' damping coefficients.

The values of the torques at hip and ankle are given by the formulae,

$$\tau_1 = -K_a\theta_1 - C_a\dot{\theta}_1 + K_h(\theta_2 - \theta_1) + C_h(\dot{\theta}_2 - \dot{\theta}_1)$$

$$\tau_2 = -K_h(\theta_2 - \theta_1) + K_hP + C_h(\dot{\theta}_2 - \dot{\theta}_1)$$

$$(3.18)$$

$$(3.19)$$

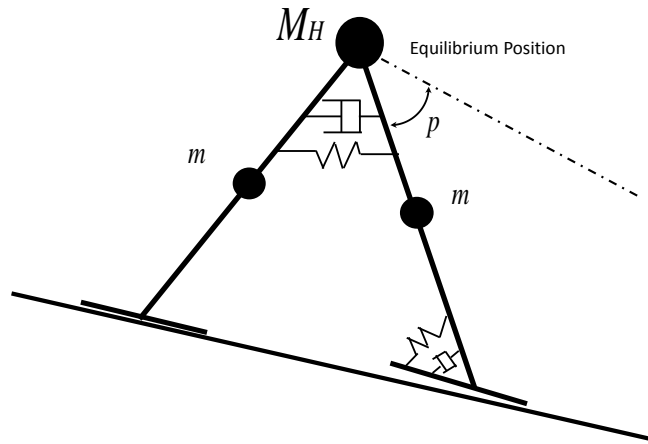


Figure 3.8: Walker with springs and dampers at ankle and hip

Here K_a, K_h are the stiffness's of both ankle spring and hip spring respectively. And C_a, C_h are the damping coefficients of both ankle damper and hip damper respectively. From simulation results it is found that the walker is efficient enough to generate stable gait when springs' stiffness and damping coefficients are high enough (approaching infinity). It is also found that for simple passive dynamic walker the power requirement is too high. Due to these reasons this model is not economical and also not preferred.

Energy losses:

- Foot to ground impact loss
- Energy dissipation by dampers

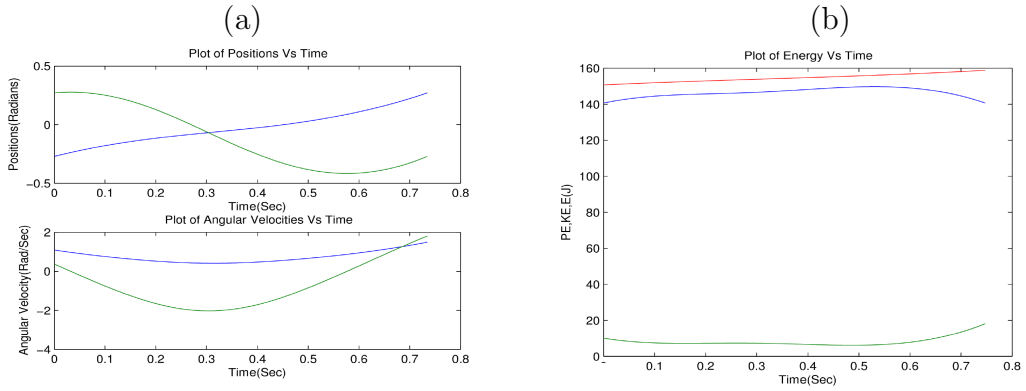


Figure 3.9: (a) Positions and velocities of legs (b) Energy changes in one step

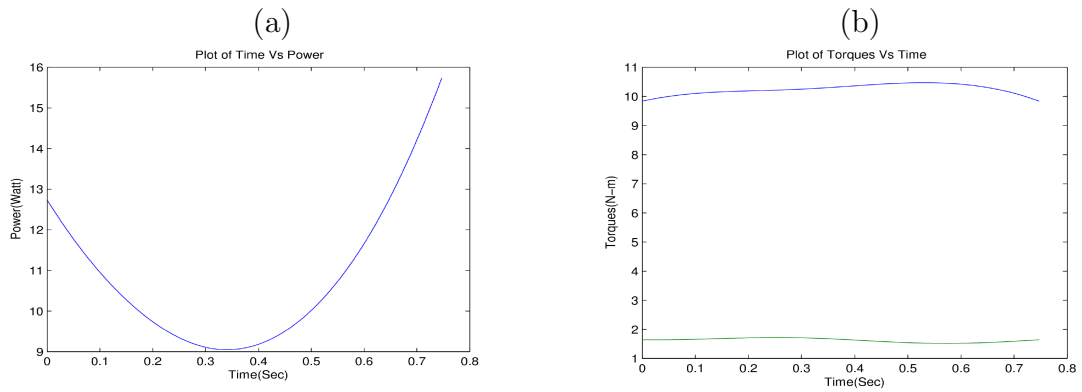


Figure 3.10: (a) Power plot (b) Torque changes

Chapter 4

Stability - Zero Moment Point

4.1 Introduction

Irrespective of their structure and number of degrees of freedoms involved, the basic characteristics of all the biped locomotion systems are as follows.

- The possibility of the rotation of the overall system about one of the foot edges caused by strong disturbances, which is equivalent to the appearance of an un powered DOF
- Gait repeatability (symmetry), which is related to regular gait only
- Regular interchangeability of single and double support phases. Double support phase is stable where single support phase is statistically unstable where only one foot of the mechanism is in contact with the ground while the other foot is being transferred from back to front positions

All of the biped mechanism joints are powered and directly controllable except for the contact between the foot and the ground (which can be considered as an additional passive DOF), where the interaction of the mechanism and environment only takes place. This contact is essential for the walk realization because the mechanism's position with respect to the environment depends on the relative position of the foot/feet with respect to the ground. The foot cannot be controlled directly but in an indirect way, by ensuring the appropriate dynamics of the mechanism above the foot. Thus, the overall indicator of the mechanism behavior is the point where the influence of all forces acting on the mechanism can be replaced by one single force. This point was termed the *Zero Moment Point* (ZMP).

ZMP plays a key role in

- In determining the proper dynamics of the mechanism above the foot to ensure a desired ZMP position.
- In determining the position of ZMP for the given mechanism motion

4.1.1 Box resting on ground

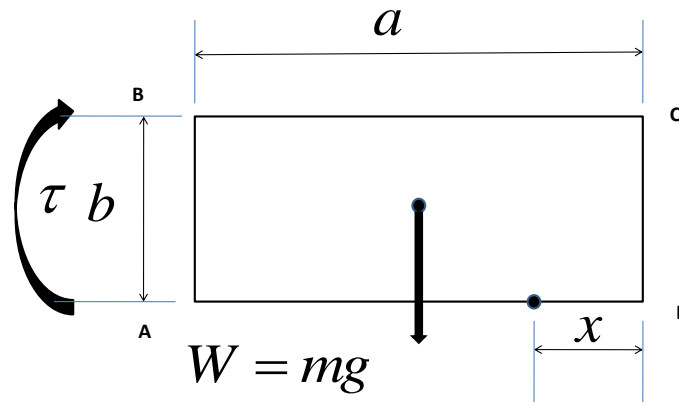


Figure 4.1: ZMP of a box resting on ground

A box of dimension 'a' and 'b' as shown in Fig. 4.1 is considered with its edge of dimension 'a' resting on ground. Now a torque is acted up on the box to rotate it in the clockwise direction. We need to find a point on the side CD where the moment of all the forces becomes zero about the point if the box is rotation about point 'D'.

We can calculate the value of 'x' from force balance and momentum balance equation at any particular position of the box.

Considering different positions of the box,

$$x = \frac{\tau}{mg} - \frac{a}{2} \quad (4.1)$$

It is concluded that the box is stable when the zero moment point lies before point 'D' the box is stable (Rotates firmly about 'D').

4.1.2 Box resting on ground acted by a force

In this case a force is acting on the box along with the torque applied in the clock wise direction as shown in Fig. 4.2. Zero moment point can be computed from force and momentum balance at point 'Z'.

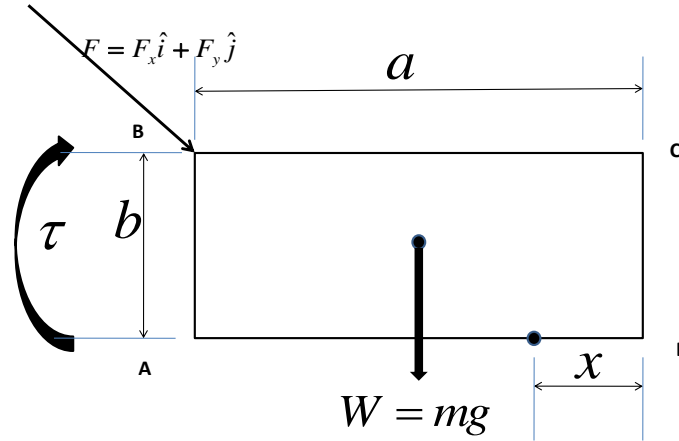


Figure 4.2: ZMP of a box resting on ground acted by a force

$$F = F_x \hat{i} + F_y \hat{j} \quad (4.2)$$

$$x = \frac{(\tau - F_x b/2 - (F_y + mga/2))}{(F_y + mg)} \quad (4.3)$$

4.1.3 Box resting on ground acted by a three dimensional force and torque

We consider a box of dimensions a , b and c as shown in Fig. 4.3. The same problem which we have solved for a rectangle above we are solving for a three dimensional box. We assume a resultant moment on the box acting in one direction τ , and also a resultant force of all the forces acting on the box as \vec{F} at a point which is at height h from the ground. The total force will be acting on the symmetric line with Y-axis and we assume it is at a distance of d in X-direction. The box has its self weight $\vec{W} = mg$. Now we are interested to find the position of ZMP(Z) of the box when it is rotated through one edge. The expressions for all the position vectors, forces and moments acting on the box,

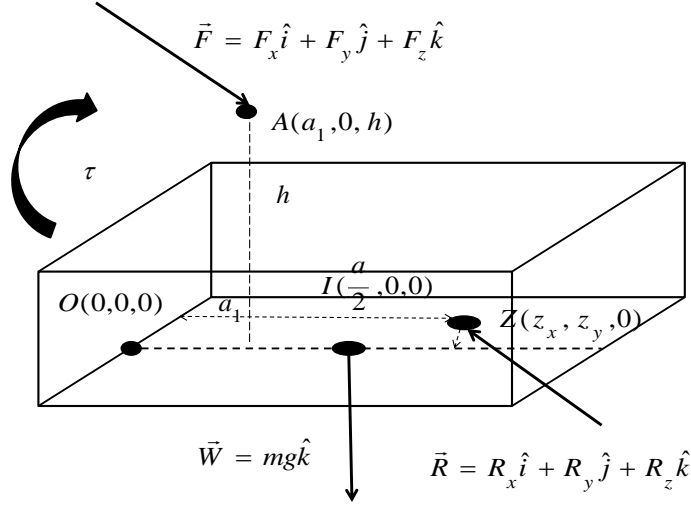


Figure 4.3: ZMP of a box resting on ground acted by a 3-D force and 3-D torque

Forces, $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$, $\vec{W} = mg \hat{k}$ and $\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$

Moment, $\tau = \tau_x \hat{i} + \tau_y \hat{j} + \tau_z \hat{k}$

We take the origin to be at O , from this we can write position vectors of all the points(A , Z and I)

$$\vec{OA} = d \hat{i} + h \hat{k}, \vec{OI} = \frac{a}{2} \hat{i}, \vec{OZ} = Z_x \hat{i} + Z_y \hat{j}$$

As we assume the ZMP lies on ground plane, it will not have Z co-ordinate.

Now we can take total force balance in all three three directions to get the reaction force components,

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

This gives,

$$R_x = F_x, R_y = F_y \text{ and } R_z = F_z + mg$$

Similarly if we can take total moment of all the forces and moments about point Z , But If Z has to be considered as ZMP, then this moment should be equated to zero.

$$\sum M_z = \tau + (\vec{ZA} \times \vec{F}) + (\vec{ZI} \times \vec{W}) = 0 \quad (4.4)$$

This gives x and y coordinates of the Z ,

$$Z_x = \frac{\tau_y - a_1 F_z - h F_x - mg \frac{a}{2}}{F_x - mg} \quad (4.5)$$

$$Z_y = \frac{\tau_x - hF_y}{F_x - mg} \quad (4.6)$$

4.1.4 ZMP for a bipedal walking model

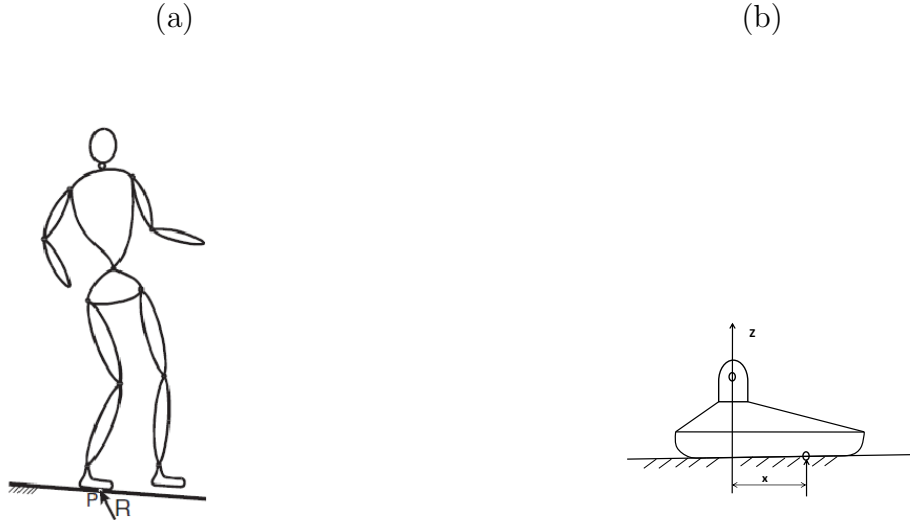


Figure 4.4: (a) Human foot resting on ground (b) ZMP direction

4.2 ZMP Analysis for the Walking Models

4.2.1 Virtual slope walking model

Now we will try to find the ZMP for the walker model with virtual slope. In the previous section we have tried to find the solution for walking condition of the model and then we found the values of the virtual slopes for which the walker achieves a stable gait. Now for the same values we will find the ZMP values to find whether the model is stable at that configuration and for that values of springs' stiffness. For 4° virtual slope with the initial conditions $(-0.2989, 0.2989, 1.1413, 0.2167)$ the variation of ZMP during one step is found after the model achieved stable gait. The minimum and maximum ZMP values came out to be -8.7cm and 8.9cm respectively. If we consider a foot that is 30cm in length and ankle exactly at halfway then this walker's ZMP is well within the foot throughout the step duration and so it is stable. The values of ZMP are evaluated for other values of virtual slope

also. It is observed that with increase in virtual slope the range of ZMP is also increased and a plot for virtual slope values from 1^0 to 5^0 is shown. However for different values of virtual slope the initial conditions and other parameters would be different. The variation is shown in Fig. 4.5.

Value of ZMP is given as

$$Z = \frac{\tau_1 + \tau_2 - g \sin \theta (m_H L + ma)}{(m_H + m)g} \quad (4.7)$$

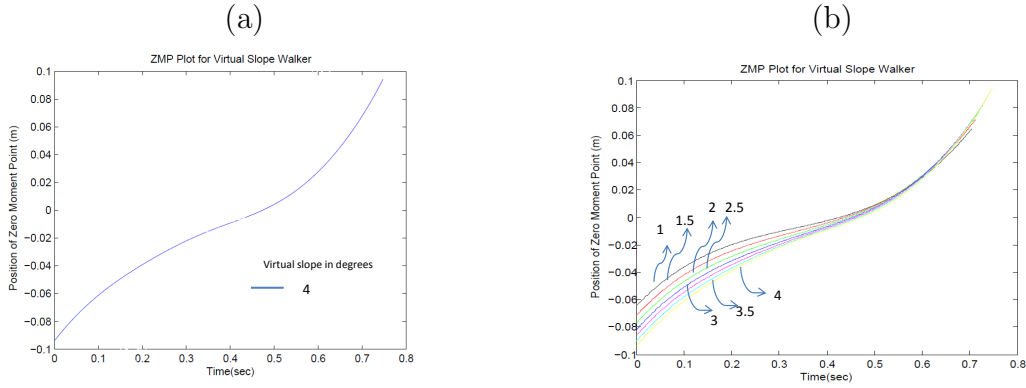


Figure 4.5: (a) Variation in ZMP for virtual slope 4^0 (b) ZMP variation for different values of virtual slope

4.2.2 Walking model with springs and dampers at hip and ankle

Now we will try to find the ZMP for the walker model with springs and dampers at ankle and hip that is explained in section 2.3.

The values of the torques at hip and ankle are given by the formulae,

$$\begin{aligned} \tau_1 &= -K_a \theta_1 - C_a \dot{\theta}_1 + K_h (\theta_2 - \theta_1) + C_h (\dot{\theta}_2 - \dot{\theta}_1) \\ \tau_2 &= -K_h (\theta_2 - \theta_1) + K_h P + C_h (\dot{\theta}_2 - \dot{\theta}_1) \end{aligned} \quad (4.8)$$

Here K_a, K_h are the stiffness's of both ankle spring and hip spring respectively. And C_a, C_h are the damping coefficients of both ankle damper and hip damper respectively.

Through programme values of ankle spring's stiffness is changed for checking if the model has a solution, if the model achieves a stable gait and if it is stable or not by finding the

ZMP. Here we assumed the foot to be 30 *cm* in length and the ankle joint exactly at half way.

Formula for finding the ZMP will be same as in the case of virtual slope walker except that the torques are found out from the spring and damper constants.

$$Z = \frac{\tau_1 + \tau_2 - g \sin \theta (m_H L + m_a)}{(m_H + m)g} \quad (4.9)$$

For one particular solution which we achieved earlier (stable gait) we tried to find the variation in ZMP throughout a step after the walker achieved steady gait. The variation comes out to be between -16.5 *cm* to 3 *cm* for $K_a = 7500$ and $K_h = 5000$. Which means that if we consider a foot and ankle with ankle after 16.5 *cm* of the foot and before 3 *cm* then the walker is stable. But we have a foot which is 30 *cm* in length and ankle half the way. So we have found out the values of ZMP for different values of ankle spring stiffness and hip spring stiffness.

If we observe in the ZMP variation plot the ZMP is tend to be more negative. The reason for this is until swing leg crosses the stance leg both the legs are behind the foot and ankle joint. The ZMP becomes positive only after stance leg crossing the vertical as the crossing of both legs is before the stance leg crossing vertical.

It is observed that with increase in the stiffness of the ankle spring the range of ZMP is reduced and the maximum and minimum values also reduced. But with increase in the stiffness of the hip spring the range of ZMP and the minimum and maximum values of the ZMP are increased. So we have made a surface plot by taking variation in both the stiffness values. Then we can find for what values of K_a and K_h the ZMP is within the foot region (-15 *cm* to 15 *cm*) as shown in Fig. 4.6.

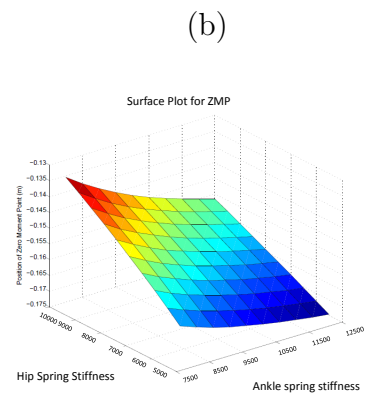
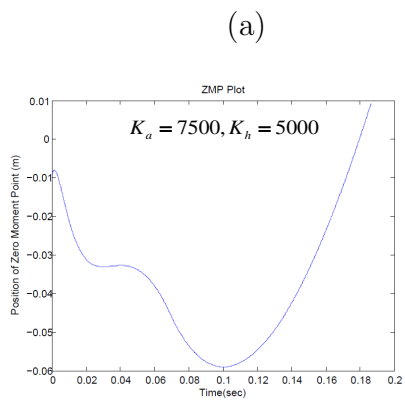


Figure 4.6: (a) Variation in ZMP for Spring damper walker (b) ZMP surface plot for different values of ankle and hip spring stiffness

Chapter 5

Friction and Contact Models - Conditions

5.1 How to Start From Zero KE Position?

Here we considering a case where the walker has stopped in between because of any disturbances or some other reasons. Let us consider it has stopped at its initial configurations with two legs making equal angles with the vertical. Now we need to find some means to make the walker walk from this position. Let us consider a simple case where walker has a

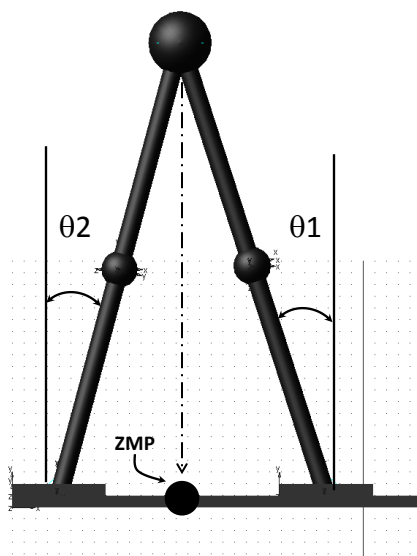


Figure 5.1: Simple walker model with massless legs

hip mass and two massless legs in the configuration shown in above figure. We can observe

that, for any values of torques at hip and ankles the zero moment point for this configuration will be outside the foot. Because the walker is static and its ZMP is same as its Center of gravity which is obviously outside the foot region. So for any torque values we cannot make the model walk as it falls back immediately. So it has to have some initial acceleration to make the ZMP lie within the foot region.

One solution of this problem can be having a torso connected to hip. This solution is well understood as this is the case in humans and some other mammals walking. Whenever the walker comes to stable position or whenever the ZMP is going out of the foot region the torso can be moved forward and backward to manipulate the ZMP to lie within foot region. But we cannot have this solution for our walker as it includes extra link and mass as torso.

Another solution is also possible. When the walker is in stable condition and we want to make it walk, instead of applying torques at hip ankle this can be done by applying small impulses (large impulsive forces) at the rear foot. This impulse results in change in momentum of the system which gives the initial conditions required for the walker to move.

5.1.1 Friction modeling

Friction models can be classified as below

1. Classical static models
 - Coulomb friction
 - Viscous friction
 - Stiction
2. Mechanics and fluid dynamics
 - Microscopical contact
 - Viscosity
3. Empirical phenomenological models
 - The Dahl model
 - The Bliman-Sorine model
 - LuGre model

- Coulomb friction model is used generally in mechanics for defining the friction between any surfaces. Coulomb friction, named after Charles-Augustin de Coulomb, is an approximate model used to calculate the force of dry friction. It is governed by the equation

$$F_f \leq \mu F_n, \text{ where}$$

F_f , is the force of friction exerted by each surface on the other. It is parallel to the surface, in a direction opposite to the net applied force.

μ , is the coefficient of friction, which is an empirical property of the contacting materials,

F_n , is the normal force exerted by each surface on the other, directed perpendicular (normal) to the surface.

The Coulomb friction F_f , may take any value from zero up to μF_n , and the direction of the frictional force against a surface is opposite to the motion that surface would experience in the absence of friction. The force of friction is always exerted in a direction that opposes movement (for kinetic friction) or potential movement (for static friction) between the two surfaces.

- But this friction model is not accurate when we perform dynamic simulations. There are other friction models. For example ADAMS/SOLVER uses a simple velocity based friction model for contacts. In this model, coefficient of friction between surfaces will be varying based on the slip velocity. The variation is shown in Fig. 5.2

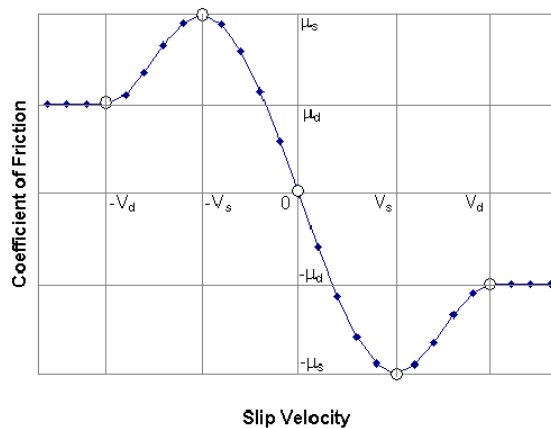


Figure 5.2: Friction model in ADAMS/SOLVER

- In the above model,

- $\mu(-V_s) = \mu_s$
- $\mu(0) = 0$
- $\mu(-V_d) = \mu_d$
- $\mu(v) = -\text{sign}(v)\mu_d$, for $|v| \geq v_d$
- $\mu(v) = -\text{step}(|v|, v_d, \mu_d, v_s, \mu_s) \cdot \text{sign}(v)$, for $v_s \leq |v| \leq v_d$
- $\mu(v) = -\text{step}(v, -v_s, \mu_s, v_s, -\mu_s)$, for $v_s \leq |v| \leq v_d$

- These are the functions in ADAMS/SOLVER where,
 - V : Slip velocity at contact point
 - V_s : Stiction transition velocity
 - V_d : Friction transition velocity
 - μ_s : Static friction coefficient
 - μ_d : Dynamic friction coefficient
- There are some other friction models based on slip velocity slip ratios they are
 - Slip Ratio-based friction model A (Linear U-Slip)
 - Slip (or Scrub) Velocity-based Friction Decay Model A
 - Slip (or Scrub) Velocity-based Friction Decay Model B
 - Slip Ratio-based model B (User-Defined μ_{Slip})

5.1.2 Contact modeling

In ADAMS, contact forces use two distinct normal force algorithms:

- *Restitution-based contact or Poisson model*: A value of zero specifies a perfectly plastic contact between the two colliding bodies, where as value of one specifies a perfectly elastic contact in which there is no energy loss.
- *Impact-function-based contact*: This requires four values which are given as
 - Stiffness: Specifies a material stiffness that is to be used to calculate the normal force for the impact model. In general, the higher the stiffness, the more rigid or hard the bodies in contact are.
 - Force Exponent: The instantaneous penetration between the contacting geometry
 - Damping : Defines the damping properties of the contacting material.
 - Penetration depth: Defines the penetration at which ADAMS/SOLVER turns on full damping.

5.1.3 Condition for slipping

- The coefficient of static friction (μ_s) for rubber is 0.8 and for steel it is 0.23. When a body of weight Mg resting on ground is acted by a force F at angle of θ with the vertical, then there will be two components of force, horizontal and vertical will be acting. The vertical force will try to move the body up or down based on the direction for which ground will have a normal reaction force

$$N = F \cos \theta + Mg.$$

The horizontal component of the force F_x will try to move the body in the direction of force component. This component will be opposed by the friction force between body and ground. This friction force

$$F_F = \mu_s(F \cos \theta + Mg)$$

If the horizontal component of the force acting on body is more than the friction force opposed, then the body starts moving. Refer Fig. 5.3(a) This condition can be shown as the inequality given below

$$F \sin \theta \geq \mu_s(F \cos \theta + Mg)$$

- Now the same condition can be applied to avoid slipping of the leg in bipedal walking. The leg angles and the hip mass positions should be modeled such that, when a torque is applied at ankle, the angle at which the force comes on to the leg should be within the friction angle (this is called as "Friction Angle").

- Now if we assume the body as massless (negligible compared to the force applied) the the inequality becomes

$$F \sin \theta \geq \mu_s F \cos \theta$$

which actually results in inequality

$$\theta \geq \tan^{-1}(\mu_s)$$

- So, for rubber $\theta \geq 38^\circ$ for slipping to occur. It means that if the angle of application force is less than 38° then there will be no slipping of body. Similarly for steel surface the angle is 16° .
- We can prove also prove that for any considerable mass this condition holds good and θ well below the angle that is calculated from the massless condition. It means that, when we have body with considerable mass, if we have the condition for no slipping in massless case satisfied, then there will be no slipping. It is depicted in Fig. 5.3

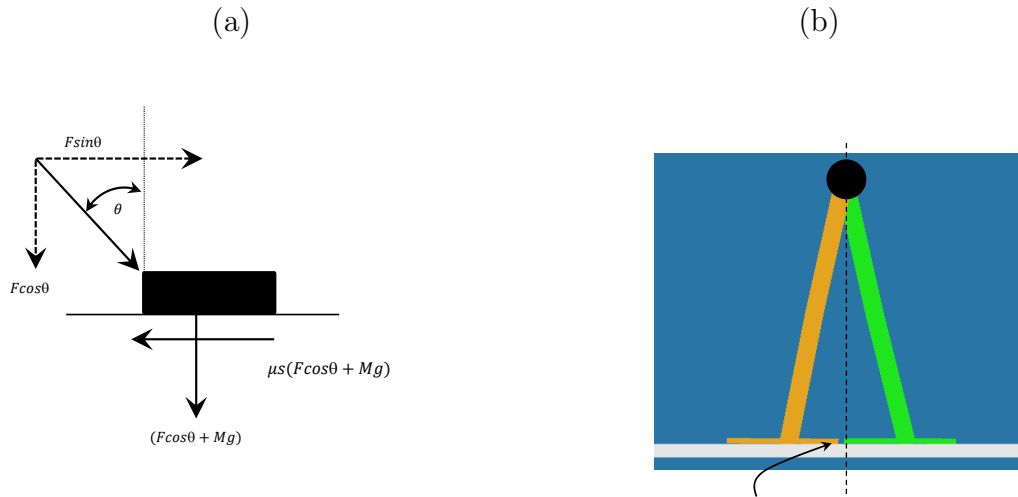


Figure 5.3: (a) Condition for slipping (b) Positions of legs

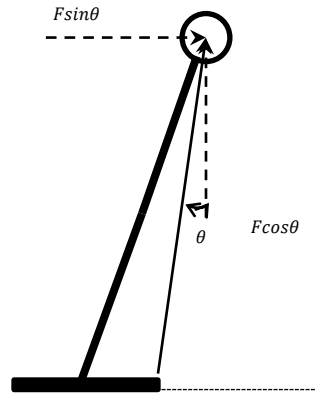


Figure 5.4: Force direction

5.1.4 Impulse - Angular momentum relation

When the walker is in static position (no movement) in the configuration shown in the Figure (a), as the model is symmetric, the reaction forces at both the feet will be equal and we can find the magnitude and direction of the reaction forces.

$$R_x = \frac{mga + \frac{MgL}{2} \tan \alpha}{L} \text{ and } R_y = mg + \frac{Mg}{2}$$

where direction and magnitude of reaction force can be obtained from the expressions

$$R = \sqrt{R_x^2 + R_y^2} \text{ and } \tan \theta = \frac{R_y}{R_x}$$

Now we assume a massless foot of length r to one leg say rear leg, and we want find out the torque that should be applied at the ankle so that the reaction force comes at the tip of the leg. We can again find out the direction and magnitude of the reaction force at the tip of the leg for the biped model to be in static equilibrium. For this, basic force balance and momentum balance principles are used.

The torque required

$$\tau = mg(r - a \sin \alpha) + \frac{Mg}{2}(r - L \sin \alpha) \quad (5.1)$$

where α is the position angle of the rear leg from vertical in radians. Now the reaction forces at the tip of the legs are

$$R_x = \frac{(mg + \frac{Mg}{2})(L \sin \alpha - r) + \tau - mgb \sin \alpha}{L \cos \alpha} \text{ and } R_y = mg + \frac{Mg}{2}$$

We can observe here that the no slipping conditions for the biped can be found from the value of the reaction forces. If μ is the co-efficient of friction for the surface then no slip condition is given by

$$R_x \leq \mu(R_y) \quad (5.2)$$

So there is a restriction for the value of torque that can be applied at the ankle because of the chance of slipping.

Now we want to apply the torque at the ankle for a very small duration, which can be called impulse. Now we need to find the change in angular velocities of the two links (front and rear legs) when the impulsive torque is applied. For this two rules are considered

- Net impulse applied is equal to the change in angular momentum of the system (Newton's second law of motion).
- Angular momentum of the second link (front leg) is conserved about a point where it is touching the ground ('E' in Figure).

From the above two statements we can find the angular velocities of the links

$$\text{Impulse} = \tau t = [-ma^2 - ML^2 - mL^2 + mLb \cos(2\alpha)]\dot{\theta}_1 + [-mLb \cos(2\alpha) + mb^2]\dot{\theta}_2 \quad (5.3)$$

$$[-ML^2 - mal + (mab + mal) \cos(2\alpha)]\dot{\theta}_1 = [mab \cos(2\alpha)]\dot{\theta}_2 \quad (5.4)$$

Chapter 6

Dynamic Simulation Using ADAMS

6.1 About ADAMS

ADAMS is Automatic dynamic analysis of mechanical systems, is a technology of MSC software corporation, that was implemented 25 years ago. Original product was ADAMS/SOLVER an application that solves nonlinear numerical equations. Later ADAMS/VIEW was released which allows users to build, simulate and examine results in a single environment. Now many industry specific Adams versions are available like ADAMS/CAR, ADAMS/ENGINE and ADAMS/RAIL etc.

Now we use basic Adams/View environment to build the model and examine the dynamic behavior of the proposed walker models. Models can be designed in other modeling packages It has a possibility of 3D model import from so widely known CAD systems as Catia, ProEngineer, SolidWorks, SolidEdge, etc. Adams can be integrated with MATLAB/EASY5 computing packages for better results.

We can even build a solid model of a mechanical system from scratch in Adams/View environment . A full library of joints and constraints is available for creating articulated mechanisms. Once the model is complete, Adams checks the model and then runs simultaneous equations for kinematic, static, quasi-static, and dynamic simulations. Results are viewable as graph, data plots, reports or colorful animations that we easily can share with others.

The steps in creating model in ADAMS/VIEW are as follows:

- Create the parts using ADAMS/VIEW library of parts or importing geometry from CAD.
- Add constraints that define how parts are attached and how they are allowed to move

relatively to each other, and apply forces that act on your model.

- Test, validate, and simulate the model.

The advantage of ADAMS vs. for example MATLAB is that mechanical dynamical systems are described geometrically. This means in ADAMS you can define mechanics by building model with ADAMS own library of parts or you can use CAD geometry to layout the system, so the deriving of complex equations of motion is eliminated.

Adams involves direct physics and so it represents actual physical behavior of any model thus save us the time and cost for building any prototypes before making the actual model. The geometry can be efficiently parameterized allowing the user to easily see the effect of changes and go through *what-if* design problems that trade off packaging against dynamic behavior. Geometric representation allows you to directly see how the mechanical system will behave instead of inferring the response from plots. With ADAMS it is easy to incorporate component flexibility into the system that is quite complex in the geometry it represent as well as how that component connects to other. Again, this is information about a system that is best described geometrically instead of mathematically.

6.2 Adams Models Description

For realizing the problems involved in practical walking, we have made different types walking models. These models were verified for the required results. The problems involved in making these models walk forms the basis of rules for the walking. After so many models we may arrive at a feasible model which serves the purpose.

A model that is shown in Fig. 6.1(A) is named as walker with knee lockers. In this model two bars were created for each leg covering the knee joint. These bars would take care of knee bending and locking. The knee is are given motions such that bending is in only one direction which is a characteristic of human walking. The bars are pushed up by applying force on them. This models is little complicated with more number of actuators required to move legs and the knee locking mechanism.

Model shown in Fig. 6.1(B) is also a simple model consisting of two legs wit knee joints and feet connected by a hip rod. This hip rod is made to facilitate the application of torque in between hip and one leg instead of between two legs. This model is similar to the lower proximity of human being with hip is considered as a rod joining two legs. Problem with this model is the slipping of front leg while trying to bring t he rear leg t o front.

Model shown in Fig. 6.1(C) is a model that is close to human being structure. All the above knee portion of human being is considered here as a torso with a mass at the end. This torso is responsible for balancing of the walker and keeping the center of gravity within the foot area to avoid falling. This imitates a human being without hands. The torso balance the entire structure and keeps the CoM with in the region to keep the system stable. The problem with this models is that only motions can be defined at different joints but the force involved could not be known.

The models shown in Fig. 6.2(A) is the systems showing the four-bar analogy of the final walking model. And in (B) is shown the final model considered for the results.

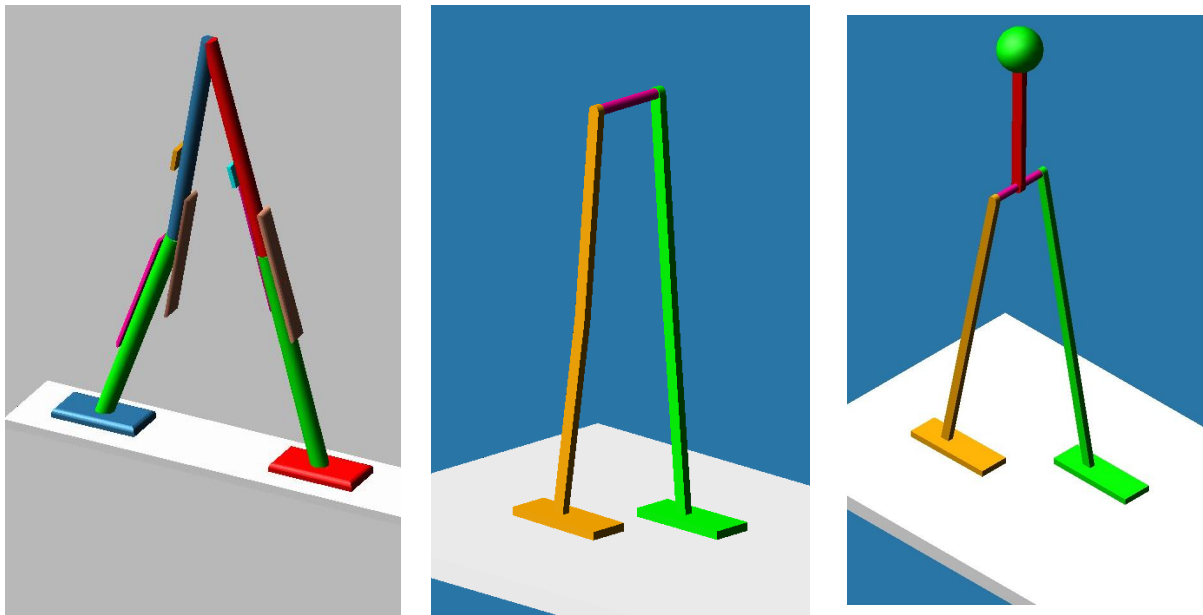


Figure 6.1: Models: (A)Walker with knee-lockers, (B)Simple dynamic walker and (C)Walker with torso

6.3 Four Bar Mechanism

In the above described final model, the walking cycle has two stages. One is the ankle push-off stage and the other being moving of the rear leg to front. The first stage is mathematically

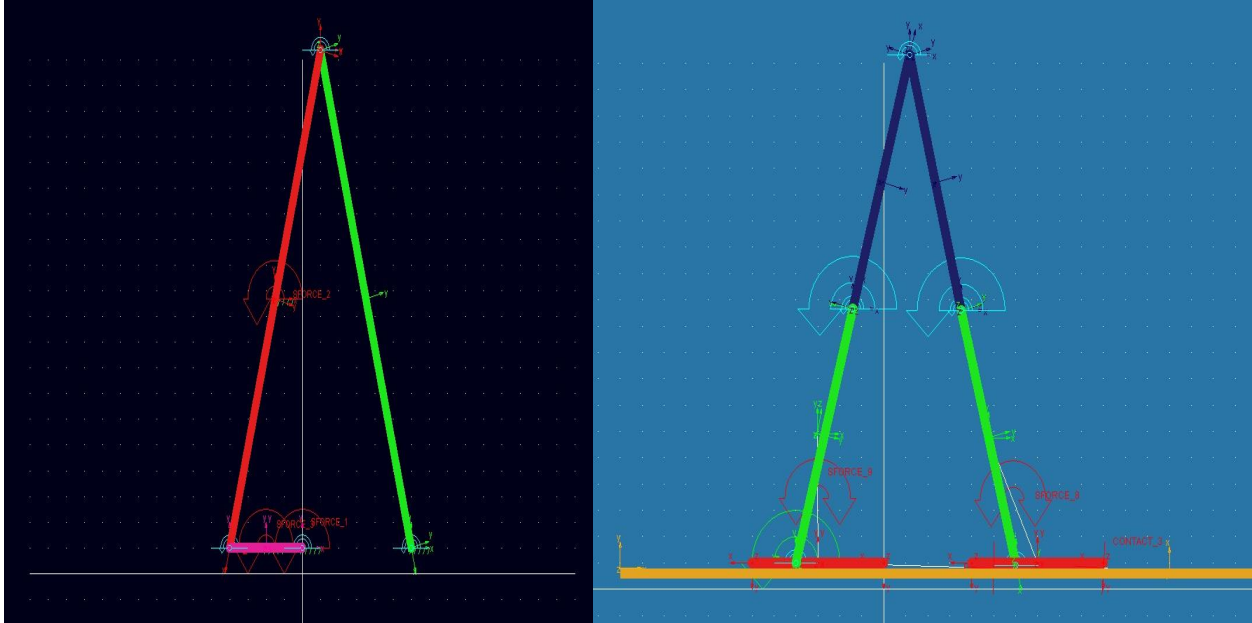


Figure 6.2: (A)Four bar mechanism (B)Similar walking model

realized as a four-bar mechanism with two ankle as the ground contact points and rear foot, forming the three links of the four and fourth is the ground. So the mathematical derivations are done and numerical results are obtained for the accurate results which will later be compared with the ADAMS results for confirmation.

6.3.1 Dynamic simulation of four bar mechanism

With the rear leg included, the bipedal model can be considered as a four bar mechanism with first link as ground, second link the rear foot, third link the rear leg and fourth link the front leg. We should remember that the system will be considered as four bar mechanism where there is no slipping at the joints(foot ground contact). When four bar mechanism is considered, the entire system is a single degree of freedom system which needs only one input to define the complete motion of the system. Now we will try to find the angular motions of the links when an impulsive torque is applied at the ankle. The procedure followed is similar to the procedure explained in four bar mechanism dynamic simulation using Lagrangian [13].

Table 6.1: Model description

Property	Value(Unit)
Leg Mass	0.5 KG
Foot Mass	2 KG
Leg Length	1 M
Foot Length	3 CM
Distance of ankle joint from rear end	10 CM
Distance of knee joint from ankle	50 CM
Static coefficient of friction	0.8
Dynamic coefficient of friction	0.76
DoF of each leg	3

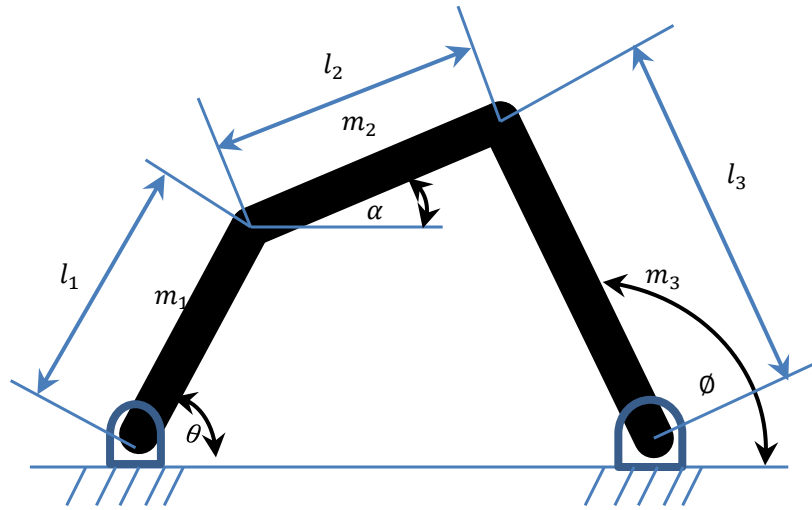


Figure 6.3: Four bar mechanism

6.3.2 Position analysis

Let us take l_2, l_3 as the lengths of rear and front legs, l_1 as the length of the foot from ankle to the tip and l_0 as the length of the step. Let us take the angles made by the first three

links with the positive horizontal as θ , α and ϕ respectively. Now from the configuration of the system we can write two loop closure equations as below.

$$-l_1 \cos \theta - l_2 \cos \alpha + l_0 + l_3 \cos \phi = 0 \quad (6.1)$$

$$-l_1 \sin \theta - l_2 \sin \alpha + l_3 \sin \phi = 0 \quad (6.2)$$

The above two equations are sufficient to write the expressions for α and ϕ in terms of θ which will then be completely depending on single variable θ . The expressions are given as

$$\phi(\theta) = 2 \tan^{-1} \left(\frac{-t_1 - \sqrt{t_1^2 + t_2^2 - t_3^2}}{t_3 - t_2} \right) \quad (6.3)$$

Where t_1, t_2 and t_3 are functions of θ given by

$$t_1 = -2l_1 l_3 \sin \theta, t_2 = 2l_3(l_0 - l_1 \cos \theta), \text{ and } t_3 = l_0^2 + l_1^2 - l_2^2 + l_3^2 - 2l_0 l_3 \cos \theta$$

$$\alpha(\theta, \phi) = \tan^{-1} \left(\frac{-l_1 \sin \theta + l_3 \sin \phi}{l_0 - l_1 \cos \theta + l_3 \cos \phi} \right) \quad (6.4)$$

Now we have defined all the variables in terms of θ .

6.3.3 Velocity analysis

By differentiating the loop closure equations with respect to time we can get expressions in terms of angular velocities of the links. In matrix form

$$\begin{bmatrix} l_2 \sin \alpha & -l_3 \sin \phi \\ -l_1 \cos \alpha & l_3 \cos \phi \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -l_1 \sin \theta \\ l_1 \cos \theta \end{bmatrix} \dot{\theta}$$

We can obtain $\dot{\alpha}$ and $\dot{\phi}$ in terms of $\dot{\theta}$ from the above equation as below

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{l_1 \sin(\phi - \theta)}{l_2 \sin(\alpha - \phi)} \\ \frac{l_1 \sin(\alpha - \theta)}{l_3 \sin(\alpha - \phi)} \end{bmatrix} \dot{\theta}$$

Now we also have all the angular velocities defined in terms of $\dot{\theta}$.

6.4 Bipedal walker four bar modeling

6.4.1 Relating biped to four bar mechanism

Here we are assuming the half portion (ankle to toe) of the rear leg of the bipedal as the first link and the two legs as the other two links of the biped. So we will have a hip mass

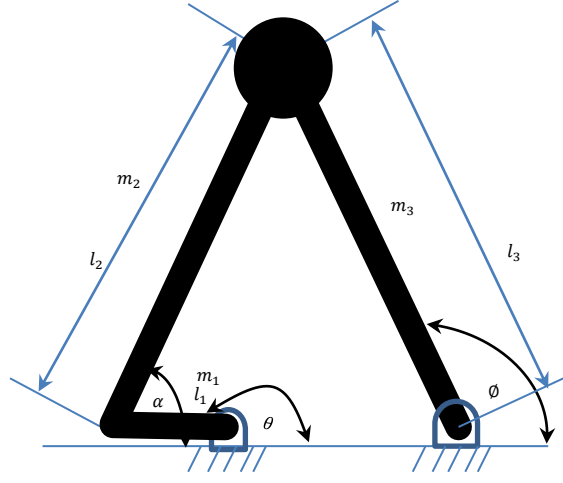


Figure 6.4: Bipedal four bar mechanism

that is fixed to the front leg. The hip mass is named as M . For position and velocity earlier analysis can be used. Now we have a new lagrangian as $L = T - V$ where,

$$T = \frac{1}{2} (m_1 v_{c1}^2 + I_1 \dot{\theta}^2) + \frac{1}{2} (m_2 v_{c2}^2 + I_2 \dot{\theta}^2) + \frac{1}{2} (m_3 v_{c3}^2 + I_3 \dot{\theta}^2) + \frac{1}{2} (M l_3^2 \dot{\phi}^2) \quad (6.5)$$

and

$$V = m_1 g y_{c1} + m_2 g y_{c2} + m_3 g y_{c3} - M g l_3 \sin \phi \quad (6.6)$$

The Lagrangian L can be obtained as

$$L = J_1 \dot{\theta}^2 + J_2 \dot{\alpha}^2 + J_3 \dot{\phi}^2 + P_1 C_1 \dot{\theta} \dot{\alpha} + G \quad (6.7)$$

where

$$J_1 = \frac{1}{2} (m_1 l_{c1}^2 + I_1 + m_2 l_1^2)$$

$$J_2 = \frac{1}{2} (m_2 l_{c2}^2 + I_2)$$

$$J_3 = \frac{1}{2} (m_3 l_{c3}^2 + I_3 + M l_3^2)$$

$$P_1 = m_2 l_1 l_{c2}$$

$$C_1(\theta, \alpha) = \cos(\theta - \alpha)$$

and

$$G(\theta, \alpha, \phi) = (-m_1 g l_{c1} - m_2 g l_1) \sin \theta - m_2 g l_{c2} \sin \alpha - m_3 g l_{c3} \sin \phi - M g l_3 \sin \phi$$

So, the full dynamic equation of motion will be

$$M\ddot{\theta} + V\dot{\theta} - N = \tau_{\theta} \quad (6.8)$$

Where

$$\begin{aligned} M &= [J_1 + J_2 S_1^2 + J_3 S_2^2 + P_1 C_1 S_1] \\ V &= \left[2J_2 S_1 \left(\frac{\partial S_1}{\partial \theta} + S_1 \frac{\partial S_1}{\partial \alpha} + S_2 \frac{\partial S_1}{\partial \phi} \right) + 2J_3 S_2 \left(\frac{\partial S_2}{\partial \theta} + S_2 \frac{\partial S_2}{\partial \alpha} + S_3 \frac{\partial S_2}{\partial \phi} \right) \right] \\ &+ \left[P_1 C_1 \left(\frac{\partial S_1}{\partial \theta} + S_1 \frac{\partial S_1}{\partial \alpha} + S_2 \frac{\partial S_1}{\partial \phi} \right) + S_1 \left(\frac{\partial C_1}{\partial \theta} + P_1 S_1 \frac{\partial C_1}{\partial \alpha} \right) \right] \text{ and} \\ N &= \left(\frac{\partial G}{\partial \theta} + S_1 \frac{\partial G}{\partial \alpha} + S_2 \frac{\partial G}{\partial \phi} \right) \end{aligned}$$

6.4.2 Method of Lagrange multipliers

Method of Lagrange multiples involves solving of equations of motion and constraint equations at a time which are called *Differential Algebraic Equations*(DAEs). Now we analyze a four-bar mechanism system and solve by method of lagrange multipliers and compare the results with results from earlier methods also from dynamic simulation softwares MSC. Adams and working model 2D. Let us consider the same four bar mechanism with link masses m_1 , m_2 and m_3 , and angles with horizontal α , β and ϕ .

Now, all the links in the system are kinematically constrained and the system has only one degree of freedom (1DOF). The system is a planar mechanism in XY-Plane and there are three links in which each link has three independent DOF (Namely X-Translation, Y-Translation and rotation about Z). All the links are pin jointed either with another link or with ground. This makes total 4 revolute joints which restricts 8 DoFs of the system. So the final DoF of the system will be 1.

Now we will try to write all the constraint equations. Let us say the system of constraint equations as Φ and system of variables as q .

$$\Phi = [\phi_1 \ \phi_2 \ \phi_3 \ \phi_4 \ \phi_5 \ \phi_6 \ \phi_7 \ \phi_8]^T = 0 \quad (6.9)$$

$$q = [q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6 \ q_7 \ q_8 \ q_9]^T = [x_1 \ y_1 \ \alpha \ x_2 \ y_2 \ \theta \ x_3 \ y_3 \ \phi]^T$$

$$\phi_1 = x_1 - \left(\frac{l_1}{2} \right) \cos \alpha = 0$$

$$\phi_2 = y_1 - \left(\frac{l_1}{2} \right) \sin \alpha = 0$$

$$\phi_3 = x_2 - l_1 \cos \alpha - \left(\frac{l_2}{2} \right) \cos \theta = 0$$

$$\begin{aligned}
\phi_4 &= y_2 - l_1 \sin \alpha - \left(\frac{l_2}{2}\right) \sin \theta = 0 \\
\phi_5 &= x_3 - l_1 \cos \alpha - l_2 \cos \theta - \left(\frac{l_3}{2}\right) \cos \phi = 0 \\
\phi_6 &= y_3 - l_1 \sin \alpha - l_2 \sin \theta - \left(\frac{l_3}{2}\right) \sin \phi = 0 \\
\phi_7 &= x_3 - l_0 - \left(\frac{l_3}{2}\right) \cos \phi = 0 \\
\phi_8 &= y_3 - \left(\frac{l_3}{2}\right) \sin \phi = 0
\end{aligned} \tag{6.10}$$

Now differentiating $\Phi = 0$ two time with respect to time,

$$\begin{aligned}
\Phi &= \Phi(q, t) = 0 \\
\dot{\Phi} &= \Phi_q \dot{q} + \Phi_t = 0 \\
\ddot{\Phi} &= \Phi_q \ddot{q} - \Gamma = 0
\end{aligned} \tag{6.11}$$

where $\Gamma = -(\Phi_q \dot{q})_q \dot{q} - 2\Phi_{qt} \dot{q} - \Phi_{tt}$ and

$$M\ddot{q} + \Phi_q^T \lambda = Q \tag{6.12}$$

Now above equations can be solved in two alternatives to eliminate Lagrange multipliers i.e. λ . The final solution of the system will look like this.

$$M\ddot{q} = Q^* \tag{6.13}$$

where Q^* stands for $Q^* = Q - \Phi_q^T (\Phi_q M^{-1} \Phi_q^T)^{-1} (\Phi_q M^{-1} Q - \Gamma)$

But numerically solving the above DAE's will not give desired results as they will have errors. To correct the errors Baumgaurte's stabilization method is used. After using this method the equation of motion will have a corrected term in it. Where the final equation of motion would look like this.

$$\Phi_q \ddot{q} = \Gamma^* \tag{6.14}$$

Where Γ^* is given by

$$\Gamma^* = \Gamma - 2\alpha \dot{\Phi} - \beta^2 \Phi$$

With these equations of motions, for the given initial conditions, we can get the solutions for positions, velocities and accelerations as well as the reaction forces at different joints. These results will be compared with the ADAMS model results that were obtained by creating and simulating the similar model.

6.5 Simulation Results - Numerical Results Comparison

For the model described in Table 6.1. We have derived the equations of motions using Newton-Euler approach and solved for the motions by applying torques. The same model is created and simulated for the same amounts in Adams. Angular positions, angular accelerations and reactions at the ground are compared for both cases. And they are found to be almost equal. The plot showing in Fig. 6.9 is condition that checks whether slip happens at the foot-ground contact are not. When $R_x - R_y$ is negative then slip does not occur. If the value is positive then there will be slipping.

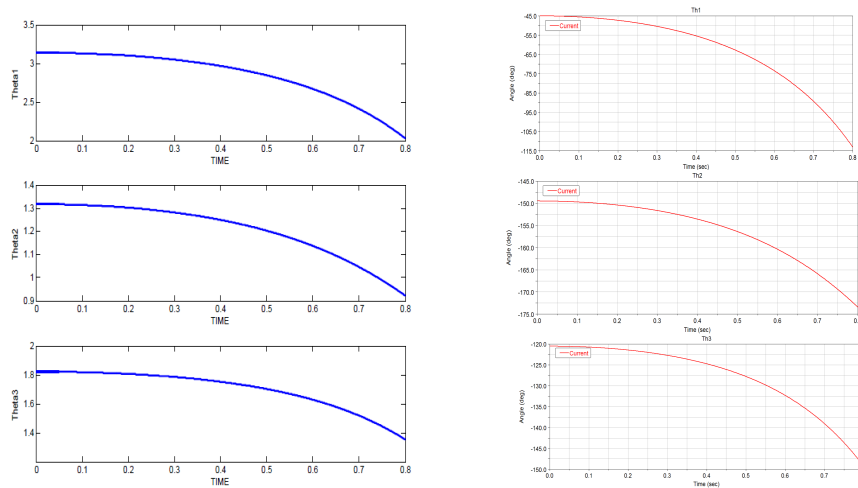


Figure 6.5: Comparing angular positions of all links

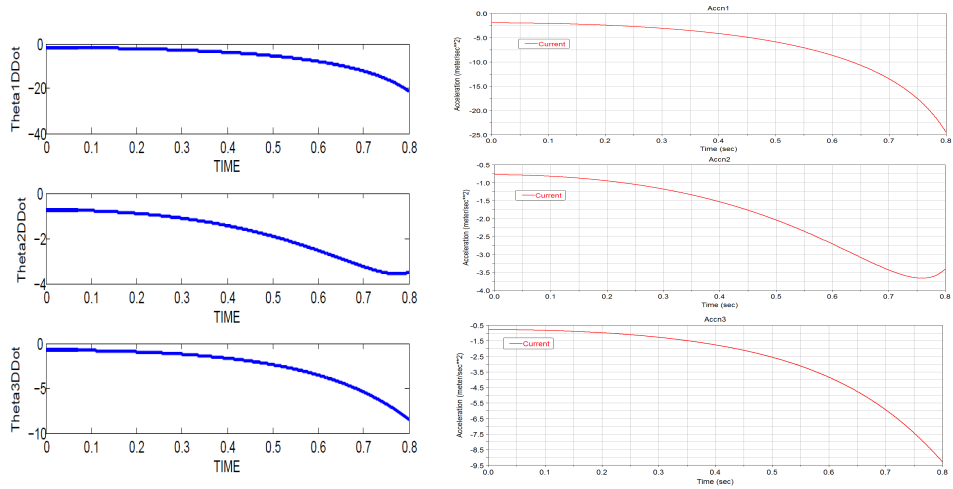


Figure 6.6: Comparing angular accelerations of all links

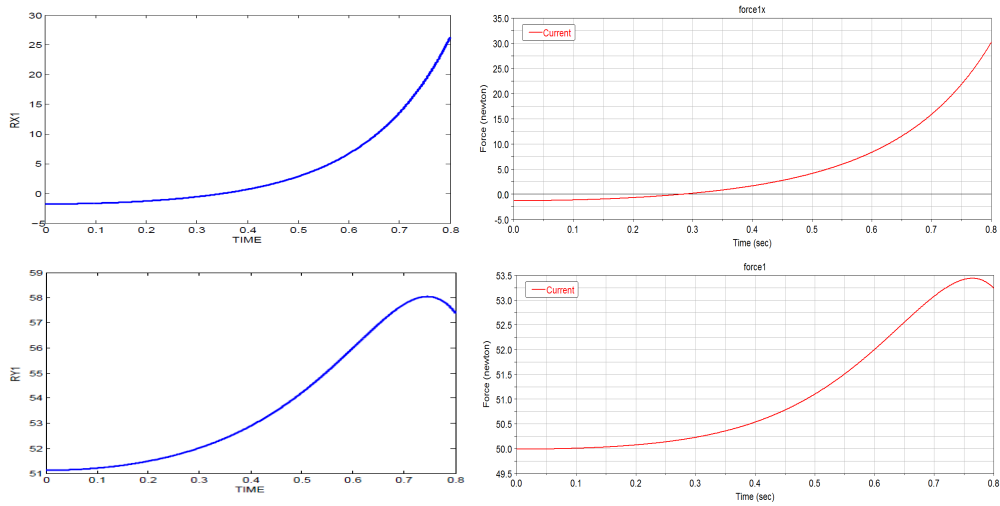


Figure 6.7: Comparing ground reactions (Vertical and horizontal) at first(Rear) contact point

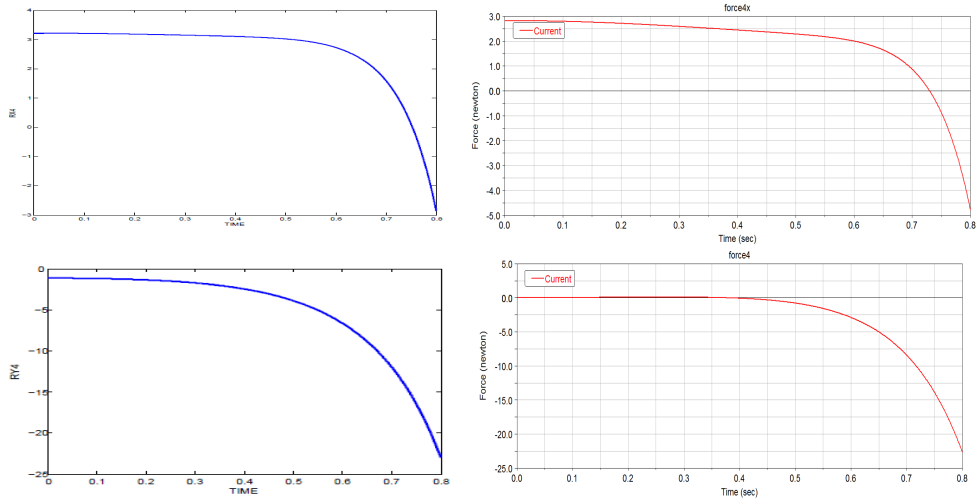


Figure 6.8: Comparing ground reactions (Vertical and horizontal) at second(Front) contact point

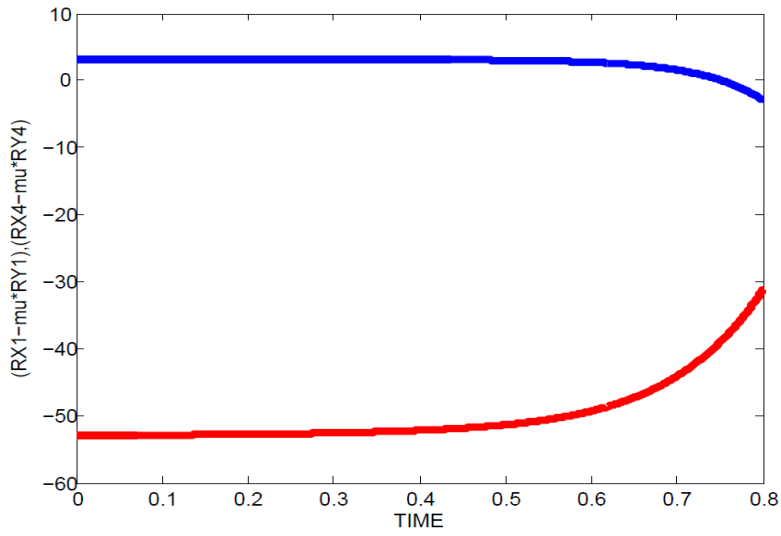


Figure 6.9: Slipping condition for both legs

Chapter 7

Results and Future Work

7.1 Summary

We have derived some walking models based on simple mechanisms. We have made a compass model from a double inverted pendulum and then made it to walk by applying required conditions. We have found various ways to make the model walk on a slope passively by giving initial conditions. After that for the same model we have tried to make it walk on level ground by supplying external energy through some means i.e torques at hip joint and ankle joints which is called active walking. For this we have followed two approaches, one is creating virtual slope and then converting it to active walking by applying torques at ankle and hip as the functions of virtual slope. Another approach was using torsional springs and dampers at hip as well as Ankles such that the torques are given in terms of springs' stiffness coefficient and damping coefficient. After this we have tried to make the walker start from stable condition. Then we have moved to deriving of rules to make the walker walk starting from stable configuration and to walk continuously.

To start the model to walk from stable conditions we have used ankle push-off concept. After the walker starts from stable condition, at some point we tried to convert it to active walking of dynamic walking derived earlier. We have used the analogy with a simple four-bar mechanism for simulation. Results are mathematically derived and compared with dynamic simulation software results (ADAMS). We have also made a model that contains torso which imitates human walking and made it to walk.

Based on the all the above models behavior we can define simple rules in such a way that, any simple walking model (less than 5 DoF) that is following these rules will walk on level ground continuously without falling.

7.1.1 Rules for walking

As discussed in chapter 5, the conditions for walker to avoid slipping of one leg when the other leg is in the air has to be satisfied. This depends on the friction model we use which are explained already. The rule is as follows.

$$F \sin \theta \geq \mu_s(F \cos \theta + Mg), \quad (7.1)$$

Similarly, the following guidelines are useful for rule based walking.

- If bipedal walker has to take a step, then the position of the rear leg should be such that the ground contact point (tip) of the rear leg should be behind the line joining the center of hip mass and ground.
- The resultant force components that act on hip mass are shown in Fig. 5.4 in Chapter 5. The angle at which the force acts on hip mass will be along the line joining tip of the rear leg (ground contact) and the center of mass of the hip mass.
- If the model has to take a step in the forward direction, then the tip of the rear leg should be behind the ground projection of the center of mass of the system. Then the vertical component of force tries to lift the hip mass, where as the horizontal component of the force pushes is forward.
- Once the ground projection of the center of mass crosses the center position between two legs, then the support will be shifted to front leg and entire system starts supporting only on front leg. Then it will be easy to lift the rear leg as it has very less mass
- If we apply the torques on system very slowly, then we need not consider ZMP for stability. We should see that at each point the ground projection center of mass of entire system is lying within the support. So once the model starts supported by front leg, we should see that the hip mass moves forward so that the ground projection of center of mass lies within the front foot area.
- Sudden changes in the motions will result in building of internal forces which will make the walking difficult. To avoid this we should see that the forces or torques are applied such that the displacements, velocities and accelerations are always continuous.

Along with these conditions, the following rules are made from the behavior of dynamic models.

1. When walker is stuck in zero KE position but standing, use ankle push-off to pump KE into the walker system.
 - Arbitrarily large impulsive torque at rear ankle for ankle push-off can lift-off the front foot from ground.
 - Large impulsive torque is also not physically realizable without using heavy actuators.
 - One solution is to apply constant angular velocity at the ankle till the front leg crosses vertical and the projection of walker CoM crosses front ankle. Then the walker starts falling forward.
2. Before the walker falls (hip touches the ground), the swing leg is brought forward quickly so that the swing foot touches the ground and prevents walker from falling forward [24].
 - Damper can be used at the stance leg ankle to slow down falling so that there is enough time to bring the swing leg forward.
 - If the walker CoM crosses front ankle too much forward, damper cannot slow down falling - tip over about the stance toe occurs.
 - Ankle push-off helps in reducing energy loss at heel strike (or swing foot landing) [21].
 - Once swing foot lands, continue with ankle push-off till projection of walker CoM crosses front ankle.

7.2 Future Work

The case of creating external disturbance while walking should be considered and algorithms or rules are to be made such that the model continues to walk by surpassing these disturbances. As all our models were planar, we can create external disturbance by applying arbitrary forces on the links. The simulations can be done by changing the external force vector. After simulating the methods to surpass the effect can be identified.

References

- [1] Vijay Kumar, 50 Years of Robotics, IEEE Robotics and Automation Magazine, pp. 1-8, Sep 2010
- [2] Jerry E. Pratt, Gill A. Pratt, Exploiting Natural Dynamics in the Control of a Planar Bipedal Walking Robot, 1998
- [3] S. Hashimoto et al., Humanoid Robots in Waseda University- Hadaly-2 and Wabain, Autonomous Robots, pp. 25-38, 2002
- [4] Jun-Ho Oh et al., Design of Android type Humanoid Robot Albert HUBO, International Conference on Intelligent Robots and Systems, Oct 2006
- [5] Grishin A.A., Formalsky A.M., Lensky A.V., Zhitomirsky S.V., Dynamical walking of a vehicle with two telescopic legs controlled by two drives, International Journal of Robotics Research, Vol. 13(2), pp. 137-147, 1994
- [6] Gurfinkel V.S., Gurfinkel E.V., Schnieder A.Y., Devjanin E.A., Lensky A.V., Shitilman L.G., Walking robot with supervisory control, Mechanism and Machine Theory, Vol. 16, pp. 313-6, 1981
- [7] Brooks R.A., A robot that walks emergent behaviors from a carefully evolved network, Neural Computation, Vol. 1, pp. 355-363, 1989
- [8] Raibert M.H., Sutherland I.E., Machines that walk, Scientific American, Vol. 248(2), pp. 44-53, 1983
- [9] Raibert M.H., Legged Robots that Balance, MIT Press, Cambridge, 1986.
- [10] Miura H., Shimoyama I., Dynamic walk of a biped, International Journal of Robotics Research, Vol. 3(2), pp. 60-74, 1984

- [11] C. Chevallereau, G. Abba, Y. Aoustin, F. Plestan, E.R. Westervelt, C. Canudas-de-Wit, J.W. Grizzle: RABBIT: a testbed for advanced control theory, *IEEE Contr. Syst. Mag.* 23(5), pp. 5779, 2003
- [12] ODEX I: The First Functionoid, by Russell, M., *Robotics Age*, vol. 5, pp. 12, 14-18, Sept 1983
- [13] Tad McGeer. Passive dynamic walking. *International Journal of Robotics Research*, 9(2):62-82, Apr 1990
- [14] H. Collins, Andy Ruina, Russ Tedrake, and Martijn Wisse. Efficient bipedal robots based on passive-dynamic walkers. *Science*, Vol. 307, pp.1082-1085, Feb 2005
- [15] Ambarish Goswami, Benoit Thuilot, and Bernard Espiau. Compass-like biped robot part I : Stability and bifurcation of passive gaits. Technical Report RR-2996, INRIA, Oct 1996
- [16] M. Vukobratovic, B. Borovac, D. Surla and D. Stokic, *Biped Locomotion - Dynamics, Stability, Control and Application* (Springer-Verlag, Berlin, 1990)
- [17] A. Goswami, Foot-rotation indicator (FRI) point: A new gait planning tool to evaluate postural stability of biped robots, in *Proc. IEEE ICRA*, Detroit, pp. 47-52, 1999
- [18] J. Adolfsson, H. Dankowicz, and A. Nordmark, 3-D passive walkers: finding periodic gaits in the presence of discontinuities *Nonlinear Dyn.*, Vol. 24, no. 2, pp. 205-229, 2001
- [19] Miomir vukobravotic, Zero-Moment Point-Thirty Five Years of its Life,*Internatiol Journal of Humanoid robotics*, Vol.1, pp. 157-173, 2004
- [20] Arthur D. Kuo, Energetics of Actively Powered Locomotion Using the Simplest Walking Model, *Journal of Biomechanical Engineering*, ASME, Vol. 124, pp. 113-120, 2002
- [21] Martijn Wisse, Arend L. Schwab, Richard Q. van der Linde, and Frans C. T. Van Der Helm, How to Keep From Falling Forward: Elementary Swing Leg

- Action for Passive Dynamic Walkers, IEEE Transactions on Robotics, Vol. 21, pp. 393-401, Jun 2005
- [22] Textbook, Springer Handbook of Robotics, Siciliano, Bruno; Khatib, Oussama (Eds.), 2008
- [23] Magazine, IEEE Robotics and Automation, pp. 52-62, Jun 2007
- [24] M. Wisse and J. Frankenhuyzen, Design and construction of mike; a 2d autonomous biped based on passive dynamic walking. Delft University of technology, 2002
- [25] Steven Hartley Collins, Andy Ruina, A Bipedal Walking Robot with Efficient and Human-Like Gait, Proceedings of the IEEE International Conference on Robotics and Automation Barcelona, Spain, pp. 1983-1988, Apr 2005
- [26] Fumihiko Asano, Masaki Yamakita, Norihiro Kamamichi, and Zhi-Wei Luo, A Novel Gait Generation for Biped Walking Robots Based on Mechanical Energy Constraint, IEEE Transactions on Robotics and Automation, Vol. 20, pp. 565-573, Jun 2004
- [27] MSC Software Corp., Getting Started Using Adams/Controls Introducing and Starting the Tutorials, Mechanical Dynamics Inc., 2008
- [28] Adams Tool Program Package MSC.ADAMS 2008
- [29] Martijin Wisse et al., IEEE Robotics and Automation Magazine, pp.52-62, June 2007