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ScienceDirect
Nuclear Physics B 980 (2022) 115764
www.elsevier.com/locate/nuclphysb

Analyses of scalar potential and lepton flavor violating decays in a model with A_4 symmetry

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Received 27 January 2022; accepted 29 March 2022
Available online 21 April 2022
Editor: Hong-Jian He

Abstract

We have considered a model, originally proposed by Ma and Wegman, where the mixing pattern in neutrino sector is explained with three Higgs doublets, six Higgs triplets and A_4 symmetry. The mixing pattern is explained with the help of vacuum expectation values (VEVs) of the above mentioned doublets and triplets. In order to study about the VEVs of the scalar fields, we construct the full invariant scalar potential of this model. After minimizing this scalar potential, we have found that two Higgs triplets can acquire zero VEVs. In order to generate non-zero VEVs to all the six Higgs triplets, we have added two more Higgs doublets to the model. Thereafter we have demonstrated that the current neutrino oscillation data can be consistently explained in our model. To study some phenomenological implications of this model, we have worked out on the branching ratios for lepton flavor violating decays.

1. Introduction

Neutrino sector can give hints about physics beyond the standard model (SM) [1]. The masses of neutrinos are tiny as compared to other fermion masses [2]. In order to explain the tiny masses for neutrinos, one has to extend the SM. In addition to the masses of neutrinos, mixing pattern in neutrino sector can also give a hint to physics beyond the SM. From the global fits to neutrino oscillation data [3], the three neutrino mixing angles are found approximately close to the tri-

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https://doi.org/10.1016/j.nuclphysb.2022.115764

bimaximal mixing (TBM) [4]. To understand this mixing pattern in neutrino sector, the SM should be extended with additional symmetries and particle content [5].

In this work, we consider the Ma-Wegman (MW) model [6], where A_4 symmetry [7] is introduced to explain the neutrino mixing pattern. For early works on A_4 symmetry, see Refs. [8]. In the MW model, the scalar sector contains three Higgs doublets and six Higgs triplets. Due to the presence of scalar Higgs triplets, neutrinos acquire masses via type II seesaw mechanism [9] in this model. The above mentioned scalar fields and lepton doublets are charged under A_4 symmetry in such a way that a realistic neutrino mixing pattern can be explained. A unique feature of the MW model is that the six Higgs triplets, which are responsible for obtaining the neutrino mixing pattern, are charged under all possible irreducible representations of A_4 symmetry. See Ref. [7] for an introduction to A_4 symmetry. In Appendix A we have summarized product rules among the irreducible representations of A_4 symmetry.

The A_4 symmetry in the MW model is spontaneously broken when the neutral component of doublet and triplet Higgs fields acquires VEVs. The VEVs of the triplet Higgses generate a mixing mass matrix for neutrino fields. After diagonalizing this mass matrix, one can obtain neutrino masses and mixing angles. In the work of MW model [6], this diagonalization has been done after making some assumptions on the VEVs of triplet Higgs fields, and thereby, it is concluded that neutrino masses can have normal ordering. This problem of diagonalizing the neutrino mass matrix of the MW model has been revisited in Ref. [10]. In the work of Ref. [10], after relaxing some of the assumptions made in Ref. [6] and also after using some approximation procedure [11], diagonalization has been done for the neutrino mass matrix of the MW model. Thereafter, it is concluded that both normal and inverted orderings for neutrino masses are possible in the MW model, apart from explaining the mixing pattern in neutrino sector.

As described above, the VEVs of scalar triplet Higgses are responsible for generating the neutrino masses and mixing angles in the MW model. One obtains the VEVs of scalar fields after minimizing the invariant scalar potential among these fields. The scalar potential in the MW model contains both the doublet and triplet Higgs fields. Minimization for this scalar potential has not been done before. On the other hand, minimization for the invariant scalar potential containing only the Higgs doublets has been done in Ref. [7], where it is shown that there exists a parameter region in which the three Higgs doublets of this model acquire the same VEV. This is known as the vacuum alignment of the Higgs doublets [12], which is necessary to achieve in order to diagonalize the charged lepton mass matrix, and thereby to explain the mixing pattern in neutrino sector.

In this work, in order to see the implications of scalar potential on neutrino masses and mixing pattern, we write the full invariant scalar potential containing the three doublet and six triplet Higgses of the MW model. After minimizing this scalar potential, we have found that the two triplets, which are charged under the non-trivial singlet representations of A_4 symmetry, acquire zero VEVs. It is to remind here that in our previous work of Ref. [10], we assumed the VEVs of all triplet Higgses be non-zero and later showed that neutrino oscillation data can be explained in the MW model. Moreover, it is stated before that we followed a specific diagonalization procedure in our previous work of Ref. [10]. Now, in this work, after finding that two Higgs triplets can acquire zero VEVs, with the diagonalization procedure of Ref. [10], we have found that the current neutrino oscillation data cannot be consistently explained. To alleviate the above mentioned problem, we add two more Higgs doublets to the MW model. After doing this, we show that at the minimum of the scalar potential, all the six Higgs triplets can acquire non-zero VEVs. As a result of this, we demonstrate that the neutrino oscillation data can be fitted in this model for both normal and inverted neutrino mass orderings. While doing the above mentioned minimization, we also address the problem on vacuum alignment of the Higgs doublets. We show that sufficient parameter region exists in this model, where the vacuum alignment of the necessary Higgs doublets can be achieved.

After analyzing the scalar potential, it is worth to study some phenomenological consequences of our model. We argue below that the scalar fields of our model can drive lepton flavor violating (LFV) processes such as $\ell \to \ell' \gamma$ and $\ell \to 3\ell'$. Here, ℓ and ℓ' are charged leptons belonging to different families. None of the above mentioned LFV decays have been observed in experiments, and as a result of that, upper limits on the branching ratios of these processes have been obtained [13]. See Refs. [14], for related studies on LFV processes in neutrino mass models. In our model, the above mentioned LFV decays are driven by the scalar fields which are charged under the A_4 symmetry. Hence, one can expect that these decays carry imprints of A_4 symmetry. In this work, one of our interests is to study signatures of A_4 symmetry in LFV decays. In a related direction to this, see Ref. [15].

The scalar triplet Higgses of our model drive LFV decays, since the Yukawa couplings for lepton doublets are flavor violating in a type II seesaw framework [16,17]. We compute branching ratios for the decays $\ell \to 3\ell'$ and $\ell \to \ell' \gamma$ in our model. The decays $\ell \to 3\ell'$ are driven by doubly charged scalar triplets at tree level, whereas, the decays $\ell \to \ell' \gamma$ are driven by doubly and singly charged scalar triplets at 1-loop level. The above mentioned LFV decays can also be driven by scalar fields of doublet Higgses, however, the contribution from these scalars has been neglected in this work. We comment about this contribution later. While computing the branching ratios for the above mentioned decays, one needs to know the mass eigenstates of the doubly and singly charged scalar triplets. These we obtain from the invariant scalar potential of our model, which we have described above. The branching ratios of the LFV decays in our work depend on Yukawa couplings and the masses of above mentioned scalar fields. We have found that for some decays the branching ratios are vanishingly small, if we assume degenerate masses for triplet scalar fields. Another fact we have found is that, due to the presence of A_4 symmetry, some of the couplings between charged scalar triplets and leptons can depend on one another. As a result of this, branching ratios for some LFV decays can depend on each other. The above mentioned facts are some of the signatures of A_4 symmetry in our model. Since the Yukawa couplings depend on neutrino oscillation observables, numerically we study the variation of these branching ratios in terms of neutrino mixing angles and the CP violating Dirac phase δ_{CP}.

The paper is organized as follows. In the next section, we briefly describe the MW model and present essential results from our earlier work [10] on this model. In Sec. 3, we construct the full invariant scalar potential of this model and give our analysis on the minimization of this potential. We study the implication of this analysis on the neutrino mixing pattern by taking into account of the results of our previous work [10]. We demonstrate that by adding two additional Higgs doublets, one can explain the neutrino mixing pattern consistently in our model. In Sec. 4, we study the LFV decays of our model. In Sec. 5, we describe future directions based on the phenomenology of our model. We conclude in the last section. In Appendix A, we have given the product rules of A_4 symmetry, which are useful for making invariant terms in our scalar potential. In Appendix B, we have listed all different quartic terms of the scalar potential, which contain only the Higgs triplets.

2. The MW model and essential results from it

In this section, we describe the MW model [6]. As stated in the previous section, the method of diagonalizing the neutrino mass matrix of this model has been improved in Ref. [10]. The

Table 1
Fields in the lepton sector of the MW model [6]. Here, $i = 1, 2, 3$ and $j = 4, 5, 6$.

Field	$L_i = (\nu_{iL}, \ell_{iL})^T$	ℓ_{1R}	ℓ_{2R}	ℓ_{3R}	Φ_i	ξ_1	ξ_2	ξ_3	ξ_j
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}''$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}''$	$\underline{3}$
$SU(2)_L$	2	1	1	1	2	3	3	3	3
$U(1)_Y$	$-\frac{1}{2}$	-1	-1	-1	$\frac{1}{2}$	1	1	1	1

essential results, related to neutrino masses and mixing angles, from the work of Ref. [10] are also presented in this section. These results are used in our study on LFV decays, which is presented in Sec. 4.

The relevant fields of the MW model, along with their charge assignments under the electroweak and A_4 symmetries are tabulated in Table 1. With the charge assignments of Table 1, the Yukawa couplings for charge leptons can be written as [7]

$$\mathcal{L} = h_{ijk}\overline{L_i}\ell_{jR}\Phi_k + h.c., \quad \Phi_k = \begin{pmatrix} \phi_k^+ \\ \phi_k^0 \end{pmatrix}. \tag{1}$$

Here, $i, j, k = 1, 2, 3$. h_{ijk} are Yukawa couplings, whose form is determined by A_4 symmetry, which can be seen in Ref. [7]. Assuming that the three Higgs doublets acquire the same VEV, after the electroweak symmetry breaking, we get a mixing mass matrix for charged leptons. This mass matrix can be diagonalized with the following transformations on the charged lepton fields [7].

$$\begin{aligned} &\Psi_L \to U_L\Psi_L, \quad \Psi_R \to U_R\Psi_R, \\ &\Psi_L = (\ell_{1L}, \ell_{2L}, \ell_{3L})^T, \quad \Psi_R = (\ell_{1R}, \ell_{2R}, \ell_{3R})^T, \\ &U_L = U_{CW} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad U_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{2}$$

Here, $\omega = e^{2\pi i/3}$.

In the neutrino sector, the invariant Lagrangian is

$$\begin{aligned} \mathcal{L} = &y_1(\overline{L_1^c}i\sigma_2\xi_1L_1 + \overline{L_2^c}i\sigma_2\xi_1L_2 + \overline{L_3^c}i\sigma_2\xi_1L_3) \\ &+y_2(\overline{L_1^c}i\sigma_2\xi_2L_1 + \omega\overline{L_2^c}i\sigma_2\xi_2L_2 + \omega^2\overline{L_3^c}i\sigma_2\xi_2L_3) \\ &+y_3(\overline{L_1^c}i\sigma_2\xi_3L_1 + \omega^2\overline{L_2^c}i\sigma_2\xi_3L_2 + \omega\overline{L_3^c}i\sigma_2\xi_3L_3) \\ &+y(\overline{L_1^c}i\sigma_2\xi_6L_2 + \overline{L_2^c}i\sigma_2\xi_4L_3 + \overline{L_3^c}i\sigma_2\xi_5L_1) + h.c., \end{aligned} \tag{3}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \xi_k = \begin{pmatrix} \frac{\xi_k^+}{\sqrt{2}} & \xi_k^{++} \\ -\xi_k^0 & -\frac{\xi_k^+}{\sqrt{2}} \end{pmatrix}, k = 1, \cdots, 6. \tag{4}$$

Here, y_1, y_2, y_3, y are dimensionless Yukawa couplings. Also, L_i^c, where $i = 1, 2, 3$, are charge conjugate doublets of L_i. The above invariant Lagrangian can be obtained from the product rules of A_4 symmetry, which are given in Appendix A. After giving VEVs to neutral component of ξ_k, from Eq. (3), we get mixing mass matrix for neutrino fields, which is given below [6].

$$\mathcal{L} = -\frac{1}{2}\overline{\Psi^c}_\nu\mathcal{M}_\nu\Psi_\nu + h.c., \quad \Psi_\nu = (\nu_{1L}, \nu_{2L}, \nu_{3L})^T,$$

$$\mathcal{M}_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}, \tag{5}$$

$$a = 2y_1 v'_1, \quad b = 2y_2 v'_2, \quad c = 2y_3 v'_3, \quad d = y v'_4, \quad e = y v'_5, \quad f = y v'_6. \tag{6}$$

Here, $\langle \xi_i^0 \rangle = v'_i$, $i = 1, \cdots, 6$. As stated in the previous section, the masses for neutrinos are very small. In order to obtain small masses for neutrinos, using the above relations, we can take either the Yukawa couplings or the VEVs of Higgs triplets to be small. In this work, we choose the VEVs of Higgs triplets to be small so that the Yukawa couplings can be $\mathcal{O}(1)$. With this choice, we can notice that LFV decays in this model are unsuppressed, and as explained in the previous section, study of LFV decays is another topic of interest in this work.

The matrix in Eq. (5) can be diagonalized after assuming $b-c, e, f$ to be small and also after applying the following transformation on the neutrino fields [10].

$$\Psi_\nu \to U_{CW} U_{TBM} U_\epsilon \Psi_\nu,$$
$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \quad U_\epsilon = \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}^* & 1 & \epsilon_{23} \\ -\epsilon_{13}^* & -\epsilon_{23}^* & 1 \end{pmatrix}. \tag{7}$$

In the unitary matrix U_ϵ [18,19], the ϵ parameters are complex and the real and imaginary parts of these are assumed [10] to be less than or of the order of $\sin\theta_{13} \sim 0.15$ [3], where θ_{13} is a neutrino mixing angle. Here, one can notice that U_ϵ is unitary only up to first order in ϵ parameters. From the above equation, we can see that U_ϵ gives a perturbation to U_{TBM}, which can produce deviation from TBM mixing pattern. There are other ways to parametrize these perturbations. However, in this work we stick to the above mentioned parametrization, which is suggested in Refs. [18,19]. Now, while diagonalizing the neutrino mass matrix in Eq. (5), terms which are of the order of $\sin^2\theta_{13} \sim \frac{m_s^2}{m_a^2} \sim 10^{-2}$ [3] have been neglected [10]. Here, m_s and m_a are the square-root of solar and atmospheric mass-square differences among the neutrino fields, respectively. The central values for these mass-square differences are given below [3].

$$m_s^2 = m_2^2 - m_1^2 = 7.5 \times 10^{-5}\ \text{eV}^2, \quad m_a^2 = \begin{cases} m_3^2 - m_1^2 = 2.55 \times 10^{-3}\ \text{eV}^2 & \text{(NO)} \\ m_1^2 - m_3^2 = 2.45 \times 10^{-3}\ \text{eV}^2 & \text{(IO)} \end{cases}. \tag{8}$$

Here, $m_{1,2,3}$ are neutrino mass eigenvalues and NO (IO) represents normal (inverted) ordering. In order to fit the above mass-square differences, the neutrino mass eigenvalues can be taken as follows.

$$\text{NO}: \quad m_1 \lesssim m_s, \quad m_2 = \sqrt{m_s^2 + m_1^2}, \quad m_3 = \sqrt{m_a^2 + m_1^2}.$$
$$\text{IO}: \quad m_3 \lesssim m_s, \quad m_1 = \sqrt{m_a^2 + m_3^2}, \quad m_2 = \sqrt{m_s^2 + m_1^2}. \tag{9}$$

As described previously, terms of the order of or higher than that of $\sin^2\theta_{13} \sim \frac{m_s^2}{m_a^2}$ are neglected in the diagonalization of M_ν of Eq. (5) [10]. As a result of this, the neutrino mass eigenvalues in terms of model parameters have been found to be [10]

$$m_1 = a + d - \frac{b+c}{2}, \quad m_2 = a+b+c, \quad m_3 = -a + d + \frac{b+c}{2}. \tag{10}$$

The above expressions are valid in NO and IO cases. The relations for other model parameters containing in M_ν depend on the neutrino mass ordering. Expressions for these are given below [10].

$$\begin{aligned}
&\text{NO}: \quad e+f=0, \quad \frac{\sqrt{3}}{2}(b-c)=m_3\epsilon_{13}^*, \quad \frac{i}{\sqrt{2}}(e-f)=m_3\epsilon_{23}^*.\\
&\text{IO}: \quad \frac{e+f}{\sqrt{2}}=-m_1\epsilon_{12}+m_2\epsilon_{12}^*, \quad \frac{\sqrt{3}}{2}(b-c)=-m_1\epsilon_{13}, \quad \frac{i}{\sqrt{2}}(e-f)=-m_2\epsilon_{23}.
\end{aligned} \tag{11}$$

Now, after diagonalizing the mass matrix M_ν, one can get expressions for neutrino mixing angles. The procedure for this is explained below. After comparing the transformations for charged leptons and neutrinos of Eqs. (2) and (7), the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix can be written as

$$U_{PMNS}=U_{TBM}U_\epsilon. \tag{12}$$

The PMNS matrix is parametrized in terms of three neutrino mixing angles and δ_{CP}, in accordance with the PDG convention [13]. Using this parametrization in Eq. (12) and after solving the relations of this matrix equation, the leading order expressions for the three neutrino mixing angles and δ_{CP} are found to be [10]

$$\begin{aligned}
&\sin\theta_{12}=\frac{1}{\sqrt{3}}+\sqrt{\frac{2}{3}}Re(\epsilon_{12}), \quad Im(\epsilon_{12})=0,\\
&\sin\theta_{23}=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{6}}Re(\epsilon_{13})+\frac{1}{\sqrt{3}}Re(\epsilon_{23}), \quad Im(\epsilon_{13})=\sqrt{2}Im(\epsilon_{23}),\\
&\sin\theta_{13}=\left(\sqrt{\frac{2}{3}}Re(\epsilon_{13})+\frac{1}{\sqrt{3}}Re(\epsilon_{23})\right)\cos\delta_{\rm CP}-\left(\sqrt{\frac{2}{3}}Im(\epsilon_{13})+\frac{1}{\sqrt{3}}Im(\epsilon_{23})\right)\sin\delta_{\rm CP},\\
&\left(\sqrt{\frac{2}{3}}Re(\epsilon_{13})+\frac{1}{\sqrt{3}}Re(\epsilon_{23})\right)\sin\delta_{\rm CP}+\left(\sqrt{\frac{2}{3}}Im(\epsilon_{13})+\frac{1}{\sqrt{3}}Im(\epsilon_{23})\right)\cos\delta_{\rm CP}=0.
\end{aligned} \tag{13}$$

Here, $Re(\epsilon_{ij})$ and $Im(\epsilon_{ij})$ are real and imaginary parts of ϵ_{ij}, $i,j=1,\cdots,3$.

From Eq. (13), we can notice that the imaginary part of ϵ_{12} is zero. Using this in the case of IO, from Eq. (11), we get $e+f\sim m_1\frac{m_s^2}{m_a^2}Re(\epsilon_{12})$. As described previously, in the approximation procedure of Ref. [10], terms higher than the order of $\sin^2\theta_{13}\sim\frac{m_s^2}{m_a^2}$ are neglected. Hence, to the leading order, in both NO and IO we get $e+f=0$. This implies $v_5'=-v_6'$, which follows from Eq. (5). Now, from Eq. (13), we can see that all ϵ parameters can be determined in terms of three neutrino mixing angles and δ_{CP}. Using this fact and from Eqs. (10) and (11), we can notice that all model parameters of M_ν are determined in terms of neutrino oscillation observables. Among these model parameters, except for a and d, rest of them depend on the neutrino mass ordering. Expressions for these parameters are given below.

$$\text{NO \& IO}: \quad a=\frac{m_1+m_2-m_3}{3}, \quad d=\frac{m_1+m_3}{2}.$$

$$\text{NO}: \quad b=\frac{m_2}{3}-\frac{m_1-m_3}{6}+\frac{m_3\epsilon_{13}^*}{\sqrt{3}}, \quad c=\frac{m_2}{3}-\frac{m_1-m_3}{6}-\frac{m_3\epsilon_{13}^*}{\sqrt{3}}, \quad e=-\frac{im_3\epsilon_{23}^*}{\sqrt{2}}.$$
$$\text{IO}: \quad b=\frac{m_2}{3}-\frac{m_1-m_3}{6}-\frac{m_1\epsilon_{13}}{\sqrt{3}}, \quad c=\frac{m_2}{3}-\frac{m_1-m_3}{6}+\frac{m_1\epsilon_{13}}{\sqrt{3}}, \quad e=\frac{im_2\epsilon_{23}}{\sqrt{2}}. \tag{14}$$

Using the above expressions in Eq. (5), we can see that all Yukawa couplings of the MW model can be determined in terms of neutrino oscillation observables and the VEVs of Higgs triplets. Here, one can notice that the coupling y can be obtained from either d or e. The fact is that v_4' and v_5' are not independent parameters. As a result of this, we can consider the following two cases in order to determine y.

$$\text{case I}: \quad y=\frac{d}{v_4'}, \quad \text{case II}: \quad y=\frac{e}{v_5'} \tag{15}$$

In case I(II), $v_4'(v_5')$ is independent parameter and $v_5'(v_4')$ is determined in terms of $v_4'(v_5')$. An interesting point is that if we choose v_4' as an independent parameter, the coupling y do not depend on neutrino mixing angles and δ_{CP}. On the other hand, in case II, y depends on neutrino mixing angles and δ_{CP}. The above mentioned cases can make a difference in the branching ratios for LFV decays of this model, which is presented in Sec. 4.

3. Analysis of scalar potential

In the MW model [6], three Higgs doublets and six Higgs triplets exist. From the previous section, we have seen that the VEVs of Higgs triplets generate masses and mixing angles for neutrino fields. The VEVs for these fields arise after minimizing the scalar potential of this model. Hence, in this section, we write the full invariant scalar potential of the MW model. Thereafter, we analyze the implications of this potential on neutrino mixing.

3.1. Scalar potential of the MW model

The invariant scalar potential in the MW model can be written as

$$V_{MW}=V_0(\Phi)+V_1(\Phi,\xi)+V_Q(\xi). \tag{16}$$

Here, $V_0(\Phi)$ is a potential which depends only on the Higgs doublets, whose form is already given in Ref. [7]. $V_1(\Phi,\xi)$ contains terms involving both Higgs doublets and triplets. $V_Q(\xi)$ contains exclusively the quartic interaction terms among the Higgs triplets. In the minimization of the scalar potential, quartic terms in $V_Q(\xi)$ give negligibly small corrections, due to the following reasons. From precision electroweak tests [13], ρ parameter gives a constraint on VEV of triplet Higgs to be less than about 1 GeV. In the MW model, since three Higgs doublets exist, we can choose the VEVs of Higgs doublets to be around 100 GeV. Hence, while doing the minimization, terms in $V_Q(\xi)$ are at least suppressed by 10^{-4} as compared to that in $V_1(\Phi,\xi)$. In our work, as stated in the previous section, we choose VEVs of Higgs triplets to be much smaller than 1 GeV, say around 0.1 eV, in order to explain the small neutrino masses. Shortly below, we give arguments for making triplet Higgs VEVs to be as small as 0.1 eV. For the above mentioned reasons, we can notice that terms in $V_Q(\xi)$ give negligibly small contributions to the minimization of the potential. Hence, we omit these terms in our analysis. However, for the sake of completeness, we present all the invariant terms of $V_Q(\xi)$ in Appendix B. In order to write the invariant terms of $V_1(\Phi,\xi)+V_Q(\xi)$, we follow the work of Ref. [20]. In Ref. [20], invariant scalar potential under the electroweak symmetry is given, for the case of one doublet and triplet Higgses. To write the terms in $V_1(\Phi,\xi)+V_Q(\xi)$, we generalize the potential given in Ref. [20], by including three Higgs doublets, six Higgs triplets and A_4 symmetry. In order to make the scalar potential invariant under A_4 symmetry, we follow the product rules of A_4 symmetry, which are given in Appendix A.

To write the scalar potential of the MW model, we define the following quantities.

$$\begin{aligned}
&(\Phi^\dagger\Phi)\equiv\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2+\Phi_3^\dagger\Phi_3,\quad (\Phi^\dagger\Phi)'\equiv\Phi_1^\dagger\Phi_1+\omega^2\Phi_2^\dagger\Phi_2+\omega\Phi_3^\dagger\Phi_3,\\
&(\Phi^\dagger\Phi)''\equiv\Phi_1^\dagger\Phi_1+\omega\Phi_2^\dagger\Phi_2+\omega^2\Phi_3^\dagger\Phi_3,\quad (\xi^\dagger\xi)\equiv\xi_4^\dagger\xi_4+\xi_5^\dagger\xi_5+\xi_6^\dagger\xi_6,\\
&(\xi^\dagger\xi)'\equiv\xi_4^\dagger\xi_4+\omega^2\xi_5^\dagger\xi_5+\omega\xi_6^\dagger\xi_6,\quad (\xi^\dagger\xi)''\equiv\xi_4^\dagger\xi_4+\omega\xi_5^\dagger\xi_5+\omega^2\xi_6^\dagger\xi_6,\\
&(\Phi^\dagger f(\xi)\Phi)\equiv\Phi_1^\dagger f(\xi)\Phi_1+\Phi_2^\dagger f(\xi)\Phi_2+\Phi_3^\dagger f(\xi)\Phi_3,\\
&(\Phi^\dagger f(\xi)\Phi)'\equiv\Phi_1^\dagger f(\xi)\Phi_1+\omega\Phi_2^\dagger f(\xi)\Phi_2+\omega^2\Phi_3^\dagger f(\xi)\Phi_3.
\end{aligned}\tag{17}$$

Here, $f(\xi)$ is a function depending on the Higgs triplet fields. Now, we have [7]

$$\begin{aligned}
V_0(\Phi)=\;&m^2(\Phi^\dagger\Phi)+\frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2+\lambda_2(\Phi^\dagger\Phi)'(\Phi^\dagger\Phi)''\\
&+\lambda_3\left[(\Phi_2^\dagger\Phi_3)(\Phi_3^\dagger\Phi_2)+(\Phi_3^\dagger\Phi_1)(\Phi_1^\dagger\Phi_3)+(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\right]\\
&+\left\{\frac{1}{2}\lambda_4\left[(\Phi_2^\dagger\Phi_3)^2+(\Phi_3^\dagger\Phi_1)^2+(\Phi_1^\dagger\Phi_2)^2\right]+h.c.\right\}.
\end{aligned}\tag{18}$$

In the above, m^2 has mass-square dimension and λ parameters are dimensionless. The invariant terms in $V_1(\Phi,\xi)$ can be written as

$$\begin{aligned}
&V_1(\Phi,\xi)=\\
&m_1^2\,\mathrm{Tr}(\xi_1^\dagger\xi_1)+m_2^2\,\mathrm{Tr}(\xi_2^\dagger\xi_2)+m_3^2\,\mathrm{Tr}(\xi_3^\dagger\xi_3)+m_0^2\,\mathrm{Tr}((\xi^\dagger\xi))+\lambda_5^{(1)}(\Phi^\dagger\Phi)\mathrm{Tr}(\xi_1^\dagger\xi_1)\\
&+\lambda_5^{(2)}(\Phi^\dagger\Phi)\mathrm{Tr}(\xi_2^\dagger\xi_2)+\lambda_5^{(3)}(\Phi^\dagger\Phi)\mathrm{Tr}(\xi_3^\dagger\xi_3)+\lambda_5^{(4)}(\Phi^\dagger\Phi)\mathrm{Tr}((\xi^\dagger\xi))\\
&+\Big\{\lambda_5^{(5)}(\Phi^\dagger\Phi)'\mathrm{Tr}(\xi_1^\dagger\xi_3)+\lambda_5^{(6)}(\Phi^\dagger\Phi)'\mathrm{Tr}(\xi_2^\dagger\xi_1)+\lambda_5^{(7)}(\Phi^\dagger\Phi)'\mathrm{Tr}(\xi_3^\dagger\xi_2)\\
&+\lambda_5^{(8)}(\Phi^\dagger\Phi)'\mathrm{Tr}((\xi^\dagger\xi)'')+\lambda_5^{(9)}\left[\Phi_2^\dagger\Phi_3\mathrm{Tr}(\xi_5^\dagger\xi_6)+\Phi_3^\dagger\Phi_1\mathrm{Tr}(\xi_6^\dagger\xi_4)+\Phi_1^\dagger\Phi_2\mathrm{Tr}(\xi_4^\dagger\xi_5)\right]\\
&+\lambda_5^{(10)}\left[\Phi_2^\dagger\Phi_3\mathrm{Tr}(\xi_6^\dagger\xi_5)+\Phi_3^\dagger\Phi_1\mathrm{Tr}(\xi_4^\dagger\xi_6)+\Phi_1^\dagger\Phi_2\mathrm{Tr}(\xi_5^\dagger\xi_4)\right]+h.c.\Big\}\\
&+\lambda_6^{(1)}(\Phi^\dagger(\xi_1^\dagger\xi_1)\Phi)+\lambda_6^{(2)}(\Phi^\dagger(\xi_2^\dagger\xi_2)\Phi)+\lambda_6^{(3)}(\Phi^\dagger(\xi_3^\dagger\xi_3)\Phi)+\lambda_6^{(4)}(\Phi^\dagger(\xi^\dagger\xi)\Phi)\\
&+\Big\{\lambda_6^{(5)}(\Phi^\dagger(\xi_1^\dagger\xi_2)\Phi)'+\lambda_6^{(6)}(\Phi^\dagger(\xi_3^\dagger\xi_1)\Phi)'+\lambda_6^{(7)}(\Phi^\dagger(\xi_2^\dagger\xi_3)\Phi)'+\lambda_6^{(8)}(\Phi^\dagger(\xi^\dagger\xi)'\Phi)'\\
&+\lambda_6^{(9)}\left(\Phi_1^\dagger\xi_6^\dagger\xi_4\Phi_3+\Phi_2^\dagger\xi_4^\dagger\xi_5\Phi_1+\Phi_3^\dagger\xi_5^\dagger\xi_6\Phi_2\right)+\lambda_6^{(10)}\Big(\Phi_1^\dagger\xi_4^\dagger\xi_5\Phi_2+\Phi_2^\dagger\xi_5^\dagger\xi_6\Phi_3\\
&+\Phi_3^\dagger\xi_6^\dagger\xi_4\Phi_1\Big)+\lambda_6^{(11)}\left(\Phi_1^\dagger\xi_6\xi_4^\dagger\Phi_3+\Phi_2^\dagger\xi_4\xi_5^\dagger\Phi_1+\Phi_3^\dagger\xi_5\xi_6^\dagger\Phi_2\right)+\lambda_6^{(12)}\Big(\Phi_1^\dagger\xi_4\xi_5^\dagger\Phi_2\\
&+\Phi_2^\dagger\xi_5\xi_6^\dagger\Phi_3+\Phi_3^\dagger\xi_6\xi_4^\dagger\Phi_1\Big)+\mu_1\left[\tilde{\Phi}_1^Ti\sigma_2\xi_1\tilde{\Phi}_1+\tilde{\Phi}_2^Ti\sigma_2\xi_1\tilde{\Phi}_2+\tilde{\Phi}_3^Ti\sigma_2\xi_1\tilde{\Phi}_3\right]\\
&+\mu_2\left[\tilde{\Phi}_1^Ti\sigma_2\xi_2\tilde{\Phi}_1+\omega\tilde{\Phi}_2^Ti\sigma_2\xi_2\tilde{\Phi}_2+\omega^2\tilde{\Phi}_3^Ti\sigma_2\xi_2\tilde{\Phi}_3\right]+\mu_3\Big[\tilde{\Phi}_1^Ti\sigma_2\xi_3\tilde{\Phi}_1
\end{aligned}$$

$$+\omega^2\tilde{\Phi}_2^T i\sigma_2\xi_3\tilde{\Phi}_2 + \omega\tilde{\Phi}_3^T i\sigma_2\xi_3\tilde{\Phi}_3\Big] + \mu\left[\tilde{\Phi}_1^T i\sigma_2\xi_5\tilde{\Phi}_3 + \tilde{\Phi}_2^T i\sigma_2\xi_6\tilde{\Phi}_1 + \tilde{\Phi}_3^T i\sigma_2\xi_4\tilde{\Phi}_2\right]$$
$$+h.c.\}\,. \tag{19}$$

In the above equation, $m^2_{0,1,2,3}$ have mass-square dimensions, μ parameters have mass dimensions and λ parameters are dimensionless. Here, $\tilde{\Phi}_k = i\sigma_2\Phi_k^*, k = 1, 2, 3$. We assume λ parameters to be $\mathcal{O}(1)$. As stated before, $\langle\phi_i^0\rangle \sim 100$ GeV. $m^2_{0,1,2,3} \sim m_T^2$ give mass scale for scalar triplet Higgses. Since we want these scalar triplet Higgses to be produced in the LHC experiment, we take $m^2_{0,1,2,3} \sim m_T^2 \sim (100\ \text{GeV})^2$. Now, after minimizing Eq. (19) with respect to triplet Higgses, naively we expect the VEVs of these fields to be $\sim \mu_T\langle\phi_i^0\rangle^2/m_T^2$. Here, μ_T represents any of the μ parameters of Eq. (19). After using the above mentioned choices of the parameters, we can see that the VEVs of triplet Higgses can be as small as 0.1 eV, provided the μ parameters are suppressed to around 0.1 eV. By suppressing the μ parameters, one can realize the hierarchy in the VEVs of doublet and triplet Higgses. See Ref. [16], for a loop induced mechanism in order to explain the smallness of μ parameters.

The minimization of $V_0(\Phi)$ has been done in Ref. [7] and it is shown that $\langle\phi_i^0\rangle = v$ can be achieved for $i = 1, 2, 3$. In this work, doublet Higgses have interactions with triplet Higgses. Since we are taking VEVs of triplet Higgses to be around 0.1 eV, the contribution from $\langle V_1(\Phi, \xi)\rangle$ is negligibly small in comparison to $\langle V_0(\Phi)\rangle$. Hence, in this work, we get $\langle\phi_i^0\rangle \approx v$ for $i = 1, 2, 3$. As stated previously, this is known as the vacuum alignment of Higgs doublets, which is necessary in order to diagonalize the charged lepton mass matrix, which is described around Eq. (2). Now, after minimizing the $V_1(\Phi, \xi)$ with respect to neutral components of ξ_2 and ξ_3, we get

$$(m_2^2 + \lambda_5^{(2)}3|v|^2)\langle\xi_2^0\rangle = 0, \quad (m_3^2 + \lambda_5^{(3)}3|v|^2)\langle\xi_3^0\rangle = 0. \tag{20}$$

From the above equations, we get $v_2' = v_3' = 0$. This implies $b = c = 0$. Using this in Eq. (11), we get $\epsilon_{13} = 0$. Thereafter, relations in Eq. (13) can be solved for $\delta_{CP} = \pi$, which is allowed for the case of NO by the current neutrino oscillation data [3]. As a result of this, at leading order, we get the following constraint relation

$$\sin^2\theta_{23} = \frac{1}{2} + \sqrt{2}\sin\theta_{13}. \tag{21}$$

The above constraint relation cannot be satisfied in the allowed 3σ regions for $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ [3].

The problem described in the previous paragraph arises due to the fact that ξ_2 and ξ_3, which transform as $\underline{1}'$ and $\underline{1}''$ respectively under A_4, acquire zero VEVs. On the other hand, the other Higgs triplets ξ_1 and ξ_j, which transform as $\underline{1}$ and $\underline{3}$ respectively under A_4, can acquire non-zero VEVs. Let us mention here that in Ref. [19] a model with ξ_1 and ξ_j is presented in order to explain neutrino mixing pattern. It is shown that the model of Ref. [19] can consistently explain neutrino mixing pattern and can predict normal ordering of masses for neutrinos. Hence, one can see that the MW model, for the case of $\langle\xi^0_{2,3}\rangle = 0$, effectively reduces to that of Ref. [19], as far as neutrino mixing is concerned. As a result of this, even if $\langle\xi^0_{2,3}\rangle = 0$, the MW model can explain neutrino mixing pattern but may only predict normal mass ordering for neutrinos. In this regard, it is worth to see if the MW model can be modified in such a way that it can explain both normal and inverted mass orderings for neutrinos. In our earlier work [10], we had shown that the above mentioned orderings are possible in the MW model, provided $\langle\xi^0_{2,3}\rangle \neq 0$. So to solve the above mentioned problem, one needs to find a mechanism which can give $\langle\xi^0_{2,3}\rangle \neq 0$ in the MW model.

One can notice that, because of the vacuum alignment of Higgs doublets, the tri-linear couplings μ_2, μ_3 of Eq. (19) do not contribute to $\langle\xi^0_2\rangle$ and $\langle\xi^0_3\rangle$ after minimizing the scalar potential. Whereas, the other tri-linear couplings μ_1, μ can contribute to the VEVs of rest of the Higgs triplets, even with the vacuum alignment of Higgs doublets. One cannot break the vacuum alignment of Higgs doublets in the MW model, since it will affect the diagonalization of charged lepton mass matrix, which in turn has an effect on the mixing pattern in neutrino sector. Hence, in order to give non-zero VEVs to $\xi_{2,3}$, one can introduce additional tri-linear couplings involving these fields. We know that $\xi_{2,3}$ are charged under $\underline{1}', \underline{1}''$ of A_4 symmetry. With $\underline{1}', \underline{1}''$, the following are the only two singlet combinations, which can represent tri-linear terms in the potential: $\underline{1}' \times \underline{1}' \times \underline{1}'$, $\underline{1}'' \times \underline{1}'' \times \underline{1}''$. Hence, in the additional tri-linear couplings containing $\xi_{2,3}$, the Higgs doublets should be charged under $\underline{1}'$ and $\underline{1}''$ of A_4 symmetry. As a result of this, we propose additional Higgs doublets Φ_5 and Φ_6 which transform as $\underline{1}'$ and $\underline{1}''$ respectively under A_4 symmetry. Now, one can see that the following terms can exist in the scalar potential, which can give non-zero VEVs to ξ_2 and ξ_3: $\tilde{\Phi}^T_6 i\sigma_2\xi_2\tilde{\Phi}_6$, $\tilde{\Phi}^T_5 i\sigma_2\xi_3\tilde{\Phi}_5$. However, the Higgs doublets Φ_5 and Φ_6 can give rise to extra terms in the scalar potential with the fields $\Phi_i, i = 1, 2, 3$. These extra terms can affect the vacuum alignment of Higgs doublets of the MW model. We study these topics in the next subsection.

3.2. Extension of the MW model with two additional Higgs doublets

As described previously, in order to get non-zero VEVs to ξ_2 and ξ_3, we add the Higgs doublets Φ_5 and Φ_6 to the MW model. Since these Higgs doublets are charged under $\underline{1}'$ and $\underline{1}''$ of A_4 symmetry, with the charge assignments given in Table 1, one can notice that they do not generate Yukawa couplings for charged leptons and neutrinos. However, the Higgs doublets $\Phi_{5,6}$ can have interactions with the other Higgs doublets $\Phi_{1,2,3}$ and also with the Higgs triplets of this model. As a result of this, the scalar potential of the MW model, which is given in Eq. (16), will change to

$$V = V_{MW} + V'_0(\Phi) + V'_1(\Phi, \xi). \tag{22}$$

Here, $V'_0(\Phi)$ and $V'_1(\Phi, \xi)$ contain terms between $\Phi_{5,6}$ and already existing scalars of the MW model. Their forms are given below.

$$\begin{aligned}
&V'_0(\Phi) = \\
&m_5^2\Phi_5^\dagger\Phi_5 + m_6^2\Phi_6^\dagger\Phi_6 + \frac{1}{2}\lambda_1^{(1)}(\Phi_5^\dagger\Phi_5)^2 + \frac{1}{2}\lambda_1^{(2)}(\Phi_6^\dagger\Phi_6)^2 + \lambda_1^{(3)}(\Phi_5^\dagger\Phi_5)(\Phi_6^\dagger\Phi_6) \\
&+\lambda_1^{(4)}(\Phi^\dagger\Phi)(\Phi_5^\dagger\Phi_5) + \lambda_1^{(5)}(\Phi^\dagger\Phi)(\Phi_6^\dagger\Phi_6) + \lambda_2^{(1)}(\Phi_5^\dagger\Phi_6)(\Phi_6^\dagger\Phi_5) \\
&+\left\{\lambda_2^{(2)}(\Phi^\dagger\Phi)'(\Phi_6^\dagger\Phi_5) + h.c.\right\} \\
&+\lambda_3^{(1)}\left[(\Phi_5^\dagger\Phi_1)(\Phi_1^\dagger\Phi_5) + (\Phi_5^\dagger\Phi_2)(\Phi_2^\dagger\Phi_5) + (\Phi_5^\dagger\Phi_3)(\Phi_3^\dagger\Phi_5)\right] \\
&+\lambda_3^{(2)}\left[(\Phi_6^\dagger\Phi_1)(\Phi_1^\dagger\Phi_6) + (\Phi_6^\dagger\Phi_2)(\Phi_2^\dagger\Phi_6) + (\Phi_6^\dagger\Phi_3)(\Phi_3^\dagger\Phi_6)\right] + \left\{\frac{1}{2}\lambda_4^{(1)}\left[(\Phi_5^\dagger\Phi_1)^2\right.\right. \\
&\left. +\omega^2(\Phi_5^\dagger\Phi_2)^2 + \omega(\Phi_5^\dagger\Phi_3)^2\right] + \frac{1}{2}\lambda_4^{(2)}\left[(\Phi_6^\dagger\Phi_1)^2 + \omega(\Phi_6^\dagger\Phi_2)^2 + \omega^2(\Phi_6^\dagger\Phi_3)^2\right]
\end{aligned}$$

$$+\lambda_4^{(3)}\left[(\Phi_5^\dagger\Phi_1)(\Phi_6^\dagger\Phi_1)+(\Phi_5^\dagger\Phi_2)(\Phi_6^\dagger\Phi_2)+(\Phi_5^\dagger\Phi_3)(\Phi_6^\dagger\Phi_3)\right]$$
$$+\lambda_4^{(4)}\left[(\Phi_5^\dagger\Phi_1)(\Phi_1^\dagger\Phi_6)+\omega(\Phi_5^\dagger\Phi_2)(\Phi_2^\dagger\Phi_6)+\omega^2(\Phi_5^\dagger\Phi_3)(\Phi_3^\dagger\Phi_6)\right]+h.c.\Big\}. \quad (23)$$

$$\begin{aligned}V_1'(\Phi,\xi) &= \Phi_5^\dagger\Phi_5\left[\lambda_5^{(11)}\mathrm{Tr}(\xi_1^\dagger\xi_1)+\lambda_5^{(12)}\mathrm{Tr}(\xi_2^\dagger\xi_2)+\lambda_5^{(13)}\mathrm{Tr}(\xi_3^\dagger\xi_3)+\lambda_5^{(14)}\mathrm{Tr}((\xi^\dagger\xi))\right]\\
&+\Phi_6^\dagger\Phi_6\left[\lambda_5^{(15)}\mathrm{Tr}(\xi_1^\dagger\xi_1)+\lambda_5^{(16)}\mathrm{Tr}(\xi_2^\dagger\xi_2)+\lambda_5^{(17)}\mathrm{Tr}(\xi_3^\dagger\xi_3)+\lambda_5^{(18)}\mathrm{Tr}((\xi^\dagger\xi))\right]\\
&+\Phi_5^\dagger\Phi_6\left[\lambda_5^{(19)}\mathrm{Tr}(\xi_1^\dagger\xi_3)+\lambda_5^{(20)}\mathrm{Tr}(\xi_2^\dagger\xi_1)+\lambda_5^{(21)}\mathrm{Tr}(\xi_3^\dagger\xi_2)+\lambda_5^{(22)}\mathrm{Tr}((\xi^\dagger\xi)'')\right]\\
&+\Phi_6^\dagger\left[\lambda_6^{(13)}\xi_1^\dagger\xi_2+\lambda_6^{(14)}\xi_3^\dagger\xi_1+\lambda_6^{(15)}\xi_2^\dagger\xi_3+\lambda_6^{(16)}(\xi^\dagger\xi)'\right]\Phi_5+\Phi_6^\dagger\left[\lambda_6^{(17)}\xi_2\xi_1^\dagger\right.\\
&\left.+\lambda_6^{(18)}\xi_1\xi_3^\dagger+\lambda_6^{(19)}\xi_3\xi_2^\dagger+\lambda_6^{(20)}(\xi\xi^\dagger)'\right]\Phi_5+\mu_4\tilde{\Phi}_6^T i\sigma_2\xi_1\tilde{\Phi}_5+\mu_5\tilde{\Phi}_6^T i\sigma_2\xi_2\tilde{\Phi}_6\\
&+\mu_6\tilde{\Phi}_5^T i\sigma_2\xi_3\tilde{\Phi}_5+\mu_7(\tilde{\Phi}_1^T i\sigma_2\xi_4+\omega^2\tilde{\Phi}_2^T i\sigma_2\xi_5+\omega\tilde{\Phi}_3^T i\sigma_2\xi_6)\tilde{\Phi}_5\\
&+\mu_8(\tilde{\Phi}_1^T i\sigma_2\xi_4+\omega\tilde{\Phi}_2^T i\sigma_2\xi_5+\omega^2\tilde{\Phi}_3^T i\sigma_2\xi_6)\tilde{\Phi}_6+h.c.\end{aligned} \quad (24)$$

In the above two equations, all λ parameters are dimensionless, μ parameters have mass dimensions and $m_{5,6}^2$ have mass-square dimensions. We choose $m_{5,6}^2\sim(100\text{ GeV})^2$ so that the VEVs of $\Phi_{5,6}$ can be of the order of VEVs of other Higgs doublets. We suppress the μ parameters in order to conceive small VEVs for Higgs triplets. Due to this suppression, one can notice that $\langle V_1'(\Phi,\xi)\rangle$ is very small in comparison to $\langle V_0'(\Phi)\rangle$.

As stated previously, terms in $V_0'(\Phi)$ can affect the vacuum alignment of Higgs doublets $\Phi_{1,2,3}$. To study these effects, we minimize $V_0(\Phi)+V_0'(\Phi)$ with respect to $\phi_1^0,\phi_2^0,\phi_3^0$ and thereby we get three relations. We solve these relations by demanding $\langle\phi_i^0\rangle=v$ for $i=1,2,3$. Thereafter we get the following relations.

$$\left[m^2+(3\lambda_1+2\lambda_3+\lambda_4+\lambda_4^*)|v|^2+(\lambda_1^{(4)}+\lambda_3^{(1)})|v_5|^2+(\lambda_1^{(5)}+\lambda_3^{(2)})|v_6|^2\right]v$$
$$+2\lambda_4^{(3)^*}v_6v_5v^*=0,$$
$$(\lambda_2^{(2)^*}+\lambda_4^{(4)})v_6v_5^*v+\lambda_4^{(1)^*}v^*v_5^2=0,\quad(\lambda_2^{(2)}+\lambda_4^{(4)^*})v_6^*v_5v+\lambda_4^{(2)^*}v^*v_6^2=0. \quad (25)$$

Here, $\langle\phi_5^0\rangle=v_5$ and $\langle\phi_6^0\rangle=v_6$. By solving the unknown parameters in the above three relations, the vacuum alignment for the Higgs doublets $\Phi_{1,2,3}$ can be achieved. Now, the VEVs of $\Phi_{5,6}$ should satisfy the following relations.

$$\left[m_5^2+3(\lambda_1^{(4)}+\lambda_3^{(1)})|v|^2+\lambda_1^{(1)}|v_5|^2+(\lambda_1^{(3)}+\lambda_2^{(1)})|v_6|^2\right]v_5+3\lambda_4^{(3)}v^2v_6^*=0,$$
$$\left[m_6^2+3(\lambda_1^{(5)}+\lambda_3^{(2)})|v|^2+\lambda_1^{(2)}|v_6|^2+(\lambda_1^{(3)}+\lambda_2^{(1)})|v_5|^2\right]v_6+3\lambda_4^{(3)}v^2v_5^*=0. \quad (26)$$

As stated before, we take $m^2,m_{5,6}^2\sim(100\text{ GeV})^2$ so that the VEVs for Higgs doublets can be chosen to be around 100 GeV. As a result of this, relations in Eqs. (25) and (26) can be solved for the unknown λ parameters, which can be $\mathcal{O}(1)$.

The VEVs of Higgs triplets can be found after minimizing the potential $V_1(\Phi,\xi)+V_1'(\Phi,\xi)$. Expressions for these are given below.

$$\left[m_1^2+3\lambda_5^{(1)}|v|^2+\lambda_5^{(11)}|v_5|^2+\lambda_5^{(15)}|v_6|^2\right]v_1'+(\lambda_5^{(19)}+\lambda_6^{(18)^*})v_5^*v_6v_3'$$
$$+(\lambda_5^{(20)^*}+\lambda_6^{(17)})v_5v_6^*v_2'-3\mu_1^*v^2-\mu_4^*v_6v_5=0, \quad (27)$$

$$\left[m_2^2+3\lambda_5^{(2)}|v|^2+\lambda_5^{(12)}|v_5|^2+\lambda_5^{(16)}|v_6|^2\right]v_2'+(\lambda_5^{(20)}+\lambda_6^{(17)^*})v_5^*v_6v_1'$$
$$+(\lambda_5^{(21)^*}+\lambda_6^{(19)})v_5v_6^*v_3'-\mu_5^*v_6^2=0, \tag{28}$$
$$\left[m_3^2+3\lambda_5^{(3)}|v|^2+\lambda_5^{(13)}|v_5|^2+\lambda_5^{(17)}|v_6|^2\right]v_3'+(\lambda_5^{(19)^*}+\lambda_6^{(18)})v_5v_6^*v_1'$$
$$+(\lambda_5^{(21)}+\lambda_6^{(19)^*})v_5^*v_6v_2'-\mu_6^*v_5^2=0, \tag{29}$$
$$\left[m_0^2+3\lambda_5^{(4)}|v|^2+\lambda_5^{(14)}|v_5|^2+\lambda_5^{(18)}|v_6|^2+(\lambda_5^{(22)}+\lambda_6^{(20)^*})v_5^*v_6\right.$$
$$\left.+(\lambda_5^{(22)^*}+\lambda_6^{(20)})v_5v_6^*\right]v_4'+(\lambda_5^{(9)}+\lambda_5^{(10)^*})|v|^2v_5'+(\lambda_5^{(9)^*}+\lambda_5^{(10)})|v|^2v_6'-\mu^*v^2$$
$$-\mu_7^*vv_5-\mu_8^*vv_6=0, \tag{30}$$
$$\left[m_0^2+3\lambda_5^{(4)}|v|^2+\lambda_5^{(14)}|v_5|^2+\lambda_5^{(18)}|v_6|^2+\omega(\lambda_5^{(22)}+\lambda_6^{(20)^*})v_5^*v_6\right.$$
$$\left.+\omega^2(\lambda_5^{(22)^*}+\lambda_6^{(20)})v_5v_6^*\right]v_5'+(\lambda_5^{(9)}+\lambda_5^{(10)^*})|v|^2v_6'+(\lambda_5^{(9)^*}+\lambda_5^{(10)})|v|^2v_4'-\mu^*v^2$$
$$-\omega\mu_7^*vv_5-\omega^2\mu_8^*vv_6=0, \tag{31}$$
$$\left[m_0^2+3\lambda_5^{(4)}|v|^2+\lambda_5^{(14)}|v_5|^2+\lambda_5^{(18)}|v_6|^2+\omega^2(\lambda_5^{(22)}+\lambda_6^{(20)^*})v_5^*v_6\right.$$
$$\left.+\omega(\lambda_5^{(22)^*}+\lambda_6^{(20)})v_5v_6^*\right]v_6'+(\lambda_5^{(9)}+\lambda_5^{(10)^*})|v|^2v_4'+(\lambda_5^{(9)^*}+\lambda_5^{(10)})|v|^2v_5'-\mu^*v^2$$
$$-\omega^2\mu_7^*vv_5-\omega\mu_8^*vv_6=0. \tag{32}$$

From Eqs. (28) and (29), we can notice that the VEVs for ξ_2 and ξ_3 can be non-zero due to the contribution from μ parameters. In fact, using Eqs. (27)–(32), one can infer that for $\mathcal{O}(1)$ λ parameters, all the VEVs of Higgs triplets can be chosen to be around 0.1 eV by suppressing the μ parameters accordingly.

We have shown that all the Higgs triplets can acquire non-zero VEVs, after adding two additional Higgs doublets to the MW model. Moreover, we have demonstrated that vacuum alignment of the Higgs doublets $\Phi_{1,2,3}$ can be achieved in this model. Hence, in a scenario like this, results described in Sec. 2 are valid. As a result of that, in this model, the neutrino masses can have either NO or IO, and moreover, this model is compatible with current neutrino oscillation data.

4. LFV decays

In this section, we compute the branching ratios for LFV decays in the scenario where we extend the MW model with the Higgs doublets $\Phi_{5,6}$. As described in Sec. 1, LFV decays can be of the following two types: $\ell\to 3\ell'$, $\ell\to\ell'\gamma$. In our scenario, decays of the form $\ell\to 3\ell'$ are driven by doubly charged triplet Higgses. On the other hand, decays of the form $\ell\to\ell'\gamma$ are driven by both doubly and singly charged scalars of this model. In order to compute the branching ratios for these decays, one needs to obtain the mass eigenstates for doubly and singly charged scalars. Below we present these mass eigenstates.

It is to be noted that neutral fields from doublet Higgses, other than the standard model Higgs, can also contribute to the above mentioned LFV decays [7]. Most of these decays are suppressed due to smallness of charged lepton Yukawa couplings. However, there are some decays, whose amplitudes are proportional to tau Yukawa coupling, can give appreciable contribution [7], provided the masses of the neutral fields are low. To study the contribution of neutral fields to LFV decays in our scenario, we have to diagonalize the mixing masses among the neutral fields of the

Higgs doublets. It is to remind here that since the VEVs of triplet Higgses are very small, the mixing between neutral fields of doublets and triplets can be neglected. In this work, we assume that the masses for the above mentioned neutral fields are high enough that their contribution to LFV decays is suppressed. In this regard, let us mention that in Ref. [21], LFV decays driven by neutral scalar fields are studied. The work done in Ref. [21] is based on some flavor models [22] containing A_4 symmetry, where neutral flavon fields induce LFV decays.

4.1. Mass eigenstates of doubly and singly charged scalars

The doubly charged scalars belong to the Higgs triplets of our model. The masses for these fields can be obtained from the scalar potential of this model, which is given in the previous section. Since six triplet Higgses exist in the model, one can expect mixing masses among the doubly charged scalars. These mixing masses are given below.

$$V \ni (\psi_1^{++})^\dagger X\psi_1^{++} + (\psi_2^{++})^\dagger Y\psi_2^{++}, \tag{33}$$

$$\psi_1^{++} = (\xi_1^{++}, \xi_2^{++}, \xi_3^{++})^T, \quad \psi_2^{++} = (\xi_4^{++}, \xi_5^{++}, \xi_6^{++})^T,$$

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{12}^* & x_{22} & x_{23} \\ x_{13}^* & x_{23}^* & x_{33} \end{pmatrix}, \quad Y = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{12}^* & y_{22} & y_{23} \\ y_{13}^* & y_{23}^* & y_{33} \end{pmatrix},$$

$$\begin{aligned}
&x_{11} = m_1^2 + 3(\lambda_5^{(1)} + \lambda_6^{(1)})|v|^2 + \lambda_5^{(11)}|v_5|^2 + \lambda_5^{(15)}|v_6|^2, \\
&x_{22} = m_2^2 + 3(\lambda_5^{(2)} + \lambda_6^{(2)})|v|^2 + \lambda_5^{(12)}|v_5|^2 + \lambda_5^{(16)}|v_6|^2, \\
&x_{33} = m_3^2 + 3(\lambda_5^{(3)} + \lambda_6^{(3)})|v|^2 + \lambda_5^{(13)}|v_5|^2 + \lambda_5^{(17)}|v_6|^2, \\
&x_{12} = (\lambda_5^{(20)*} + \lambda_6^{(13)})v_5 v_6^*, \\
&x_{13} = (\lambda_5^{(19)} + \lambda_6^{(14)*})v_5^* v_6, \quad x_{23} = (\lambda_5^{(21)*} + \lambda_6^{(15)})v_5 v_6^*, \\
&y_{11} = m_0^2 + 3(\lambda_5^{(4)} + \lambda_6^{(4)})|v|^2 + \lambda_5^{(14)}|v_5|^2 + \lambda_5^{(18)}|v_6|^2 \\
&+[(\lambda_5^{(22)*} + \lambda_6^{(16)})v_5 v_6^* + h.c.], \\
&y_{22} = m_0^2 + 3(\lambda_5^{(4)} + \lambda_6^{(4)})|v|^2 + \lambda_5^{(14)}|v_5|^2 + \lambda_5^{(18)}|v_6|^2 \\
&+[\omega^2(\lambda_5^{(22)*} + \lambda_6^{(16)})v_5 v_6^* + h.c.], \\
&y_{33} = m_0^2 + 3(\lambda_5^{(4)} + \lambda_6^{(4)})|v|^2 + \lambda_5^{(14)}|v_5|^2 + \lambda_5^{(18)}|v_6|^2 + \\
&+[\omega(\lambda_5^{(22)*} + \lambda_6^{(16)})v_5 v_6^* + h.c.], \\
&y_{12} = (\lambda_5^{(9)} + \lambda_5^{(10)*} + \lambda_6^{(9)} + \lambda_6^{(10)})|v|^2, \quad y_{13} = y_{12}^*, \quad y_{23} = y_{12}.
\end{aligned} \tag{34}$$

From Eq. (33) we can notice that there is no mixing between $\xi_{1,2,3}^{++}$ and $\xi_{4,5,6}^{++}$. However, from the quartic terms of the potential, which are given in Appendix B, there may be mixing between the above mentioned doubly charged scalars. One can expect this mixing to be proportional to square of the VEVs of Higgs triplets, which in our case is very small. Hence, we neglect the above mentioned mixing. After diagonalizing the matrices X, Y of Eq. (33), we get mass eigenstates for doubly charged scalars, which are defined below.

$$\xi_i^{++} = \sum_{k=1}^{3} U_{ik}^{++}\xi_k^{(m)++}, \quad \xi_{i+3}^{++} = \sum_{k=1}^{3} V_{ik}^{++}\xi_{k+3}^{(m)++}. \tag{35}$$

Here, $i = 1, 2, 3$ and $\xi_k^{(m)++}$, where $k = 1, \cdots, 6$, are the mass eigenstates of the doubly charged scalars. The unitary matrices U^{++}, V^{++} diagonalize X, Y as

$$(U^{++})^\dagger X U^{++} = \mathrm{diag}(M^2_{++(1)}, M^2_{++(2)}, M^2_{++(3)}), \tag{36}$$
$$(V^{++})^\dagger Y V^{++} = \mathrm{diag}(M^2_{++(4)}, M^2_{++(5)}, M^2_{++(6)}). \tag{37}$$

In analogy to doubly charged scalars, mass eigenstates for singly charged scalars can be obtained. Singly charged scalars belong to both doublet and triplet Higgses. In our scenario, due to smallness of VEVs of Higgs triplets, we can neglect the mixing among singly charged scalars between doublet and triplet Higgses. Singly charged scalars of doublet Higgses can drive LFV decays $\ell \to \ell'\gamma$ through charged lepton Yukawa couplings. One can expect this contribution to be small unless these decays are induced by tau Yukawa coupling. To simplify our analysis we assume the masses for the singly charged scalars of doublet Higgses are high enough that their contribution to LFV decays is suppressed. As a result of this, in this model, the above mentioned LFV decays are dominantly driven by singly charged scalars of triplet Higgses. For these reasons, below we present the mass eigenstates for singly charged scalars from triplet Higgses. These scalars can have mixing masses, which can be written as

$$V \ni (\psi_1^+)^\dagger X' \psi_1^+ + (\psi_2^+)^\dagger Y' \psi_2^+, \tag{38}$$
$$\psi_1^+ = (\xi_1^+, \xi_2^+, \xi_3^+)^T, \quad \psi_2^+ = (\xi_4^+, \xi_5^+, \xi_6^+)^T,$$
$$X' = \begin{pmatrix} x'_{11} & x'_{12} & x'_{13} \\ x'^*_{12} & x'_{22} & x'_{23} \\ x'^*_{13} & x'^*_{23} & x'_{33} \end{pmatrix}, \quad Y' = \begin{pmatrix} y'_{11} & y'_{12} & y'_{13} \\ y'^*_{12} & y'_{22} & y'_{23} \\ y'^*_{13} & y'^*_{23} & y'_{33} \end{pmatrix},$$
$$x'_{11} = x_{11} - \frac{3}{2}\lambda_6^{(1)}|v|^2, \quad x'_{22} = x_{22} - \frac{3}{2}\lambda_6^{(2)}|v|^2, \quad x'_{33} = x_{33} - \frac{3}{2}\lambda_6^{(3)}|v|^2,$$
$$x'_{12} = x_{12} - \frac{1}{2}(\lambda_6^{(13)} - \lambda_6^{(17)})v_5 v_6^*, \quad x'_{13} = x_{13} - \frac{1}{2}(\lambda_6^{(14)*} - \lambda_6^{(18)*})v_5^* v_6,$$
$$x'_{23} = x_{23} - \frac{1}{2}(\lambda_6^{(15)} - \lambda_6^{(19)})v_5 v_6^*,$$
$$y'_{11} = y_{11} - \frac{3}{2}\lambda_6^{(4)}|v|^2 - \frac{1}{2}[(\lambda_6^{(16)*} - \lambda_6^{(20)*})v_5^* v_6 + h.c.],$$
$$y'_{22} = y_{22} - \frac{3}{2}\lambda_6^{(4)}|v|^2 - \frac{1}{2}[\omega(\lambda_6^{(16)*} - \lambda_6^{(20)*})v_5^* v_6 + h.c.],$$
$$y'_{33} = y_{33} - \frac{3}{2}\lambda_6^{(4)}|v|^2 - \frac{1}{2}[\omega^2(\lambda_6^{(16)*} - \lambda_6^{(20)*})v_5^* v_6 + h.c.],$$
$$y'_{12} = y_{12} - \frac{1}{2}(\lambda_6^{(9)} + \lambda_6^{(10)} - \lambda_6^{(11)*} - \lambda_6^{(12)*})|v|^2, \quad y'_{13} = (y'_{12})^*, \quad y'_{23} = y'_{12}. \tag{39}$$

From Eq. (38), in analogy to doubly charged scalars, we can notice that there is no mixing between $\xi^+_{1,2,3}$ and $\xi^+_{4,5,6}$ at leading order. Now, we can define the mass eigenstates for singly charged scalars as

$$\xi_i^+ = \sum_{k=1}^{3} U^+_{ik}\xi_k^{(m)+}, \quad \xi^+_{i+3} = \sum_{k=1}^{3} V^+_{ik}\xi^{(m)+}_{k+3}. \tag{40}$$

Here, $i = 1, 2, 3$ and $\xi_k^{(m)+}$, where $k = 1, \cdots, 6$, are the mass eigenstates of singly charged scalars. The unitary matrices U^+, V^+ diagonalize X', Y' as

$$(U^+)^\dagger X' U^+ = \mathrm{diag}(M^2_{+(1)}, M^2_{+(2)}, M^2_{+(3)}), \tag{41}$$

$$(V^+)^\dagger Y' V^+ = \mathrm{diag}(M^2_{+(4)}, M^2_{+(5)}, M^2_{+(6)}). \tag{42}$$

4.2. Branching ratios of $\ell \to 3\ell'$

In this subsection, we compute the branching ratios for decays of the form $\ell \to 3\ell'$. Since these decays are driven by doubly charged scalars at tree level, we need to obtain couplings between doubly charged scalars and charged leptons. These couplings are determined by the Lagrangian of Eq. (3), where all the scalars and fermions of this Lagrangian are in flavor states. For charged leptons, by applying the transformations in Eq. (2), we get the corresponding mass eigenstates. For doubly charged scalars, the mass eigenstates have been described in the previous subsection. After using the above mentioned mass eigenstates in Eq. (3), we get the desired couplings needed for the decays $\ell \to 3\ell'$. These are given below.

$$\begin{aligned}
&\mathcal{L} \ni -\sum_{k,l=1,k\le l}^{3} \ell_k^{(m)T} C \frac{1-\gamma_5}{2}\left[\sum_{j=1}^{3} f_{1j}^{k,l}\xi_j^{(m)++} + f_{2j}^{k,l}\xi_{j+3}^{(m)++}\right]\ell_l^{(m)},\\
&f_{1j}^{1,1} = y_1 U_{1j}^{++}, \quad f_{2j}^{1,1} = \frac{y}{3}(V_{1j}^{++} + V_{2j}^{++} + V_{3j}^{++}),\\
&f_{1j}^{2,3} = 2f_{1j}^{1,1}, \quad f_{2j}^{2,3} = -f_{2j}^{1,1},\\
&f_{1j}^{2,2} = y_2 U_{2j}^{++}, \quad f_{2j}^{2,2} = \frac{y}{3}(V_{1j}^{++} + \omega^2 V_{2j}^{++} + \omega V_{3j}^{++}),\\
&f_{1j}^{1,3} = 2f_{1j}^{2,2}, \quad f_{2j}^{1,3} = -f_{2j}^{2,2},\\
&f_{1j}^{3,3} = y_3 U_{3j}^{++}, \quad f_{2j}^{3,3} = \frac{y}{3}(V_{1j}^{++} + \omega V_{2j}^{++} + \omega^2 V_{3j}^{++}),\\
&f_{1j}^{1,2} = 2f_{1j}^{3,3}, \quad f_{2j}^{1,2} = -f_{2j}^{3,3}.
\end{aligned} \tag{43}$$

Here, C is the charge conjugation matrix and $\ell_j^{(m)}$ is a mass eigenstate of charged lepton. From the above equation, we can notice that some of the couplings between doubly charged scalars and charged leptons are related to one another. This is a result due to A_4 symmetry of the model. This result has implications on the branching ratios of the decays of the form $\ell \to 3\ell'$. We will explain these implications shortly later.

Using the couplings in Eq. (43), we compute the branching ratios for $\ell \to 3\ell'$, after neglecting the masses of final state charged leptons. Branching ratios for τ decays are found to be

$$\mathrm{Br}(\tau \to \bar{\ell}_i \ell_j \ell_k) = \frac{S}{32G_F^2}\left|\sum_{n=1}^{3}\frac{(f_{1n}^{j,k})^* f_{1n}^{i,3}}{M^2_{++(n)}} + \frac{(f_{2n}^{j,k})^* f_{2n}^{i,3}}{M^2_{++(n+3)}}\right|^2 \mathrm{Br}(\tau \to \mu\bar{\nu}\nu). \tag{44}$$

Here, G_F is the Fermi constant and $\mathrm{Br}(\tau \to \mu\bar{\nu}\nu) = 0.1739$ [13]. Moreover, the indices $i, j, k = 1, 2$ are for electron and muon fields. $S = 1(2)$ if $\ell_j \neq \ell_k (\ell_j = \ell_k)$. In the above equation, one should use $f_{1n}^{j,k} = f_{1n}^{k,j}$ and $f_{2n}^{j,k} = f_{2n}^{k,j}$. These relations follow from the Lagrangian of Eq. (43). The branching ratio for $\mu \to 3e$ is

$$\mathrm{Br}(\mu \to \bar{e}ee) = \frac{1}{16G_F^2}\left|\sum_{n=1}^{3}\frac{(f_{1n}^{1,1})^* f_{1n}^{1,2}}{M^2_{++(n)}} + \frac{(f_{2n}^{1,1})^* f_{2n}^{1,2}}{M^2_{++(n+3)}}\right|^2. \tag{45}$$

In this work, we have assumed $\mathrm{Br}(\mu \to e\bar{\nu}\nu) = 100\%$.

As stated before, some relations exist among the couplings in the Lagrangian of Eq. (43). An implication of this is there can exist relations among branching ratios for some decays. From Eq. (44), we can see that

$$\mathrm{Br}(\tau \to \bar{e}ee) = \mathrm{Br}(\tau \to \bar{\mu}\mu\mu). \tag{46}$$

From Eqs. (44) and (45), after assuming degenerate values for $M^2_{++(4)}, M^2_{++(5)}, M^2_{++(6)}$, we get

$$\mathrm{Br}(\tau \to \bar{\mu}e\mu) = 2\mathrm{Br}(\mu \to \bar{e}ee)\mathrm{Br}(\tau \to \mu\bar{\nu}\nu). \tag{47}$$

On the other hand, in the limit where the masses for all doubly charged scalars are degenerate, Eqs. (44) and (45) imply that the branching ratios for following decays go to zero: $\tau \to \bar{e}ee$, $\tau \to \bar{e}e\mu$, $\tau \to \bar{\mu}\mu\mu$, $\tau \to \bar{\mu}e\mu$, $\mu \to \bar{e}ee$. Relations among the branching ratios described in Eqs. (46) and (47) are due to the A_4 symmetry of our model. In the work of Ref. [21], which is based on A_4 symmetry, a similar kind of relations among various branching ratios for $\ell \to 3\ell'$ have been derived. Since the flavor models considered in Ref. [21] are different from our model, the relations for branching ratios given in Ref. [21] are different from Eqs. (46) and (47). We can notice here that searching for LFV decays in experiments can distinguish various flavor models. Moreover, these searches can give some hints about A_4 symmetry.

Among various decays of the type $\ell \to 3\ell'$, branching ratio for $\mu \to \bar{e}ee$ is severely constrained. From experiments, we have $\mathrm{Br}(\mu \to \bar{e}ee) < 1.0 \times 10^{-12}$ [23]. In order to satisfy this constraint in our work, we study the branching ratio of $\mu \to \bar{e}ee$. From Eq. (45), we can see that $\mathrm{Br}(\mu \to \bar{e}ee)$ depends on masses of doubly charged scalars and on couplings between doubly charged scalars and charged leptons. These couplings, which can be seen from Eq. (43), depend on neutrino Yukawa couplings and the unitary matrices which diagonalize the mixing masses for doubly charged scalars. Hence, the masses for doubly charged scalars and the above mentioned unitary matrices are determined from the parameters of the scalar potential. On the other hand, the neutrino Yukawa couplings are determined from the VEVs of Higgs triplets and neutrino oscillation observables. This fact can be seen from Eqs. (6), (14) and (13). From these equations, one can notice that the neutrino Yukawa couplings depend on θ_{13} and θ_{23}, but not on θ_{12}.

As described above, $\mathrm{Br}(\mu \to \bar{e}ee)$, in our work, depend on neutrino oscillation observables, VEVs of Higgs triplets and parameters of scalar potential. It is interesting to see the variation of $\mathrm{Br}(\mu \to \bar{e}ee)$ in terms of neutrino oscillation observables. Hence, we have fixed VEVs of Higgs triplets and parameters of scalar potential to some specific values in our analysis. The details of our analysis have been described below.

To simplify our numerical analysis, we take all the independent Higgs triplet VEVs to be same as v_T. It is to remind here that the VEVs v'_4, v'_5 are not independent. It is discussed in Sec. 2 that the Yukawa coupling y can be determined in terms of v'_4 or v'_5. As a result of this, from Eq. (15), we can see that in case I(II) $v'_4(v'_5)$ is independent parameter. As for the masses of doubly charged scalars, they are determined after diagonalizing the X, Y matrices of Eq. (33). Since there are several λ parameters exist in X, Y, for the sake of illustration, we choose all these λ parameters to be 0.1. We take the mass-square parameters of X, Y as $m_1^2 = m_2^2 = m_3^2 = m_0^2 = (850\ \mathrm{GeV})^2$. We have taken the VEVs for doublet Higgses as $v = v_5 = v_6 = 174/\sqrt{5}$ GeV. With the above mentioned parameters, we have found the masses for all doubly charged scalars to be slightly above 850 GeV. These mass values for doubly charged scalars satisfy the lower bound on them, which is obtained by the LHC experiment [24].

In Figs. 1 and 2, we have given the plots for branching ratios of $\mu \to \bar{e}ee$, in the cases of NO and IO respectively. As already described above, the neutrino Yukawa couplings of our

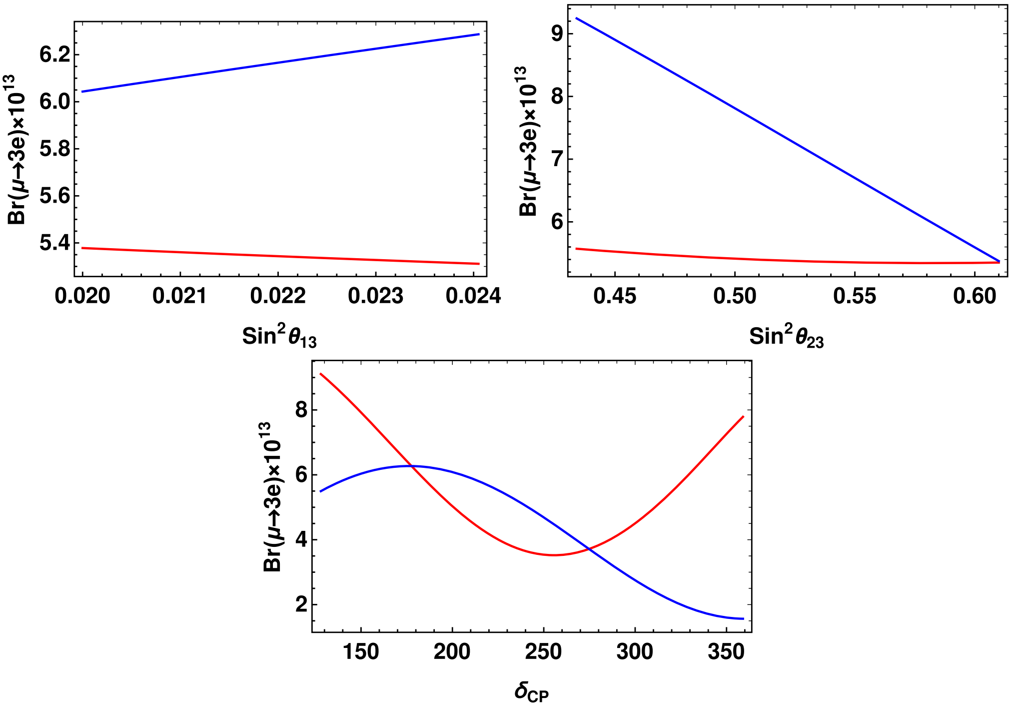

Fig. 1. Branching ratios for $\mu \to \bar{e}ee$ in the case of NO. Here, red and blue lines are for the cases I and II respectively. δ_{CP} is expressed in degrees. In these plots, $v_T = 0.08$ eV. In the top-left plot, $\sin^2\theta_{23}$ and δ_{CP} are fixed to the best fit values of Table 2. In the top-right plot, $\sin^2\theta_{13}$ and δ_{CP} are fixed to the best fit values of Table 2. In the bottom plot, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are fixed to the best fit values of Table 2. In all these plots, lightest neutrino mass is taken to be zero and the other neutrino masses are computed from Eqs. (8) and (9). For details related to masses of doubly charged scalars, see the text.

Table 2
Values of the neutrino oscillation parameters [3], which are used in this work.

Parameter	best fit	3σ range
$\sin^2\theta_{13}/10^{-2}$ (NO)	2.200	2.000 - 2.405
$\sin^2\theta_{13}/10^{-2}$ (IO)	2.225	2.018 - 2.424
$\sin^2\theta_{23}/10^{-1}$ (NO)	5.74	4.34 - 6.10
$\sin^2\theta_{23}/10^{-1}$ (IO)	5.78	4.33 - 6.08
δ_{CP}/o (NO)	194	128 - 359
δ_{CP}/o (IO)	284	200 - 353

model, up to the leading order, do not depend on the mixing angle θ_{12}. Hence, in Figs. 1 and 2, $\mathrm{Br}(\mu \to \bar{e}ee)$ is plotted against $\sin^2\theta_{13}$, $\sin^2\theta_{23}$ and δ_{CP}. The allowed ranges and best fit values for the neutrino mixing angles and δ_{CP}, which are used in this work, are tabulated in Table 2. In Figs. 1 and 2, in the plot between $\mathrm{Br}(\mu \to \bar{e}ee)$ and $\sin^2\theta_{13}$, we have fixed the best fit values for $\sin^2\theta_{23}$ and δ_{CP}, which are given in Table 2. Similar kind of things have been done in other plots of Figs. 1 and 2. In the plots of both these figures, we have taken the lightest neutrino mass to be zero and the other neutrino masses are computed from Eqs. (8) and (9). In Figs. 1 and 2, we

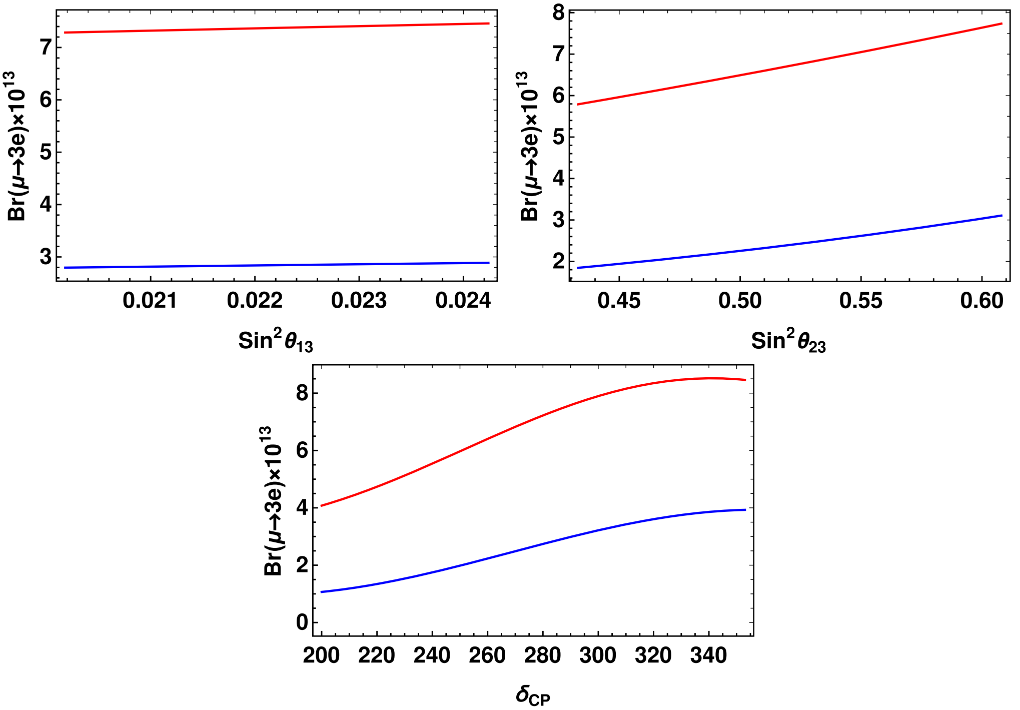

Fig. 2. Branching ratios for $\mu \to \bar{e}ee$ in the case of IO. Here, red and blue lines are for the cases I and II respectively. δ_{CP} is expressed in degrees. In these plots, $v_T = 0.14$ eV. In the top-left plot, $\sin^2\theta_{23}$ and δ_{CP} are fixed to the best fit values of Table 2. In the top-right plot, $\sin^2\theta_{13}$ and δ_{CP} are fixed to the best fit values of Table 2. In the bottom plot, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are fixed to the best fit values of Table 2. In all these plots, lightest neutrino mass is taken to be zero and the other neutrino masses are computed from Eqs. (8) and (9). For details related to masses of doubly charged scalars, see the text.

have taken v_T to be 0.08 eV and 0.14 eV respectively. If we decrease v_T below than the above mentioned values, the value for Br($\mu \to \bar{e}ee$) may exceed the experimental limit on this in the plots of Figs. 1 and 2. One can notice, in each plot of these figures we get two lines, which is due to the fact that the Yukawa coupling y can be determined either in terms of v_4' or v_5'. Depending on our choice of free parameter between v_4' and v_5', the branching ratio for $\mu \to \bar{e}ee$ can be different in this model, which is evident from Figs. 1 and 2. Which of these two choices is true is something we may tell after measuring the branching ratio for this decay in experiments.

4.3. *Branching ratios of $\ell \to \ell'\gamma$*

As stated before, decays of the form $\ell \to \ell'\gamma$ are driven by both doubly and singly charged triplet scalars. Interaction terms between doubly charged scalars and charged leptons, which are given in Eq. (43), drive $\ell \to \ell'\gamma$ at 1-loop level. In addition to this contribution, singly charged triplet scalars interacting with charged leptons and neutrinos also contribute to $\ell \to \ell'\gamma$ at 1-loop level. To obtain these interaction terms, which involve singly charged scalars, we use the transformations for left-handed charged leptons and neutrinos of Eqs. (2) and (7) in Eq. (3), apart from using Eq. (40). As a result of this, we get the following interaction terms for singly charged triplet scalars with charged leptons and neutrinos.

$$
\begin{aligned}
\mathcal{L} \ni & -\sum_{j,k=1}^{3} \nu_k^{(m)^T} C \frac{1-\gamma_5}{2} \left[g_{jk}^{1,1} \xi_j^{(m)+} + g_{jk}^{1,2} \xi_{j+3}^{(m)+} \right] \ell_1^{(m)} \\
& -\sum_{j,k=1}^{3} \nu_k^{(m)^T} C \frac{1-\gamma_5}{2} \left[g_{jk}^{2,1} \xi_j^{(m)+} + g_{jk}^{2,2} \xi_{j+3}^{(m)+} \right] \ell_2^{(m)} \\
& -\sum_{j,k=1}^{3} \nu_k^{(m)^T} C \frac{1-\gamma_5}{2} \left[g_{jk}^{3,1} \xi_j^{(m)+} + g_{jk}^{3,2} \xi_{j+3}^{(m)+} \right] \ell_3^{(m)} + h.c., \\
& g_{jk}^{1,1} = \sqrt{2}[y_1 U_{1j}^{+} (U_{PMNS})_{1k} + y_2 U_{2j}^{+} (U_{PMNS})_{3k} + y_3 U_{3j}^{+} (U_{PMNS})_{2k}], \\
& g_{jk}^{1,2} = \frac{y}{3\sqrt{2}} [2(V_{1j}^{+} + V_{2j}^{+} + V_{3j}^{+})(U_{PMNS})_{1k} - (V_{1j}^{+} + \omega V_{2j}^{+} + \omega^2 V_{3j}^{+})(U_{PMNS})_{2k} \\
& -(V_{1j}^{+} + \omega^2 V_{2j}^{+} + \omega V_{3j}^{+})(U_{PMNS})_{3k}], \\
& g_{jk}^{2,1} = \sqrt{2}[y_1 U_{1j}^{+} (U_{PMNS})_{3k} + y_2 U_{2j}^{+} (U_{PMNS})_{2k} + y_3 U_{3j}^{+} (U_{PMNS})_{1k}], \\
& g_{jk}^{2,2} = \frac{y}{3\sqrt{2}} [-(V_{1j}^{+} + \omega V_{2j}^{+} + \omega^2 V_{3j}^{+})(U_{PMNS})_{1k} \\
& +2(V_{1j}^{+} + \omega^2 V_{2j}^{+} + \omega V_{3j}^{+})(U_{PMNS})_{2k} - (V_{1j}^{+} + V_{2j}^{+} + V_{3j}^{+})(U_{PMNS})_{3k}], \\
& g_{jk}^{3,1} = \sqrt{2}[y_1 U_{1j}^{+} (U_{PMNS})_{2k} + y_2 U_{2j}^{+} (U_{PMNS})_{1k} + y_3 U_{3j}^{+} (U_{PMNS})_{3k}], \\
& g_{jk}^{3,2} = \frac{y}{3\sqrt{2}} [-(V_{1j}^{+} + \omega^2 V_{2j}^{+} + \omega V_{3j}^{+})(U_{PMNS})_{1k} - (V_{1j}^{+} + V_{2j}^{+} + V_{3j}^{+})(U_{PMNS})_{2k} \\
& +2(V_{1j}^{+} + \omega V_{2j}^{+} + \omega^2 V_{3j}^{+})(U_{PMNS})_{3k}].
\end{aligned} \tag{48}
$$

In the above equation, $\nu_k^{(m)}$, where $k = 1, 2, 3$, are mass eigenstates for neutrinos.

Using the interaction terms of Eqs. (43) and (48), the total amplitude for the decay $\ell \to \ell' \gamma$ can be written as

$$
\mathcal{M} = -\frac{Q_e^2}{24\pi^2} (a_{++}^{\ell,\ell'} + \frac{1}{8} a_{+}^{\ell,\ell'}) \epsilon_\mu^*(q) \bar{u}_{\ell'}(p-q) \left[m_{\ell'} \frac{1-\gamma_5}{2} + m_\ell \frac{1+\gamma_5}{2} \right] i\sigma^{\mu\nu} q_\nu u_\ell(p). \tag{49}
$$

Here, $m_{\ell'}$ and m_ℓ are masses for the charged leptons ℓ' and ℓ respectively. Q_e is the magnitude of charge of electron. The quantities $a_{++}^{\ell,\ell'}$ and $a_{+}^{\ell,\ell'}$ depend on masses of triplet charged scalars and their couplings with leptons. Their forms are given below.

$$
\begin{aligned}
& a_{++}^{\ell,\ell'} = \sum_{j=1}^{3} \frac{a_{1j}^{++(\ell,\ell')}}{M_{++(j)}^2} + \frac{a_{2j}^{++(\ell,\ell')}}{M_{++(j+3)}^2}, \quad a_{+}^{\ell,\ell'} = \sum_{j=1}^{3} \frac{a_{1j}^{+(\ell,\ell')}}{M_{+(j)}^2} + \frac{a_{2j}^{+(\ell,\ell')}}{M_{+(j+3)}^2}, \\
& a_{nj}^{++(\mu,e)} = (f_{nj}^{1,1})^* f_{nj}^{1,2} + \frac{1}{2} (f_{nj}^{1,3})^* f_{nj}^{2,3} + (f_{nj}^{1,2})^* f_{nj}^{2,2}, \quad n = 1, 2, \\
& a_{1j}^{+(\mu,e)} = \sum_{k=1}^{3} (g_{jk}^{1,1})^* g_{jk}^{2,1}, \quad a_{2j}^{+(\mu,e)} = \sum_{k=1}^{3} (g_{jk}^{1,2})^* g_{jk}^{2,2}, \\
& a_{nj}^{++(\tau,\mu)} = \frac{1}{2} (f_{nj}^{1,2})^* f_{nj}^{1,3} + (f_{nj}^{2,2})^* f_{nj}^{2,3} + (f_{nj}^{2,3})^* f_{nj}^{3,3}, \quad n = 1, 2,
\end{aligned}
$$

$$
\begin{aligned}
a_{1j}^{+(\tau,\mu)} &= \sum_{k=1}^{3} (g_{jk}^{2,1})^* g_{jk}^{3,1}, \quad a_{2j}^{+(\tau,\mu)} = \sum_{k=1}^{3} (g_{jk}^{2,2})^* g_{jk}^{3,2}, \\
a_{nj}^{++(\tau,e)} &= (f_{nj}^{1,1})^* f_{nj}^{1,3} + \frac{1}{2} (f_{nj}^{1,2})^* f_{nj}^{2,3} + (f_{nj}^{1,3})^* f_{nj}^{3,3}, \quad n = 1, 2, \\
a_{1j}^{+(\tau,e)} &= \sum_{k=1}^{3} (g_{jk}^{1,1})^* g_{jk}^{3,1}, \quad a_{2j}^{+(\tau,e)} = \sum_{k=1}^{3} (g_{jk}^{1,2})^* g_{jk}^{3,2}.
\end{aligned} \tag{50}
$$

Using the amplitude in Eq. (49), we find the branching ratios for the decays of the form $\ell \to \ell' \gamma$, where we have neglected the mass of ℓ'. Expressions for these are given below.

$$
\begin{aligned}
\mathrm{Br}(\tau \to \ell' \gamma) &= \frac{\alpha}{12\pi G_F^2} \left| a_{++}^{\tau,\ell'} + \frac{1}{8} a_{+}^{\tau,\ell'} \right|^2 \mathrm{Br}(\tau \to \mu \bar{\nu} \nu), \\
\mathrm{Br}(\mu \to e\gamma) &= \frac{\alpha}{12\pi G_F^2} \left| a_{++}^{\mu,e} + \frac{1}{8} a_{+}^{\mu,e} \right|^2 .
\end{aligned} \tag{51}
$$

Here, $\alpha = \frac{Q_e^2}{4\pi}$ and $\ell' = e, \mu$.

In the previous subsection, we have shown in Eqs. (46) and (47) that branching ratios for different decays of the form $\ell \to 3\ell'$ can relate to each other. We have explained that this is due to an implication of A_4 symmetry, under which the couplings of doubly charged scalars can relate to one another. We have found that even for the decays of the form $\ell \to \ell' \gamma$, there can exist relations among branching ratios of different decays, under some particular conditions. If $M_{++(j)}^2, M_{+(j)}^2$ are degenerate for $j = 4, 5, 6$, from Eq. (51) we get

$$
\mathrm{Br}(\tau \to \mu\gamma) = \mathrm{Br}(\mu \to e\gamma) \mathrm{Br}(\tau \to \mu \bar{\nu} \nu) \tag{52}
$$

On the other hand, if $M_{++(j-3)}^2, M_{++(j)}^2, M_{+(j)}^2$ are degenerate for $j = 4, 5, 6$, we get

$$
\mathrm{Br}(\tau \to \mu\gamma) = \mathrm{Br}(\tau \to e\gamma) = \mathrm{Br}(\mu \to e\gamma) \mathrm{Br}(\tau \to \mu \bar{\nu} \nu) \tag{53}
$$

We can also notice that in the limit where all the masses of doubly and singly charged scalar triplets are degenerate, the branching ratios in Eq. (51) go to zero. Verifying the relations of Eqs. (52) and (53) in experiments can give some hints about A_4 symmetry of this model. Notice here that, in a related work of Ref. [21], similar kind of relations among the branching ratios for the decays $\ell \to \ell' \gamma$ have been given.

Among the various decays of the form $\ell \to \ell' \gamma$, branching ratio for $\mu \to e\gamma$ is severely constrained and we have $\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [25]. From the expression given for $\mathrm{Br}(\mu \to e\gamma)$ in Eq. (51), one can see that this depends on the masses and couplings of both doubly and singly charged triplet Higgses. The couplings of doubly and singly charged triplets are given in Eqs. (43) and (48). These couplings depend on neutrino Yukawa couplings and also on parameters of scalar potential. Now, from the discussion given for the case of $\mathrm{Br}(\mu \to \bar{e}ee)$, one can realize that $\mathrm{Br}(\mu \to e\gamma)$ in our work is determined by neutrino oscillation observables, VEVs of Higgs triplets and parameters of the scalar potential. From the same discussion, one can also realize that $\mathrm{Br}(\mu \to e\gamma)$ in our work do not depend on the mixing angle θ_{12}, at the leading order. Since it is interesting to study variation of $\mathrm{Br}(\mu \to e\gamma)$ with respect to neutrino oscillation observables, we have fixed VEVs of Higgs triplets and parameters of the scalar potential to some specific values, which will be described below. It should be noticed that both $\mathrm{Br}(\mu \to e\gamma)$ and

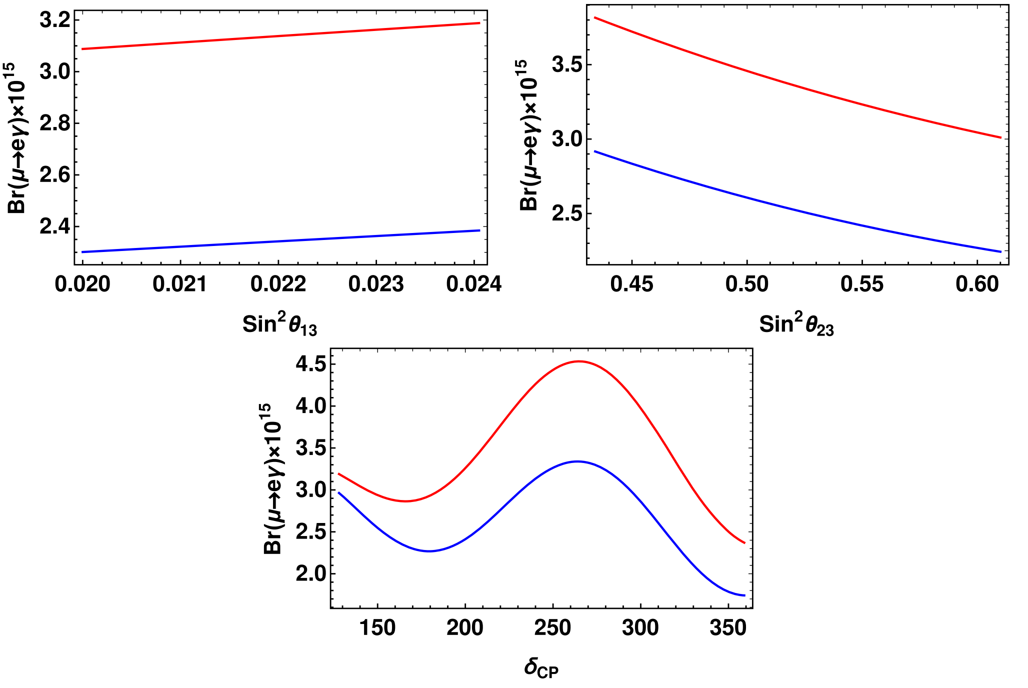

Fig. 3. Branching ratios for $\mu \to e\gamma$ in the case of NO. Here, red and blue lines are for the cases I and II respectively. δ_{CP} is expressed in degrees. In these plots, $v_T = 0.08$ eV. The neutrino oscillation parameters in these plots are taken to be same as for Fig. 1. For details related to charged triplet scalar masses, see the text.

Br($\mu \to \bar{e}ee$) are determined by a common set of parameters, since doubly charged triplet Higgses contribute to both of the above observables. In addition to this common set of observables, Br($\mu \to e\gamma$) is determined by parameters related to singly charged triplet Higgses.

We have computed Br($\mu \to e\gamma$) in our model for the cases of NO and IO, which are presented in Figs. 3 and 4 respectively. While computing the Br($\mu \to e\gamma$), we have used the same set of parameters which are described for the computation of Br($\mu \to \bar{e}ee$). Now the additional parameters which govern the decay $\mu \to e\gamma$ are due to the singly charged triplet scalar fields. The masses and couplings of these singly charged scalars are determined after diagonalizing the mass matrices for these, which are given in Eq. (38). There is a common set of parameters in the mass matrices for singly and doubly charged scalar fields. This common set of parameters is same as what we have used for the computation of Br($\mu \to \bar{e}ee$). The additional λ parameters in the mass matrices of singly charged triplet scalars are taken to be 0.1 in this analysis. As a result of this, the masses for both doubly and singly charged triplets are slightly above 850 GeV. After using the above mentioned parameters for the computation of Br($\mu \to e\gamma$), from Figs. 3 and 4, we can see that the branching ratio for this decay is around 10^{-15}. This value of branching ratio is two orders lower than that for $\mu \to \bar{e}ee$, whose results can be seen from Figs. 1 and 2. The reason for this suppression in the branching ratio is due to the fact that the decays $\mu \to e\gamma$ and $\mu \to \bar{e}ee$ take place at 1-loop and tree level respectively. As a result of this, a loop suppression factor of $\alpha \sim 10^{-2}$ exists in the Br($\mu \to e\gamma$), which gives the above mentioned suppression.

In the upcoming MEG II experiment, the sensitivity to probe Br($\mu \to e\gamma$) is around 10^{-14} [26]. Hence, the parameter region of Figs. 3 and 4 may not be reachable in the upcoming MEG II

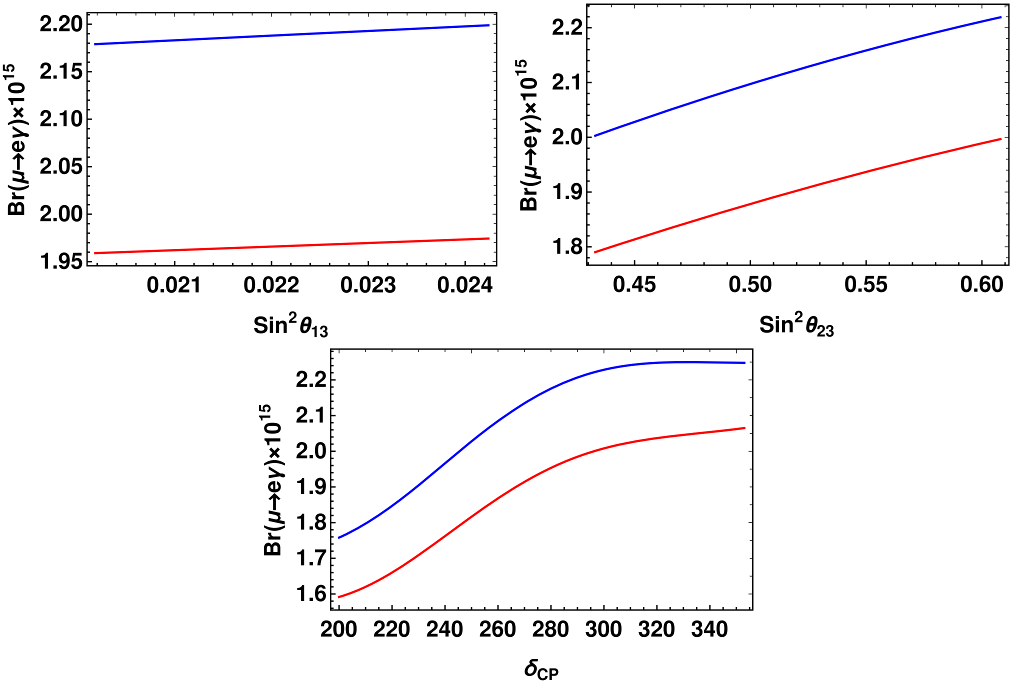

Fig. 4. Branching ratios for $\mu \to e\gamma$ in the case of IO. Here, red and blue lines are for the cases I and II respectively. δ_{CP} is expressed in degrees. In these plots, $v_T = 0.14$ eV. The neutrino oscillation parameters in these plots are taken to be same as for Fig. 2. For details related to charged triplet scalar masses, see the text.

experiment. We can get $\mathrm{Br}(\mu \to e\gamma) \sim 10^{-14}$ in this analysis, by decreasing the values of either v_T or the masses for charged triplet fields. However, in such cases the branching ratio for $\mu \to \bar{e}ee$ may exceed the experimental limit on this decay. Moreover, it is to be noted that we have chosen the parametric values of $m_1^2 = m_2^2 = m_3^2 = m_0^2 = (850\ \mathrm{GeV})^2$ in such a way that the doubly charged scalar fields have masses above 850 GeV. The current stringent lower bound on the doubly charged scalar mass is around 850 GeV [24]. By decreasing the values for above mentioned mass-square parameters, one needs to ensure that the lower bound on the doubly charged scalar masses is satisfied. One can do a detailed study on the above mentioned topic, nevertheless, we can notice that probing LFV decays in experiments can reveal something about our model, which is based on the MW model. Finally, in each plot of Figs. 3 and 4, the two lines correspond to the choice of the free parameter between v_4' and v_5', which is described around Eq. (15). Depending on this choice of parameter, the branching ratio for $\mu \to e\gamma$ can be different. After this decay is observed in experiments, by matching the theoretical formula for $\mathrm{Br}(\mu \to e\gamma)$ with the observed data, we may tell which of the above mentioned parameters can be chosen free.

It is mentioned previously that contribution from the neutral scalar fields to the LFV decays is neglected in this work. Even after including this contribution, it is still an interest to know the results about LFV decays, in the limit where the masses of these fields are heavy enough that the contribution can be neglected. In this work, we have analyzed the above mentioned case. On the other hand, depending on the masses and coupling strengths of these neutral scalar fields, the results mentioned in this work can be altered. It is worth to study this contribution, however, it is stated that only the neutral scalars which interact with tau lepton may give appreciable contribution. Before studying this contribution, one has to diagonalize the mixing masses among the neutral scalar fields, which is an involved work and we postpone it to future.

5. Future directions and phenomenology of our model

The model presented in this work contains additional scalar fields which are five Higgs doublets and six Higgs triplets. After the electroweak symmetry breaking, the following fields remain in the theory: six doubly charged scalars, ten singly charged scalars, twenty one neutral scalars. One of these neutral scalars can be identified as the Higgs boson, which is discovered in the LHC. All the above mentioned scalars have gauge interactions. Hence, it is possible to produce them at the LHC, and after production, they can subsequently decay into standard model fields via their Yukawa or gauge interactions. So the model presented in this work can be tested at the LHC. We have shown that this model can make certain predictions in LFV decays, which are given in Eqs. (46), (47), (52) and (53). Among these, testing the LFV relation in Eq. (46) is the best way to check this model in experiments, since this relation is independent on the assumptions made on the masses of charged scalars.

From the context of LFV decays, the model presented in this work can be distinguished from the original MW model. Our model is an extension of MW model with additional Higgs doublets $\Phi_{5,6}$. Hence, by putting $\langle\Phi_{5,6}\rangle = 0$ in our results of LFV, one can get corresponding results in the MW model. After using $\langle\Phi_{5,6}\rangle = 0$ in the mixing mass matrices of doubly and singly charged triplets, which are given in Sec. 4.1, one can notice that doubly and singly charged scalars of $\xi_{1,2,3}$ are already in mass eigenstates. On the other hand, doubly and singly charged scalars of $\xi_{4,5,6}$ can mix non-trivially. As a result of this, in the MW model, LFV decays are driven by only the doubly and singly charged scalars of $\xi_{4,5,6}$, in contrast to the fact that these decays are driven by all charged triplet Higgses in our model. Hence, the rate of LFV decays in the MW model can be different from that in our model. This can be one source to distinguish our model from the MW model in experiments. Another source to distinguish our model from other A_4 symmetry models is the study of collider implications in the scalar sector.

From the plots of Figs. 1 to 4, we can see that the LFV decays in our work depend on neutrino oscillation observables. However, due to large number of parameters in our model, we have simplified the numerical analysis by choosing some specific values for the parameters in the scalar potential. Hence, the plots in Figs. 1 to 4 are for some specific benchmark points of our model, where we have taken all λ parameter to 0.1. An extensive numerical analysis on LFV decays in our model is still possible. Since in our model, neutrinos are Majorana particles, the neutrino oscillation observables can get additional constraints due to neutrino-less double beta decay. From the non-observation of this decay, upper bounds have been set on the effective Majorana mass m_{ee} [13], which is expressed in terms of neutrino masses and elements of the first row of U_{PMNS}. The most stringent upper bound on m_{ee} is 61–165 meV [27]. Using this bound on m_{ee}, allowed regions for LFV decays in our work can be studied. Apart from the above mentioned bounds, precision electroweak observables [13] can also give additional constraints on the model.

The singly and doubly charged scalars of our model can drive $H \to \gamma\gamma$ at 1-loop level. Here, H is a neutral scalar of our model, which represents Higgs boson of standard model. The decay rate for $H \to \gamma\gamma$ in our model depends on the tri-linear couplings of H with singly and doubly charged scalars. These couplings are determined by the parameters of the scalar potential of our model. Since the signal strength for $H \to \gamma\gamma$ at the LHC [13] agrees with the standard model prediction for Higgs boson, there can be additional constraints on the above mentioned tri-linear couplings in our model.

In Sec. 3.2, we have given the minimization conditions for the doublet and triplet Higgses of our model. These conditions can represent a possible minimum for the scalar potential of our model. This minimum may or may not be a global minimum of our scalar potential. We may expect additional conditions to be imposed on the parameters of the scalar potential in order to make this minimum to be global. For related studies in this direction, see Refs. [28].

In this work, we have studied mixing pattern in lepton sector by introducing additional Higgs doublets and triplets. It is interesting to know about masses and mixing pattern of quarks in our framework with A_4 symmetry. In this direction, in Refs. [29], breaking of A_4 symmetry is suggested for obtaining realistic mixing pattern in quark and lepton sectors. Following these ideas, one can study quark masses and mixing pattern in our model.

6. Conclusions

In this work, we have considered the MW model [6], where the mixing pattern in neutrino sector is explained with three Higgs doublets, six Higgs triplets and with the additional symmetry A_4. The VEVs of Higgs triplets play a part in explaining the neutrino mixing pattern, apart from the fact that the VEVs of Higgs doublets should be same in order to diagonalize the charged lepton mass matrix. To study the pattern of VEVs of scalar fields of the MW model, in this work, we have constructed the invariant scalar potential of this model. After minimizing this scalar potential, we have found that among the six Higgs triplets two of them acquire zero VEVs. As a result of this, after using the results from the diagonalization procedure of our previous work [10], we have found that the neutrino mixing angles cannot be consistently explained. In order to see if we can get a consistent picture with the diagonalization procedure of our previous work [10], we have added two additional Higgs doublets to the MW model. Thereafter, we have shown that all the Higgs triplets acquire non-zero VEVs and the current neutrino oscillation data can be explained in this model. After adding extra Higgs doublets to the model, we have demonstrated that enough parameter space exists, where the above mentioned vacuum alignment of Higgs doublets can be achieved.

To study some phenomenological consequences of the model under consideration, we have computed branching ratios for the LFV decays of the form $\ell \to 3\ell'$ and $\ell \to \ell'\gamma$. We have found that A_4 symmetry of this model can bring some relations among the couplings between charged triplet scalars and lepton fields. As a result of this, relations can exist among branching ratios for different decays. Relation shown in Eq. (46) is independent of any assumption on the masses of charged triplet scalars. However, relations in Eqs. (47), (52) and (53) are valid under some assumptions made on the masses of charged triplet scalars. Apart from this, branching ratios for the LFV decays in our work depend on the neutrino mixing angles θ_{13} and θ_{23} and also on the CP violating Dirac phase δ_{CP}. We have plotted branching ratios for these decays in both the cases of NO and IO. From these plots, we have found that the choice of free parameters among the VEVs of Higgs triplets can have implications on the branching ratios for the LFV decays of this model.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Product rules of A_4 symmetry

The discrete symmetry A_4 has 12 elements which constitute the following 4 irreducible representations: $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{3}$. Product rules for these irreducible representations are

$$\begin{aligned}&\underline{1}'\times\underline{1}'=\underline{1}'',\quad \underline{1}''\times\underline{1}''=\underline{1}',\quad \underline{1}'\times\underline{1}''=\underline{1},\\ &\underline{1}'\times\underline{3}=\underline{3},\quad \underline{1}''\times\underline{3}=\underline{3},\quad \underline{3}\times\underline{3}=\underline{1}+\underline{1}'+\underline{1}''+\underline{3}_1+\underline{3}_2.\end{aligned}\tag{54}$$

Let (x_1, x_2, x_3) and (y_1, y_2, y_3) be two triplets under A_4. Then we have [30]

$$\begin{aligned}&\underline{1}=x_1y_1+x_2y_2+x_3y_3,\quad \underline{1}'=x_1y_1+\omega^2x_2y_2+\omega x_3y_3,\\ &\underline{1}''=x_1y_1+\omega x_2y_2+\omega^2x_3y_3,\\ &\underline{3}_1=(x_2y_3,x_3y_1,x_1y_2),\quad \underline{3}_2=(x_3y_2,x_1y_3,x_2y_1).\end{aligned}\tag{55}$$

Let $u\sim\underline{1}'$ and $v\sim\underline{1}''$. Then we have [30]

$$\underline{1}'\times\underline{3}=u(x_1,\omega x_2,\omega^2x_3),\quad \underline{1}''\times\underline{3}=v(x_1,\omega^2x_2,\omega x_3).\tag{56}$$

Appendix B. Quartic terms in the scalar potential

Quartic terms in the scalar potential, which contain only Higgs triplets, can be categorized into three classes. To write some of the invariant terms, we define the following quantities.

$$\begin{aligned}&(\xi\xi)\equiv\xi_4\xi_4+\xi_5\xi_5+\xi_6\xi_6,\quad (\xi\xi)'\equiv\xi_4\xi_4+\omega^2\xi_5\xi_5+\omega\xi_6\xi_6,\\ &(\xi\xi)''\equiv\xi_4\xi_4+\omega\xi_5\xi_5+\omega^2\xi_6\xi_6,\quad (\xi\xi^\dagger)\equiv\xi_4\xi_4^\dagger+\xi_5\xi_5^\dagger+\xi_6\xi_6^\dagger,\\ &(\xi\xi^\dagger)'\equiv\xi_4\xi_4^\dagger+\omega^2\xi_5\xi_5^\dagger+\omega\xi_6\xi_6^\dagger,\quad (\xi\xi^\dagger)''\equiv\xi_4\xi_4^\dagger+\omega\xi_5\xi_5^\dagger+\omega^2\xi_6\xi_6^\dagger,\\ &(\xi^\dagger\xi^\dagger)\equiv(\xi\xi)^\dagger,\quad (\xi^\dagger\xi^\dagger)'\equiv((\xi\xi)'')^\dagger,\quad (\xi^\dagger\xi^\dagger)''\equiv((\xi\xi)')^\dagger.\end{aligned}\tag{57}$$

Below we list all the distinct quartic terms in the scalar potential, which are formed with only Higgs triplets of the MW model. If a term is not self-adjoint, hermitian conjugate of that should be included in the potential.

$$\begin{aligned}&[\mathrm{Tr}(\xi_1^\dagger\xi_1)]^2,\ [\mathrm{Tr}(\xi_2^\dagger\xi_2)]^2,\ [\mathrm{Tr}(\xi_3^\dagger\xi_3)]^2,\ [\mathrm{Tr}((\xi^\dagger\xi))]^2,\ \mathrm{Tr}(\xi_1^\dagger\xi_1)\mathrm{Tr}(\xi_2^\dagger\xi_2),\\ &\mathrm{Tr}(\xi_1^\dagger\xi_1)\mathrm{Tr}(\xi_3^\dagger\xi_3),\ \mathrm{Tr}(\xi_1^\dagger\xi_1)\mathrm{Tr}((\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_2^\dagger\xi_2)\mathrm{Tr}(\xi_3^\dagger\xi_3),\ \mathrm{Tr}(\xi_2^\dagger\xi_2)\mathrm{Tr}((\xi^\dagger\xi)),\\ &\mathrm{Tr}(\xi_3^\dagger\xi_3)\mathrm{Tr}((\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_1^\dagger\xi_2)\mathrm{Tr}(\xi_1^\dagger\xi_3),\ \mathrm{Tr}(\xi_1^\dagger\xi_2)\mathrm{Tr}(\xi_2^\dagger\xi_1),\ \mathrm{Tr}(\xi_1^\dagger\xi_2)\mathrm{Tr}(\xi_3^\dagger\xi_2),\\ &\mathrm{Tr}(\xi_1^\dagger\xi_2)\mathrm{Tr}((\xi^\dagger\xi)''),\ \mathrm{Tr}(\xi_3^\dagger\xi_1)\mathrm{Tr}(\xi_1^\dagger\xi_3),\ \mathrm{Tr}(\xi_3^\dagger\xi_1)\mathrm{Tr}(\xi_3^\dagger\xi_2),\ \mathrm{Tr}(\xi_3^\dagger\xi_1)\mathrm{Tr}((\xi^\dagger\xi)''),\\ &\mathrm{Tr}(\xi_2^\dagger\xi_3)\mathrm{Tr}(\xi_3^\dagger\xi_2),\ \mathrm{Tr}(\xi_2^\dagger\xi_3)\mathrm{Tr}((\xi^\dagger\xi)''),\ \mathrm{Tr}((\xi^\dagger\xi)')\mathrm{Tr}((\xi^\dagger\xi)''),\\ &\mathrm{Tr}(\xi_5^\dagger\xi_6)\mathrm{Tr}(\xi_6^\dagger\xi_5)+\mathrm{Tr}(\xi_6^\dagger\xi_4)\mathrm{Tr}(\xi_4^\dagger\xi_6)+\mathrm{Tr}(\xi_4^\dagger\xi_5)\mathrm{Tr}(\xi_5^\dagger\xi_4),\\ &[\mathrm{Tr}(\xi_5^\dagger\xi_6)]^2+[\mathrm{Tr}(\xi_6^\dagger\xi_4)]^2+[\mathrm{Tr}(\xi_4^\dagger\xi_5)]^2.\end{aligned}\tag{58}$$

$$\begin{aligned}&\mathrm{Tr}(\xi_1^\dagger\xi_1^\dagger)\mathrm{Tr}(\xi_1\xi_1),\ \mathrm{Tr}(\xi_1^\dagger\xi_1^\dagger)\mathrm{Tr}(\xi_2\xi_3),\ \mathrm{Tr}(\xi_1^\dagger\xi_1^\dagger)\mathrm{Tr}((\xi\xi)),\ \mathrm{Tr}(\xi_2^\dagger\xi_3^\dagger)\mathrm{Tr}(\xi_2\xi_3),\\ &\mathrm{Tr}(\xi_2^\dagger\xi_3^\dagger)\mathrm{Tr}((\xi\xi)),\ \mathrm{Tr}((\xi^\dagger\xi^\dagger))\mathrm{Tr}((\xi\xi)),\ \mathrm{Tr}(\xi_1^\dagger\xi_3^\dagger)\mathrm{Tr}(\xi_1\xi_3),\ \mathrm{Tr}(\xi_1^\dagger\xi_3^\dagger)\mathrm{Tr}(\xi_2\xi_2),\\ &\mathrm{Tr}(\xi_1^\dagger\xi_3^\dagger)\mathrm{Tr}((\xi\xi)''),\ \mathrm{Tr}(\xi_2^\dagger\xi_2^\dagger)\mathrm{Tr}(\xi_2\xi_2),\ \mathrm{Tr}(\xi_2^\dagger\xi_2^\dagger)\mathrm{Tr}((\xi\xi)''),\ \mathrm{Tr}((\xi^\dagger\xi^\dagger)')\mathrm{Tr}((\xi\xi)''),\\ &\mathrm{Tr}(\xi_1^\dagger\xi_2^\dagger)\mathrm{Tr}(\xi_1\xi_2),\ \mathrm{Tr}(\xi_1^\dagger\xi_2^\dagger)\mathrm{Tr}(\xi_3\xi_3),\ \mathrm{Tr}(\xi_1^\dagger\xi_2^\dagger)\mathrm{Tr}((\xi\xi)'),\ \mathrm{Tr}(\xi_3^\dagger\xi_3^\dagger)\mathrm{Tr}(\xi_3\xi_3),\end{aligned}$$

$$\begin{aligned}
&\mathrm{Tr}(\xi_3^\dagger\xi_3^\dagger)\mathrm{Tr}((\xi\xi)'),\ \mathrm{Tr}((\xi^\dagger\xi^\dagger)'')\mathrm{Tr}((\xi\xi)'),\\
&\mathrm{Tr}(\xi_5^\dagger\xi_6^\dagger)\mathrm{Tr}(\xi_5\xi_6)+\mathrm{Tr}(\xi_6^\dagger\xi_4^\dagger)\mathrm{Tr}(\xi_6\xi_4)+\mathrm{Tr}(\xi_4^\dagger\xi_5^\dagger)\mathrm{Tr}(\xi_4\xi_5).
\end{aligned}\tag{59}$$

$$\begin{aligned}
&\mathrm{Tr}(\xi_1^\dagger\xi_1\xi_1\xi_1^\dagger),\ \mathrm{Tr}(\xi_1^\dagger\xi_1\xi_2^\dagger\xi_2),\ \mathrm{Tr}(\xi_1^\dagger\xi_1\xi_2\xi_2^\dagger),\ \mathrm{Tr}(\xi_1\xi_1^\dagger\xi_2^\dagger\xi_2),\ \mathrm{Tr}(\xi_1\xi_1^\dagger\xi_2\xi_2^\dagger),\ \mathrm{Tr}(\xi_1^\dagger\xi_1\xi_3^\dagger\xi_3),\\
&\mathrm{Tr}(\xi_1^\dagger\xi_1\xi_3\xi_3^\dagger),\ \mathrm{Tr}(\xi_1\xi_1^\dagger\xi_3^\dagger\xi_3),\ \mathrm{Tr}(\xi_1\xi_1^\dagger\xi_3\xi_3^\dagger),\ \mathrm{Tr}(\xi_1^\dagger\xi_1(\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_1^\dagger\xi_1(\xi\xi^\dagger)),\\
&\mathrm{Tr}(\xi_1\xi_1^\dagger(\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_1\xi_1^\dagger(\xi\xi^\dagger)),\ \mathrm{Tr}(\xi_2^\dagger\xi_2\xi_2\xi_2^\dagger),\ \mathrm{Tr}(\xi_2^\dagger\xi_2\xi_3^\dagger\xi_3),\ \mathrm{Tr}(\xi_2^\dagger\xi_2\xi_3\xi_3^\dagger),\\
&\mathrm{Tr}(\xi_2\xi_2^\dagger\xi_3^\dagger\xi_3),\ \mathrm{Tr}(\xi_2\xi_2^\dagger\xi_3\xi_3^\dagger),\ \mathrm{Tr}(\xi_2^\dagger\xi_2(\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_2^\dagger\xi_2(\xi\xi^\dagger)),\ \mathrm{Tr}(\xi_2\xi_2^\dagger(\xi^\dagger\xi)),\\
&\mathrm{Tr}(\xi_2\xi_2^\dagger(\xi\xi^\dagger)),\ \mathrm{Tr}(\xi_3^\dagger\xi_3\xi_3\xi_3^\dagger),\ \mathrm{Tr}(\xi_3^\dagger\xi_3(\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_3^\dagger\xi_3(\xi\xi^\dagger)),\ \mathrm{Tr}(\xi_3\xi_3^\dagger(\xi^\dagger\xi)),\\
&\mathrm{Tr}(\xi_3\xi_3^\dagger(\xi\xi^\dagger)),\ \mathrm{Tr}((\xi^\dagger\xi)(\xi\xi^\dagger)),\ \mathrm{Tr}((\xi^\dagger\xi)(\xi^\dagger\xi)),\ \mathrm{Tr}(\xi_1^\dagger\xi_2\xi_1^\dagger\xi_3),\ \mathrm{Tr}(\xi_1^\dagger\xi_2\xi_3\xi_1^\dagger),\\
&\mathrm{Tr}(\xi_2\xi_1^\dagger\xi_1^\dagger\xi_3),\ \mathrm{Tr}(\xi_2\xi_1^\dagger\xi_3\xi_1^\dagger),\ \mathrm{Tr}(\xi_1^\dagger\xi_2\xi_1\xi_2^\dagger),\ \mathrm{Tr}(\xi_2\xi_1^\dagger\xi_2^\dagger\xi_1),\ \mathrm{Tr}(\xi_1^\dagger\xi_2\xi_3^\dagger\xi_2),\ \mathrm{Tr}(\xi_1^\dagger\xi_2\xi_2\xi_3^\dagger),\\
&\mathrm{Tr}(\xi_2\xi_1^\dagger\xi_3^\dagger\xi_2),\ \mathrm{Tr}(\xi_2\xi_1^\dagger\xi_2\xi_3^\dagger),\ \mathrm{Tr}(\xi_1^\dagger\xi_2(\xi^\dagger\xi)''),\ \mathrm{Tr}(\xi_1^\dagger\xi_2(\xi\xi^\dagger)''),\\
&\mathrm{Tr}(\xi_2\xi_1^\dagger(\xi^\dagger\xi)''),\ \mathrm{Tr}(\xi_2\xi_1^\dagger(\xi\xi^\dagger)''),\ \mathrm{Tr}(\xi_3^\dagger\xi_1\xi_3\xi_1^\dagger),\ \mathrm{Tr}(\xi_1\xi_3^\dagger\xi_1^\dagger\xi_3),\ \mathrm{Tr}(\xi_3^\dagger\xi_1\xi_3^\dagger\xi_2),\\
&\mathrm{Tr}(\xi_3^\dagger\xi_1\xi_2\xi_3^\dagger),\ \mathrm{Tr}(\xi_1\xi_3^\dagger\xi_3^\dagger\xi_2),\ \mathrm{Tr}(\xi_1\xi_3^\dagger\xi_2\xi_3^\dagger),\ \mathrm{Tr}(\xi_3^\dagger\xi_1(\xi^\dagger\xi)''),\ \mathrm{Tr}(\xi_3^\dagger\xi_1(\xi\xi^\dagger)''),\\
&\mathrm{Tr}(\xi_1\xi_3^\dagger(\xi^\dagger\xi)''),\ \mathrm{Tr}(\xi_1\xi_3^\dagger(\xi\xi^\dagger)''),\ \mathrm{Tr}(\xi_2^\dagger\xi_3\xi_2\xi_3^\dagger),\ \mathrm{Tr}(\xi_3\xi_2^\dagger\xi_3^\dagger\xi_2),\ \mathrm{Tr}(\xi_2^\dagger\xi_3(\xi^\dagger\xi)''),\\
&\mathrm{Tr}(\xi_2^\dagger\xi_3(\xi\xi^\dagger)''),\ \mathrm{Tr}(\xi_3\xi_2^\dagger(\xi^\dagger\xi)''),\ \mathrm{Tr}(\xi_3\xi_2^\dagger(\xi\xi^\dagger)''),\ \mathrm{Tr}((\xi^\dagger\xi)'(\xi^\dagger\xi)''),\\
&\mathrm{Tr}((\xi^\dagger\xi)'(\xi\xi^\dagger)''),\ \mathrm{Tr}((\xi\xi^\dagger)'(\xi^\dagger\xi)''),\ \mathrm{Tr}((\xi\xi^\dagger)'(\xi\xi^\dagger)''),\ \mathrm{Tr}(\xi_1\xi_1\xi_2^\dagger\xi_3^\dagger),\ \mathrm{Tr}(\xi_1\xi_1\xi_3^\dagger\xi_2^\dagger),\\
&\mathrm{Tr}(\xi_1\xi_1(\xi^\dagger\xi^\dagger)),\ \mathrm{Tr}(\xi_2\xi_3(\xi^\dagger\xi^\dagger)),\ \mathrm{Tr}(\xi_3\xi_2(\xi^\dagger\xi^\dagger)),\ \mathrm{Tr}((\xi\xi)(\xi^\dagger\xi^\dagger)),\ \mathrm{Tr}(\xi_1\xi_2\xi_3^\dagger\xi_3^\dagger),\\
&\mathrm{Tr}(\xi_2\xi_1\xi_3^\dagger\xi_3^\dagger),\ \mathrm{Tr}(\xi_1\xi_2(\xi^\dagger\xi^\dagger)''),\ \mathrm{Tr}(\xi_2\xi_1(\xi^\dagger\xi^\dagger)''),\ \mathrm{Tr}(\xi_3\xi_3(\xi^\dagger\xi^\dagger)''),\\
&\mathrm{Tr}((\xi\xi)'(\xi^\dagger\xi^\dagger)''),\ \mathrm{Tr}[(\xi_5^\dagger\xi_6)^2+(\xi_6^\dagger\xi_4)^2+(\xi_4^\dagger\xi_5)^2],\\
&\mathrm{Tr}[\xi_5\xi_5^\dagger\xi_6\xi_6^\dagger+\xi_6\xi_6^\dagger\xi_4\xi_4^\dagger+\xi_4\xi_4^\dagger\xi_5\xi_5^\dagger],\ \mathrm{Tr}[\xi_5^\dagger\xi_5\xi_6^\dagger\xi_6+\xi_6^\dagger\xi_6\xi_4^\dagger\xi_4+\xi_4^\dagger\xi_4\xi_5^\dagger\xi_5],\\
&\mathrm{Tr}[\xi_6\xi_5\xi_6^\dagger\xi_5^\dagger+\xi_4\xi_6\xi_4^\dagger\xi_6^\dagger+\xi_5\xi_4\xi_5^\dagger\xi_4^\dagger],\ \mathrm{Tr}[\xi_5\xi_5\xi_6^\dagger\xi_6^\dagger+\xi_6\xi_6\xi_4^\dagger\xi_4^\dagger+\xi_4\xi_4\xi_5^\dagger\xi_5^\dagger].
\end{aligned}\tag{60}$$

References

[1] C. Quigg, arXiv:hep-ph/0404228 [hep-ph];
J. Ellis, Nucl. Phys. A 827 (2009) 187C–198C, https://doi.org/10.1016/j.nuclphysa.2009.05.034, arXiv:0902.0357 [hep-ph].

[2] M.C. Gonzalez-Garcia, M. Maltoni, Phys. Rep. 460 (2008) 1–129, https://doi.org/10.1016/j.physrep.2007.12.004, arXiv:0704.1800 [hep-ph].

[3] P.F. de Salas, D.V. Forero, S. Gariazzo, P. Martínez-Miravé, O. Mena, C.A. Ternes, M. Tórtola, J.W.F. Valle, J. High Energy Phys. 02 (2021) 071, https://doi.org/10.1007/JHEP02(2021)071, arXiv:2006.11237 [hep-ph].

[4] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167, https://doi.org/10.1016/S0370-2693(02)01336-9, arXiv:hep-ph/0202074 [hep-ph];
P.F. Harrison, W.G. Scott, Phys. Lett. B 535 (2002) 163–169, https://doi.org/10.1016/S0370-2693(02)01753-7, arXiv:hep-ph/0203209 [hep-ph];
Z.Z. Xing, Phys. Lett. B 533 (2002) 85–93, https://doi.org/10.1016/S0370-2693(02)01649-0, arXiv:hep-ph/0204049 [hep-ph].

[5] G. Altarelli, arXiv:hep-ph/0611117 [hep-ph];
S.F. King, C. Luhn, Rep. Prog. Phys. 76 (2013) 056201, https://doi.org/10.1088/0034-4885/76/5/056201, arXiv:1301.1340 [hep-ph];
M.H. Rahat, P. Ramond, B. Xu, Phys. Rev. D 98 (5) (2018) 055030, https://doi.org/10.1103/PhysRevD.98.055030, arXiv:1805.10684 [hep-ph];
M.J. Pérez, M.H. Rahat, P. Ramond, A.J. Stuart, B. Xu, Phys. Rev. D 101 (7) (2020) 075018, https://doi.org/10.1103/PhysRevD.101.075018, arXiv:2001.04019 [hep-ph].

[6] E. Ma, D. Wegman, Phys. Rev. Lett. 107 (2011) 061803, https://doi.org/10.1103/PhysRevLett.107.061803, arXiv:1106.4269 [hep-ph].
[7] E. Ma, G. Rajasekaran, Phys. Rev. D 64 (2001) 113012, https://doi.org/10.1103/PhysRevD.64.113012, arXiv:hep-ph/0106291 [hep-ph].
[8] E. Ma, Phys. Rev. D 70 (2004) 031901, https://doi.org/10.1103/PhysRevD.70.031901, arXiv:hep-ph/0404199 [hep-ph];
G. Altarelli, F. Feruglio, Nucl. Phys. B 720 (2005) 64–88, https://doi.org/10.1016/j.nuclphysb.2005.05.005, arXiv:hep-ph/0504165 [hep-ph].
[9] M. Magg, C. Wetterich, Phys. Lett. B 94 (1980) 61–64, https://doi.org/10.1016/0370-2693(80)90825-4;
J. Schechter, J.W.F. Valle, Phys. Rev. D 22 (1980) 2227, https://doi.org/10.1103/PhysRevD.22.2227;
R.N. Mohapatra, G. Senjanovic, Phys. Rev. D 23 (1981) 165, https://doi.org/10.1103/PhysRevD.23.165;
G. Lazarides, Q. Shafi, C. Wetterich, Nucl. Phys. B 181 (1981) 287–300, https://doi.org/10.1016/0550-3213(81)90354-0.
[10] R.S. Hundi, I. Sethi, Phys. Rev. D 102 (5) (2020) 055007, https://doi.org/10.1103/PhysRevD.102.055007, arXiv:2003.09809 [hep-ph].
[11] J. Ganguly, R.S. Hundi, Phys. Rev. D 103 (3) (2021) 035007, https://doi.org/10.1103/PhysRevD.103.035007, arXiv:2005.04023 [hep-ph].
[12] F. Feruglio, C. Hagedorn, L. Merlo, J. High Energy Phys. 03 (2010) 084, https://doi.org/10.1007/JHEP03(2010)084, arXiv:0910.4058 [hep-ph];
K.S. Babu, S. Gabriel, Phys. Rev. D 82 (2010) 073014, https://doi.org/10.1103/PhysRevD.82.073014, arXiv:1006.0203 [hep-ph];
S.F. King, C. Luhn, J. High Energy Phys. 09 (2011) 042, https://doi.org/10.1007/JHEP09(2011)042, arXiv:1107.5332 [hep-ph].
[13] P.A. Zyla, et al., Particle Data Group, PTEP 2020 (8) (2020) 083C01, https://doi.org/10.1093/ptep/ptaa104.
[14] A. Masiero, S.K. Vempati, O. Vives, Nucl. Phys. B 649 (2003) 189–204, https://doi.org/10.1016/S0550-3213(02)01031-3, arXiv:hep-ph/0209303 [hep-ph];
F. Deppisch, S. Albino, R. Ruckl, AIP Conf. Proc. 903 (1) (2007) 307–310, https://doi.org/10.1063/1.2735186, arXiv:hep-ph/0701014 [hep-ph];
A. Abada, A. Figueiredo, J. Romao, A. Teixeira, J. High Energy Phys. 10 (2010) 104, https://doi.org/10.1007/JHEP10(2010)104, arXiv:1007.4833 [hep-ph];
R.S. Hundi, Eur. Phys. J. C 73 (4) (2013) 2396, https://doi.org/10.1140/epjc/s10052-013-2396-8, arXiv:1209.6482 [hep-ph];
R.S. Hundi, Phys. Rev. D 93 (2016) 015008, https://doi.org/10.1103/PhysRevD.93.015008, arXiv:1510.02253 [hep-ph].
[15] R. Korrapati, J. More, U. Rahaman, S.U. Sankar, Eur. Phys. J. C 81 (5) (2021) 382, https://doi.org/10.1140/epjc/s10052-021-09171-z, arXiv:2009.00865 [hep-ph].
[16] E.J. Chun, K.Y. Lee, S.C. Park, Phys. Lett. B 566 (2003) 142–151, https://doi.org/10.1016/S0370-2693(03)00770-6, arXiv:hep-ph/0304069 [hep-ph].
[17] M. Kakizaki, Y. Ogura, F. Shima, Phys. Lett. B 566 (2003) 210–216, https://doi.org/10.1016/S0370-2693(03)00833-5, arXiv:hep-ph/0304254 [hep-ph];
E.K. Akhmedov, W. Rodejohann, J. High Energy Phys. 06 (2008) 106, https://doi.org/10.1088/1126-6708/2008/06/106, arXiv:0803.2417 [hep-ph];
A.G. Akeroyd, M. Aoki, H. Sugiyama, Phys. Rev. D 79 (2009) 113010, https://doi.org/10.1103/PhysRevD.79.113010, arXiv:0904.3640 [hep-ph];
T. Fukuyama, H. Sugiyama, K. Tsumura, J. High Energy Phys. 03 (2010) 044, https://doi.org/10.1007/JHEP03(2010)044, arXiv:0909.4943 [hep-ph].
[18] H. Ishimori, S. Khalil, E. Ma, Phys. Rev. D 86 (2012) 013008, https://doi.org/10.1103/PhysRevD.86.013008, arXiv:1204.2705 [hep-ph];
E. Ma, A. Natale, A. Rashed, Int. J. Mod. Phys. A 27 (2012) 1250134, https://doi.org/10.1142/S0217751X12501345, arXiv:1206.1570 [hep-ph];
S. Bhattacharya, E. Ma, A. Natale, D. Wegman, Phys. Rev. D 87 (1) (2013) 013006, https://doi.org/10.1103/PhysRevD.87.013006, arXiv:1210.6936 [hep-ph].
[19] H. Ishimori, E. Ma, Phys. Rev. D 86 (2012) 045030, https://doi.org/10.1103/PhysRevD.86.045030, arXiv:1205.0075 [hep-ph].
[20] E. Ma, M. Raidal, U. Sarkar, Nucl. Phys. B 615 (2001) 313–330, arXiv:hep-ph/0012101 [hep-ph].

[21] S. Pascoli, Y.L. Zhou, J. High Energy Phys. 10 (2016) 145, https://doi.org/10.1007/JHEP10(2016)145, arXiv:1607.05599 [hep-ph].
[22] S. Pascoli, Y.L. Zhou, J. High Energy Phys. 06 (2016) 073, https://doi.org/10.1007/JHEP06(2016)073, arXiv:1604.00925 [hep-ph].
[23] U. Bellgardt, et al., SINDRUM, Nucl. Phys. B 299 (1988) 1–6, https://doi.org/10.1016/0550-3213(88)90462-2.
[24] M. Aaboud, et al., ATLAS, Eur. Phys. J. C 78 (3) (2018) 199, https://doi.org/10.1140/epjc/s10052-018-5661-z, arXiv:1710.09748 [hep-ex].
[25] A.M. Baldini, et al., MEG, Eur. Phys. J. C 76 (8) (2016) 434, https://doi.org/10.1140/epjc/s10052-016-4271-x, arXiv:1605.05081 [hep-ex].
[26] A.M. Baldini, et al., MEG II, Eur. Phys. J. C 78 (5) (2018) 380, https://doi.org/10.1140/epjc/s10052-018-5845-6, arXiv:1801.04688 [physics.ins-det].
[27] A. Gando, et al., KamLAND-Zen, Phys. Rev. Lett. 117 (8) (2016) 082503, Addendum: Phys. Rev. Lett. 117 (10) (2016) 109903, arXiv:1605.02889 [hep-ex].
[28] X.J. Xu, Phys. Rev. D 94 (11) (2016) 115025, https://doi.org/10.1103/PhysRevD.94.115025, arXiv:1612.04950 [hep-ph];
X.J. Xu, Phys. Rev. D 95 (11) (2017) 115019, https://doi.org/10.1103/PhysRevD.95.115019, arXiv:1705.08965 [hep-ph].
[29] E. Ma, Mod. Phys. Lett. A 17 (2002) 627–630, https://doi.org/10.1142/S0217732302006722, arXiv:hep-ph/0203238 [hep-ph];
K.S. Babu, E. Ma, J.W.F. Valle, Phys. Lett. B 552 (2003) 207–213, https://doi.org/10.1016/S0370-2693(02)03153-2, arXiv:hep-ph/0206292 [hep-ph];
X.G. He, Y.Y. Keum, R.R. Volkas, J. High Energy Phys. 04 (2006) 039, https://doi.org/10.1088/1126-6708/2006/04/039, arXiv:hep-ph/0601001 [hep-ph].
[30] E. Ma, New J. Phys. 6 (2004) 104, https://doi.org/10.1088/1367-2630/6/1/104, arXiv:hep-ph/0405152 [hep-ph];
J. Berger, Y. Grossman, J. High Energy Phys. 02 (2010) 071, https://doi.org/10.1007/JHEP02(2010)071, arXiv:0910.4392 [hep-ph].