Hadronic Decay of B_c meson to Pseudoscalar and Vector mesons using the Isgur-Scora-Grinstein-Wise Model

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Certificate

This is to certify that the thesis entitled *'Hadronic Decay of B_c meson to Pseudoscalar and Vector mesons using the Isgur-Scora-Grinstein-Wise Model'* being submitted in partial fulfilment of the requirements for the award of the Masters of Sciences in IIT Hyderabad, by ***Raya Dastidar*** embodies the research work done by under my supervision.

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APPROVAL SHEET

This thesis entitled *'Hadronic Decay of B_c meson to Pseudoscalar and Vector mesons using the Isgur-Scora-Grinstein-Wise Model'* by ***Raya Dastidar*** for the degree of Master of Sciences from IIT Hyderabad.

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ABSTRACT

The extensive study of B meson decays has led to precise measurements of the Standard Model parameters. The study of B_c meson decays is gaining ground these days as it is the only quark-antiquark bound system ($\bar{b}$c), discovered at Fermilab, composed of heavy quarks (b,c) with different flavours, and are thus flavour asymmetric. In this paper, we have investigated $B_c \to PV$ decays, where P is pseudoscalar meson and V is vector meson, using the ISGW model. Using the 'naive' factorization approach and the form factors derived from the ISGW model, we have calculated the amplitudes of these decays and finally predicted the branching ratios of these decays. Also, we investigated the CP asymmetry factor for a decay involving two Feynman diagrams and a possible variation of this parameter weak phase.

Contents

1 Introduction

The reason we are interested in studying the decays of B mesons is that they may shed light on one of the major mysteries of the Universe, namely the source of the observed matter-antimatter asymmetry. In the early Universe, tha amount of matter and anti-matter was the same. But at the present stage, the observable Universe($\sim 10Mpc$) consists of matter. As the Universe expanded and cooled, baryons annihilated into photons. But $1/10^9$ of the baryons did not annihilate.
A.Sakharov predicted the necessary conditions for the evolution of asymmetric Universe[1]:
-baryon number violation
-thermal non-equilibrium
-CP and C symmetry violation(CPV).

Thus, CPV is one of the necessary condition for a matter's dominance over anti-matter.

The CP transformation combines charge conjugation C with parity P. Under C, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, e.g.,$Q \to -Q$ for electromagnetic charge. Under P, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. Thus e_L^- is transformed under CP into a right-handed positron, e_R^+.

If CP were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. CP violation is one of the least understood aspects of particle physics. The Standard Model provides a simple description of this phenomenon through the complex Cabibbo-Kobayashi-Maskawa (CKM) matrix, which is consistent with present particle physics experiments. However, the baryon asymmetry of the universe clearly requires additional sources of CP violation; therefore CP violation might be the road to new physics.

One of the main aims of B factories is the observation of CP violation. The B_u, B_d and B_s meson decays and CP violation has been discussed extensively. The decays of B_c meson ($\bar{b}c$ and $b\bar{c}$ bound states) seems to be another valuable window for probing the origin of CP violation. Since large number of B_c meson is produced at hadronic colliders like LHC, to examine the features of the B_c meson decays and CP violation becomes more and more interesting for both experimental efforts and theoretical studies.

The B_c meson discovery by the collider detector at Fermilab(CDF)[2] paved the way for a series of investigations which sheds light on the structure of strong and weak interactions. The B_c is the only heavy meson consisting of two heavy quarks with different flavours. This difference of quarks flavour forbids annihilation into gluons. A peculiarity of the B_c decays, with respect to the decays of B and B_s mesons, is that both the quark may involve in its weak decays. A considerable amount of theoretical works has been done studying various leptonic, semileptonic and hadronic decay channels of B_c mesons in different models. As per the B_c decay rates estimated in the literature, it can be said that the c quark give dominant contribution as compared to b-quark decays.

In the present paper, we have calculated the branching ratio of $B_c \to PV$ decays. Using the naive factorization approach , we have factorized the amplitude into a from involving form factors and decay constants. We have used the ISGW model and the definition of the form factors given in the literature [5] to calculate the form factors for $B_c \to PV$ decays. Finally we have predicted the branching ratios of some decays.

The present paper is organized as follows. We begin with a brief summary of CP violation in the Standard Model. This is followed by a brief introduction to nonleptonic B_c decays, the formalism used, a short discussion of the factorization hypothesis and the ISGW model. Finally, the amplitude calculation is shown, followed by the prediction of branching ratios, summary and conclusions.

2 CP violation in the Standard Model

2.1 Weak Decays and CP violation

The rich phenomenology of weak decays have always been a good source of information about the nature of elementary particle interactions. The form of the effective flavour changing interactions came to light long back, through the β- and μ- decay experiments. Physicists unravelled the properties of the nuclei by studying the consequences of the weak interaction decays. Upto this day, particle physicists employ weak decays of quarks for precise measurement of the Standard Model parameters. They offer the most direct way to determine the weak mixing angles and to test the unitarity of the Cabibbo-Kobayashi-Maskawa matrix. Again, the non-perturbative confinement forces of strong interaction physics, are also studied by the help of weak transitions. For a precise measurement of weak mixing angles and CP violating phase, it is necessary to have an understanding of the connection between quark and hadron properties. So these two tasks complements each other, leading to more precise measurements of the parameters[7].

The simplest processes involves a minimum number of hadrons, i.e., a semileptonic decay with a single hadron (or hadron resonance) in the final state, or a non-leptonic decay with two hadrons in the final state. These kind of transitions have been studied extensively in the recent years and now physicists have a fair understanding of these transitions. Simple bound-state models are able to describe, in a semi-quantitative way, the current matrix elements occuring in semileptonic decay amplitudes. A model independent approach to determine parameters is most preferable. One may use the factorization prescription to reduce the complicacy of expressions of the hadronic matrix elements, which may shed light onto the dynamics of nonleptonic processes, that was extremely difficult to comprehend before. More recently, the extensive study of symmetries of QCD arising in the limit of infinite quark mass constituted a further step forward.

2.2 Cabibbo Angle

On the most fundamental material scale, that we are aware of, matter is formed from fermions. These take two forms, leptonic matter and quark matter. The leptons do not have any strong interactions, that is they lack a property called "colour charge", which allows quark to bind together either mesonically or baryonically[9].

Quarks and leptons can be ordered in flavour doublets, each column being called a family,

$$Quarks: \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

The structure of the charged currents,

$$j^{\pm}_{\mu} = \bar{\chi}_L \gamma_\mu \tau_\pm \chi_L \tag{1}$$

allows transitions within a single doublet, e.g. $d \to u, c \to s, t \to b$, but not between different doublets. This would imply that the lightest particle of each doublet should be stable (the electromagnetic and strong interactions do not allow flavour changing processes, since photons and gluons do not carry any flavour quantum numbers), a fact which is in contradiction with the observation that our universe is composed almost exclusively of particles of the first family, consisting of the lightest particles.

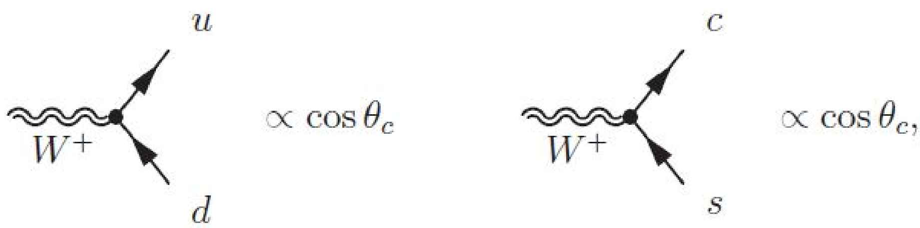
If we assume that the weak eigenstates of the d-type quarks are linear combinations of the mass eigenstates, we can reproduce the observed phenomenology. Let us first consider the case of two quark families for simplicity. We have the weak eigenstate doublets,

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$

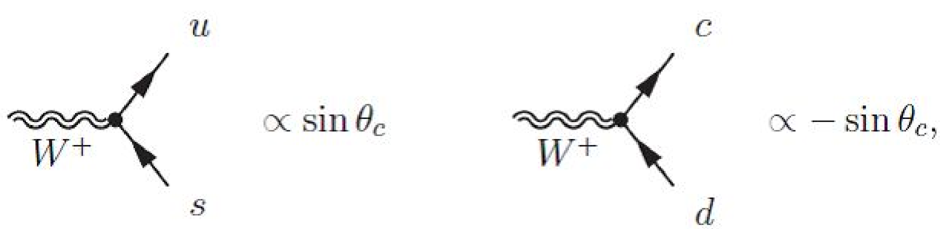
and we assume that the weak eigenstates $|d^{'}\rangle$ and $|s^{'}\rangle$ are linear combinations of the mass eigenstates $|d\rangle$ and $|s\rangle$,

$$|d^{'}\rangle = cos\,\theta_c|d\rangle + sin\,\theta_c|s\rangle$$
$$|s^{'}\rangle = -sin\,\theta_c|d\rangle + cos\,\theta_c|s\rangle$$

where θ_c is called the Cabibbo angle. Since decaying particles and decay products are mass eigenstates, this strategy allows transitions between different families. Using Eq. (1), we can write vertex factors between mass eigenstates,



called Cabibbo preferred decays, and,



called Cabibbo suppressed decays. If the weak and mass eigenstates would be the same, $\theta_c = 0$ and the second series of decay could not occur.

The introduction of the Cabibbo angle also destroys the universality of the Fermi constant,

$$G_F^{n\rightarrow pe^-\bar{\nu}_e} = cos\theta_c G_F^{\mu^-\rightarrow pe^-\bar{\nu}_e} \tag{2}$$

with the experimentally measured value,

$$cos\theta_c \approx 0.974. \tag{3}$$

The Lagrangian for the charged current coupling to quarks now takes the form,

$$i\mathcal{L}^{W^\pm,q} = -i\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{u} & \bar{c}\end{pmatrix}\gamma_\mu\frac{1-\gamma_5}{2}U\begin{pmatrix}d\\ s\end{pmatrix}W^{+\mu} - i\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{d} & \bar{s}\end{pmatrix}U^T\gamma_\mu\frac{1-\gamma_5}{2}\begin{pmatrix}u\\ c\end{pmatrix}W^{-\mu} \tag{4}$$

$$U = \begin{pmatrix} cos\theta_c & sin\theta_c\\ -sin\theta_c & cos\theta_c\end{pmatrix} \in U(2) \tag{5}$$

where U ∈ O(2) implying that $U^\dagger = U^T$.

2.3 Cabibbo-Kobayashi-Maskawa matrix

Even before the observation of c,b and t quarks, in 1973, the existence of three families and its implications were already hypothesized. Analogous to Eq. (4), we write for three families,

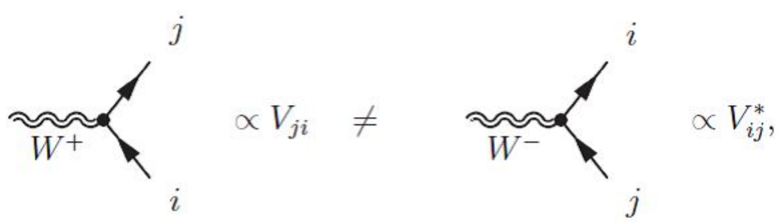
$$i\mathcal{L}^{W^\pm,q} = -i\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{u} & \bar{c} & \bar{t}\end{pmatrix}\gamma_\mu\frac{1-\gamma_5}{2}V\begin{pmatrix}d\\ s\\ b\end{pmatrix}W^{+\mu} - i\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{d} & \bar{s} & \bar{b}\end{pmatrix}V^\dagger\gamma_\mu\frac{1-\gamma_5}{2}\begin{pmatrix}u\\ c\\ t\end{pmatrix}W^{-\mu} \tag{6}$$

where $V \in$ U(3). For a matrix $V \in$ U(N):

- V contains N^2 real parameters ($2N^2$ entries minus N^2 from the unitarity condition $V^\dagger V = \mathbb{1}$
,
- 2N-1 relative phases can be factorized by a phase redefinition of the quantum fields. Thus V contains $N^2 - (2N-1) = (N-1)^2$ independent real parameters. On the other hand, a matrix $O \in O(N)$ is determined by $\frac{1}{2}N(N-1)$ independent real parameters(Euler angles).

Comparing V and O, we have, $N_a = \frac{1}{2}N(N-1)$ real angles and $N_p = (N-1)^2 - N_a = \frac{1}{2}(N-1)(N-2)$ complex phases for $N \geq 3$, implying $V^* \neq V$.

Looking at the vertex factors connected through a CP-transformation,



we conclude that the weak interaction violates CP invariance for $N \geq 3$ through complex phases in the CKM matrix V.

3 Nonleptonic B_c Decays

The confining colour forces among the quarks strongly influences the dynamics of weak decays. In case of semi-leptonic decays we described the long distance QCD effects in terms of few hadronic form factors appearing in the hadronic current part, but the nonleptonic processes are complicated by the phenomenon of quark rearrangement due to the exchange of hard and soft gluons. The theoretical description involves matrix elements of local four-quark operators, which are much harder to deal with than current operators.[8]

Due to the strong interaction effects, the structure of nonleptonic decays was least understood for a long time. Consider the $|\Delta I| = \frac{1}{2}$ rule in strange particle decays. It is only in the recent years, the physicists have been able to develop successful theoretical descriptions for the decay, although the selection rule is known for the last four decades. The strong colour force between quarks in colour-antitriplet states has been identified as the dominant mechanism responsible for the dramatic enhancement of $|\Delta I| = \frac{1}{2}$ processes.

With the inclusion of heavy charm and bottom quarks to the particle zoo, the possibility to study a great variety of new decay channels has greatly enhanced. A large amount of experimental data is now available on many exclusive and inclusive decay modes due to the incessant effort of various experimental groups. In many respects, nonleptonic decays of heavy mesons are instrumental in exploring the most interesting aspect of QCD, i.e., its non-perturbative long range character. The initial state consists of an isolated heavy particle, and the weak transition operator is known and exhibits a particularly simple structure. Thus a detailed analysis of decays into particles with different spin and flavour quantum numbers provides valuable information about the nature of the long range forces influencing these processes.

In energetic transitions,where large energy is carried by the W^- ,hadronization of the decay products does not occur until they travelled some distance away from each other. Once the quarks have grouped into colour-singlet pairs, soft gluons are ineffective in rearranging them. This is the reason we can make the assumption that the decay amplitudes factorize into products of hadronic matrix elements of colour-singlet quark currents. The factorization approximation works reasonably well for the heavy B and D mesons. It relates the complicated nonleptonic decay amplitudes to products of meson decay constants and hadronic matrix element of current operators. The decay constants f_π, f_D etc. are a measure of the strength of the quark-antiquark attraction inside a hadronic state. Some of them are directly accessible in leptonic or electromagnetic processes. Their extraction from nonleptonic transitions provides important information on the confinement forces of QCD.

The B_c meson discovered at fermilab is the only quark-antiquark bound system ($\bar{b}$c) composed of heavy quarks (b,c) with different flavours, and are thus flavour asymmetric. The CDF collaboration has announced an accurate determination of the B_c meson mass and its lifetime. The investigation of the B_c meson properties (mass spectrum,decay rates, etc.) is therefore of special interest compared to symmetric heavy quarkonium ($\bar{b}$b, $\bar{c}$c) states. The decay processes of the B_c meson can be broadly divided into two classes: involving the decay of b quark, c quark, besides the relatively suppressed annihilation of b and $\bar{c}$.

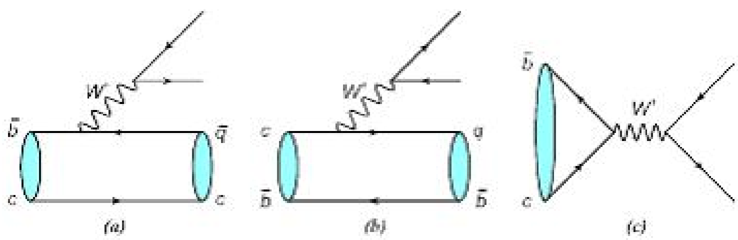


Figure 1: Typical Feynman diagrams for three types of B_c decays: (a) $\bar{b}$ weak decay modes with q=c or u, (b) c weak decay modes with q=s or d, and (c) pure weak annihilation channels, respectively.[6]

The mass and lifetime of B_c meson as given in Particle Data Group are the following:

$$m_{B_c} = (6274.5 \pm 1.8) MeV$$

$$\tau_{B_c} = (0.452 \pm 0.033) ps$$

3.1 Formalism

In the Standard Model, non leptonic B_c decays are described by the effective Hamiltonian obtained by integrating out the heavy W boson and top quark. For the case of $b \to c, u$ transitions, one gets

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}[c_1(\mu)O_1^{cb} + c_2(\mu)O_2^{cb}] + \frac{G_F}{\sqrt{2}} V_{ub}[c_1(\mu)O_1^{ub} + c_2(\mu)O_2^{ub}] + ...$$

The Wilson coefficients $c_{1,2}(\mu)$ are evaluated perturbatively at the W scale and then they are evolved down to the renormalization scale $\mu \approx m_b$ by the renormalization-group equations. The local four-quark operators O_1 and O_2 are given by[9]

$$O_1^{qb} = [(\bar{d}u)_{V-A} + (\bar{s}c)_{V-A}](\bar{q}b)_{V-A}$$

$$O_2^{qb} = (\bar{q}u)_{V-A}(\tilde{d}b)_{V-A} + (\bar{q}c)_{V-A}(\tilde{s}b)_{V-A}$$

where the rotated antiquark fields are

$$\tilde{d} = V_{ub}\bar{d} + V_{us}\bar{s}, \quad \tilde{s} = V_{cd}\bar{d} + V_{cs}\bar{s}$$

and for the hadronic current the following notation is used

$$(q\bar{q}^{'})_{V-A} = \bar{q}\gamma_\mu(1-\gamma_5)q^{'} \equiv J_\mu^W$$

The QCD modified weak Hamiltonian generating the b-quark decays in CKM enhanced modes ($\Delta b = 1, \Delta C = 1, \Delta S = 0; \Delta b = 1, \Delta C = 0, \Delta S = -1$) is thus given by,

$$H_W^{\Delta b=1} = \frac{G_F}{\sqrt{2}}[V_{cb}V_{ud}^*[c_1(\mu)(\bar{c}b)(\bar{d}u) + c_2(\mu)(\bar{c}u)(\bar{d}b] + V_{cb}V_{cs}^*[c_1(\mu)(\bar{c}b)(\bar{s}c) + c_2(\mu)(\bar{c}c)(\bar{s}b)]] \quad (7)$$

where $\bar{q}q \equiv \bar{q}\gamma_\mu(1-\gamma_5)q$, G_F is the Fermi constant and V_{ij} are the CKM matrix elements, c_1 and c_2 are the standard perturbative QCD coefficients.

In addition to the bottom changing decays, bottom conserving decay channel is also available for the B_c meson, where the charm quark decays to an s or d quark. The weak Hamiltonian generating the c-quark decays in CKM enhanced mode ($\Delta b = 0, \Delta C = -1, \Delta S = -1$) is given by,

$$H_W^{\Delta C=1} = \frac{G_F}{\sqrt{2}}[V_{ud}V_{cs}^*[c_1(\mu)(\bar{u}d)(\bar{s}c) + c_2(\mu)(\bar{u}c)(\bar{s}d] \quad (8)$$

One may expect this channel to be suppressed kinematically due to small phase space available. However, the kinematic suppression is compensated by the CKM element V_{cs}, which is larger than V_{cb} appearing in the bottom changing decays.
In the present paper, we have considered bottom changing decays only.

3.2 Decay rate

The decay rate is given by[11]

$$\Gamma(B_c \to PV) = \frac{p_V^3}{8\pi m_V^2}|A(|B_c \to PV)|^2,$$

where the three-momentum p_V in the rest frame of B_c is given by

$$p_V = \frac{1}{2m_{B_c}}[[m_{B_c}^2 - (m_P + m_V)^2][m_{B_c}^2 - (m_P - m_V)^2]]^{\frac{1}{2}}$$

3.3 Factorization Hypothesis

The factorization approach, which is extensively used for the calculation of two-body nonleptonic decays, such as $B_c \to M_2M_3$, assumes that two body hadronic decays of B mesons can be expressed as the product of two independent hadronic currents, one describing the formation of a meson from the converted b quark and the light spectator quark and the other describing the production of meson by the hadronization of the virtual W^- and finally, the nonleptionic decay amplitude reduces to the product of a form factor and a decay constant.

This assumption, in general, cannot be exact. However it is expected that factorization can hold for the energetic decays, where the final M_1 meson is heavy and the M_2 meson is light. A justification of this assumption is usually based on the issue of colour transperancy. The colour transperancy argument suggests that a quark-antiquark pair remains in a state of small size until it is far from other decay products. In these decays the final hadrons are produced in the form of point-like colour singlet objects with a large relative momentum.Thus, the hadronization of decay product occur after they are too far away for strongly interacting with each other, and hence final state interactions is negligible in these cases.

While calculating the hadronic matrix elements, we need to cope well with the physical scales around $\sqrt{\Lambda_{QCD}m_b}$. Scales below this factorization scale are treated as the nonperturbative physics, described by the transition form factors, while scales above the factorization scale are considered perturbative physics.

In the standard factorization scheme, the decay amplitude is obtained by sandwiching the QCD modified weak Hamiltonian:[10]

$$A(B_c \to PV) \propto \langle P|J^{\mu}|0\rangle\langle V|J^{\dagger}_{\mu}|B_c\rangle + \langle V|J^{\mu}|0\rangle\langle P|J^{\dagger}_{\mu}|B_c\rangle$$

where the weak current J_{μ} is given by

$$J_{\mu} = (\bar{u}\,\bar{c}\,\bar{t})\gamma_{\mu}(1-\gamma_5)\begin{pmatrix} d^{'} \\ s^{'} \\ b^{'} \end{pmatrix},$$

and $d^{'}$, $s^{'}$,$b^{'}$ are mixture of the d, s, and b quarks, as given by the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Matrix elements of the currents are defined as, [5]

$$\langle P|J^{\mu}|0\rangle = -if_P P_{\mu}$$

$$\langle V|J_{\mu}|0\rangle = \in^{*}_{\mu} f_V m_V$$

$$\langle P|J^{\mu}|B_c\rangle = f_+(P_B+P_P)^{\mu} + f_-(P_B-P_P)^{\mu}$$

$$\langle V|V_{\mu}|B_c\rangle = ig \in_{\mu\nu\rho\sigma} \epsilon^{*\nu}(P_B+P_V)^{\rho}(P_B-P_V)^{\sigma}$$

$$\langle V|A_{\mu}|B_c\rangle = f\epsilon^{*}_{\mu} + (\epsilon^{*}.P_B)[a_+(P_B+P_V) + a_-(P_B-P_V)]$$

where ϵ_{μ} denotes the polarization vector of the outgoing vector meson.

Sandwiching the weak Hamiltonian between the initial and final states, the decay amplitudes for various $B_c \to PV$ decay modes can be obtained for the following categories:

1. Class I transitions: contain those decays which are caused by colour favoured diagram and the decay amplitudes are proportional to a_1, where $a_1(\mu) = c_1(\mu) + \frac{1}{N_c}c_2(\mu)$, and N_c is the number of colours.

2. Class II transitions: consists of those decays which are caused by colour suppressed diagrams. The decay amplitude in this class is proportional to a_2 i.e. for the colour suppressed modes $a_2(/mu) = c_2(\mu) = \frac{1}{N_c}c_1(\mu)$.

3. Class III transitions: these decays are caused by the interference of colour singlet and colour neutral currents and consists of both colour suppressed and colour favoured diagrams i.e the amplitudes a_1 and a_2 interfere.

For numerical calculations, we follow the convention of taking $N_c = 3$ to fix the QCD coefficients a_1 and a_2, where we use:

$$c_1(\mu) = 1.12,\; c_2(\mu) = -0.26\,at\,\mu \approx m_b^2$$

.

3.4 ISGW Model

Isgur and Wise formulated a theoretical breakthrough for determining the form factors of exclusive semileptonic B decays in the late 1980's. In the limit of infinitely heavy quark masses QCD possess additional flavour and spin symmetries and since the heavier b and c quarks have masses much heavier than the scale of the QCD coupling constant, they are heavy enough to possess this symmetry. But since the quark mass is not infinite corrections had to be made. Isgur and Wise showed there was a systematic method of making these corrections by expanding a series in terms of the inverse quark mass[4]. This theory is known as the Heavy Quark Effective Theory(HQET).

The ISGW model is regarded as the first step to heavy quark symmetry. It is a model which respects the symmetry in the heavy quark limit near zero recoil. In their paper [5], they have discussed extensively on the inapplicabilty of the free quark decay model (and its derivatives) for the end point region in b→ u semi leptonic decay. Out of this discussion, the model was developed and was shaped in such a way as to provide the minimum reasonable prediction for the decay rate in the end point spectrum region for fixed V_{ub}.

As it has been pointed out earlier, the ISGW model is applicable only to heavy-light mesons in which the heavy quark mass, m_Q is much greater than Λ_{QCD}. In that case, the four velocity of the heavy quark is essentially the same as the four velocity of the meson. In the rest frame of the meson, the heavy quark appears as a static colour source to the light degrees of freedom. The heavy quark mass suppresses the spin interaction between the heavy quark and the light degrees of freedom.

So if a weak current changes the heavy quark into a heavy quark of different flavour, or flips its spin, without changing its velocity, then it still appears the same to the light degrees of freedom. This is the reason for spin-flavour symmetries of HQET.

The ISGW model breaks the problem of computing a current matrix element of a transition from a state H of mass, momentum and spin m, p, s to H' with m', p', s' into kinematical and dynamical parts. It first makes the usual mechanical Lorentz invariant decomposition of the matrix element into Lorentz tensors and invariant form factors $f_i (i = 1, 2, ..., N)$ which depend only on the four momentum transfer variable $(t_m - t)$ where $t = (p' - p)^2$ and where $t_m = (m' - m)^2$ is the maximum momentum transfer. The variable $(t_m - t)$ is used since it is at the 'zero recoil point' where H' is left at rest in the rest frame of H.

Isgur, Scora, Grinstein, Wise used the mock meson method to calculate the form factors $f_+(t)$ and $f_-(t)$, where $t = q^2$, in a quark model. A "mock" meson is a loosely bound state of a quark and an antiquark with a mass approximately equal to the sum of the constituent masses of the quark and the antiquark. They argue that at maximum momentum transfer their calculation of "mock" form factors $\tilde{f}_+(\tilde{t})$ and $\tilde{f}_-(\tilde{t})$ ("mock" quantities are characterised by tilde) is trustworthy since the initial and the final "mock" mesons are stationary. One then assumes that the "physical" $f_+(t)$ and $f_-(t)$ are equal to the "mock" $\tilde{f}_+(\tilde{t})$ and $\tilde{f}_-(\tilde{t})$ at maximum momentum transfer.

Table 1: Quark Model parameters

Parameter	ISGW	X	β_X
m_c	1.82 GeV	B_c	0.92
m_b	5.12 GeV	η_c	0.88
m_s	0.55 GeV	J/ψ	0.62
		D_s^*	0.44

The form factors in the ISGW model for $B \to PV$ involving $b \to q$ transition with c as spectator at maximum momentum transfer $t_m = t$ has the form:

$$f_+ = F_3 \sqrt{\frac{3}{8}} \frac{m_b}{\mu_+} [\frac{\beta_B^2 - \beta_P^2}{\beta_B^2 + \beta_P^2} + \frac{m_q m_c}{3\mu_- \tilde{m}_P} \frac{\beta_B^2}{\beta_{BP}^2} (\frac{7\beta_P^2 - 3\beta_B^2}{4\beta_{BP}^2})]$$

$$f = \sqrt{6} F_3 \tilde{m}_B [\frac{\beta_B^2 - \beta_V^2}{\beta_B^2 + \beta_V^2}]$$

$g=\sqrt{\frac{3}{8}}F_3[(\frac{\beta_B^2-\beta_V^2}{\beta_B^2+\beta_V^2})(\frac{1}{m_q}-\frac{m_c}{2\mu_-\tilde{m_V}}\frac{\beta_B^2}{\beta_V^2})+\frac{m_c}{3\mu_-\tilde{m}_V}\frac{\beta_B^2\beta_V^2}{\beta_{BV}^4}]$

$$a_+=\frac{F_5}{\sqrt{6}\tilde{m}_V}[\frac{3\tilde{m}_B\beta_{BV}^2}{2m_b\beta_B\beta_V}(1-\frac{m_c^2m_b}{4\tilde{m}_B^2\mu_-}\frac{\beta_V^4}{\beta_{BV}^4})-\frac{3m_c}{2m_b}\frac{\beta_V}{\beta_B}+\frac{5m_c\beta_B\beta_V}{2m_b\beta_{BV}^2}-\frac{3\tilde{m}_B}{2m_b}\frac{\beta_B}{\beta_V}+\frac{7m_c^2\beta_B}{8\tilde{m}_B\mu_-\beta_V}\frac{\beta_V^4}{\beta_{BV}^4}]$$

where

$F_n=\sqrt{\frac{\tilde{m}_X}{\tilde{m}_B}}(\frac{\beta_B\beta_X}{\beta_{BX}^2})^{\frac{n}{2}}$

$\beta_{BX}^2=\frac{1}{2}(\beta_B^2+\beta_X^2)$

$\mu_\pm=[\frac{1}{m_c}-\frac{1}{m_b}]^{-1}$

Table 2: Decay constants and mass of mesons used in calculation.

Meson	Decay Constant(in GeV)	Mass(in GeV)
J/ψ	0.411	3.097
η_c	0.400	2.984
D^{*-}	0.273	2.1123
π	0.131	0.13957
ρ	0.221	0.77526
D^-	0.273	1.869

4 Amplitude Calculation

When B_c decays to P meson and the V meson hadronizes from vacuum, the decay rate is proportional to:

$|\langle P|J^\mu|B_c\rangle\langle V|J_\mu^\dagger|0\rangle|^2$

$=(f_+(P_B+P_P)^\mu+f_-(P_B-P_P)^\mu)(f_+(P_B+P_P)^\nu+f_-(P_B-P_P)^\nu)f_V^2m_V^2\epsilon^{\mu*}\epsilon^\nu$

$=[f_+^2(P_B+P_P)^\mu(P_B+P_P)^\nu+2f_+f_-(P_B+P_P)^\mu f_+(P_B-P_P)^\nu+f_-^2(P_B+P_P)^\mu(P_B+P_P)^\nu][-g_{\mu\nu}+\frac{P_V^\mu P_V^\nu}{m_V^2}]f_V^2m_V^2$

Here we neglect f_- as it has a very small value, so that we are left with:

$$f_+^2f_V^2m_V^2[-(P_B+P_P)^2+\frac{\{(P_B+P_P).P_V\}^2}{m_V^2}]$$

$$=(-m_B^2-m_P^2-2m_BE_P+\frac{m_BE_V+E_PE_V-p^2}{m_V^2})f_+^2f_V^2m_V^2$$

where we have used,

$$\sum_{all\,polarizations}\epsilon^{\mu*}\epsilon^\nu=-g_{\mu\nu}+\frac{P_V^\mu P_V^\nu}{m_V^2}$$

Thus, the overall expression for decay rate is:

$$\Gamma_1 = \frac{G_F^2}{2}|V_{cb}|^2|V_{q\bar{q}}|^2[c_1(-m_B^2 - m_P^2 - 2m_B E_P + \frac{m_B E_V + E_P E_V - p^2}{m_V^2})f_+^2 f_V^2 m_V^2]$$

where we have considered only $b \to c$ transition with c as spectator, and $q\bar{q} \equiv$ cs or ud.

When B_c decays to V meson and the P meson hadronizes from vacuum, the decay rate is proportional to:

$$|\langle P|J^\mu|0\rangle|^2|\langle V|J_\mu|B_c\rangle|^2$$

$$= |\langle P|J^\mu|0\rangle|^2[|\langle V|V_\mu|B_c\rangle - \langle V|A_\mu|B_c\rangle|^2]$$

$$= |\langle P|J^\mu|0\rangle|^2|\langle V|V_\mu|B_c\rangle|^2 + |\langle P|J^\mu|0\rangle|^2|\langle V|A_\mu|B_c\rangle|^2 - 2|\langle P|J^\mu|0\rangle|^2|\langle V|V_\mu|B_c\rangle\langle V|A_\mu|B_c\rangle|$$

Therefore,

$$|\langle P|J^\mu|0\rangle|^2|\langle V|J_\mu|B_c\rangle|^2 = |\langle P|J^\mu|0\rangle|^2|\langle V|V_\mu|B_c\rangle|^2+|\langle P|J^\mu|0\rangle|^2|\langle V|A_\mu|B_c\rangle|^2-2|\langle P|J^\mu|0\rangle|^2|\langle V|V_\mu|B_c\rangle\langle V|A_\mu|B_c\rangle| \quad (9)$$

The second term in equation (9) is simplified as:

$$g^2 f_P^2|\langle P|J^\mu|0\rangle|^2|\langle V|A_\mu|B_c\rangle|^2$$

$$= g^2 f_P^2 P_P^\mu P_P^\alpha \in_{\mu\nu\rho\sigma}\in_{\alpha\beta\gamma\delta} \epsilon^{\nu*}\epsilon^\beta(P_B + P_V)^\rho(P_B + P_V)^\gamma(P_B - P_V)^\sigma(P_B - P_V)^\delta$$

$$= g^2 f_P^2[-g^{\nu\beta} + \frac{P_V^\nu P_V^\beta}{m_V^2}]P_P^\mu P^\alpha \in_{\mu\nu\rho\sigma}\in_{\alpha\beta\gamma\delta} g^{\rho\gamma}g^{\sigma\delta}(P_B + P_V)^2(P_B - P_V)^2$$

$$= g^2 f_P^2[-g^{\nu\beta} + \frac{P_V^\nu P_V^\beta}{m_V^2}]P_P^\mu P^\alpha \in_{\mu\nu\rho\sigma}\in_{\alpha\beta}^{\rho\sigma} (m_B^2 - m_V^2)^2$$

$$= g^2 f_P^2[-g^{\nu\beta} + \frac{P_V^\nu P_V^\beta}{m_V^2}]P_P^\mu P^\alpha \in_{\mu\nu\rho\sigma}\in^{\phi\chi\rho\sigma} g_{\alpha\phi}g_{\beta\chi}(m_B^2 - m_V^2)^2$$

$$= -2g^2 f_P^2[-g^{\nu\beta} + \frac{P_V^\nu P_V^\beta}{m_V^2}]P_P^\mu P^\alpha[\delta_\mu^\phi\delta_\nu^\chi - \delta_\nu^\phi\delta_\mu^\chi]g_{\alpha\phi}g_{\beta\chi}(m_B^2 - m_V^2)^2$$

$$= -2g^2 f_P^2[-g^{\nu\beta} + \frac{P_V^\nu P_V^\beta}{m_V^2}][P_P^\phi P^\alpha\delta_\nu^\chi - P_P^\chi P_P^\alpha\delta_\nu^\phi]g_{\alpha\phi}g_{\beta\chi}(m_B^2 - m_V^2)^2$$

$$= -2g^2 f_P^2[-g^{\nu\beta} + \frac{P_V^\nu P_V^\beta}{m_V^2}][P_P^2 g_{\nu\beta} - P_{P\beta}P_P^\alpha g_{\alpha\nu}](m_B^2 - m_V^2)^2(m_B^2 - m_V^2)^2$$

$$= -2g^2 f_P^2[-4m_P^2 + g^{\nu\beta}g_{\alpha\nu}P_{P\beta}P_P^\alpha + P_P^2 - (\frac{P_V.P_P}{m_V})^2](m_B^2 - m_V^2)^2$$

$$= 2g^2 f_P^2 [2m_P^2 + (\frac{E_V E_P - p^2}{m_V})^2](m_B^2 - m_V^2)^2$$

where we have used

$$g^{\nu\beta} g_{\alpha\nu} = \delta_\alpha^\beta$$

The first term in equation (9) is simplified as:

$|\langle P|J^\mu|0\rangle|^2 |\langle V|V_\mu|B_c\rangle|^2$

$= f_P^2 P_P^\mu P_P^\nu [f^2 \epsilon_\mu^* \epsilon_\nu + (\epsilon^*.P_B)^2 [a_+^2 (P_B + P_V)_\mu (P_B + P_V)_\nu + a_-^2 (P_B - P_V)_\mu (P_B - P_V)_\nu +$

$2a_+ a_- (P_B + P_V)_\mu (P_B - P_V)_\nu] + 2f\epsilon_\nu (\epsilon^*.P_B)[a_+ (P_B + P_V)_\mu + a_- (P_B - P_V)_\mu]]$

First term:

$f_P^2 P_P^\mu P_P^\nu f^2 \epsilon_\mu^* \epsilon_\nu = f_P^2 f^2 P_P^\mu P_P^\nu (-g_{\mu\nu} + \frac{P_{V\mu} P_{V\nu}}{m_V^2})$

$= f_P^2 f^2 (-P_P^2 + (\frac{P_P.P_V}{m_V})^2))$

$= f_P^2 f^2 (-P_P^2 + (\frac{E_V E_P - p^2}{m_V})^2)$

Second term:

$f_P^2\, P_P^\mu\, P_P^\nu\, (\epsilon^*.P_B)^2\, a_+^2\, (P_B + P_V)_\mu\, (P_B + P_V)_\nu$

$= f_P^2\, a_+^2\, P_P^\mu\, P_P^\nu\, \epsilon_\alpha^*\, P_B^\alpha\, \epsilon_\beta^*\, P_B^\beta (P_B + P_V)_\mu\, (P_B + P_V)_\nu$

$= f_P^2\, a_+^2\, [P_P.(P_B + P_V)]^2\, [-g_{\alpha\beta} + \frac{P_{V\alpha} P_{V\beta}}{m_V^2}]\, P_B^\alpha\, P_B^\beta$

$= f_P^2\, a_+^2\, [E_P(m_B + E_V) - p^2]^2\, [-P_B^2 + (\frac{P_B.P_V}{m_V})^2]$

$= f_P^2\, a_+^2\, (\frac{m_B}{m_V})^2\, [E_P(m_B + E_V) - p^2]^2\, (E_V^2 - m_V^2)$

Third, fourth and last term: We may neglect these terms as a_- is negligibly small.

Fifth term:

$2\, f_P^2\, P_P^\mu\, P_P^\nu\, f\, \epsilon_\nu\, (\epsilon^*.P_B)\, a_+\, (P_B + P_V)_\mu$

$= 2\, f_P^2\, a_+\, f\, P_P^\mu\, P_P^\nu\, \epsilon_\alpha^* P_B^\alpha\, (P_B + P_V)_\mu$

$= 2\, f_P^2\, a_+\, f\, [-g_{\alpha\nu} + \frac{P_{V\alpha} P_{V\nu}}{m_V^2}] P_B^\alpha P_P.(P_B + P_V) P_P^\nu$

$= 2\, f_P^2\, a_+\, f\, [-P_P.P_B\; +\; \frac{P_B.P_V\, P_P.P_V}{m_V^2}][E_P(m_B + E_V) - p^2]$

$= 2\, f_P^2\, a_+\, f\, [-m_B\, E_P + \frac{m_B\, E_V(E_P\, E_V - p^2)}{m_V^2}(E_P\, m_B\; + E_V\, E_P - p^2)]$

where we have used : $\in_{\mu\nu\rho\sigma}\in^{\alpha\beta\rho\sigma} = -2[\delta_\mu^\alpha \delta_\nu^\beta - \delta_\mu^\beta \delta_\nu^\alpha]$

Thus, the overall expression for decay rate is:

$\Gamma_2 = \frac{G_F^2}{2}|V_{cb}|^2|V_{q\bar{q}}|^2 f_P^2[c_2(f^2(-P_P^2+(\frac{E_V E_P - p^2}{m_V})^2)+(a_+^2\,(\frac{m_B}{m_V})^2\,[E_P(m_B+E_V)-p^2]^2\,(E_V^2-m_V^2))+$

$(-m_B\, E_P + \frac{m_B\, E_V(E_P\, E_V - p^2)}{m_V^2}(E_P\, m_B + E_V\, E_P - p^2)))]$

5 Numerical Branching Ratios

The decay width is a measure of the probability of a specific decay process occuring within a given amount of time in the parent particles' rest frame. The total decay width for B_c decay is $\Gamma = \frac{\hbar}{\tau_{B_c}} = 1.456 \times 10^{-12} GeV$.

The branching ratio is the probability of decay by individual modes when multiple modes are available.

$$B_i = \frac{\Gamma_i}{\Gamma_{total}}$$

Since the dimension of Γ is inverse of time, in the system of natural units, it is the same as the dimension of mass (or energy). When the mass of an elementary particle is measured, the total rate shows up as the irreducible width of he shape of the distribution. Hence the name decay width.

The form factors calculated from the ISGW model for each decay is given in Table 3. Using the form factors derived from the ISGW model, we finally predict the branching ratios which are given in Table 4. We use $G_F = 1.0639 \times 10^{-5}$, $V_{cs} = 1.006$, $V_{cb} = 0.0409$ and $V_{ud} = 0.97427$.

1. The first decay we have looked at is $B_c^- \to \eta_c \rho^-$. The Feynman diagram for this decay is shown below:

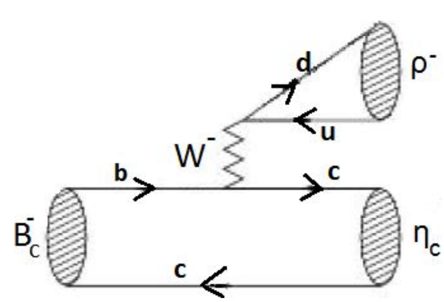


Figure 2: Feynman diagram for $B_c^- \to \eta_c \rho^-$ decay.

This is a colour favoured diagram, so the amplitude will contain only the c_1 part. The decay width for this channel is given by:

$$\Gamma_1 = \frac{G_F^2}{2} |V_{ub}|^2 |V_{ud}|^2 c_1^2 |\langle \eta_c | J^\mu | B_c \rangle \langle \rho | J_\mu | 0 \rangle|^2$$
$$\approx \frac{G_F^2}{2} |V_{ub}|^2 |V_{ud}|^2 c_1^2 f_+^2 f_\rho^2 m_\rho^2 (m_{B_c}^2 + m_\rho^2 + 2 m_{B_c} \sqrt{p_\rho^2 + m_\rho^2})$$

2. Next, we investigated $B_c^- \to \pi^- J/\psi$, which is similar to the above case. The only difference is that in this case the vector meson originates from B_c while the pseudoscalar meson hadronizes from vacuum.

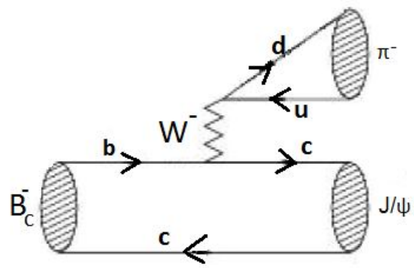


Figure 3: Feynman diagram for $B_c^- \to \pi^- J/\psi$ decay.

3. The third and the last decay channel is interesting as it involves both colour favoured and colour suppressed decays. So the decay widths will have contributions from both the diagrams. In B meson decays, the experimental data favours constructive interference between the colour favoured and colour suppressed diagrams. This yields the $a_1 = 1.10$ and $a_2 = 0.20$.
The two possible diagrams are shown in the figure below:

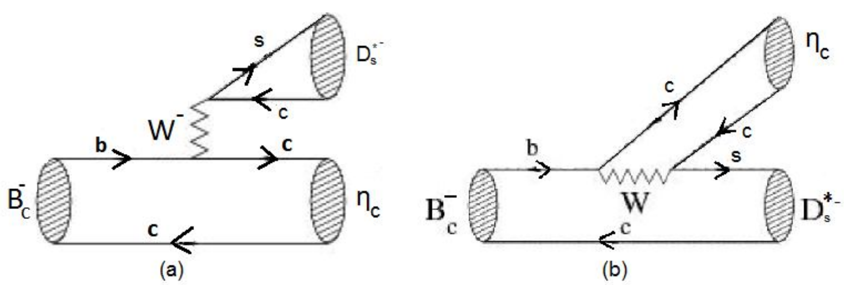


Figure 4: Feynman diagram for $B_c^- \to \eta_c D_s^{-*}$ decay: (a) Colour favoured diagram, (b)Colour suppressed diagram.

Table 3: Form Factors from the ISGW model at maximum momentum transfer.

Decay Mode	f_+	f	g	a_+
$B_c^- \to \eta_c \rho^-$	0.2318	-	-	-
$B_c^- \to \pi^- J/\psi$	-	4.0001	0.0705	-0.0196
$B_c^- \to \eta_c D_s^{-*}$	0.2318	4.2520	0.0336	-0.0407
$B_c^- \to J/\psi D_s^-$	0.4775	4.0001	0.0705	-0.0196

Table 4: Branching Ratios

Decay Modes	Decay Type	BR's%
$B_c^- \to \eta_c \rho^-$	$\Delta b = 1, \Delta c = 1, \Delta S = 0$	5.58
$B_c^- \to \pi^- J/\psi$	$\Delta b = 1, \Delta c = 1, \Delta S = 0$	7.1434×10^{-2}
$B_c^- \to \eta_c D_s^{-*}$	$\Delta b = 1, \Delta c = 0, \Delta S = -1$	0.56
$B_c^- \to J/\psi D_s^-$	$\Delta b = 1, \Delta c = 0, \Delta S = -1$	4.7934×10^{-2}

6 CP violation and Interfering amplitudes

As we know, there are two phases: weak phase and strong phase. Weak phases change sign under CP while the strong phase does not change. Consider for instance that

$$\langle f|T|i\rangle = Ae^{i}(\delta + \phi)$$

$$\langle \bar{f}|T|\bar{i}\rangle = Ae^{i}(\delta - \phi + \theta)$$

. Here, A is a real positive number, ϕ is a weak (CP-odd)phase, δ is strong (CP even) phase, and θ is an arbitrary CP-transformation phase which has no effect on CP violation. For the case of the first two decay considered in this paper, there is only one possible Feynman diagram, so the quantity $|\langle f|T|i\rangle - \langle \bar{f}|T|\bar{i}\rangle| = A - A$ vanishes and CP is conserved.[12]
But for the third decay considered, two Feynman diagrams contribute to the amplitude, so that we may write

$$\langle f|T|i\rangle = A_1 e^{i(\delta_1 + \phi_1)} + A_2 e^{i(\delta_2 + \phi_2)}$$

$$\langle \bar{f}|T|\bar{i}\rangle = A_1 e^{i}(\delta_1 - \phi_1 + \theta) + A_2 e^{i}(\delta_2 - \phi_2 + \theta)$$

These are the two interfering amplitudes; they have moduli A_1 and A_2, CP even or strong phases δ_1 and δ_2, and CP odd or weak phases ϕ_1 and ϕ_2, respectively.
The CP asymmetry is quantified by the quantity:

$$\frac{|\langle f|T|i\rangle|^2 - |\langle \bar{f}|T|\bar{i}\rangle|^2}{|\langle f|T|i\rangle|^2 + |\langle \bar{f}|T|\bar{i}\rangle|^2} = \frac{-2A_1A_2 sin(\delta_1 - \delta_2) sin(\phi_1 - \phi_2)}{A_1^2 + A_2^2 + 2A_1A_2 cos(\delta_1 - \delta_2) cos(\phi_1 - \phi_2)}.$$

In order to have maximum asymmetry, three conditions are necessary:

1. The difference between the weak phases of the two interfering amplitudes should be close to $\pi/2$, so that $cos(\phi_1 - \phi_2) \approx 0$;

2. the strong phases of the interfering amplitudes must also differ by $\pi/2$, so that the other cosine term also vanishes.

In that case, we obtain A_{CP} for the decay $B_c^- \to \eta_c D_s^{-*}$ is : $A_{CP} = \frac{-2A_1A_2}{A_1^2 + A_2^2} \approx 30\%$

The variation the CP asymmetry parameter with the weak phase is plotted in the graph below:

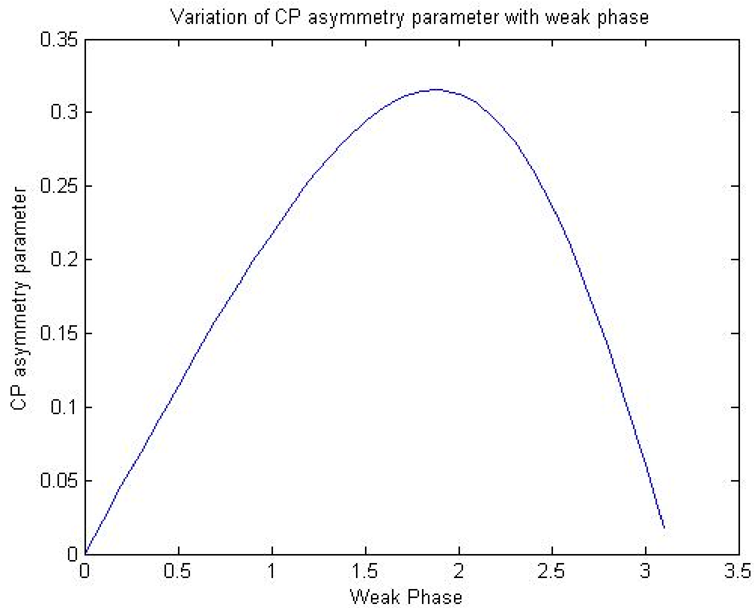


Figure 5: A_{CP} vs Weak Phase

7 Summary and Conclusions

In the present paper, we investigated four $B_c \to PV$ decays. Since, it is a hadronic decay, the hadronic part becomes complex. To overcome the complicacy, we have used the 'naive' factorization approximation, whereby, we simplify the expression by factorizing the amplitude into two parts, which involves form factors and decay constants. Then, we used the ISGW model to determine the form factors for each deacy channel.

The four decay channels considered in the paper are in some way different from each other. In the first decay, $B_c^- \to \eta_c \rho^-$, the P meson η_c originated from B_c while the vector meson hadronized from the virtual W^-. In the second decay, the V meson originated from B_c, while the P meson hadronized from the virtual W^-. There is only one Feynman diagram possible for the above two decays, hence CP will be conserved in these decays. However, for the last two decay channels considered in this paper, $B_c^- \to \eta_c D_s^{-*}$ and $B_c^- \to J/\psi D_s^-$, there is contribution from two Feynman diagrams, so CP asymmetry is observed.

So, B decays seems to be a contributor to the baryon asymmetry of the Universe, which we were looking for in the first place, notwithstanding the fact that the possible asymmetry B physics brings into picture is not enough to explain the baryon asymmetry of the Universe. We conclude the paper by stating that more extensive searches for CP violation is required in the near future. The Standard Model predictions of the CP violation is not enough to explain the baryon asymmetry, so we may have to go beyond the Standard model and look into the realm of new physics.

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