NEUTRINO PHYSICS :Leading pathway To New Discovery

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Certificate

This is to certify that the thesis entitled *'Neutrino Physics:Leading Pathway to New Discovery"* being submitted to the Indian Institute of Technology Hyderabad in partial fulfilment of the requirements for the award of the M.Sc in Physics degree , embodies the research work done by *Mangalika Sinha (ph12m1008)* under my supervision at IIT-Hyderabad.

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The thesis entitled *'Neutrino Physics:Leading Pathway to New Discovery"* being submitted to the Indian Institute of Technology Hyderabad,by*Mangalika Sinha (ph12m1008)* is approved for the partial fulfilment of the degree of Masters of Science,from IIT-Hyderabad.

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Abstract

In this project I have discussed about neutrino oscillation in matter and its implications on astrophysical objects e.g neutron star. I have proposed about the mechanism in which a rotating neutron stars receive a substantial kick during its birth. According to this mechanism these stars receive a kick due to asymmetric neutrino emission.Since neutron stars have strong magnetic field,this alters the shape of the neutrinosphere during core collapse supernova which results in an anisotropy of the emitting neutrinos.I have calculated this fractional asymmetry in the momentum for active-active neutrino transformation & active-sterile neutrino transformation.

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1 INTRODUCTION

Neutrinos are one of the most important fundamental particles that universe is composed of. Having no charge and zero mass(according to the SM) these particles were formed within a few seconds after the creation of the Big Bang,so they play a big role in the study of the origin of the universe. They are very difficult to detect since they have feeble interaction power. Billions of neutrinos are always passing through our bodies,still we are not aware of it-this makes them more interesting. The source of the production of neutrinos are not limited. They are produced in large amount in the fusion reaction occuring at the core of the sun. During Supernova explosions also,neutrinos are emitted and they carry a large percent of energy. Other than this,neutrinos are also produced in atmosphere,accelerators,nuclear power stations.Last but not the least with the continuous expansion of the universe,the production of neutrinos are also increasing and they remain within the universe. Though neutrinos were largely available throughtout the universe,its existence was not known until **Pauli** postulated it in 1930.The history goes as follows.
While interpreting β-spectrum two facts were revealed-
1)Presence of discrete energy levels.
2)Presence of continuous energy spectrum.
The nuclear-β decay has the form,

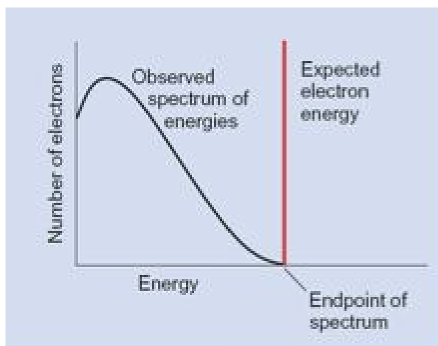

Figure 1: Continuous energy distribution of β-decay(Source-Google images)

$$n \rightarrow p + \beta^- \tag{1}$$

The physicists had to face a lot of difficulties while interpreting the β-spectrum. These can be listed as-
1) The existence of continuous energy spectrum appeared as a **violation of the law of energy conservation**.From the reaction it was clear that the β-particle is emitted with a certain value of energy but the observations conclude something different
.2) Since the mother nucleus is at rest, so the β particle must emit opposite to the direction of the daughter product for the conservation of momentum. But it is observed that the β particle is not emitted in opposite direction,rather it emits it an arbitrary direction leading to the **violation of the law of conservation of linear momentum**.
3)All the particles associated with the β decay are spin $\frac{1}{2}$ particles. Thus the products have a net spin 1,on the other hand the mother nucleus is spin $\frac{1}{2}$. Thus,there is also **violation of the law of conservation of angular momentum**.
To avoid the violation of these conservation laws Pauli postulated the existence of a new spin $\frac{1}{2}$ particle,having zero mass & charge carrying a little kinetic energy.Thus,it was explained that the continuous energy distribution arises from the variable manner in which the total energy is shared among β particle and neutrinos.
The name of this new particle was later coined by **Enrico Fermi** as "neutrino" meaning-"little neutral one".At the same time he developed the theory of β-decay. According to his theory, if N(p) is the probability that a β particle is emitted with momentum between p & p+dp,then,

$$N(p) = CF(Z,p)p^2(E_0 - E_e)^2$$

where F(Z,p) is the Fermi factor and $(E_0 - E_e)$ is the kinetic energy of the neutrino. If we plot $\sqrt{\frac{N(p)}{p^2 F(Z,p)}}$ vs E_e ,we obtain a straight line under the condition that neutrinos are massless. This is known as **Kurie plot**. If the neutrinos were massive then the plot would not be a straight line. However, Kurie plot confirms the correctness of the β-decay.

It took about 20 years for the physicists to detect neutrinos experimentally. In 1956,Cowan & Reines

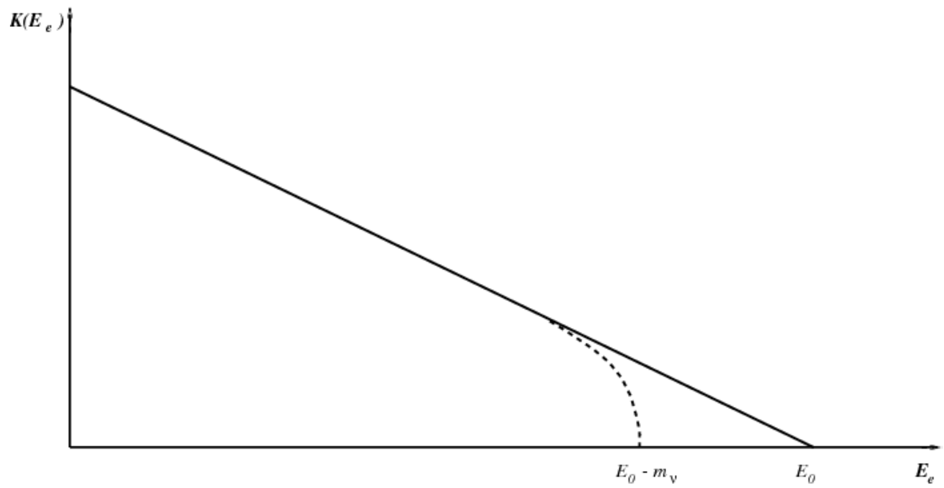

Figure 2: Kurie Plot(Source-Inspirehep-net)

detected anti-neutrinos emitted from a nuclear reactor at Savannah River in U.S.A. They also got the Nobel prize for their discovery in the year 1995.However,the discovery lead to a great leap in the study of neutrinos.Since then,several experiments on neutrinos were performed. The experiment measuring the flux of the atmospheric neutrinos observed that the ν_μ disappeared while propagating over a large distance. Not only this,the most important is the experiment performed by Davis on solar neutrino flux. It was observed that the ν_e's present in the solar flux also vanished while propagating over a large distance.

Initially,it was thought that there existed some anomaly for which the detected flux was found to be lesser than the observed flux. Later,the puzzle was solved by the term "**Neutrino Oscillation**". In 1968 Gribov & Pontecarvo postulated that flavor oscillation may occur if neutrinos are massive and mixed and they got the idea from the K_0 oscillation. Thus,when Davis showed the deficit of solar neutrino flux the possible solution that was suggested was"Neutrino Oscillation'. But the idea does not work in all cases.We also have to take into account how oscillations will get modified in presence of any field or medium. Not only this we have to know that how the modification occurs with different types of medium or field.For e.g since the density of sun or atmosphere is not uniform,the matter effect has a large implication on the phenomenon of neutrino oscillation on a matter of varying density. This was realized by Mikheyev,Smirnov & Wolfenstein and was named after them.Basically Wolfenstein observed the effect of oscillation due to matter interaction,while Mikheyev & Smirnov postulated the resonant amplification of oscillation when neutrinos pass through a matter(medium) of non-uniform density.If we have a magnetized medium,it will have a different implication on the phenomenon of neutrino oscillation. When there is an external magnetic field ,the background particles gets modified and they influences the behavior of the medium. As the neutrinos interact with the modified background particles,the properties of neutrinos are not same as those of vacuum and also as that in non-magnetized medium.In presence of a magnetic medium the neutrino self energy term appears in the modified Dirac equation which depends on the temperature and magnetic field.

We know that most of the stellar objects for e.g,neutron stars,white dwarf have very strong magnetic field and are rich sources of neutrinos. Thus,due to the presence of this strong magnetic field the neutrinos emitting during the formation of neutron star have modified properties and the phenomenon of resonant neutrino oscillation can be explained by MSW effect. It can also solve the well-known astrophysical puzzle-**high kick velocity of the rotating neutron star**.

We know that stars keep on evolving until their core is formed of any stable element like iron. In this

process of evolution when an iron core is formed no more burning of core occurs,rather it absorbs a certain amount of energy leading to the increase in the collapse rate of the star. Due to this increase in collapse rate the core becomes smaller and denser, and the electrons get absorbed by the protons to form neutrinos and neutrons. As the collapse rate increases the mass of the core increases and after a certain time the stars bursts leading to supernova explosion,along with neutrinos are emitted carrying most of the energy and the denser core which is neutron rich remains and is named as neutron star. During their birth they have a proper motion with a velocity of the order of 1000 km/s. Along with the proper motion it was observed that it experience certain kicks with an average velocity of 450 ± 50 km/s.The physical origin of these kick velocities are not properly understood. A number of mechanisms were proposed to explain this phenomenon. Two of the mechanisms were found to be more logical to explain this. The mechanisms goes like this-a kick can be caused due to the asymmetric neutrino emission or due to asymmetric mass ejection during the core collapse.
We know that the magnetic field of these stars are of the order of ($> 10^{15}G$). So,the kicks can be due to the effect of magnetic field on neutrino emitting during the supernova explosion,for the dependence of neutrino scattering in a magnetic medium. In my report I have presented the calculation of the asymmetry in neutrino emission due to the presence of magnetic field,though some approximations are taken into account.

2 THE STANDARD MODEL

After Maxwell's success of unifying electricity & magnetism in a single theory physicists had been trying to unify all the forces existing in nature. Ultimately, Glashow-Wienberg & Salam were successful in unifying the electromagnetic & weak interactions which brought about a new revolution in the world of physics. The model they constructed was known as the GWS model. The Standard Model of Particle physics, constructed in 1970 stands upon the GWS model & the strong interactions. It has been a successful model since it can describe most of the phenomena of the elementary particles,though it has some exceptions too. The SM is based on the gauge group,

$$SU(3) \times SU(2) \times U(1)$$

where SU(3),SU(2) & U(1) are related to the strong,weak & electromagnetic interactions. All these interactions are mediated by the exchange of a particle.These mediators transfer the forces between quarks & the leptons. The mediators for different interactions are given in figure(3). The SM is composed mostly of fundamental spin half particles and they are classified as-[1]
1) Six Quarks: up(u),down(d),strange(s), charm(c),bottom(b) & top(t)

Interaction	Mediator	Spin/parity
strong	gluon, G	1^-
electromagnetic	photon, γ	1^-
weak	$W^{\pm}, Z^0$	$1^-, 1^+$

2) Six Leptons: Electron(e^-) ,Muon(μ^-), tau(τ^-) and three flavors of neutrinos($\nu_e, \nu_\mu, \& \nu_\tau$)

Besides the mediators are also a part of the standard model which are generally bosons. Last but not the least Higgs Boson is another important particle present in the SM. The quarks present in the SM are obtained in three generations. The 1st generation consists of up & down quarks,2nd generation consists of strange & charm quarks and the 3rd generation consists of top & bottom quarks. The higher generation quarks are mostly unstable. Similarly, for leptons electron is the fundamental particle and muon & tau are heavier version of electron. Each of these particles also have antiparticles too.

After knowing the particle contents now we can discuss the various properties of these particles for e.g mass,charge & quantum numbers associated with them. Beginning with charge, leptons can be classified into two category on the basis of charge-

1)Charged leptons like (e^-),Muon(μ^-),tau (τ^-) carry integral charge.
2) Neutral leptons like neutrinos have zero electric charge.

On the other hand quarks have fractional charges. The (u,c,t) carry a charge of($+\frac{2e}{3}$) while (d,s,b) carry ($-\frac{1e}{3}$).
As one goes from 1st generation to 3rd generation the masses of the quarks keeps on increasing. Again,in case of leptons electron is the lightest lepton while muon and tau are much heavier than electron thus they are unstable.The most important information that SM gives us is that neutrinos are massless! Another important aspect is the quantum numbers associated with these particles. For example, strangeness is associated with a strange quark,i.e a particle composed of strange quark has S=-1. Similarly, there are other quantum numbers associated with charm,bottom & top quarks and they have same name. Now the question arises what is the quantum no. associated with the rest two quarks?
The up & down quarks possesses one of the two component of the isospin vector $I = \frac{1}{2}$ which is analogous to the spin vector. Thus the Standard Model of the elementary particle physics along with a total of 61 particles is one of the most successful model describing the interactions & other particle phenomena, mainly explaining the various low energy processes.

2.1 Neutrinos In The Standard Model

In SM there are 3 neutrinos present ,each corresponding to the charged current interaction **CC** and the Lagrangian takes the form,[2]

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \Sigma_l \bar{\nu_l}_L \gamma^\mu l_L^- W_\mu^+ + h.c \tag{2}$$

and the neutral current interactions are given by,

$$-\mathcal{L}_{\mathcal{NC}} = \frac{g}{2\cos\theta_w} \Sigma_l \bar{\nu_l}_L \gamma^\mu l_L^- Z_\mu^0 + h.c \tag{3}$$

The equation (2) & (3) describe all the neutrino interactions.

In the SM all the fermions have their left & right handed pairs except the neutrinos and this makes the Standard Model to predict that neutrinos are massless. According to the SM the fermions acquire their masses by means of Yukawa interaction which couples a right & left handed fermion via Higgs field which,due to breaking of symmetry gives the mass to the fermions. The mass terms for up & down quarks and the charged leptons are given by,[3]

$$-\mathcal{L}_{\mathcal{MASS}} = \Sigma_{a,b}[\bar{u}_{aL} M^u_{ab} u_{bR} + \bar{d}_{aL} M^d_{ab} d_{bR} + \bar{l}_{aL} M^l_{ab} l_{bR}] \tag{4}$$

where,

$$M^f_{ab} = \frac{h^f_{ab} v}{\sqrt{2}}$$

f=u,d or l and h is the coupling matrices.
Since the SM contains no RH neutrinos so they cannot couple via Higgs field and thus remains massless.

We can think that neutrino masses can rise from higher order loop correction. But this can't happen because the term for which mass can arise is $\bar{l}_l l_l^c$ and this term violates the total lepton number by 2 units and since this is a global symmetry so the L-violating terms can't be induced by loop correction.

2.2 Emergence of New Physics:Massive Neutrinos

The SM of particle physics predicts neutrinos to be massless since it has no right handed pair like other fermions. But various experiments,for e.g, Solar neutrino anomaly shows the evidence of non-zero neutrino mass. Thus in order to add mass to the neutrinos one can do two things-

1) Either introduce new particles which are beyond the SM particle content.
2) Abandon gauge invariance which is the basis of the SM.

But if one abandons the gauge invariance the SM fails to describe many phenomena of particle physics so it is better to extend the particles contents. If one adds to the SM, RH neutrinos then they can also acquire masses like other fermions. These RH neutrinos which have no gauge interactions are also known as the sterile neutrinos. With this addition the Lagrangian describing the neutrino interaction changes, as mass terms are involved.

2.2.1 Introduction to the Dirac & Majorana Masses

We know that the massive spin $\frac{1}{2}$ fermions obeys the Dirac equation and can be described by the four component complex spinors. These four components can be reduced to two component spinors and in terms of these two component spinor we obtain a set of two coupled equations given by,[4]

$$p^0 u_R(p) - \vec{\sigma}.\vec{p} u_R(p) = m u_L(p) \tag{5}$$

&

$$p^0 u_L(p) + \vec{\sigma}.\vec{p} u_L(p) = m u_R(p) \tag{6}$$

One thing is to be observed is that as the mass goes to zero, the equations are decoupled but still the nature of the equation is invariant under Lorentz transformation. The only thing that is not invariant is parity and these equations are known as **Weyl equations**. Since SM says that neutrinos are massless so they must obey the Weyl equation. But due to sufficient number of evidence of the fact that neutrinos have non-zero mass,we can consider neutrinos as Dirac particles only.

Another important feature which makes the neutrinos different from the other fermions is that they have zero charge. So,it is possible that a neutrino particle is its own antiparticle, inspite of violation of lepton number.These particles are known as Majorana particles and neutrinos can be regarded as Majorana particles too.

2.2.2 Dirac Masses

The most important tool of particle physics is the Quantum field theory.According to this theory,the equation of motion are derived from the Lagrangian of the field operators. Since neutrinos are fermions,they obeys the well known Dirac equation. Thus the Lagrangian from which the Dirac equation can be obtained is given by,

$$\mathcal{L} = \bar{\nu}(i\gamma^\mu \partial_\mu - m_D)\nu \tag{7}$$

Thus, the Dirac mass term is given by,

$$\mathcal{L_D} = m_D \bar{\nu}\nu \tag{8}$$

The term $\bar{\nu}\nu$ has to be Lorentz invariant. Now,we evaluate the $\bar{\nu}\nu$ term by writing each field in terms of its

left and right handed components.
$\bar{\nu}\nu = (\bar{\nu_L} + \bar{\nu_R})(\nu_L + \nu_R) = \bar{\nu_L}\nu_R + \bar{\nu_R}\nu_L$
Thus,the Dirac mass term is,

$$L_D = m_D(\bar{\nu_L}\nu_R + \bar{\nu_R}\nu_L) \tag{9}$$

2.3 Majorana masses

As,I have discussed before that the only difference between neutrinos and other fermions is the electric charge. For neutrinos the electric charge is zero,which makes it similar to its charge conjugate pair.Thus,in the Lagrangian one can include ν^c terms. On adding the charge conjugate terms one can obtain different combinations of spinor which behaves as Lorentz invariant. Some of the combinations are : $\bar{\nu^c}\nu^c, \bar{\nu}\nu^c \& \bar{\nu^c}\nu$ where the 1st term is same as $\bar{\nu}\nu$ & the last two terms are hermitian conjugate of each other.With this we have an additional Hermitian mass term which is given by,
$\mathcal{L_M} = \frac{1}{2}(m_M\bar{\nu}\nu^c + m_M^*\bar{\nu^c}\nu)$
where m_M is the Majorana mass.If we use the chiral projections,then, two hermitian mass terms are obtained,

$$\mathcal{L_L} = \frac{1}{2}m_L(\bar{\nu_L}\nu_R^c + \bar{\nu_R^c}\nu_L) = \frac{1}{2}m_L\bar{\nu_L}\nu_R^c + h.c$$

$$\mathcal{L_R} = \frac{1}{2}m_R(\bar{\nu_L}^c\nu_R + \bar{\nu_R}\nu_L^c) = \frac{1}{2}m_L\bar{\nu_L^c}\nu_R + h.c$$

Here, $m_L \& m_R$ are real Majorana masses.

2.4 Dirac & Majorana Masses:Seesaw Mechanism

From the preceding sections we got the expressions of Dirac & Majorana masses,thus the total Lagrangian takes the form,[5]

$$2\mathcal{L} = m_D(\bar{\nu_L}\nu_R + \bar{\nu_L^c}\nu_R^c) + m_L\bar{\nu_L}\nu_R^c + m_R\bar{\nu_L^c}\nu_R^c + h.c \tag{10}$$

$$= \begin{pmatrix} \bar{\nu_L} & \bar{\nu_L^c} \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} + h.c$$

where,$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$ is the mass matrix and this is real when we consider CP to be conserved. We now,diagonalize the mass matrix M,to find its eigenvalues and eigen states. To find the eigen value we solve the characteristic equation and get the eigen values,

$$\phi_\pm = \frac{1}{2}(m_L + m_R) \pm \frac{1}{2}\sqrt{(m_L + m_R)^2 - (4m_Lm_R - m_D^2)}$$

Now,we consider the following cases,
1) If $m_L = m_R = 0$ then from the eigen value we obtain $\phi_\pm = m_D$,i.e we are left with the Dirac field only.
2)If $m_D = 0$ we are left with pure Majorana field only.
3) When $m_R \gg m_D$,$m_L = 0$.The two mass eigen values that are obtained is given by,
$\phi_- = m_\nu = \frac{m_D^2}{m_R}$ & $\phi_+ = m_N = m_R(1 + \frac{m_D^2}{m_R^2}) \simeq m_R$
We know that the Dirac mass term is similar to the charged lepton masses and occurs generally at lower energies($10^2 Gev$),on the other hand the Majorana mass term is possible at much higher energy($10^{19}Gev$).Thus it is clear that the left handed neutrino masses are less than the charged lepton masses by a factor of $\frac{m_D}{m_R}$. Thus,from the two eigen values we can say that if one of the eigen value increases another decreases and vice versa. This phenomena is known as Seesaw mechanism. To know in details we have to use SU(5) model.

3 NEUTRINO OSCILLATIONS IN VACUUM

Neutrinos are mostly produced by the charged current weak interaction along with a charged lepton of $e^-, \mu^- \& \tau^-$ flavor.In that sense we can name these neutrinos as the flavor eigenstates. In analogy with the quarks and the CKM matrix it is possible that the flavor eigenstates i.e states with definite flavor are not similar to the mass eigenstates i.e states which have definite mass. The flavor eigenstates are the linear combinations of the mass eigenstates and they are related via a mixing matrix,known as PMNS matrix which is equivalent to CKM matrix as in case of the quarks.

$$|\nu_\alpha\rangle = \Sigma_i U^*_{\alpha 1} \nu_1 \tag{11}$$

Suppose at the source point a neutrino of a certain flavor ν_α is generated which is the linear combination of different mass eigen states.As the states have different masses then the phases between the states will change as the distance changes and it is detected that, we obtain a different flavor state which was not present initially. As it travels through a certain distance,after a time t(say),evolution of the neutrino flavor occurs and it is given by,

$$|\nu_\alpha(t)\rangle = \Sigma_i U^*_{\alpha i} |\nu_i(t)\rangle \tag{12}$$

Now using the approximation that ν_i is a plane wave i.e it takes the form,

$$|\nu_i(t)\rangle = e^{(-iE_i t)} |\nu_i(0)\rangle \tag{13}$$

and also we assume that the momentum vector $\vec{p}$ of the different components of the neutrino beam are same. Considering the neutrinos to be relativistic particle we assume that $p \simeq E$. Thus the relativistic energy-momentum relation is given by,

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p + \frac{m_i^2}{2E} \tag{14}$$

In calculating this we have assumed that the neutrinos generated at the source point are stable. Now, the amplitude of finding another flavor of neutrino ν_β after a time t,in the original ν_α flavor is,

$$\langle \nu_\beta | \nu_\alpha(t) \rangle = \Sigma_{i,j} U^*_{\alpha i} U_{\beta j} \langle \nu_j | \nu_i(t) \rangle \tag{15}$$

Using equation (13) and taking into account the fact that the mass eigen states are orthogonal to each other i.e

$$\langle \nu_j | \nu_i \rangle = \delta ij$$

the transition amplitude takes the form,

$$\langle \nu_\beta | \nu_\alpha(t) \rangle = \Sigma_i U^*_{\alpha i} U_{\beta i} e^{(-iE_i t)} \tag{16}$$

Thus at time t,the probabilty of finding another flavor of neutrino ν_β after a time t,in the original ν_α flavor is,[6]

$$P_{\alpha,\beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = |\Sigma_i U^*_{\alpha i} U_{\beta i} e^{(-iE_i t)}|^2 = \Sigma_{i,j} [U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}] e^{i(E_j - E_i)t} \tag{17}$$

Now using equation (14) we get the exponential term having the form,

$$e^{i(E_j - E_i)t} = e^{2i\frac{\Delta m^2}{4E}t}$$

where

$$\Delta m^2 = m_j^2 - m_i^2$$

Thus, putting these and finally using De Moivre's theorem,

$$e^{in\theta} = \cos n\theta + i \sin n\theta$$

we get ultimately the transition probability to be,

$$P_{\alpha,\beta} = \delta_{\alpha\beta} - 4\Sigma_{i,j} Re[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin^2 Y_{ij} + 2\Sigma_{i,j} Img[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}] \sin 2Y_{ij} \tag{18}$$

The 1st term is due to the fact that at t=0 the amplitude is $\delta_{\alpha\beta}$,using the unitarity of the matrix U. The real part of the probability is CP conserving and the imaginary part is CP violating. Moreover,

$$Y_{ij} = \frac{\Delta m^2 L}{4E} = 1.27 \frac{\Delta m_{ij}^2}{eV^2} \frac{L/E}{m/Mev} \tag{19}$$

Since neutrinos are moving with the speed of the light then we can write t=L. The transition probability that we obtain has an oscillatory behavior and the oscillation length is,

$$L_{oscillation} = \frac{4\pi E}{\Delta m_{ij}^2} \tag{20}$$

Thus,in order to undergo oscillations,neutrinos must have different masses i.e

$$\Delta m_{ij}^2 \neq 0$$

and they should mix i.e non-zero mixing matrix.

This is the general treatment of neutrino oscillation. Now,considering,two-flavor case,the mixing matrix now depends on only one parameter,

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{21}$$

& since only two-flavors are associated so there is only one mass-difference square term. Thus, the transition probability takes the form,

$$P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 Y \tag{22}$$

where Y is the oscillation phase & $\sin^2 2\theta$ is the oscillation amplitude.

Though in reality two flavor oscillation does not occur but still for simplified calculation it is used and one of its main advantage is that there is a reduction of the number of neutrino oscillation parameter in this case. (For two-flavor case only two parameters are there-only is the vacuum mixing angle θ & one mass square difference term.)

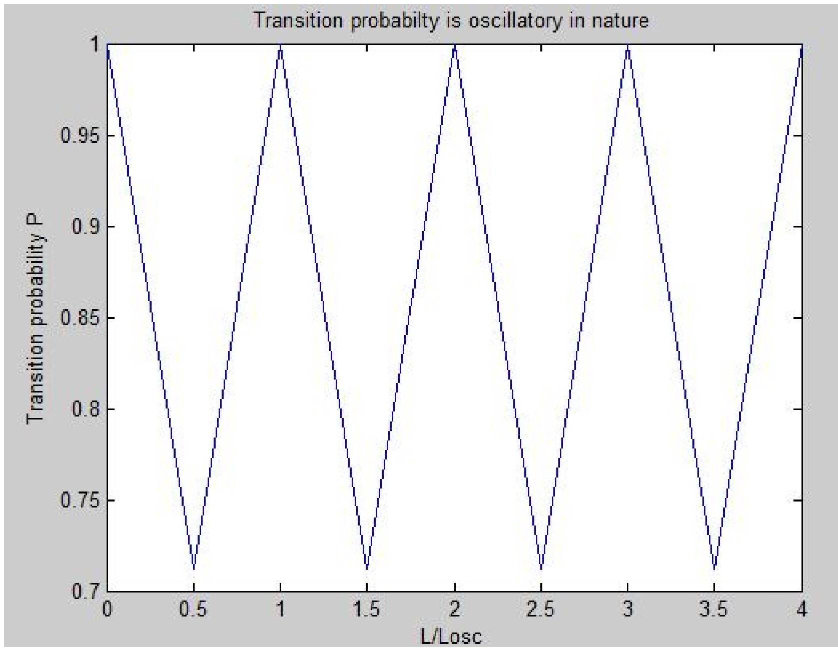

Figure 3: Oscillatory Behavior of Probability

4 NEUTRINO OSCILLATION IN MATTER

Neutrino oscillation phenomena is strongly affected when a beam of neutrinos propagates through a medium or matter. On propagation of neutrinos through a medium, modification of different flavors occurs due to scattering of neutrinos. The scattering due to which this matter effect arises can be classified as-

1) Incoherent scattering
2)Coherent scattering

In case of incoherent scattering,scattering occurs in all possible direction. Suppose,we have a N-body entangled state which cannot be factorized into N individual wave-function.These entangled states have effects only on the incoherent scattering. Moreover,the interaction cross-section is extremely small,

$$\sigma \simeq 10^{-43} cm^2 (\frac{E}{Mev})^2$$

.Since they are proportional to the square of the Fermi coupling constant G_F, so their effects are not considered.Thus in the standard formalism for the calculation of the neutrino oscillation in matter the contribution of incoherent scattering is absent.
But in case of coherent scattering,the entanglement does not have any effect, so the medium remains unchanged. Coherent scattering always occurs in the forward direction. In this case the incident & the scattered state of waves have same momentum, differing only by a phase factor.Here interference of scattered and the un-scattered neutrino waves occurs which increases the matter effect.The standard formalism is valid in case of coherent scattering. In this formalism we can decouple the evolution equation of the neutrinos from the equation of the medium.Generally,the medium effect is described by an effective potential and this potential depends on the density of the medium.

4.1 Calculation Of the Effective Potential

Neutrino-matter interaction occurs by means of scattering via neutral current(**N**C) & charged current (**C**C) mechanism. Generally,matter is mostly composed of electrons. It does not contain muon or tau. For this reason different flavor of neutrino undergoes different type of interaction. For e.g an electron neutrino interacts with matter via both charged & neutral current ,while ν_μ & ν_τ undergoes interaction via neutral current only.The Feynman diagrams of the following processes are given in figure (4).

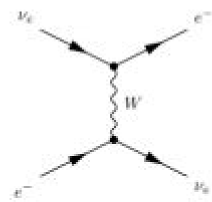

Figure 4: Feynman diagram of CC interaction

The effective Hamiltonian of the CC scattering of neutrinos due to electrons is given by,[8]

$$H_{CC} = \frac{G_F}{\sqrt{2}}[\bar{\nu_e}\gamma^\mu(1-\gamma^5)e][\bar{e}\gamma_\mu(1-\gamma^5)\nu_e] \tag{23}$$

To separate the contribution of electrons & neutrinos we apply the Fierz transformation. We get,

$$H_{CC} = \frac{G_F}{\sqrt{2}}[\bar{\nu_e}\gamma^\mu(1-\gamma^5)\nu_e][\bar{e}\gamma_\mu(1-\gamma^5)e] \tag{24}$$

Now,averaging over the electron spinors and summing over all electrons in the medium,the effective Hamiltonian takes the form,

$$\bar{H}_{CC} = \frac{G_F}{\sqrt{2}}[\bar{\nu_e}\gamma^\mu(1-\gamma^5)\nu_e]\int d^3p_e f(E_e,T)\times\Sigma_{s_e=\pm1}\langle e^-(p_e,s_e)|\bar{e}\gamma_\mu(1-\gamma^5)e|e^-(p_e,s_e)\rangle \tag{25}$$

where $s_e\&p_e$ remains same for the initial & final states as the scattering is coherent in nature. Here, $f(E_e,T)$ is the energy distribution function of the homogeneous medium.
Now,we have to calculate the averaging part of the expression. Here the axial current vector transforms to spin in the non-relativistic limit. on averaging the spin reduces to zero. On the other hand,we have the vector current term,whose space component cancels because the medium is homogeneous & thus isotropic. Thus the only term that remains is the time component of the vector current. So,we have,

$$\int d^3p_e f(E_e,T)\times\Sigma_{s_e=\pm1}\langle e^-(p_e,s_e)|\bar{e}\gamma_0 e|e^-(p_e,s_e)\rangle = N_e \tag{26}$$

where N_e is the electron density in the medium.
Thus the effective Hamiltonian due to CC interaction takes the form,

$$H_{CC} = \sqrt{2}G_F N_e \bar{\nu_{eL}}\gamma_0\nu_{eL} \tag{27}$$

where $\nu_{eL} = \nu_e(1-\gamma^5)$ & the effective potential for CC interaction is $V_{CC} = \sqrt{2}G_F N_e$
Now,the effective Hamiltonian for the NC interactions is given by,

$$H_{NC} = \frac{G_F}{\sqrt{2}} \Sigma_{x=e,\mu,\tau}[\bar{\nu_x}\gamma^\mu(1-\gamma^5)\nu_x]\Sigma_{f=e,p,n}[\bar{f}\gamma_\mu(g_V^f - g_A^f\gamma^5)f] \tag{28}$$

where,

$$(g_V^f - g_A^f\gamma^5) = g_V^f(1 - \frac{g_A^f}{g_V^f}\gamma^5) \simeq g_V^f(1-\gamma^5)$$

Thus,we see that the expression is same as that for the Hamiltonian for the CC scattering. Obviously,the effective potential due to NC interaction will be of the form,

$$V_{NC}^f = \sqrt{2}G_F N_f g_V^f \tag{29}$$

where,

$$g_v^e = -\frac{1}{2} + 2\sin^2\theta_w$$

$$g_v^p = \frac{1}{2} - 2\sin^2\theta_w$$

$$g_V^n = -\frac{1}{2}$$

Putting these values and considering the medium to be uniform i.e the density of electrons is equal to the density of protons. We get,

$$V_{NC} = \sqrt{2}G_F N_n g_V^n = -\frac{1}{2}G_F N_n \tag{30}$$

4.2 Evolution of Neutrino Flavors

We know that the mass eigen states & the flavor eigen states of the neutrino are related by a mixing matrix and is of the form,

$$|\nu_\alpha\rangle = \Sigma U_{\alpha 1}^* \nu_1$$

The energy equation is given by.

$$H_o|\nu_k\rangle = E_k|\nu_k\rangle \tag{31}$$

where,

$$E_k\sqrt{\vec{p}^2 + m_k^2}$$

But due to the presence of the medium,the Hamiltonian becomes modified & takes the form,$H = H_o + H_1$ where,

$$H_1|\nu_\alpha\rangle = V_\alpha|\nu_\alpha\rangle \tag{32}$$

Thus,after a certain a certain time,say t, a neutrino of flavor ν_α changes to another flavor, ν_β and the evolution equation is given by,

$$i\frac{d}{dt}|\nu_\alpha(t)\rangle = H|\nu_\alpha(t)\rangle \tag{33}$$

For simplicity,let us consider the two flavor case. Let the mass eigen states for $\nu_e \& \nu_\mu$ are $\nu_1 \& \nu_2$ respectively.In the mass-eigen basis the evolution equation can be written in the form,

$$i\frac{d}{dt}\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = H^{'}\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} \tag{34}$$

Similarly,in the flavor basis,the evolution equation is of the form,

$$i\frac{d}{dt}\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = H\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} \tag{35}$$

where,

$$H^{'}|\nu_1\rangle = E + \frac{m_1^2}{2E} + \sqrt{2}G_F N_e - \frac{1}{2}G_F N_n$$

&

$$H^{'}|\nu_2\rangle = E + \frac{m_2^2}{2E} - \frac{1}{2}G_F N_n$$

This is due to the fact that an electron type neutrino undergoes both CC & NC type of interaction while muon type undergoes only NC interaction. We know that the flavor basis and the mass basis are related by a mixing matrix U. So,the Hamiltonian between these two basis are also related by U,i.e

$$H = UH^{'}U^\dagger \tag{36}$$

$$H = E - \frac{1}{\sqrt{2}}G_F N_n + \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \frac{m_1^2}{2E} + \sqrt{2}G_F N_e & 0 \\ 0 & \frac{m_2^2}{2E} \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{37}$$

$$= E - \frac{1}{\sqrt{2}}G_F N_n + \begin{pmatrix} \sqrt{2}G_F N_e & 0 \\ 0 & 0 \end{pmatrix} + \frac{m_1^2}{4E}\begin{pmatrix} 1+\cos 2\theta & 0 \\ 0 & 1-\cos 2\theta \end{pmatrix} + \frac{m_2^2}{4E}\begin{pmatrix} 1-\cos 2\theta & 0 \\ 0 & 1+\cos 2\theta \end{pmatrix}$$

$$= E + \frac{m_1^2 + m_2^2}{4E} - \frac{1}{\sqrt{2}}G_F N_n + \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E_2}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}$$

Now,we have to find the eigen values of the Hamiltonian H to study mixing in matter. Before that for our convenience we choose$A = 2\sqrt{2}G_F N_e E$.Thus,to find the eigen value we have to solve the characteristic equation,

$$|H - \lambda I| = 0$$

$$\lambda = E - \frac{1}{\sqrt{2}} G_F N_n + \frac{1}{4E}[(m_1^2 + m_2^2 + A) \pm \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + \Delta m^2 \sin 2\theta^2}] \tag{38}$$

The effective mixing angle in matter is given by,

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A} \tag{39}$$

Thus the mixing angle in matter is maximum when, $\theta_M = \frac{\pi}{4}$,i.e the two neutrino flavor have maximum mixing. Thus,this is the point of **Resonance**. When $\theta_M = \frac{\pi}{4}$,then $\tan 2\theta_M \to \infty$. This leads to the following condition,

$$A = \Delta m^2 \cos 2\theta$$

. From here,we get the density of electron to be,

$$N_e = \frac{\Delta m^2 \cos 2\theta}{\sqrt{2} G_F E}$$

All these calculations are done when the medium is considered to be uniform. But when the medium is not uniform another phenomena occurs.

4.3 Evolution of Neutrino in a Non-Uniform medium

We know that neutrino oscillation in a medium depends on the density of the medium. Thus,if the density of the medium is non-uniform it can have peculiar effect..The phenomenon of neutrino oscillation in a medium of varying density is known as the **MSW Effect**. When A=0,from equation(39),we see that the mixing angle corresponds to that in vacuum. Thus,the lighter mass eigen state ν_1 is pure ν_e,and the heavier eigenstate,ν_2 is almost pure ν_μ. When,the density of the medium becomes high,i.e N_e is high,then $A \gg \Delta m^2 \cos 2\theta$,so the matter mixing angle $\theta_M \to \frac{\pi}{2}$ i.e the lighter eigen state becomes ν_μ.Thus,for a neutrino beam propagating through a matter of varying density the dominant flavor component of a particular mass eigen state changes while passing through the region with $A = A_{resonance}$. This phenomenon is known as the **Level Crossing** In case of matter with non-uniform density,The flavor eigen state are related to the mass eigen state as,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{pmatrix} \begin{pmatrix} \tilde{\nu_1} \\ \tilde{\nu_2} \end{pmatrix}$$

The evolution of the matter eigen state components is of the form;

$$i\frac{d}{dx} \begin{pmatrix} \tilde{\nu_1} \\ \tilde{\nu_2} \end{pmatrix} = \begin{pmatrix} \frac{\tilde{m_1^2}}{2E} & i\frac{d\theta_M}{dx} \\ -i\frac{d\theta_M}{dx} & \frac{m_2^2}{2E} \end{pmatrix} \begin{pmatrix} \tilde{\nu_1} \\ \tilde{\nu_2} \end{pmatrix} \tag{40}$$

where,

$$H_M = \begin{pmatrix} \frac{\tilde{m_1^2}}{2E} & i\frac{d\theta_M}{dx} \\ -i\frac{d\theta_M}{dx} & \frac{m_2^2}{2E} \end{pmatrix}$$

is a time dependent matrix. The equation (40) cannot be solved by just taking a trial solution. In this case we have to take on approximate solutions. If we take,

$$\gamma = |\frac{\Delta\tilde{m}^2}{2E\frac{d\theta_M}{dx}}| \gg 1 \tag{41}$$

then the mass eigen states in matter $\tilde{\nu_1}$ & $\tilde{\nu_2}$ propagates unchanged with the proportion of ν_e&ν_μ changing adiabatically according to the density of electron at that particular point. Adiabatic process occurs only when the matter density is varying slowly. γ is the adiabaticity parameter. The adiabatic evolution of the matter eigen state components are given as,

$$\tilde{\nu_1}(t) = \tilde{\nu_1}(0)$$

&

$$\tilde{\nu_2}(t) = \tilde{\nu_2}(t)\exp[i\Gamma t]$$

where,

$$\Gamma(t) = \int_0^t \frac{\Delta\tilde{m}^2}{2E} dt$$

is the phase factor.If the initial state is given by $|\nu_e\rangle$ then, $\tilde{\nu_1}(0) = \cos\theta_M$ & $\tilde{\nu_2}(0) = \sin\theta_M$. Thus the transition probability is given by,

$$P_{ex} = |\langle\nu_x|\nu_e(t)\rangle|^2 = \frac{1}{2}[1 + \cos 2\theta_M \cos 2\theta + \sin 2\theta_M \sin\theta \cos\Gamma(t)]$$

[7]

The adiabatic propagation of neutrinos has many applications,and the most important of them is that it is used in the treatment of the neutrino oscillation in sun.

5 SOLAR NEUTRINOS

The nuclear reaction taking place in the core of the sun is one of the major source of neutrinos ands these neutrinos are known as solar neutrinos.The main nuclear fusion reaction which shows the production of neutrinos is-

$$4p \rightarrow \alpha + 2\nu_e + 2e^+ + 28Mev. \tag{42}$$

The fact is that different nuclear reaction produces neutrinos having a variety of energies.Knowing the cross-section of the neutrino-electron scattering we can have and idea about the mean free path of a neutrino [generally $\sigma \simeq 10^{-43}cm^2$] which is of the order of 10^{17} cm.Observations shows that the approximate neutrino flux coming to earth is, $6 \times 10^{10}cm^{-2}s^{-1}$,which gives us the opportunity to study solar physics. Most of the neutrinos produced in a nuclear reaction are found to be in pp-I chain.The chain goes as[3]-

$$p + p \rightarrow d + e^+ + \nu_e(0.42Mev) \tag{43}$$

$$p + e^- + p \rightarrow d + \nu_e(1.44Mev) \tag{44}$$

$$d + p \rightarrow \gamma + He^3 \tag{45}$$

$$He^3 + He^3 \rightarrow He^4 + p + p \tag{46}$$

There are many other chains which produces neutrinos-
1) Hep chain- contributes a fraction of 10^{-7} of the neutrino flux.
2)pp-II chain-contributes 14% neutrino flux.
3)pp-III chain-contributes a fraction of 10^{-5} of the neutrino flux.

CNO cycle also contributes to the flux of neutrinos but this cycle is predominant only at a very high temperature$\simeq 10^7K$. The study of solar neutrinos began with the experiments performed by Davis and his collaborators using radiochemical method. Since then several solar neutrino experiments were performed. Like Bahcall & group started this experiment around mid 1960s and with the ongoing time they improved their calculations and by 1988 they arrived at a capture rate $7.9 \pm 0.9SNU$. In the mean time Saclay group calculated it to be $5.8 \pm 1.3SNU$. Later,it was noticed that Saclay group used a low cross section reaction,leading to over-correction. If those factors were taken into account the results of both the group were found to be same. In this way different experiments were carried out and in every experiment the main conclusion was found to be same-the detected neutrino fluxes were lower than the prediction of the standard solar model. This came to be known as **Solar Neutrino Puzzle**.

Physicists thought that this puzzle may indicate some unknown properties of the sun since in calculating neutrino flux several assumptions were taken into account.In spite of these assumptions & arguments it was regarded that the modification of the solar model could not solve the puzzle. So,one need to consider some new properties of neutrinos which are beyond the standard model to explain this solar ν puzzle.

The explanation of the puzzle was at last concluded from the experimental results of **SNO(Sudbury Neutrino Observatory)**. They published results from different channels- charged current (**CC**), electron scattering (**ES**) & neutral current (**NC**). It was observed that the flux observed at CC detection was lower than that of ES. This difference indicates that the ν flux contains a $\nu_\mu \tau$ component also.The neutral current results also detects all ν flavors with same efficiency.So one can conclude that the flavors are not constant while propagating. This phenomenon was nothing but **neutrino oscillation** which was realized by Pontecorvo & Gribov long time back.Since then various experiments were performed to figure out the solar neutrinos parameters and many of them are successful too.

6 ATMOSPHERIC NEUTRINOS

The interaction of the primary cosmic radiation(composed of protons,alpha particles & massive nuclei) in the earth's atmosphere results in the formation of *secondary* cosmic*radiation*. The secondary cosmic radiation produced is mostly composed of mesons and a little amount of electron & positrons.The mesons while propagating in the atmosphere,decay to produce mu and muon-type neutrinos.

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_\mu(\bar{\nu_\mu}) \tag{47}$$

At low energies the muons always decay before reaching the earth,

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_e(\bar{\nu_e}) + \bar{\nu_\mu}(\nu_\mu) \tag{48}$$

On the other hand,at higher energies(about 5 Gev and higher) the μ^- reach the earth before decaying and on interaction lose their energy.
Though the detection of atmospheric neutrinos was first done in 1960's by Reines but modern techniques were not applied until 1980's.In the beginning of 1980's,two different detection techniques were employed:
1) Water Cherenkov detector.
2) Iron calorimeter.
Both of the detection techniques were able to measure the scattering angle of the scattered charged lepton and energy distribution.After that many projects were developed for the study of atmospheric neutrinos,like,Soudan2(which employed iron calorimeter detector),MACROS & Kamiokande.The evidence of the atmospheric neutrino anomaly was first observed by IMB & Kamiokande collaboration. In the recent years,the evidence of the atmospheric neutrino anomaly became stronger with high precision data from the Super-Kamiokande experiment.In 1998,at Neutrino98 conference, they talked about the evidence of ν_μ oscillation on the basis of the angular distribution for their event sample data. The main objective of the Super-Kamiokande experiment was to observe the e^- & μ^- like events for different zenith angle which corresponds different path length of the neutrinos. However,the following things were observed-

1)The distribution of the electron neutrinos are observed to be same as that obtained from Monte Carlo simulations but their was a deficit in case of ν_μ.

2)When $\cos\theta$ is larger,μ like events occur very less i.e as the distance from the production point to the detection point increases the deficit also increases.Moreover,when the event is of higher energy the effect is observed to be more prominent.The reason of this observation is that the direction of the charged lepton is more aligned with the direction of the neutrino.

3)The muons that are produced along with the muon type neutrino are also observed to be lesser after propagating ,which indicates that neutrinos having larger energy has less probability to disappear.

All the effects that are mentioned above are also confirmed by other experiments which lead to the conclusion that it was not the fault of the water detector,it was the neutrino oscillation phenomenon which was responsible for these.

As I have discussed earlier, we find that 2 $(\nu_\mu + \bar{\nu_\mu})$ and 1 $(\nu_e + \bar{\nu_e})$ are produced for every charged-pion decay. Since the energies of these neutrinos are almost equal, we can see that the flux ratio of $(\nu_\mu + \bar{\nu_\mu})$ and $(\nu_e + \bar{\nu_e})$ should be approximately 2. This flux ratio acts as a signal for neutrino oscillations, since this ratio should deviate from the predicted ratio if neutrinos oscillate.
An important feature of the atmospheric neutrino flux is the up-down symmetry. The neutrinos which are produced in meson decay enters the Earth at a point with a certain zenith angle should exit the Earth at a point with a zenith angle,and the two angles are related by 180 degree. Since the cosmic ray enters into the atmosphere with approximately equal rate in every position in the Earth,the two processes occur with an equal rate. Thus one can conclude that the flux is up-down symmetric. But it was observed from the Super-Kamiokande data that an up-down asymmetry is present and it is given by,[2]

$$A_\mu = \frac{U - D}{U + D} = -0.29 \pm 0.03 \tag{49}$$

where,U(D) are the multi-Gev μ^- like events with the zenith angle,$-1 < \cos\theta < -0.2$ ($1 < \cos\theta < 0.2$).This is the thing which was discussed in 2nd point. We know,that the PMNS matrix can be parameterized in terms of three mixing angles θ_{12},θ_{23}& θ_{13} and a CP violating phase δ_{CP}. Moreover,if we consider neutrinos to be Majorana particles,two Majorana phases will also be present.There are other parameters,for e.g the mass-squared difference term Δm^2_{12}, Δm^2_{23} Δm^2_{31} on which the neutrino oscillation depends. The atmospheric neutrino oscillation parameters θ_{23} & Δm^2_{23} are constrained by the Super-kamiokande,MINOS & T2K data.The super-Kamiokande series of experiments provides us the best fit values of these parameters to be,$\Delta m^2_{23} = 2.30 \times 10^{-3}$ & $\sin^2 2\theta_{23} = 0.99$. Many experiments are still going on to study aspects of the atmospheric neutrino oscillation phenomenon.

7 REACTOR NEUTRINOS

Fission reactors are rich source of neutrinos or better to say anti-neutrinos.We know that during fission of 235^U,neutron rich fragments are produced,these undergoes β-decay to produce anti-neutrinos. A fission reaction releases about 200 Mev of energy,of which 5% of the energy is radiated away as the anti neutrinos.We know that the number of decays is related to the thermal power of the reactor,so if we know the energy of the $\bar{\nu}$ emitted in the decay,the reactor $\bar{\nu}$ flux can be calculated with accuracy. Now,to study the neutrino oscillation phenomena of the reactor neutrinos,we should have a knowledge of the initial flux and the energy spectrum of the anti-neutrinos(and this depends on the nuclear composition of the fuel).The kamLAND experiments have rigorously studied the phenomenon of reactor neutrino oscillation and have also able to get the parameters involved.[16]
In a scintillator experiment,inverse beta decay occurs as the anti-neutrinos are captured by the proton,

$$p + \bar{\nu_e} \rightarrow n + e^+ \tag{50}$$

The positron produced in the capture process has a kinetic energy of about 1.8 MeV which is less than the energy of the anti-neutrino. This positron annihilates a electron , releasing a total energy of about 0.78 MeV (this is detected in the detector used). The neutron captures a few hundred microseconds later, providing a coincidence signal.This delayed coincidence signal acts as a tool for distinguishing anti-neutrinos from backgrounds produced by other particles. The relationship between the positron energy and the energy of the anti-neutrino makes measuring anti-neutrino spectra relatively easy. While passing through the earth, anti-neutrinos undergo oscillations,that is some fraction of the electron anti-neutrinos that are produced in the nuclear reactors change to muon or tau anti-neutrinos with time.The spectrum of anti-neutrino that is obtained from the scintillator experiments is consistent with neutrino oscillation and a fit provides the values of the parameters. The most precise and accurate oscillation parameters are obtained by combining the results from solar experiments and KamLAND which gives $\Delta m^2 = (7.9 + 0.6) \times 10^5 eV^2$ and $\tan 2\theta = 0.40 + 0.10$. Another famous experiment using reactor anti-neutrinos is the **Daya Bay Reactor Neutrino Experiment**. This experiment provides us the value of the mixing angle θ_{13}. On March 2012,they announced that $\theta_{13} \neq 0$ and, $\sin^2 2\theta_{13} = 0.092 \pm 0.016$. Thus,with this all the parameters of PMNS matrix are found except the CP violating phases.

8 SUPERNOVA NEUTRINOS

Stars keep on evolving to get their energy burning hydrogen to helium and so on until iron is formed in the core. As the iron core grows, the mass of the core exceeds the **Chandrashekhar Limit** and at that moment the electron Fermi pressure could not balance the gravitational pull and the core keeps on becoming smaller and denser. Since the binding energy of iron is very high it will not lead to the formation on any new nucleus on further burning rather it will disintegrate absorbing an energy of about 124.4 Mev. This increases the rate of collapse and also increases the temperature to such an extent that the electrons get absorbed by protons leading to the formation of neutrinos. After this collapse the core mass becomes very high forming a neutron star as along with ν neutrons are also formed.
During this process of collapse,the core releases extra energy and the most of the energy are carried by these neutrinos.These neutrinos are known are supernova neutrinos and are of immense importance for astrophysical studies.In the following section I will discuss about one of the implication of these type of neutrinos on astrophysical objects.

9 NEUTRON STAR

9.1 Kicks In Neutron Star

We know that the rotating neutron stars are known as Pulsars and these stellar objects rotates with a velocity of more than hundreds of kilometers per sec. It is observed that along with the proper motion the pulsar may receive a "kick" velocity during its birth. The origin of this kick velocity is not yet experimentally confirmed. However, two theories are proposed which can explain the origin of this kick velocities[9].These are-

1) Due to the asymmetries in the supernova explosion where they are born.
2)Due to asymmetries in the neutrino emission due to neutrino oscillation.

In this thesis I have suggested a methodology to explain the origin of this kick velocity using the second mechanism.We know that stars keep on evolving until the core of the star becomes a stable one. Due to the formation of a stable core,instead of the formation of a new nucleus the core element disintegrate absorbing a certain amount of energy. This process increases the rate of the collapse of the core,also the temperature to such extent that electrons get absorbed by protons resulting in the formation of neutrinos.In this process of collapse when the mass of the core is of the order of 1.4-2 solar masses neutron star is formed and the rest of the mass of the star is ejected in the form of supernova releasing the extra energy in the form of neutrinos. In a dense neutron star ν_e has a shorter mean free path compared to other types of neutrinos since ν_e undergoes both charged current and neutral current interactions due to the presence of electrons in the medium while the other form of neutrinos undergoes only neutral current interaction due to the absence of μ^- & τ^- in that medium.Suppose ν_τ undergoes a resonant oscillation into ν_e above the τ neutrinosphere but below the electron neutrinosphere then it will be absorbed by the medium and the ν_τ- sphere will be determined by the point of resonance,resulting in an effective ν_τ sphere. The effective sphere that is formed is not actually a sphere rather it has the shape of an ellipse,since the point of resonance is determined by the magnetic field present in the neutron star and also on the relative orientation of $\vec{B}$ & $\vec{k}$.[10] This dependence is due to the fact that in a magnetized medium the Dirac equation gets modified with the introduction of the ν self-energy term which is a function of temperature as well as the magnetic field. Thus the ν_τ that is emitted in the direction of $\vec{B}$ has a different temperature to that of those which are emitted in the opposite direction of $\vec{B}$. Not only temperature,they also carry out different momenta in different direction resulting in the **asymmetry**.

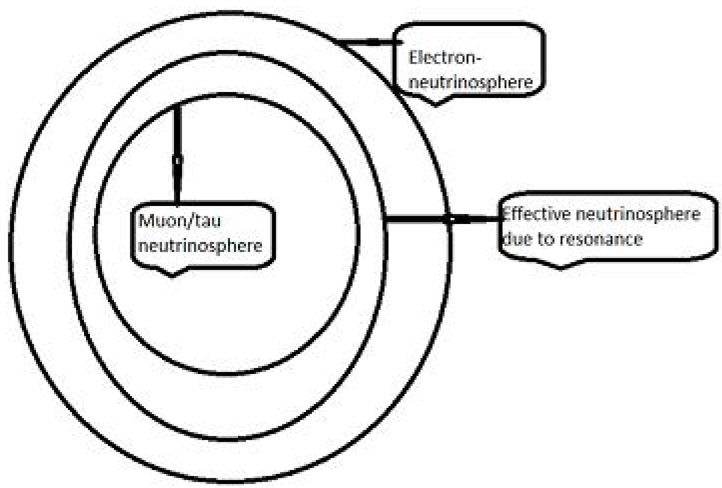

Figure 5: Effective neutrinosphere due to resonant transformation of active neutrinos in magnetised medium

9.2 Calculation Of Asymmetry

The Pulsars having a mass of $1.0 - 1.5\ M_\odot$ have a velocity of the order of $450 \pm 90\ km - s^{-1}$,thus the corresponding momentum is of the order of $1.2{\times}10^{41}\ g - cm - s^{-1}$. Again the energy carried by the ν in the supernova explosion is of the order of 3×10^{53} erg. Assuming that $p_\nu = E_\nu$,we get the corresponding neutrino momentum to be $10^{43}\ g - cm - s^{-1}$. Comparing both the momentum we can observe that a few % assymetry in the distribution of the emitted ν would be responsible for the kick.
An important fact is that,inside the ν sphere the neutrinos are opaque while it is transparent outside it. .Moreover, we know that the mean free path of ν_τ & ν_e has a difference in their magnitude near the ν_e sphere where the density is nearly 10^{11} - 10^{12} gcm^{-3}. For this reason the ν_τ escape from the interior of the star where the temperature is higher & the mean energy of them is greater than that of the ν_e. Now, I will apply the ν- oscillation formulation considering two-flavour for simplicity with $\Delta m^2 = m^2(\nu_\tau) - m^2(\nu_e) \simeq 0$, considering small mixing. To find the momentum asymmetry first we have to calculate the condition for resonance.Let us begin with that,
We know that the flavour eigen states of neutrinos are related to the mass eigenstates via a matrix,known as PMNS matrix i.e $\nu_\alpha = \Sigma U_{\alpha 1} \nu_{\ i}$. Since we are considering two-flavour oscillation the mixing matrix is of the

form,U= $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \sin\theta \end{pmatrix}$,which is almost similar in all cases. For a magnetized medium the Hamiltonian of the system is given by

$$H = k + \frac{m^2}{2k} - b_L + c_L \frac{\vec{k}.\vec{B}}{k} \tag{51}$$

The expression of the Hamiltonian is given considering the ultra-relativistic approximation and in the mass eigen-basis the 2nd term is given by

$$\frac{m^2}{2k} = \begin{pmatrix} \frac{m_1^2}{2k} & 0 \\ 0 & \frac{m_2^2}{2k} \end{pmatrix} \tag{52}$$

The expression of b_L & c_L are taken from literature[11],since at this moment it is not possible to explain these terms. They are,

$$b_L = b_L^W \begin{pmatrix} \cos^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin^2\theta \end{pmatrix} + b_L^Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{53}$$

$$c_L = c_L^W \begin{pmatrix} \cos^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin^2\theta \end{pmatrix} + c_L^Z \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{54}$$

Putting these values in the equation (53) & (54) we get the complete expression of the Hamiltonian. To find the matter eigenstates and the condition of resonance from the effective mixing angle first of all we have to find the eigenvalues. For simplicity I am considering

$$a = \frac{m_1^2}{2k} - b_L^W \cos^2\theta - b_L^Z + c_L^W \cos^2\theta \frac{\vec{k}.\vec{B}}{k} + c_L^Z \frac{\vec{k}.\vec{B}}{k} \tag{55}$$

$$b = \frac{m_2^2}{2k} - b_L^W \sin^2\theta - b_L^Z + c_L^W \sin^2\theta \frac{\vec{k}.\vec{B}}{k} + c_L^Z \frac{\vec{k}.\vec{B}}{k} \tag{56}$$

$$c = b_L^W \sin\theta\cos\theta - c_L^W \sin\theta\cos\theta \frac{\vec{k}.\vec{B}}{k} \tag{57}$$

Now solving the characteristic equation

$$\lambda^2 - \lambda(a+b) + (ab - c^2) = 0 \tag{58}$$

Solving this equation we get

$$\lambda = \frac{(a+b)}{2} \pm \sqrt{a^2 + b^2 - 2ab + 4c^2} \tag{59}$$

Putting the values of a,b & c ultimately we get the final result of the form

$$\lambda = \frac{m_1^2 + m_2^2}{2k.2} - \frac{(b_L^W - c_L^W \frac{\vec{k}.\vec{B}}{k})}{2} - \frac{(b_L^Z - c_L^Z \frac{\vec{k}.\vec{B}}{k})}{2} \pm \frac{\sqrt{(\frac{\Delta m^2}{2k}\cos 2\theta + b_L^W - c_L^W \frac{\vec{k}.\vec{B}}{k})^2 + (\frac{\Delta m^2}{2k}\sin 2\theta)^2}}{2} \tag{60}$$

Thus the effective eigenvalue is

$$\omega = k + \frac{m_1^2 + m_2^2}{2k.2} - \frac{(b_L^W - c_L^W \frac{\vec{k}.\vec{B}}{k})}{2} - \frac{(b_L^Z - c_L^Z \frac{\vec{k}.\vec{B}}{k})}{2} \pm \frac{\sqrt{(\frac{\Delta m^2}{2k}\cos 2\theta + b_L^W - c_L^W \frac{\vec{k}.\vec{B}}{k})^2 + (\frac{\Delta m^2}{2k}\sin 2\theta)^2}}{2}$$

With the eigenvalues we can easily determine the eigenstates with an mixing angle given by

$$\sin 2\theta_m = \frac{\frac{\Delta m^2}{2k}\sin 2\theta}{\sqrt{(\frac{\Delta m^2}{2k}\cos 2\theta + b_L^W - c_L^W \frac{\vec{k}.\vec{B}}{k})^2 + (\frac{\Delta m^2}{2k}\sin 2\theta)^2}} \tag{61}$$

From the above equation it is clear that in presence of a magnetic field the resonant condition will occur if the following relation is true

$$\frac{\Delta m^2}{2k}\cos 2\theta = -b_L^W + c_L^W \frac{\vec{k}.\vec{B}}{k} \tag{62}$$

For a system of degenerate gas,we can obtain the values of b_L^W & c_L^W from the literature,leading to the condition of the form,

$$\frac{\Delta m^2}{2k}\cos 2\theta = \sqrt{2}G_F N_e + \frac{eG_F}{\sqrt{2}}(\frac{3N_e}{\pi^4})^{\frac{1}{3}}\frac{\vec{k}.\vec{B}}{k} \tag{63}$$

Now,let us consider that in presence of the magnetic field the surface of resonance is given by,

$$r(\phi) = r_o + \delta\cos\phi \tag{64}$$

where r_o is the position of resonance in the absence of the magnetic field and $\pm\delta$ is the shift of the resonance co-ordinate for the neutrino emitted parallel/antiparallel to the magnetic field. We can find the shift in the co-ordinate δ from equation (62). Since the density of electron is the function of the distance.Differentiating equation (62) we get,

$$\sqrt{2}G_F\frac{dN_e}{dr} \simeq \frac{c_{eff}}{dr}B \simeq \frac{c_{eff}}{\delta}$$

where we can assume the shift in co-ordinate $\delta \simeq dr$. Now,putting the value of c_{eff} we get,

$$2\frac{dN_e}{dr}\delta \simeq e(\frac{3N_e}{\pi^4})^{\frac{1}{3}}B \tag{65}$$

Now,we are going to estimate the asymmetry and for that we can assume that approximately each species of neutrinos carries equal amount of energy.Thus,the total energy is distributed among six species. Also,assuming the object to be a black body whose luminosity is proportional to T^4,we get,

$$\frac{\Delta k}{k} \propto \frac{T^4(r_o-\delta) - T^4(r_o+\delta)}{T^4(r_o)} \tag{66}$$

Now,using the differential formula we get,

$$\frac{\Delta k}{k} \simeq -\delta\frac{T^4(r_o-\delta) - T^4(r_o)}{T^4(r_o)\delta} - \delta\frac{T^4(r_o+\delta) - T^4(r_o)}{T^4(r_o)\delta}$$

$$= -2\delta\frac{4T^3}{T^4}\frac{dT}{dr} \simeq -2\delta\frac{4}{T}\frac{dT}{dr}$$

$$\frac{\Delta k}{k} \simeq \frac{4}{3}\frac{1}{T}\frac{dT}{dr}R\delta$$

where R is the geometrical scattering factor which takes into account the fact that the resonance condition is not affected by the neutrinos emitted in the direction perpendicular to the magnetic field,and it is taken to be $R = \frac{1}{2}$. Now,using the chain rule $\frac{dT}{dr} = \frac{dT}{dN_e}\frac{dN_e}{dr}$ and then putting the value of δ we get the final expression to be,

$$\frac{\Delta k}{k} \simeq \frac{e}{3\pi^2}\eta\frac{dT}{dN_e}B$$

where $\eta = \frac{\mu_e}{T}$ is the degeneracy parameter. Now,we have to determine the term $\frac{dT}{dN_e}$,for that we have to differentiate

$$N_e = 2\int\frac{d^3p}{2\pi^3}\frac{1}{1+e^{-\frac{p-\mu}{T}}}$$

.

$$= 2\int\frac{4\pi p^2 dp}{(2\pi)^3(1+e^{\frac{p-\mu}{T}})}$$

In the relativistic limit we can consider $p \simeq E$. Now using the Sommerfeld expansion [15] we can obtain the expression.The calculation goes as,

$$N_e = \frac{2}{8\pi^3}\int_0^\infty\frac{4\pi E^2 dE}{1+e^{\frac{E-\mu}{T}}}$$

$$= \frac{1}{\pi^2}[\int_\mu^\infty\frac{E^2 dE}{1+e^{\frac{E-\mu}{T}}} + \int_0^\mu\frac{E^2 dE}{1+e^{\frac{E-\mu}{T}}}]$$

$$= \frac{1}{\pi^2}[\int_\mu^\infty\frac{E^2 dE}{1+e^{\frac{E-\mu}{T}}} + \int_0^\mu E^2 dE - \int_0^\mu E^2 dE(1-\frac{1}{1+e^{\frac{E-\mu}{T}}})]$$

$$= \frac{1}{\pi^2}[\int_\mu^\infty\frac{E^2 dE}{1+e^{\frac{E-\mu}{T}}} + \int_0^\mu E^2 dE - \int_0^\mu\frac{E^2 dE}{1+e^{-\frac{E-\mu}{T}}}]$$

Now,we assume,$x = \frac{E-\mu}{T}$ & $x' = -\frac{E-\mu}{T}$,then the expression becomes,

$$N_e = \frac{1}{\pi^2}[\int_0^\mu E^2 dE + \int_0^\infty\frac{(xT+\mu)^2 T dx}{1+e^x} - \int_{\frac{\mu}{T}}^0\frac{(\mu - Tx')^2 T d(-x')}{1+e^{x'}}]$$

Considering degeneracy parameter to be very large i.e $\eta = \frac{\mu}{T} \rightarrow \infty$,the last term can be ignored,so the expression takes the form,

$$N_e = \frac{1}{\pi^2}[\frac{\mu^3}{3} + 2T^2\mu\int_0^\infty\frac{x^2 dx}{1+e^x}]$$

Now,using the integral formula,

$$I(p) = \int_0^\infty\frac{x^{p-1}dx}{1+e^x} = (1-\frac{1}{2^{p-1}})\zeta(p)\Gamma(p)$$

We get,

$$N_e = \frac{1}{\pi^2}[\frac{\mu^3}{3} \pm 3.66\mu T^2] \quad (67)$$

where,the Riemann-zeta function $\zeta(3) = 1.202$ & the Euler-beta or Gamma function $\Gamma(3) = 2$

On differentiating N_e obtained with respect to temperature and putting the value of $\pi^2 = 9.86$,we get,

$$\frac{dN_e}{dT} \simeq \frac{2}{3}T^2\eta$$

. Thus,

$$\eta\frac{dT}{dN_e} \simeq \frac{1.50}{T^2}$$

. The fractional asymmetry in momentum is then,

$$\frac{\Delta k}{k} = \frac{1.50}{T^2}B \quad (68)$$

Generally,during the emission of neutrino their energy is of the order of 10 Mev which is equivalent to a temperature of 3 Mev. If the magnetic field is of the order of $B \simeq 3 \times 10^{14}G$,then the asymmetry is of the order of 0.018.

Thus,we can easily conclude that the change in shape of the neutrinosphere due to the neutrino oscillation in the magnetic field leads to the asymmetry in the neutrino-flux which is responsible for the kicks in the pulsar. Now,we have to reveal some facts regarding this phenomena.We have considered one assumption that the distribution of temperature is not affected by the position of the resonant. Moreover,we know that the magnetic fields at the surface of the neutron star is of the order of $10^{12} - 10^{13}$ G,which we have considered for our calculation. Actually,the magnetic field inside the pulsars are much higher,of the order of $\simeq 10^{16}G$. Such a strong magnetic field had a different implication on the neutron star. First of all it can lead to the formation of certain spots(similar to sun spots which is due to the presence of strong magnetic field),which causes the asymmetric emission of neutrinos. Secondly,in a strong magnetic field several weak processes becomes important which are responsible for production of neutrinos with a finite asymmetry[12]. Though the asymmetry in production does not cause asymmetry in emission but due to this weak processes,other particles are formed with a small scattering cross-section,which escapes from the neutron star with an asymmetry equal to the production asymmetry leading to a recoil in the neutron star.This recoil of the neutron star leads to the additional velocity known as the kick velocity.

9.3 *Is it possible to explain pulsar kick with active neutrinos alone?*

The phenomenon of the oscillation of active neutrinos only could explain the formation of pulsar kicks but that would require one of the neutrino mass to be of the order of $\simeq 10^2 eV$ which is beyond the present limits available from the experimental data. To get out of this,the possible remedy one can think is the resonant transformation of active-sterile neutrino in the neutron star.Considering this, we observe the active neutrinos to be trapped within the neutrinosphere and the sterile neutrinos escape in different directions depending on the temperature gradient and the magnetic field in a asymmetric way.The concept will remain same as discussed previously,the only things that will change is the resonant condition. The resonant conversion of the active to sterile neutrino depends on the mass and the mixing angle [13]as before,but since we are considering that $\nu_{\mu,\tau} \rightarrow \nu_s$ transformation takes place so the resonant condition takes the form,

$$\frac{\Delta m^2}{2k}\cos 2\theta = \frac{1}{\sqrt{2}}G_F N_n + \frac{eG_F}{\sqrt{2}}(\frac{3N_e}{\pi^4})^{\frac{1}{3}}\frac{\vec{k}.\vec{B}}{k} \quad (69)$$

As we can notice that their is a change in the 1st term,this is due to the fact that $\nu_{\mu,\tau}$ undergoes only neutral current interaction so the term b_L^Z will come instead of the term b_L^W in the resonant condition and the term c_L^W has almost same expression as c_L^Z differing only by a numerical factor. The term b_L is nothing but the effective potential and for sterile neutrinos this effective potential term is zero.As before,we can consider the surface of resonance in presence of magnetic field.But the shift of co-ordinate in this case will be different since we have different neutrinosphere for the sterile neutrinos. Considering the density of neutral particles as the function of distance,we obtain,

$$\frac{dN_n}{dr}\delta \simeq e(\frac{3N_e}{\pi^4})^{\frac{1}{3}}B \tag{70}$$

Now,we can consider that the core of neutron star having density of the order of $\simeq 10^{14}gm/cm^3$ emit a black-body radiation luminosity in sterile neutrinos. So,the fractional asymmetry in momentum is of the same form

$$\frac{\Delta k_s}{k_s} \simeq \frac{1}{3}\frac{T^4(r_o-\delta)-T^4(r_o+\delta)}{T^4(r_o)} \simeq \frac{4}{3}\frac{1}{T}\frac{dT}{dr}2\delta \tag{71}$$

Here the factor of one-third is the geometrical factor which I have discussed before and the value is taken from the literature.Now,using the chain rule

$$\frac{dT}{dr} = \frac{dT}{dN_n}\frac{dN_n}{dr}$$

and then putting the value in the l expression,we get,

$$\frac{\Delta k_s}{k_s} \simeq \frac{2e}{3\pi^2}\frac{\mu_e}{T}\frac{dT}{dN_n}B$$

where,$\mu_e = (3\pi^2 N_e)^{\frac{1}{3}}$ is the chemical potential of the degenerate relativistic electron gas.
Assuming thermal equilibrium,the relation between the density of neutron gas with temperature is,

$$N_n = \frac{(m_n T)^{\frac{3}{2}}}{2\sqrt{2}\pi^2}\int \frac{\sqrt{z}dz}{e^{z-\frac{\mu_n}{T}}+1} \tag{72}$$

where

$$z = \beta\epsilon$$

. On computing this expression using Sommerfeld expansion[15] we get

$$N_n \simeq N_n(T=0) + \frac{m_n{}^{\frac{3}{2}}}{2\sqrt{2}\pi^2}\frac{\pi^2}{6}T^2\frac{1}{2}\mu_n{}^{-\frac{1}{2}}$$

. This on differentiating wrt temperature and then putting in the fractional asymmetry term we get finally,

$$\frac{\Delta k_s}{k_s} = \frac{8e\sqrt{2}}{\pi^2}\frac{\mu_e \mu_n{}^{\frac{1}{2}}}{m_n{}^{\frac{3}{2}}T^2}B \tag{73}$$

We have assumed that only one neutrino undergoes oscillation to transform into a sterile neutrino but in reality the total energy is shared among 3 neutrinos & 3 anti-neutrinos.So,the net fractional asymmetry is[14],

$$\frac{\Delta k_s}{k_s} = \frac{4e\sqrt{2}}{3\pi^2}\frac{\mu_e \mu_n{}^{\frac{1}{2}}}{m_n{}^{\frac{3}{2}}T^2}B \tag{74}$$

Now,considering $B \simeq 10^{16}G$,$T = 20Mev$,& the chemical potentials of the order of 100 Mev we observe that,

$$\frac{\Delta k_s}{k_s} = 0.01(\frac{\mu_e}{100Mev})^2(\frac{\mu_n}{80Mev})^{\frac{1}{2}}(\frac{B}{10^{16}G})^2(\frac{T}{20Mev})^2$$

. Thus,the asymmetry in momentum observed, can be responsible for the pulsar kicks. The treatment for resonant conversion of active to sterile neutrino is same as that of active-active transformation,only the conditions are a bit different. One thing we have to keep in mind is that we can consider blackbody radiation luminosity of sterile neutrinos only if the resonant transformation is occuring at the core and is of very high density.It is also possible that the MSW resonance takes place out of the core where density is $\simeq 10^{11} g/cm^3$,but it that case the method of calculation of the fractional asymmetry will be different. In this case the neutrinos pass between $r_- = r_o - \delta \cos\phi$ and $r_+ = r_o + \delta \cos\phi$,with one side as sterile neutrino and the other side as the active neutrinos.Due to the interaction of the active neutrinos,they obtain some extra momentum on the side where they are passing between r_- & r_+. This asymmetric momentum found in the outer layers of the core can be responsible for pulsar kicks.

10 CONCLUSION

Resonant production of the neutrinos is the main reason behind the pulsar kick.Neutrino oscillations in presence of the magnetic fields alters the shape of the neutrinosphere at the position of its resonance. This position of resonance depend on the direction of the magnetic field as well as the direction of the neutrino momentum. Thus,the neutrinos oscillating across the neutrinosphere escape from different directions,depending on their densities. The presence of the temperature gradient leads to the variation of the average neutrino energy with the density as well as the depth,so the momentum distribution of the neutrinos becomes anisotropic in nature. The fractional asymmetry of momentum in the weak field limit is calculated to be about 1%. In this report I have shown how active-active neutrino transformation can be responsible for the pulsar kick mechanism.But in reality such large neutrino masses for active neutrino is not possible,as a result of which we can consider resonant transformation of active-sterile neutrino.Considering this we can easily explain how the fractional asymmetry in momentum is coming into the picture in the strong field limit with the mass of the sterile neutrinos in keV range.

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