Relic Abundance of Inert Fermion Doublet Dark Matter

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A Thesis Submitted to

Indian Institute of Technology Hyderabad

In Partial Fulfillment of the Requirements for

The Degree of Master of Science

Department of Physics

April 2013

Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

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Acknowledgements

First and foremost I offer my sincerest gratitude to my supervisor, Dr Narendra Sahu , who has supported me throughout my thesis with his patience and knowledge whilst allowing me the room to work in my own way. I attribute the level of my Masters degree to his encouragement and effort and without him this thesis, too, would not have been completed or written. One simply could not wish for a better, friendlier and cooler supervisor.

Furthermore I would like to thank Dr. Anjan Giri for introducing me to the basics of Particle Physics and Field Theory, as well for the support on the way. Thanks to the small group of my gritty , cheerful classmates who always made life simpler , happier and better during past two years.

Dedication

To My Supervisor

Dr. Narendra Sahu

Abstract

The nature of dark matter (DM) and the mechanism that provides its measured relic abundance are currently unknown and demand physics beyond the Standard Model(SM) of particle physics. In this thesis after giving a brief introduction to the theory of DM we consider a particle physics model in which a vector-like fermion doublet and a triplet scalar are added to the SM spectrum. We also impose Z_2 symmetry under which the vector-like fermion doublet is odd, while all other fields are even. As a result the resulting model explains the relic abundance of DM and neutrino masses simultaneously. The mass of DM particle is calculated by taking into account of observed relic abundance of DM. From the observed DM abundance we also obtain a correlation between DM and scalar triplet mass in a certain parameter space.

Contents

Chapter 1

Introduction

With recent discovery of higgs-like particle in CMS-ATLAS experiment [1], the Standard Model(SM) description of particle physics seems to be complete. But still it fails to explain certain major problems like neutrino oscillations, CP violation, baryogenesis and so on. One of the major problems is it is inadequate to explain the existence of dark matter(DM) and dark energy in our universe. The recent experimental evidence such as WMAP(Wilkinson Microwave Anisotropy Probe), revealed that the visible matter comprises only a fraction (4.56 ± 0.15%)[2] of universe's total energy and rest of the total energy constitute DM and dark energy. From these data we also know that the DM constituent in the present universe is 22.8 ± 1.3%[2] and rest 72.1 ± 1.3%[2] is dark energy.

There are various experimental evidences of DM in our universe, like flatness of galaxy rotation curves,gravitational lensing and so on. A further detailed discussion on these evidences can be found in chapter1 . All these evidences suggest the existence of DM and it should be neutral,massive and stable on cosmological time scale. It is usually assumed that DM is a weakly interacting massive neutral particle(WIMP) and its annihilation cross section $\langle \sigma |v| \rangle$ is of the order of $3.2 \times 10^{-26} cm^3/s$ which can satisfy the requirement of relic abundance[3].

With these assumptions in hand we discussed a possible mechanism to calculate the relic abundance of DM in chapter3. Assuming the universe to be isotropic and homogeneous, the non-equilibrium thermodynamics in the early universe is considered and Boltzmann equation is solved to calculate the relic density of cold DM (CDM). The result shows that relic density is inversely proportional to cross-sections of reactions through which a WIMP annihilates. Regarding this the brief thermal history of the universe is also described to have an idea about energy,time and temperature relations at different epochs of the universe.

Being DM is a long lived particle it is absent in the current particle spectrum of the SM of particle physics. The only information about DM known till date is its relic abundance measured by WMAP which is given by $\Omega_{DM} h^2 = 0.11$[2]. However the underlying mechanism which gives the relic abundance is unknown. Therefore in this thesis we discuss a model in the physics beyond SM scenario. So as a part of the thesis we briefly recapitulate the relevant aspects of SM of particle physics which is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(1)_Y$ in chapter 3. In particular we discuss the whole SM Lagrangian based on the electroweak group $SU(2)_L \times SU(1)_Y$ .We also describe briefly about concept of spontaneous symmetry breaking(SSB).

With all the preliminary discussions in hand the main idea and works of this project are described in chapter 4. SM is extended with a scalar triplet and vector-like fermion doublet to explain the abundance of DM. We also impose a Z_2 symmetry on top of the SM gauge group under which the vector-like fermion doublet is odd while rest of the fields are even. As a result the massive neutral component of vector like fermion doublet is considered as candidate of WIMP. The possible couplings of the triplet with SM leptons and new doublets are taken into account. All the possible decay modes of candidate WIMP are considered and reaction cross-sections are calculated elaborately . A description of which can be found in Appendix. Considering the experimental data of relic density from WMAP a constraint on the DM mass is obtained. These theoretical calculations are checked with LANHEP [4], a tool through which one can generate new particle physics model for MICROMEGAS [5] which helps us to calculate the relic density of DM in the new user defined models. A close interplay between the coupling constants and the mass of triplet is examined. The constraints on the values of coupling constants are fixed,considering the neutrino mass via Type-2 see-saw[6]. Then keeping the relic density fixed for the cold DM, at the experimental value found from WMAP a contour plot is drawn between mass of triplet and mass of DM at two different couplings. All the results and analysis of the plots can be found in chapter 4.

We conclude our results in chapter 5. We found that a minimal extension of SM with scalar triplet and fermion doublet is enough to explain the relic abundance of DM and neutrino mass generation via type-2 see-saw mechanism[6]. This model is also capable of explaining the matter-antimatter asymmetry of the universe which is given by baryon to photon ratio $\eta \equiv \frac{n_B}{n_\gamma} = 6.15 \times 10^{-10}$ [2]. However it is beyond the scope of this thesis work.

Chapter 2

Evidence of Dark Matter : Physics Beyond the Standard Model

There are lots of experimental evidences in support of existence of DM in our universe and one of them is the flatness of the galactic rotation curves which were first explained by Fritz Zwicky in 1933 by invoking the concept of missing mass which is later came to be known as DM. If the visible stars and gas provided all the mass in the galaxy then one would expect that the rotation curve should fall according to the Keplerian relation ${v_c}^2 = \frac{GM_{obs}}{r}$. On the other hand observation tells that $v_c(r)$ remains constant up to much larger radii . As much of the visible mass comprises of hot X-ray gas, the discrepancy of the observed result with the experimental evidence can be explained by considering greater part(80%) of the galaxy mass is due to DM[7].

Another evidence for existence of DM is gravitational lensing which is exhibited by galaxy clusters. The gravitational field of the cluster bends the light around on their way to our telescopes. For a lensing cluster with total mass M and impact parameter d the deflection angle is of the order $\alpha \approx (\frac{GM}{d})^{\frac{1}{2}}$[8]. Thus from measurements of deflection angle and impact parameter, one can infer that the total mass M of a cluster is much larger than the observed baryonic mass.

Very recently the view of bullet cluster from NASA's Chandra Observatory is taken as the direct proof of the existence of DM[9]. The two colliding galaxies and their constituents are imaged in X-ray and the hot gases are found to interact electromagnetically but DM was detected indirectly by the gravitational lensing of the background objects. These observations suggests that the DM should be massive and electrically neutral. Moreover the existence of large scale structure tells that the DM should be stable on cosmological time scale.

Though the relic abundance of DM is known from WMAP data[2] but the reason which gives rise to the relic abundance is still unknown. The properties of DM suggested from the above observations ensure that such a particle is absent in the SM spectrum and that motivates us to study physics beyond the SM. In our work we have taken a weakly interacting massive particle(WIMP) as a candidate of DM where the annihilation cross-section satisfies the requirement of relic abundance through thermal freeze-out mechanism[3], which we describe in the next section. A large part of our analytical calculation of cross-section are based on the SM Lagrangian formalism. So we need to look at the basics of SM Lagrangian.

2.1 A Brief Description of SM Lagrangian

The unified description of electromagnetic and weak interaction by Glashow, Salam and Weinberg along with strong interaction is known as SM of particle physics. The gauge theory that describes the strong,weak and electromagnetic interactions of the quarks and leptons is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The $SU(3)_C$ part describes the strong interaction and $SU(2)_L \times U(1)_Y$ describes the electroweak interaction. Here C refers to color, L refers to left, and Y refers to weak hypercharge, related to to the weak isospin and electric charge through the Gell-Mann-Nishijima formula $Q = T_3 + \frac{Y}{2}$, where T_3 is the third component of weak isospin[10]. $SU(N)$ is the group of special unitary transformations of N objects.

The most interesting feature of SM is the spontaneous symmetry breaking(SSB) which is supposed to be accomplished by a scalar field famously known as Higgs Field. The $SU(3)_C$ part of the gauge symmetry remains unbroken while the electroweak part, described by $SU(2)_L \times U(1)_Y$, breaks down to electromagnetic part described by $U(1)_{EM}$. The symmetry is broken in such a way that the vacuum i.e the ground state does not respect the full gauge symmetry. Because of Higgs mechanism the vector bosons become massive and the short range of weak interaction is easily explained. On the other hand the vacuum remains invariant under $U(1)_{EM}$. So photons remain massless. This explains the long range nature of electromagnetic force.

Another interesting feature of the standard model is chirality [10][11]. The field that participate in electroweak interaction are the left and right handed components of the quark and lepton fields: $\psi_L = \frac{1-\gamma_5}{2}\psi$, $\psi_R \frac{1+\gamma_5}{2}\psi$ and $\psi = \psi_L + \psi_R$, where a left handed fermion is one whose spin is anti-parallel to its momentum vector and vice versa. Remarkably, under the electroweak gauge group left and right handed projections of a fermion transform differently. The left handed fields of SM are doublets under $SU(2)_L$ while the right handed particles are singlets. Each multiplets of $SU(2)_L$ has a unique value of weak hypercharge quantum number corresponding to $U(1)_Y$. The left handed matter doublets have $Y = -1$, while the right handed singlets carry a hypercharge of $Y = -2$. As the electron in the standard model is massive so its fields can be decomposed into right and left handed component. Since the right handed neutrinos are abscent in the SM spectrum neutrinos are massless upto all orders of purturbations. However neutrino oscillation experiment found it to be wrong. Like lepton families the quark families can also be described by similar structures but the only difference is as all quark fields are massive, there will be right handed singlet components for every quark field as opposed to the absence of ν_R in the leptonic sector.

Now we describe in brief the complete Lagrangian density for the theory which has the local gauge invariance $SU(2)_L \times U(1)_Y$. First we start with the gauge invariant Lagrangian density for the gauge fields

$$L_G = -\frac{1}{4}F^a{}_{\mu\nu}F^{\mu\nu a} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.1}$$

where the field strength tensors for the fields are defined as

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu Y_\nu - \partial_\nu Y_\nu \\ F^a{}_{\mu\nu} &= \partial_\mu W^a{}_\nu - \partial_\nu W^a{}_\mu - g\epsilon^{abc}W^b{}_\mu W^c{}_\nu \end{aligned} \tag{2.2}$$

. We have taken that g, g' are the coupling constants for the gauge interactions associated with the groups $SU(2)_L$ and $U(1)_Y$ respectively. In order to maintain the gauge invariance under the $SU(2)_L \times U(1)_Y$, one should introduce the covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i\frac{g}{2} W^i{}_\mu \Gamma_i + i\frac{g'}{2} Y B_\mu \tag{2.3}$$

.Here Γ's are Pauli spin matrices which are generators of $SU(2)_L$ group. The gauge invariant Lagrangian density for the matter fields can be obtained through minimal coupling. Since the left-handed fermions belong to an isospin doublet while the right handed fermions are isospin singlets, the Lagrangian density for the fermions take the form

$$L_f = i\overline{\psi_L}\gamma^\mu(\partial_\mu + i\frac{g}{2} W^i{}_\mu \Gamma_i + i\frac{g'}{2} Y B_\mu)\psi_L + \overline{\psi_R} i\gamma^\mu(\partial_\mu + i\frac{g'}{2} Y B_\mu)\psi_R \tag{2.4}$$

The $SU(2)_L$ invariance forbids the mass term $m\overline{\psi_L}\psi_R$ for the matter fields. In other words all the matter fields are massless before SSB. The gauge fields i.e $W_\mu^{\pm}, Z_\mu, A_\mu$ also remain massless until SSB . So now we add a scalar doublet ϕ to break gauge symmetry spontaneously. This is popularly called as Higgs mechanism. When the Higgs acquire a vacuum expectation value(vev) the gauge symmetry $SU(2)_L \times U(1)_Y$ breaks down to $U(1)_{EM}$. The scalar Lagrangian which is responsible for this is

$$L_H = \partial_\mu \phi^+ \partial^\mu \phi - \mu^2 \phi^+ \phi + \lambda(\phi^+ \phi)^2 \tag{2.5}$$

So with the introduction of new scalar field the symmetry of the gauge group breaks and W and Z boson acquire mass but photon remains mass-less as one should expect because $U(1)_{EM}$ remains as a symmetry of the theory. Now to give masses to the fermions Yukawa interaction of the fermions with the scalar fields are included, which is described by the Lagrangian density as

$$-L_Y = h\overline{\psi_R}\phi^+ \psi_L + h^* \overline{\psi_L}\phi\psi_R \tag{2.6}$$

where h is the Yukawa coupling. So now collecting all the terms the complete weak interaction Lagrangian density describing the family of electron can be written as

$$L = L_G + L_f + L_H + L_Y \tag{2.7}$$

. As our thesis deals with beyond standard model physics and we extend the leptonic sector with a fermion doublet the above described Lagrangian will be very useful for the remaining discussion. In this work we carefully omit the quark sector Lagrangian description here.

Chapter 3

Theory of Dark Matter

Particle physicists so far have relied on the use of powerful accelerators to study the interactions of particles at high energy. From elementary quantum theory it follows that to probe smaller and smaller distances we need higher and higher energies. The high energy labs at CERN can produce particles of energies of the order of 10^3 GeV. On the contrary the early universe provides a natural platform to study the fundamental interactions of elementary particles as the temperature of the thermal bath is very high.

As the main aim of our project is to find the relic abundance of DM where the DM particles decouple from thermal bath to produce the relic abundance, we need to find the cross sections through which the particle annihilates before the electroweak symmetry breaking. It is very important to describe the general procedure to find the relic abundance and freeze out mechanism which are described in detail in the next section. Before that we should describe the thermal history of universe in brief, keeping in mind the the most significant events.

The earliest epoch in the history of BigBang cosmology is the Planck era with temperature at $10^{19}\, GeV$. Before then the quantum mechanical aspects of gravity are expected to be important, so one would need a quantum theory of gravity to describe this period, a theory which is still unknown to us [3]. The energy scale10^{16} GeV which came after 10^{-36}Sec later big-bang is known as Grand unified energy scale where it is believed that electromagnetic, weak, and strong force are of equal strength and unify to one force [12]. As universe expands and temperature drops down we reach the most important era, the electroweak symmetry breaking. At this epoch the energy drops down to 1TeV scale and all the reaction through which DM particle annihilates are taken to happen before this symmetry breaking. The W and Z boson acquire mass after spontaneous symmetry breakdown(SSB) by higgs phenomenon and photon remains mass-less. At around energy scale 1MeV i.e few seconds after big-bang the heavier isotopes of hydrogen, helium formed during the period known as Big-Bang Nucleosynthesis [3]. It is believed that light elements like deuterium, helium, lithium formed in this era but the further heavier elements formed in the interior of stars much later in the history of universe[3]. Earlier universe was radiation dominated but as temperature drops down matter starts dominating and at around 5000Yr after big-bang matter takes over radiation with temperature around 10^5 K . We are now at a temperature around 2.7K with the universe filled with cosmic microwave background radiation.

So with these basic things we move towards the core work of our thesis. First we discuss the non-equilibrium thermodynamics to calculate the relic abundance and then in the next section we will go beyond the SM to introduce a new model which will satisfy the relic abundance data and also find a solution for the neutrino mass problem.

3.1 Thermal Equilibrium in Early Universe

Regarding the calculation of relic abundance we started our study with the equilibrium thermodynamics at the early universe. The universe is assumed to be homogeneous and isotropic, i.e every point in the space is equivalent to other point and there is no preferred direction i.e no observer occupies a special position in the universe. The expansion rate of the universe is determined by Hubble parameter $H = \frac{\dot{R}}{R}$, where $R(t)$ is the scale factor of expansion in the Friedmann-Robertson -Walker cosmology(FRW). For a particle species in kinetic equilibrium, the phase space occupancy is given by familiar Fermi-Dirac(FD) or Bose-Einstein(BE) distribution. From this equilibrium distribution the number density n is given by -

$$n = \frac{g}{2\pi^2} \int_m^\infty (E^2 - m^2)^{\frac{1}{2}} f(E) E dE \tag{3.1}$$

$$\text{where } f(E) = \frac{1}{\exp\left(\frac{E-\mu}{T}\right) \pm 1} \tag{3.2}$$

μ is the chemical potential. In the relativistic limit $T >> m$, n is proportional to T^3 for B.E as well as F.D statistics. But in the non relativistic limit the number density is given by

$$n = g \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} \exp - \left(\frac{E - \mu}{T}\right) \tag{3.3}$$

where g is internal degrees of freedom of the particle concerned. Now we will see that whenever the interaction rate of certain particle species drops below the expansion rate of universe, the particle decouples from the thermal bath. This is called 'Freeze Out'. The departures from thermal equilibrium have led to important relics- the light elements, the neutrino background, weakly interacting massive(WIMP) particles and so on.

In the following discussion we will study freeze out epoch of a WIMP, which is taken to be a candidate of DM.

3.2 Freeze out of DM

We assume that DM is massive and was in thermal equilibrium in the early universe through its weak interaction process. As the temperature falls below the mass scale of DM, it decouples from thermal bath. The details of decoupling history can be tracked by solving the relevant Boltzmann equation which can be written in operator form as

$$\hat{L}[f] = \hat{C}[f] \tag{3.4}$$

where $\hat{L}$ is Liouville operator and $\hat{C}$ is collision operator. For the isotropic homogenous FRW model the distribution function can be taken to depend only on E, such that $f = f(E, t)$. The covariant relativistic generalization of Liouville's operator is

$$\hat{L} = p^{\alpha} \frac{\partial}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\beta\gamma} p^{\beta} p^{\gamma} \frac{\partial}{\partial p^{\alpha}} \tag{3.5}$$

So $\alpha = 0$ component reduces the Liouville operator into

$$\hat{L} = E \frac{\partial f}{\partial t} - \frac{\dot{R}}{R} p^2 \frac{\partial f}{\partial E} \tag{3.6}$$

Now the number density in terms of phase space density is given by

$$n = \frac{g}{2\pi} \int f(E, t) d^3 p \tag{3.7}$$

Differentiating equation (3.7) we get

$$\frac{dn}{dt} = \frac{g}{(2\pi)^3} \int d^3 p \frac{\partial f}{\partial t} \tag{3.8}$$

From equation (3.6) we replace $\frac{\partial f}{\partial t}$ and use it in equation (3.8) to have

$$\frac{dn}{dt} = \frac{g}{(2\pi)^3} \int d^3 p \frac{\hat{L}[f]}{E} + \frac{g}{(2\pi)^3} \frac{\dot{R}}{R} \int \frac{p^2}{E} \frac{\partial f}{\partial E} d^3 p \tag{3.9}$$

Now using Boltzmann equation (3.4) we have

$$\frac{dn}{dt} = \frac{g}{(2\pi)^3} \int d^3 p \frac{\hat{C}[f]}{E} + \frac{g}{(2\pi)^3} \frac{\dot{R}}{R} \int \frac{p^2}{E} \frac{\partial f}{\partial E} d^3 p \tag{3.10}$$

Considering the second integral, changing variable from p to E; $p^2 = E^2 - m^2 \Rightarrow pdp = EdE$. So the second integral transforms into

$$\int \frac{E^2 - m^2}{E} \frac{\partial f}{\partial E} \frac{EdE}{(E^2 - m^2)^{\frac{1}{2}}} \tag{3.11}$$

Now taking $\frac{\partial f}{\partial E}$ as the first function we have from integration by parts

$$(E^2 - m^2)^{\frac{3}{2}} \int \frac{\partial f}{\partial E} dE]^{\infty}_{m} - \int \left(\frac{3}{2}(E^2 - m^2)^{\frac{1}{2}} 2E \int \frac{\partial f}{\partial E} dE \right) dE \tag{3.12}$$

The first integral vanishes in the given limit, and the second integral reduces to

$$-3 \int \frac{(E^2 - m^2)^{\frac{1}{2}} EdE}{exp \frac{(E-\mu)}{T} \pm 1} \tag{3.13}$$

Using equation (3.1) we get ultimately

$$\frac{dn}{dt} + 3\frac{\dot{R}}{R} n = \frac{g}{(2\pi)^3} \int \hat{C}[f] d^3 p \tag{3.14}$$

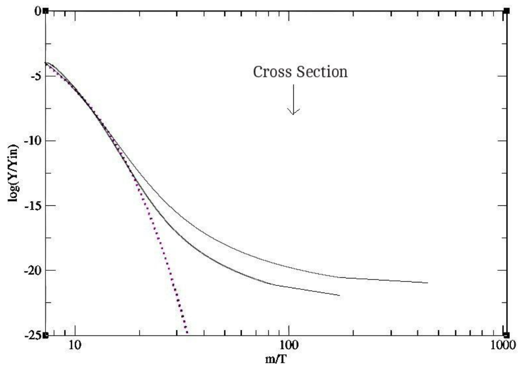

Figure 3.1: The solution of Boltzmann equation for two different cross-sections. The dotted line shows equilibrium distribution which falls exponentially while the non equilibrium solutions decouple from the thermal bath to give rise to relic abundance. The graph also shows that with increasing cross section relic abundance decreases.

Thus the remaining is to find the collision cross section for a particular reaction. For a given cross section for a particular process we can solve this Boltzmann equation analytically.

We consider a reaction of type $\chi\bar{\chi} \leftrightarrow X\bar{X}$, where χ is a candidate for DM. Here we assume that all the species to which $\chi, \bar{\chi}$ annihilate have thermal distributions with zero chemical potential.The annihilation rate should be $\propto \sigma(\chi\bar{\chi} \to X\bar{X})\,|v|\,n_\chi^2$. At the same time $\chi\bar{\chi}$ are also produced in the process $X\bar{X} \to \chi\bar{\chi}$ with a rate proportional to n_χ^2. So we get from equation (3.14)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\left\langle \sigma(\chi\bar{\chi} \to X\bar{X})\,|v|\right\rangle [n^2_\chi - n^2_{\chi eq}] \tag{3.15}$$

Where we have used the definition of collision operator for a process $1 + 2 \longleftrightarrow 3 + 4$ as

$$\frac{g}{2\pi^3}\int d^3p\frac{\hat{C}[f]}{2E} = \int dp_1\,dp_2\,dp_3\,dp_4\,\delta^4(p_1 + p_2 - p_3 + p_4)\,|M|^2$$

$$[f_3f_4(1 \pm f_1)(1 \pm f_2) - f_1f_2(1 \pm f_3)(1 \pm f_4)]$$

Now using the definition of $\langle\sigma\,|v|\rangle$ which is given by

$$\langle\sigma\,|v|\rangle = \frac{1}{n_1n_2}\int dp_1\,dp_2\,dp_3\,dp_4 \exp\left(-\frac{E_1 + E_2}{T}\right)\delta^4(p_1 + p_2 - p_3 + p_4)\,|M|^2 \tag{3.16}$$

we found out equation (3.15).

In order to scale out the effect of the universe expansion we define a new variable $Y_\chi \equiv \frac{n_\chi}{s}$, where s is the entropy density given by

$$s = \frac{2\pi^2}{45}g_{*s}T^3 \tag{3.17}$$

where g_{*s} is the number of relativistic degrees of freedom in the thermal bath and is given by

$$g_{*s} = \sum_{bosons} g_i \left(\frac{T_i}{T_\gamma}\right)^3 + \frac{7}{8} \sum_{fermions} g_i \left(\frac{T_i}{T_\gamma}\right)^3 \tag{3.18}$$

Therefore we have

$$\dot{Y}_\chi = \frac{\dot{n}_\chi}{s} - \frac{n_\chi}{s^2}\dot{s} = \frac{\dot{n}_\chi - n_\chi \frac{\dot{s}}{s}}{s} \tag{3.19}$$

Assuming the adiabatic expansion of the universe where the total entropy is constant sR^3 = constant, so we have

$$\dot{s}R^3 + 3R^2 \dot{R} s = 0; \Rightarrow \frac{\dot{s}}{s} = -3H \tag{3.20}$$

Using equation (3.20) we rewrite equation (3.19) as

$$\dot{Y}_\chi = \frac{(\dot{n}_\chi + 3H n_\chi)}{s} \tag{3.21}$$

So the Boltzmann equation (3.15) can be written as

$$s\dot{Y}_\chi = \langle\sigma |v|\rangle [n_\chi^2 - (n_{\chi eq})^2] \tag{3.22}$$

Now using the definition of Y we have

$$\frac{\dot{Y}_\chi}{Y_{\chi eq}} = \langle\sigma |v|\rangle \left[\left(\frac{Y_\chi}{Y_{\chi eq}}\right)^2 - 1\right] sY_{\chi eq} \tag{3.23}$$

The relation between the temperature and time is $t = 0.30 \frac{M_{Pl}}{T^2 g_*^{\frac{1}{2}}}$. Now we define another new variable as $x \equiv \frac{m}{T}$, where m is the mass of DM and rewrite dt in terms of dx in order to change variables in the Boltzmann equation. So ultimately the Boltzmann equation takes the form

$$\frac{dY}{dx} = -\frac{x\langle\sigma |v|\rangle s}{H(m)}[Y^2 - Y_{eq}^2] \tag{3.24}$$

where $H(m)$ is related to Hubble's constant as $H(T) = \frac{H(m)}{x^2}$.

where in the non relativistic limit $(x >> 3)$ the equilibrium value Y_{eq}, has simple limiting form

$$Y_{eq}(x) = 0.145 \frac{g}{g_{*s}} x^{\frac{3}{2}} exp(-x) \tag{3.25}$$

where g_{*s} is given by equation (3.18). Analytically Y can be solved and it is given by

$$Y = \frac{3.79(n+1)(\frac{g_*}{g_{*s}})x_f}{m m_{pl} \langle\sigma |v|\rangle} \tag{3.26}$$

Here m_{pl} is a constant known as Planck mass and given by 1.211×10^{19} Gev. The interesting thing about the relic abundance expression is that it is inversely proportional with annihilation cross section and the mass of the DM.

The annihilation rate Γ varies as n_{eq} times the thermally averaged annihilation cross section $\langle \sigma |v| \rangle$. In the non-relativistic regime $n = (mT)^{\frac{3}{2}} exp\left(-\frac{m}{T}\right)$, where as in relativistic regime $\Gamma \propto T^3$. So in both these regimes Γ decreases as T decreases and when $\Gamma \approx H$ which for definiteness occur for $x = x_f$ called "freeze out" epoch. So we expect that for $x \leq x_f, Y \approx Y_{eq}$,while for $x \geq x_f$ the abundance "freezes in" $Y(x \geq x_f) = Y_{eq}(x_f)$.

We numerically solve the Boltzmann equation with two different given cross sections(for a weakly interacting process)$10^{-26} cm^2$ and $10^{-28} cm^2$ and found the freeze out epoch(x_f) are 26.43 and 27.41 respectively. The standard freeze out epoch comes within 25 to 35. The Boltzmann equation is the primary theoretical tool to deal with non-equilibrium in the expanding universe, and we used it to calculate the relic abundance of a WIMP.

3.3 Possible Modes of Detecting DM

We know that SM cannot explain the existence of DM. So we need to go beyond the SM physics and introduce different models where DM fits in. In order to distinguish the model and confirm the identity of DM particle we need various techniques to detect DM. We can distinguish between three different kind of detections of DM. one possibility is that the DM particle can be produced in pairs at colliders, which is known as collider detection. Another possibility is to measure the interactions of DM background in the laboratory known as direct detection. One can also measure the energy of end products of astrophysical DM annihilation or decay, this is called indirect detection.

Let us consider the direct detection first. Since DM is everywhere and we should be able to see its interactions with the baryonic matter at terrestrial laboratories. As the DM interactions are weak, sensitive dedicated experiments are required to detect this. The problem with direct detection is there are many similar type signals which cause the background for WIMP detectors. So the WIMP detectors are continuously detecting something. Hence one possible way is to look for annual modulation in the signal. The WIMPS if they exist, have a particular velocity distribution related to the gravitational well of our galaxy[7][13]. The earth is moving with respect to this velocity distribution, and the annual change in the direction of earth's motion should result in a corresponding variation in the detection rate.

Indirect detection is very different than direct detection process. In this method the signature of the DM is searched in the cosmic ray shower. In particular certain experiments in the current days are looking for positron and anti proton excess through the possible reaction $\psi + \overline{\psi} \longrightarrow f + \overline{f}$. Since the antiparticles are secondary, their number density is expected to decrease as the energy increases. If any experiment detects rise of positron flux with energy then it may hints the signature of DM.

Chapter 4

Relic Abundance of Inert fermion Doublet Dark Matter

4.1 Model Description and Lagrangian

As already mentioned, the inadequacy of SM in explaining the neutrino mass and existence of DM leads to several models beyond the SM of particle physics. In this work a very economical model is introduced to explain the above mentioned two problems.

We extend the SM Lagrangian by adding a vector-like fermion doublet $\psi^T = (\psi^0, \psi^-)$ and impose a Z_2 symmetry under which ψ is odd, while all other fields are even. As a result the neutral component of ψ behaves as a candidate of DM [6][14][15]. We also add a scalar triplet $\Delta \equiv (\Delta^{++}, \Delta^+, \Delta^0)$ with hypercharge $Y = 2$. Since Δ has $Y = 2$ it can couples symmetrically to two $SU(2)_L$ doublets [14] . As a result the Lagrangian is given by

$$-L \supset +\frac{1}{\sqrt{2}}(f_H M_\Delta \Delta^\dagger H H + f_L \Delta L L + f_\psi \Delta\psi\psi + \text{h.c}) \tag{4.1}$$

The bilinear DM coupling $\Delta\psi\psi$ can be expressed as

$$\begin{aligned} \frac{1}{\sqrt{2}} g\Delta\psi\psi &\equiv \frac{1}{\sqrt{2}}\overline{\psi^c} i\tau_2 \Delta\psi \\ &= \frac{1}{2} g[\sqrt{2}(\overline{\psi^c}_-\psi_-\Delta^{++}) + (\overline{\psi^c}_-\psi_0 + \overline{\psi^c}_0\psi_-)\Delta^+ - \sqrt{2}(\overline{\psi^{c0}}\psi_0\Delta^0)] \end{aligned} \tag{4.2}$$

. Where the scalar triplet is used in the given matrix form

$$\begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \tag{4.3}$$

The scalar potential involving Δ with the Higgs field is given by

$$V(\Delta, H) = M_\Delta^2 \Delta^+ \Delta + \frac{\lambda_\Delta (\Delta^+ \Delta)^2}{2} - M_H^2 H^+ H + \frac{\lambda_H (H^+ H)^2}{2}$$
$$+ \lambda_{\Delta H} H^+ H \Delta^+ \Delta + \frac{\mu_H \Delta^+ HH + \text{h.c}}{\sqrt{2}}$$

. When H acquires a vacuum expectation value (vev) it gives rise to an induced vev for Δ, given by

$$\langle \Delta \rangle = -\frac{\mu_H v^2}{{M_\Delta}^2} \tag{4.4}$$

where $v = \langle H \rangle \approx 246$ GeV.

As a result the coupling $\Delta L_\alpha L_\beta$ gives rise to neutrino mass and the mass is given by

$$\begin{aligned} M_\nu &= f_L \langle \Delta \rangle \\ &= f_L \mu_H \frac{v^2}{{M_\Delta}^2} \end{aligned} \tag{4.5}$$

Let us define $f_H = \frac{\mu_H}{M_\Delta}$ so the above equation further reduces to

$$M_\nu = f_L f_H \frac{v^2}{M_\Delta} \tag{4.6}$$

In the usual type 2 see-saw $M_\Delta \approx 10^{14}$ GeV and $v = 246$ GeV [6]. As a result taking $f_L \approx f_H \approx O(1)$, we can explain sub-ev neutrino mass $M_\nu = O(0.1 eV)$. However in our case we assume Δ in electroweak scale. So to explain neutrino mass we take $f_H \approx 10^{-14}$ while keeping $M_\Delta \approx O(100 GeV)$. As a result neutrino mass remains same, while the theory gives rich phenomenology in the DM sector as we describe below. The most stringent bound on DM mass comes from the invisible Z-decay width. This gives $M_{DM} \geq \frac{M_Z}{2} = 45$GeV.

4.2 Relic Abundance

Since in our model ψ^0 is the candidate of DM, it will annihilate to the SM fermions through the following channels -

- $\psi^0 + \overline{\psi^0} \overset{Z}{\longleftrightarrow} f + \overline{f}$;where f is the SM fermion.
- $\psi^0 + \psi^- \overset{W}{\longleftrightarrow} \ _1 + \overline{f_2}$ where f_1 and f_2 are SM fermions.

$+ \nu$

f

- $\psi^0 + \psi^0 \overset{\Delta}{\longleftrightarrow} \ \alpha \quad \beta$
- $\psi^0 + \psi^- \overset{\Delta^-}{\longleftrightarrow} \nu_\alpha + f_\beta^-$
  Here we also note that
  $\psi^0 + \psi^0 \overset{\Delta}{\longleftrightarrow} h + h$ channel gives negligible contribution as $f_h \ll 1$ as required to explain neutrino masses keeping M_Δ at electrweak scale.

We describe about the cross sections for all the above mentioned channels in the next section and calculates relic abundance of DM.

4.3 Results and Discussions

After defining all the interaction terms the collision cross sections are calculated. As the DM density found from WMAP data suggests a freeze out cross section which is $3.26 \times 10^{-26} cm^3/sec$, the DM mass at first is estimated from this, taking into account of W and Z channels only.

The cross section through Z channel is

$$\langle\sigma|v|\rangle_{cm} = \frac{24 \times g^4}{256\pi(4m_D{}^2 - M_z{}^2)^2 + (\Gamma_Z M_Z)^2}\left[(c_v{}^2 + c_A{}^2)2m_D{}^2 + (c_v{}^2 - c_A{}^2)m_f{}^2\right]\left(1 - \frac{m_f{}^2}{m_D{}^2}\right)^{\frac{1}{2}} \tag{4.7}$$

where c_v is the correction to the vector weak charge and c_A is the correction to the axial vector weak charge , and m_1 is the mass of DM and M_Z is the mass of Z boson, and g is the coupling constant which can be defined in terms of electromagnetic coupling constant (g_e) as $g = \frac{g_e}{sin\theta_w}$. In SM model θ_w is the weak mixing angle or Weinberg angle whose value is taken as $\theta_w = 28.7^0$.

The cross section through W channel is

$$= \frac{g^4}{64\pi[(m_1 + m_2)^2 - M_W{}^2)]^2 + \Gamma_W{}^2 M_W{}^2}\sqrt{m_3{}^2 + p_3{}^2}\sqrt{m_4{}^2 + p_4{}^2} \times \left(1 - \frac{m_3{}^2}{(m_1 + m_2)^2} - \frac{m_4{}^2}{(m_1 + m_2)^2}\right) \times (V_{ud}^2 + V_{us}^2 + V_{ub}^2) \times 9 \tag{4.8}$$

The detailed calculations are done in Appendix. The formula is a generalized one where up quark is one of the outgoing particles, it will change accordingly for charm or top quarks also. The V_{ud}'s are the CKM matrix elements. M_W is the mass of W boson, δm describes the mass splitting between the neutral and charged part of the members of inert fermionic doublet (ψ^0, ψ^-) . We have assumed that the charged part of this doublet is more massive than the neutral part. For the leptonic sector though there is no mixing effect and the cross section formula will just be multiplied with 3 for 3 pairs of lepton doublets. The DM mass for the above mentioned cross-section when added with Z channel process, comes out to be 530GeV . This mass is used to find a close interplay between the mass of triplet and the product of coupling constants f_l and f_ψ for the fixed cross section mentioned above.

The cross section through Δ channel is :

$$\langle\sigma|v|\rangle = \frac{1}{16\pi}\frac{(f_\psi f_l)^2}{(4m^2 - M_\Delta{}^2)^2} m^2 (1 - \frac{m_f^2}{m^2}) \times 6 \tag{4.9}$$

The factor 6 comes because there are 6 outgoing channels which are (ν_e, ν_e), (ν_μ, ν_μ), (ν_τ, ν_τ), (ν_e, ν_μ), (ν_e, ν_τ), (ν_μ, ν_τ). Now this cross section formula blows up at Δ pole. So we introduced a regulator in the denominator which is essentially relevant near the resonance and otherwise the effect of it is very small. If the coupling of the Δ with higgs is given by f_h then the
cross section formula ultimately takes the form

$$\langle\sigma|v|\rangle = \frac{1}{16\pi}\frac{(f_\psi f_l)^2}{(4m^2 - M_\Delta{}^2)^2 + [\frac{1}{8\pi}(f_l{}^2 + f_h{}^2 + f_\psi{}^2)M_\Delta{}^2]^2} m^2 (1 - \frac{m_f^2}{m^2}) \times 6 \tag{4.10}$$

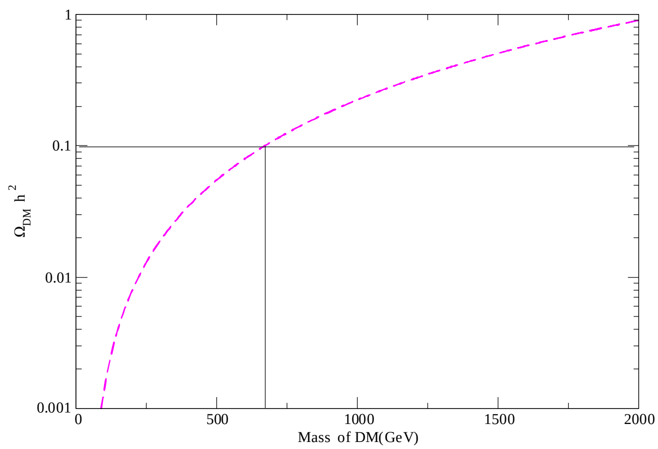

Figure 4.1: The graph shows the analytical results found while calculating the relic abundance of DM. As DM abundance is fixed from WMAP data $\Omega_{DM} h^2 = 0.11$, the mass of candidate DM which satisfies this is 670GeV.

Now this formula seems to depend upon various factors. The mass of DM m, the mass of M_Δ and the coupling constants f_l and $f_{\psi'}$,f_h . Where f_L describes the coupling of the Δ with the leptons, $f_{\psi'}$ describes the coupling of Δ with the added vector-like fermion doublet, and f_h describes about the coupling of Δ with the SM Higgs. We have already drawn an analytical curve of dark matter density with the mass of DM for the decay through W and Z channel only, which shows that the mass of DM is 670GeV for which we get that above abundance.

We have fixed the dark matter mass and vary the triplet mass to see the behavior of coupling constants for the fixed desired cross-section which will produce the above mentioned abundance. The drop in the product of coupling constants near about 1070GeV is easy to explain because near that point 2m = M_Δ condition is satisfied so coupling constants have to go down to reduce the cross section which we kept fixed.

The third graph though has something more to tell than the second one. If we go for higher coupling say around 0.5 then $\langle\sigma |v|\rangle_{cm}$ is of the order of 10^{-7} which is much higher than the relevant cross-section for the DM abundance, again if we consider the coupling constants values around 0.2 then cross-section is of the order of 10^{-10} which will produce much greater relic abundance as cross-section is very less.

All these analysis we are doing for a triplet mass around 100 − 200GeV because that is our point of interest. A recent paper[16] suggested that the scalar-triplet beyond standard model can explain the $h \to \gamma\gamma$ if the quartic coupling mixing of triplet with SM higgs is large and the mass of triplet is as small as 100GeV. From the graph it seems that when the individual couplings of triplet with inert fermion doublet and with the SM leptons are around 0.3 then 100GeV triplet is a very good candidate to give the relevant cross section for DM abundance.

The variation of cross-section with different couplings near the resonance point is independent of the couplings. This suggests that the triplet coupling with the higgs i.e.f_h is very small otherwise the cross-section would depend on that coupling near the resonance point.

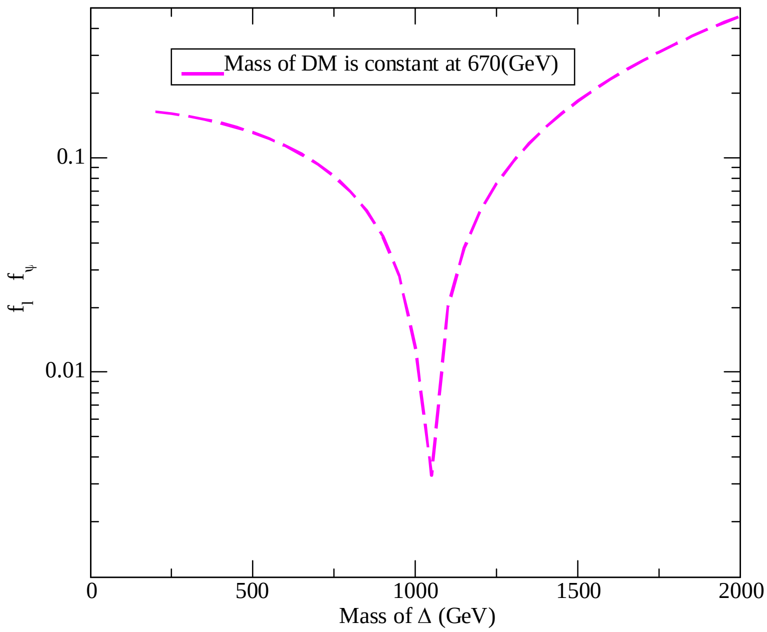

Figure 4.2: The graph shows the variation of coupling constants with the mass of triplet. The sharp decrease in the product suggests the pole of **Δ.**

The fourth graph plots the viable region of DM mass with triplet mass for two different couplings where we have kept $\Omega_{DM} h^2$ fixed at the relic density of DM predicted from recent data [2]. We have already discussed the behavior of cross-section with triplet mass for different couplings.

For very small triplet mass 0.2 coupling value gives very less cross section, which will produce very high abundance. So here the dominant processes to produce the required abundance are W and Z channel decay so the mass of DM lies in the region which will give the relevant abundance for SM model decay modes only and we have already seen that is around 670GeV. Now when the triplet mass is increasing the cross-section increases(from figure 3) and that slowly starts playing role in the DM mass. The maximum effect of triplet mass on DM mass is around 1000GeV which is very near to the delta pole.

Now when we increase the coupling from 0.2to0.3 the scenario is completely different from the previous one. We have seen from the last plot that around 0.3 value of coupling the cross section lies in the required range for DM abundance even for low triplet mass. Now the delta channel starts dominating compare to the W and Z channel and to restrict the DM abundance in the relevant range the only possible option is to increase the mass of DM. As cross-section is inversely proportional with the square of the dark matter mass from equation(10). The effect of triplet mass is more prominent compare to the previous coupling because as triplet mass increases the denominator of equation (10) decreases as the factor$(4m^2 - M_{\Delta}{}^2)$ decreases for fixed DM mass, so to reduce the delta channel dominance and restrict the DM abundance in the specified range one have to increase the DM mass which is basically shown in this plot.

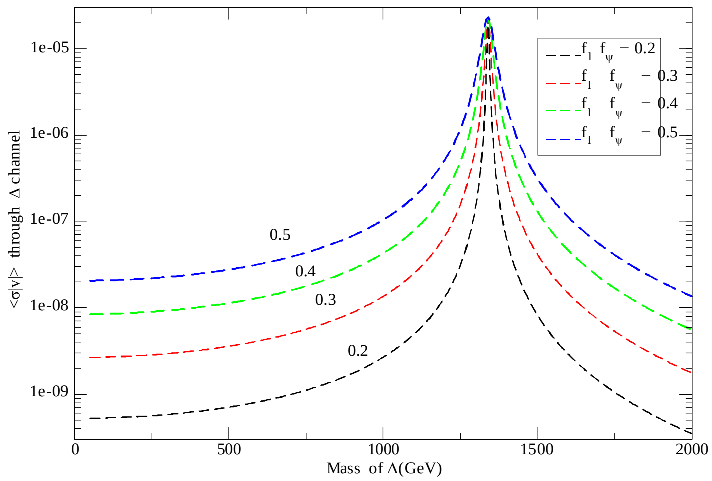

Figure 4.3: This figure shows the variation of annihilation cross-section with the mass of scalar triplet for different coupling constants.

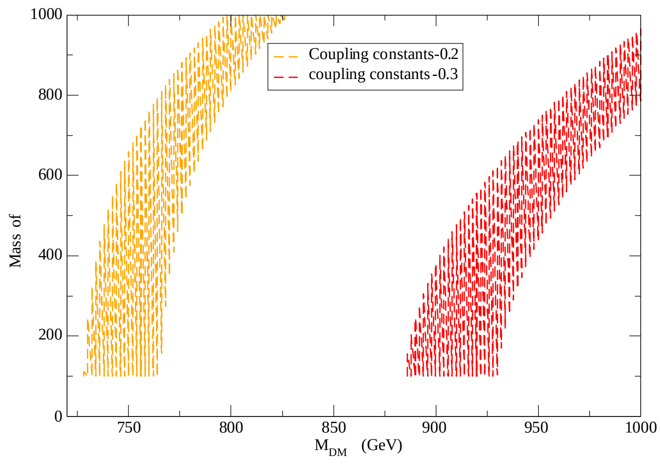

Figure 4.4: The contour plot shows the viable region of DM mass and triplet mass for fixed DM density for two different couplings.

Chapter 5

Conclusion

So after all the discussions the conclusion of the work is drawn as follows. Existence of DM and neutrino mass suggests physics beyond the SM of particle physics. Though there exists many such theories beyond SM physics but in this work a simple extension of SM is done with a vector-like massive lepton doublet, whose neutral component can play the role of DM. Not only that the added triplet scalar gives rise to neutrino mass through type-2 see-saw. The theoretical and analytical calculations (using MICROMEGAS and LANHEP) are compared and our analytical estimation agrees with the numerical findings.

- The candidate DM particle mass is 530GeV in our model which gives the required relic abundance.
- Once the candidate DM particle mass is found we use it in the Δ mediated process to find a close interplay between the coupling constants and mass of triplet. As our special interest is a triplet of 100GeV mass it is found that when both f_l and f_ψ is around 0.3 and $f_H \approx (o(10^{-14}))$ we get the required cross section for relic abundance of DM.
- It is also shown that the above mentioned coupling constants values and mass of triplet gives rise to required neutrino mass through type 2 see-saw.

Appendix A

A.1 Z channel Cross Section

The vertex factor for Z^0 is $\frac{ig_z\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)}{2}$ where g_z is defined as

$$g_z = \frac{g_e}{\sin\theta_w \cos\theta_w} \tag{A.1}$$

So to generate the matrix element we use feynmann rules and after a bit simplification it takes the form

$$M = -\frac{g^2}{8(q^2 - M_z{}^2)cos^2\theta_w}[\overline{u(4)}\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)v(3)][\overline{v(2)}\gamma_\mu(1-\gamma^5)u(1)] \tag{A.2}$$

To find the modulus of the matrix element square we need to apply the trace theorem which is as follows

$$\sum_{\text{all spins}} [\overline{u(a)}\Gamma_1 u(b)][\overline{u(a)}\Gamma_2 u(b)]^* = \text{Tr}[\Gamma(\not{p}_b + m_b)\Gamma(\not{p}_a + m_a)] \tag{A.3}$$

Applying this theorem and multiplying $\frac{1}{4}$ factor as the average over all initial spins,we have

$$\sum |M|^2 = \left[\frac{g^2}{16(q^2 - M_z{}^2)cos^2\theta_w}\right]^2 \text{Tr}[\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)(\not{p}_3 - m_f)\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)(\not{p}_4 + m_f)]$$
$$\times\ \text{Tr}[\gamma_\mu(c_v{}^\psi - c_A{}^\psi\gamma^5)(\not{p} + m_D)\gamma_\nu(c_v{}^\psi - c_A{}^\psi\gamma^5)(\not{p} - m_D)] \tag{A.4}$$

Now we consider the second trace

$$\text{Tr}[\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)(\not{p}_3 - m_f)\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)(\not{p}_4 + m_f)] =$$
$$\text{Tr}[\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_3\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)(\not{p}_4 + m_f) - \gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)m_f\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)(\not{p}_4 + m_f)] \tag{A.5}$$

This can be further reduced to

$$
\begin{aligned}
&= \mathrm{Tr}[\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_3\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_4 \\
&+ \gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_3\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)m_f \\
&- \gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)m_f\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_4 \\
&- \gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)m_f{}^2\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)]
\end{aligned} \tag{A.6}
$$

The middle two terms go away because of odd no. of gammas. We are left with two terms only.

$$
\begin{aligned}
&\mathrm{Tr}[\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_3\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5)\not{p}_4 \\
&- m_f{}^2\gamma^\mu(c_v{}^f - c_A{}^f\gamma^5)\gamma^\nu(c_v{}^f - c_A{}^f\gamma^5 \\
&)]
\end{aligned} \tag{A.7}
$$

let us define for simplicity $x = \frac{c_A}{c_v}$ .Now if we simply consider the second term of the previous equation

$$
\begin{aligned}
&c_v{}^2 m_f{}^2\mathrm{Tr}[\gamma^\mu(1 - x\gamma^5)\gamma^\nu(1 - x\gamma^5)] \\
&= c_v{}^2 m_f{}^2[\mathrm{Tr}(\gamma^\mu\gamma^\nu) - x\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu) - x\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5 \\
&) \\
&+ x^2\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)]
\end{aligned} \tag{A.8}
$$

Now we use the trace theorems to further reduce it

$$
\begin{aligned}
&\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \\
&\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu) = 0 \\
&\mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\gamma^5) = -(\gamma^5\gamma^5\gamma^\mu\gamma^\nu) = \\
&-4g^{\mu\nu}
\end{aligned} \tag{A.9}
$$

So using these three trace theorems middle two terms go away and the trace reduces to

$$
T_2 = -c_v{}^2 m_f{}^2 4g^{\mu\nu}(1 - x^2) \tag{A.10}
$$

Now if we consider the first trace with the above simplification it reduces to

$$
c_v{}^2\mathrm{Tr}[\gamma^\mu(1 - x\gamma^5)\not{p}_3\gamma^\nu(1 - x\gamma^5)\not{p}_4] \tag{A.11}
$$

$$
\begin{aligned}
&= c_v{}^2[\mathrm{Tr}(\gamma^\mu\not{p}_3\gamma^\nu\not{p}_4) - x\mathrm{Tr}(\gamma^\mu\not{p}_3\gamma^\nu\gamma^5\not{p}_4) \\
&- x\mathrm{Tr}(\gamma^\mu\gamma^5\not{p}_3\gamma^\nu\not{p}_4) \\
&+ x^2\mathrm{Tr}(\gamma^\mu\gamma^5\not{p}_3\gamma^\nu\gamma^5\not{p}_4)]
\end{aligned} \tag{A.12}
$$

The last term of the above equation reduces to

$$
\mathrm{Tr}(\gamma^\mu\gamma^5\not{p}_3\gamma^\nu\gamma^5\not{p}_4) = \mathrm{Tr}(\gamma^\mu\not{p}_3\gamma^\nu\not{p}_4) \tag{A.13}
$$

Now if we take the first term that simply reduces in a similar form what we have already derived in equation(16). Similarly the second term reduces in a similar manner like equation(18) .

So now all together the first trace reduces to

$$c_v{}^2[4(1+x^2)(p_3{}^\mu p_4{}^\nu - g^{\mu\nu}(p_3\cdot p_4) + p_3{}^\nu p_4{}^\mu \quad - 8i\epsilon^{\mu\nu\lambda\sigma}(p_{3\lambda}p_{4\sigma})] \quad) \tag{A.14}$$

This is actually very similar to the equation(19) with $x = 1$. So ultimately equation (36) reduces to

$$4c_v{}^2[(1+x^2)(p_3{}^\mu p_4{}^\nu - g^{\mu\nu}(p_3\cdot p_4) + p_3{}^\nu p_4{}^\mu) - 2i\epsilon^{\mu\nu\lambda\sigma}x(p_{3\lambda}p_{4\sigma}) + m_f{}^2 g^{\mu\nu}(1-x^2)] \tag{A.15}$$

Now using $x = \frac{c_A}{c_v}$ the above equation reduces to

$$= 4(c_v{}^2 + c_A{}^2)(p_3{}^\mu p_4{}^\nu - g^{\mu\nu}(p_3\cdot p_4) + p_3{}^\nu p_4{}^\mu) - 4(c_v{}^2 - c_A{}^2)m_f{}^2 g^{\mu\nu} - 8i\epsilon^{\mu\nu\lambda\sigma}(p_{3\lambda}p_{4\sigma})c_v c_A \tag{A.16}$$

Similarly if we consider the second trace that can also be reduced in a similar manner and ultimately gives

$$= 8(p_{1\,\mu}p_{2\,\nu} - g_{\mu\nu}(p_1\cdot p_1) + p_{1\,\nu}p_{2\mu}) - 8i\epsilon^{\mu\nu\theta\Gamma}(p_1{}^\theta p_2{}^\Gamma) \tag{A.17}$$

Now we have to multiply these two traces

$$\begin{aligned}&[4(c_v{}^2 + c_A{}^2)(p_3{}^\mu p_4{}^\nu - g^{\mu\nu}(p_3\cdot p_4) + p_3{}^\nu p_4{}^\mu)\\ &+ 4(c_v{}^2 - c_A{}^2)m_f{}^2 g^{\mu\nu} - 8i\epsilon^{\mu\nu\lambda\sigma}(p_{3\lambda}p_{4\sigma})c_v c_A]\\ &\times [4(c_{v1}{}^2 + c_{A1}{}^2)(p_{1\,\mu}p_{2\,\nu} - g_{\mu\nu}(p_1\cdot p_1) + p_{1\,\nu}p_{2\,\mu})\\ &+ 4(c_{v1}{}^2 - c_{A1}{}^2)m_1{}^2 g^{\mu\nu} - 8i\epsilon^{\mu\nu\theta\Gamma}(p_1{}^\theta p_2{}^\Gamma)c_{v1}c_{A1}]\end{aligned} \tag{A.18}$$

Now the terms after multiplying takes the form

$$\begin{aligned}&+32(c_v{}^2 + c_A{}^2)[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_2)(p_3\cdot p_4)]\\ &-32(c_v{}^2 - c_A{}^2)m_f{}^2(p_1\cdot p_2)\\ &+32(c_v{}^2 + c_A{}^2)[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_2)(p_3\cdot p_4)]\\ &-32(c_v{}^2 - c_A{}^2)m_f{}^2(p_1\cdot p_2)\\ &-32(c_v{}^2 + c_A{}^2)[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_2)(p_3\cdot p_4) - 4(p_1\cdot p_2)(p_3\cdot p_4)]\\ &-128c_v c_A[(p_1\cdot p_3)(p_2\cdot p_4) - (p_1\cdot p_4)(p_2\cdot p_3)]\end{aligned} \tag{A.19}$$

Now after simplification it reduces to(in C.M frame)

$$+ 64({c_v}^2 + {c_A}^2)[(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3$$ (A.20)

$$)]$$
$$+ 64({c_v}^2 - {c_A}^2){m_f}^2{m_D}^2$$

In C.M reference frame the first term further reduces to $64({c_v}^2 + {c_A}^2)[2{m_D}^4]$ So equation(4) reduces to

$$\sum |M|^2 = \frac{g^4}{4\cos^4\theta_w}\frac{1}{(4m_D{}^2 - M_z{}^2)^2}[(c_v{}^2 + c_A{}^2)2m_D{}^4 + (c_v{}^2 - c_A{}^2)m_f{}^2 m_D{}^2] \quad \text{(A.21)}$$

So, the cross section times velocity in C.M frame for this process reduces to

$$\langle\sigma|v|\rangle_{cm} = \frac{1}{256\pi}\frac{g^4}{(4m_D{}^2 - M_z{}^2)^2}[(c_v{}^2 + c_A{}^2)2m_D{}^2 + (c_v{}^2 - c_A{}^2)m_f{}^2](1 - \frac{m_f{}^2}{m_D{}^2})^{\frac{1}{2}} \quad \text{(A.22)}$$

Now for the weak neutral current the process is like $\psi^0 + \overline{\psi^0} \leftrightarrow f + \overline{f}$ where f's are the standard model fermions. so f can be $e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau, u, d, c, s, t, b$. So total no channels will be 6 for leptons and 6 × 3 for quarks(3 accounts for the three color associated with each quark). so the whole expression will be multiplied with 24 .

$$\langle\sigma|v|\rangle_{cm} = \frac{24 \times g^4}{256\pi(4m_D{}^2 - M_z{}^2)^2}[(c_v{}^2 + c_A{}^2)2m_D{}^2 + (c_v{}^2 - c_A{}^2)m_f{}^2](1 - \frac{m_f{}^2}{m_D{}^2})^{\frac{1}{2}} \quad \text{(A.23)}$$

As it stands the formula blows up at the Z^0 pole - that is when the total energy hits the value M_Z. The problem is that we have treated Z^0 as a stable particle though it has a finite lifetime. So this can be accounted by modifying the propagator as

$$\frac{1}{q^2 - M_Z{}^2} \to \frac{1}{q^2 - M_Z{}^2 - iM_Z\Gamma_Z} \quad \text{(A.24)}$$

Where Γ_Z is the decay rate given by $\Gamma_Z = 2.4952 \pm 0.0023$ GeV [17]. With this adjustment the cross-section finally takes the form

$$\langle\sigma|v|\rangle_{cm} = \frac{24 \times g^4}{256\pi(4m_D{}^2 - M_z{}^2)^2 + (\Gamma_Z M_Z)^2}[(c_v{}^2 + c_A{}^2)2m_D{}^2 + (c_v{}^2 - c_A{}^2)m_f{}^2]\left(1 - \frac{m_f{}^2}{m_D{}^2}\right)^{\frac{1}{2}} \quad \text{(A.25)}$$

A.2 W Channel Cross Section

The vertex factor for the W channel is $-\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1 - \gamma^5)$ and the propagator is $-\frac{i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_w{}^2}\right)}{q^2 - M_w{}^2}$. Similarly as before we calculate the $\sum |M|^2$ over all spin states, the formula simplifies to

$$\sum |M|^2 = \left[\frac{g^2}{16(q^2 - M_w{}^2)}\right]^2 \text{Tr}[\gamma^\mu(1 - \gamma^5)(\not{p}_4 + m_4)\gamma^\nu(1 - \gamma^5)(\not{p}_3 - m_3)]$$
$$\times \text{Tr}[\gamma^\mu(1 - \gamma^5)(\not{p}_1 + m_1)\gamma^\nu(1 - \gamma^5)(\not{p}_2 - m_2)] \quad \text{(A.26)}$$

Now if we consider the first trace we have

$$\text{Tr}[\gamma^\mu(1 - \gamma^5)(\not{p}_4 + m_4)\gamma^\nu(1 - \gamma^5)(\not{p}_3 - m_3)] = \text{Tr}[\gamma^\mu(1 - \gamma^5)\not{p}_4\gamma^\nu(1 - \gamma^5)(\not{p}_3 - m_3) + \gamma^\mu(1 - \gamma^5)m_4\gamma^\nu(1 - \gamma^5)(\not{p}_3 - m_3)] \quad \text{(A.27)}$$

Which on further multiplication gives

$$= \mathrm{Tr}[\gamma^\mu(1-\gamma^5)\not{p}_4\gamma^\nu(1-\gamma^5)\not{p}_3 - \gamma^\mu(1-\gamma^5)\not{p}_4\gamma^\nu(1-\gamma^5)m_3 + \gamma^\mu(1-\gamma^5)m_4\gamma^\nu(1-\gamma^5)\not{p}_3 - \gamma^\mu(1-\gamma^5)m_4 m_3\gamma^\nu(1-\gamma^5)] \tag{A.28}$$

The middle two terms go away because of odd number of gammas. So we are left with

$$= \mathrm{Tr}[\gamma^\mu(1-\gamma^5)\not{p}_4\gamma^\nu(1-\gamma^5)\not{p}_3 - m_4\, m_3\, \gamma^\mu(1-\gamma^5)\gamma^\nu(1-\gamma^5)] \tag{A.29}$$

If we only consider the first term we have

$$\begin{aligned} &= \mathrm{Tr}[\gamma^\mu\not{p}_4\gamma^\nu\not{p}_3] - \mathrm{Tr}[\gamma^\mu\gamma^5\not{p}_4\gamma^\nu\not{p}_3] - \mathrm{Tr}[\gamma^\mu\not{p}_4\gamma^\nu\gamma^5\not{p}_3] \\ &+ \mathrm{Tr}[\gamma^\mu\gamma^5\not{p}_4\gamma^\nu\gamma^5\not{p}_3] \end{aligned} \tag{A.30}$$

Now we reduce the previous expression term by term

$$\begin{aligned} \mathrm{Tr}(\gamma^\mu\not{p}_3\gamma^\nu\not{p}_4) &= \mathrm{Tr}(\gamma^\mu p_{3\lambda}\gamma^\lambda\gamma^\nu p_{4\sigma}\gamma^\sigma) \\ &= (p_3)_\lambda(p_4)_\sigma \mathrm{Tr}(\gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\sigma) \\ &= 4(p_3)_\lambda(p_4)_\sigma(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\nu}g^{\lambda\sigma} + g^{\mu\sigma}g^{\lambda\nu}) \\ &= 4(p_3^\mu p_4^\nu - g^{\mu\nu}(p_3\cdot p_4) + p_3^\nu p_4^\mu) \end{aligned} \tag{A.31}$$

The last term similarly can also be reduced by using

$$\mathrm{Tr}(\gamma^\mu\gamma^5\not{p}_3\gamma^\nu\gamma^\nu\gamma^5\not{p}_4) = \mathrm{Tr}(\gamma^\mu\not{p}_3\gamma^\nu\not{p}_4) \tag{A.32}$$

But the second and third term reduces in a different manner

$$\begin{aligned} \mathrm{Tr}(\gamma^\mu\not{p}_3\gamma^\nu\gamma^5\not{p}_4) &= \mathrm{Tr}(\gamma^\mu\gamma^5\not{p}_3\gamma^\nu\not{p}_4) \\ &= -p_{3\lambda}p_{4\sigma}\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\sigma) \\ &= -4i\epsilon^{\mu\lambda\nu\sigma}p_{3\lambda}p_{4\sigma} \end{aligned} \tag{A.33}$$

So in equation(13) the first trace term ultimately reduces to

$$8(p_3{}^\mu p_4{}^\nu + p_3{}^\nu p_4{}^\mu - g^{\mu\nu}(p_3\cdot_4) - i\epsilon^{\mu\nu\lambda\sigma}p_{3\lambda}p_{4\sigma}) \tag{A.34}$$

Now in equation(13) if we concentrate on the second term that reduces to

$$\begin{aligned} m_4\, m_3\, \mathrm{Tr}[\gamma^\mu(1-\gamma^5)\gamma^\nu(1-\gamma^5)] &= m_4\, m_3\,[\mathrm{Tr}(\gamma^\mu\gamma^\nu) - \mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu) \\ &= -\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5) + \mathrm{Tr}(\gamma^\mu\gamma^5\gamma^\nu\gamma^5)] \\ &= 0; \text{Because of odd number of gammas} \end{aligned} \tag{A.35}$$

So the first trace of equation(10) i.e equation(13)ultimately reduces to

$$8(p_3{}^\mu p_4{}^\nu + p_3{}^\nu p_4{}^\mu - g^{\mu\nu}(p_3\cdot p_4) + i\epsilon^{\mu\nu\lambda\sigma}p_{3\lambda}p_{4\sigma}) \tag{A.36}$$

In an exact similar manner the second trace of equation(10) reduces to

$$8(p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu} - g_{\mu\nu}(p_1 \cdot p_2) - i\epsilon_{\mu\nu\theta\tau} p_1{}^{\theta} p_2{}^{\tau}) \tag{A.37}$$

Now if we multiply equation (36) and (37) we have

$$\begin{aligned} &= 8^2 [(p_3 \cdot p_1)(p_2 \cdot p_4) + (p_3 \cdot p_2)(p_1 \cdot p_4) - (p_1 \cdot p_2)(p_3 \cdot p_4)] \\ &8^2 [(p_3 \cdot p_1)(p_2 \cdot p_4) + (p_3 \cdot p_2)(p_1 \cdot p_4) - (p_1 \cdot p_2)(p_3 \cdot p_4)] \\ &8^2 [(p_3 \cdot p_4)(p_1 \cdot p_2) + (p_1 \cdot p_2)(p_3 \cdot p_4) - 4(p_1 \cdot p_2)(p_3 \cdot p_4)] \\ &- 128[(p_3 \cdot p_1)(p_2 \cdot p_4) - (p_3 \cdot p_2)(p_1 \cdot p_4)] \end{aligned} \tag{A.38}$$

So ultimately simplifying it the product of the traces reduces to in C.M frame

$$256 m_1 m_2 \sqrt{m_3{}^2 + p_3{}^2} \sqrt{m_4{}^2 + p_4{}^2} \tag{A.39}$$

which helps to reduce equation(26) in the following form

$$\sum |M|^2 = \frac{g^4 m_1 m_2}{(q^2 - M_W{}^2)^2} \sqrt{m_3{}^2 + p_3{}^2} \sqrt{m_4{}^2 + p_4{}^2} \tag{A.40}$$

Now using the differential cross section formula in center of mass reference frame we have integrating over the whole solid angle

$$\begin{aligned} \langle \sigma |V| \rangle_{cm} &= \frac{g^4}{64\pi[(m_1 + m_2)^2 - M_W{}^2)]^2} \sqrt{m_3{}^2 + p_3{}^2} \sqrt{m_4{}^2 + p_4{}^2} \\ &\times \left(1 - \frac{m_3{}^2}{(m_1 + m_2)^2} - \frac{m_4{}^2}{(m_1 + m_2)^2}\right) \end{aligned} \tag{A.41}$$

The calculation seems over, but the formula will be slightly modified as we know there will be mixing between the quark states and CKM matrix carries the information about the strength of flavor changing weak decays which is of the form $\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$. So for the leptonic sector there will be three outgoing channels through W channel and they are $(\nu_e, e^-), (\nu_\mu, \mu^-), (\nu_\tau, \tau^-)$ and the quark sector will be modified as if u quark is one of the outgoing particles then the whole cross section formula will be modified as - The factor 9 takes care of the color factor. Similarly if we take c or t quark as outgoing particles the formula will be modified accordingly. The diagonal terms are very close to one and off diagonal terms are almost zero in CKM matrix suggesting that same quark family transition probability is much more than the other. Exactly as the previous Z boson case we need to take into account of finite lifetime of W boson also. The full decay width of W boson is $\Gamma_W = 2.085 \pm 0.0042$ GeV [17]. So the cross-section formula reduces to

$$\begin{aligned} &= \frac{g^4}{64\pi[(m_1 + m_2)^2 - M_W{}^2)]^2 + \Gamma_W{}^2 M_W{}^2} \sqrt{m_3{}^2 + p_3{}^2} \sqrt{m_4{}^2 + p_4{}^2} \\ &\times \left(1 - \frac{m_3{}^2}{(m_1 + m_2)^2} - \frac{m_4{}^2}{(m_1 + m_2)^2}\right) \times (V_{ud}^2 + V_{us}^2 + V_{ub}^2) \times 9 \end{aligned} \tag{A.42}$$

A.3 Cross Section Through Delta Channel

The calculation of cross-section through Δ channel is rather simple because it is a scalar particle. The coupling is defined as $\overline{L_4{}^C} i\gamma_2 \Delta L_4$. The coupling constant of delta particle with inert fermions is defined as f_ψ and that of with leptons as f_l. so now the matrix element takes the form

$$-iM = i f_\psi [\overline{u(4)} \frac{1+\gamma_5}{2} v(3)] i \frac{i f_l}{q^2 - M_\Delta{}^2 + i\tau M_\Delta} [\overline{v(2)} \frac{1+\gamma_5}{2} u(1)] \tag{A.43}$$

Now to find $|M|^2$ over all spin states we use trace theorem, and it reduces to

$$\sum |M|^2 = \frac{1}{4} \frac{f_\psi f_l^2}{(q^2 - M_\Delta{}^2)^2} \mathrm{Tr}[\frac{1+\gamma_5}{2}(\not{p}_4 + m_4)\frac{1-\gamma_5}{2}(\not{p}_3 - m_3)] \times \mathrm{Tr}[\frac{1+\gamma_5}{2}(\not{p}_2 - m_2)\frac{1-\gamma_5}{2}(\not{p}_1 + m_1)] \tag{A.44}$$

The traces yield $(p_2 \cdot p_1), (p_3 \cdot p_4)$ with some multiplivative factor. IN non-relativistic limit $(p_1 \cdot p_2)$ yields m^2 and $(p_3 \cdot p_4)$ yields $2m^2 \left(1 - \frac{M_f^2}{2m^2}\right)$

Where m is the mass of DM. Where M_f is the mass of outgoing leptons. As the momentum of the light leptons are very high compared to their masses the previous equation reduces to

The differential cross section in center of mass frame

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{cm}} = \frac{1}{2E_A 2E_B |V_A - V_B|} \left(\frac{|P_f|}{4\pi^2 4E_{cm}}\right) |M|^2 \tag{A.45}$$

finally reduces to

$$\langle\sigma |v|\rangle = \frac{1}{16\pi} \frac{(f_\psi f_l)^2}{(4m^2 - M_\Delta{}^2)^2} m^2 (1 - \frac{m_f{}^2}{m^2}) \times 6 \tag{A.46}$$

The factor 6 comes because there are 6 outgoing channels which are (ν_e, ν_e), $(\nu_\mu, \nu_\mu), (\nu_\tau, \nu_\tau), (\nu_e, \nu_\mu), (\nu_e, \nu_\tau), (\nu_\mu, \nu_\tau)$. Now this cross section formula blows up at Δ pole. So we introduced a regulator in the denominator which is essentially relevant near the resonance and otherwise the effect of it is very small. If the coupling of the Δ with higgs is given by f_h then the

cross section formula ultimately takes the form

$$\langle\sigma |v|\rangle = \frac{1}{16\pi} \frac{(f_\psi f_l)^2}{(4m^2 - M_\Delta{}^2)^2 + [\frac{1}{8\pi}(f_l{}^2 + f_h{}^2 + f_\psi{}^2) M_\Delta{}^2]} m^2 (1 - \frac{m_f{}^2}{m^2}) \times 6 \tag{A.47}$$

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