Coherent State and its Application

A Dissertation Submitted to
Indian Institute of Technology Hyderabad
In Partial Fulfillment of the Requirements for
The Degree of Master of Science

Dr.Anjan Kumar Giri

By
Sanjeev Kumar

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Department of Physics

April, 2013

Declaration

I declare that this written submission represents my ideas in my own words, and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

Sanjeev Kumar

PH11M006

Approval Sheet

This thesis entitled **Coherent State and its Application** by **Mr.Sanjeev Kumar** is approved for the degree of Master of Science from Indian Institute of Technology Hyderabad.

-Name and affiliation-
Examiner

-Name and affiliation-
Examiner

-Name and affiliation-
Adviser

-Name and affiliation-
Co-Adviser

-Name and affiliation-
Chairman

Approval Sheet

This thesis entitled **Coherent State and its Application** by **Mr.Sanjeev Kumar** is approved for the degree of Master of Science from Indian Institute of Technology Hyderabad.

Vandana Sharma
VANDANA SHARMA, IIT-H
-Name and affiliation-
Examiner

-Name and affiliation-
Examiner

-Name and affiliation-
Adviser

-Name and affiliation-
Co-Adviser

-Name and affiliation-
Chairman

(MANISH NIRANJAN)

iii

Acknowledgement

I express my profound gratitude to my supervisor, **Dr. Anjan Kumar Giri**, for his encouraging, cooperation and inspiring guidance throughout my project.

Dedicated to

My Parents, Brother, and My Sir

Abstract

In this Thesis we construct coherent states and its application in future. Pure Coherent states are known as the most Classical state in Quantum mechanics. These states minimize the quantum mechanical uncertainty between x and p obey the classical equation of motion for the harmonic oscillator. And other is that coherent state in quantum computation. This dissertation discusses mainly transmission of coherent state qubits, generation of cat states and entanglement purification of any stabilizer state. A quantum computer is any device for computation that makes direct use of distinctively quantum mechanical phenomena, such as superposition and entanglement, to perform on operation on data.

The elementary carriers in quantum computation and information are the quantum bits or qubits. In contrast to classical bits, qubits can be in every superposition of the states $|0\rangle$ and $|1\rangle$. This means that a vector describing a qubit may be any vector in a two dimensional Hilbert space. We review a method for constructing a linear optical quantum computer using coherent state of light as the qubits developed by Ralph, Gilchrist, Milburn, Munro and Clancy. We show how an universal set of logic operations can be performed using coherent states, beam splitter, photon counters and a source of superposition of coherent state called “cat states”. We also discuss the behaviour of teleportation, when a non-maximally entangled Bell state is used.

Contents

Chapter 1

Coherent state of the harmonics oscillator

In this chapter the concept of coherent states will be introduced, inspired on section of [1]. First we will investigate the harmonics oscillator in quantum mechanics. It will turn out that coherent states represent the equation of motion of the classical harmonic oscillator.

1.1. The harmonics oscillators

In classical mechanics we can talk about the position of a particle at any given time $x(t)$. The quantum mechanics analog to this is a particles wave function: $|\psi(x,t)\rangle$. This wave function has a statistical interpretation , $|\psi(x,t)|^2$ gives the probability of finding the particle at position x and time t.More precisely we could say that $\int_a^b |\psi(x,t)|^2 dx$ is the probability of finding of the particle between a and b at time t. The wave function can be obtained by the Schrodinger equation.

Definition 1.1.*The following equation is called the one dimensional Schrodinger equation*

$$ih\frac{\partial|\psi\rangle}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2|\psi\rangle}{\partial x^2} + V|\psi\rangle.$$

Here, i is the square root of -1 and $\hbar = \frac{h}{2\pi}$ with h the Planck constant.

The paradigm for a classical harmonic oscillator is a mass m attached to a spring of force constant κ. Ignoring friction the potential energy is given by $\omega = \sqrt{\frac{\kappa}{m}}$. The quantum problem is to solve the one dimensional Schrodinger equation for the potential $V(x) = \frac{1}{2}m\omega^2 x^2$. Because the potential is not the time dependent we can solve the Schrodinger equation by the method of separation of variables. For more detail about this method see section 2.1 of [3]. It suffices to solve the time independent Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2|\psi\rangle}{dx^2}+V(x)|\psi\rangle = E|\psi\rangle.$$

We can rewrite this equation with the help of the momentum operator $p=\frac{\hbar}{i}\frac{d}{dx}$, which results in

$$(\frac{p^2}{2m}+V(x))\,|\psi\rangle = E|\psi\rangle$$

$$H|\psi\rangle = E|\psi\rangle$$

Where H is called the Hamiltonian. The Hamiltonian of the harmonics oscillator is given by

$$H=\frac{1}{2m}[p^2+(m\omega x)^2].$$

The wave function of the harmonic oscillator can be determined using ladder operators:

Definition 1.2 *The following quantities are called ladder operators*:

$$a=\frac{1}{\sqrt{2\hbar m\omega}}(ip+m\omega x),$$

$$a^+=\frac{1}{\sqrt{2\hbar m\omega}}(-ip+m\omega x).$$

Here a is called the lowering operator and a^+ is called the raising.

The commentator of a and a^+ can be calculated directly from their definition

$$[a,a^+]=\frac{i}{\hbar}[p,x]=1$$

Here we used that commentator of p and x is equal to $-i\hbar$ which follows from equation (1) the operator x and p expressed in terms of the ladder operators are

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^+)$$

$$p = \frac{1}{i}\sqrt{\frac{\hbar m\omega}{2}}(a - a^+).$$

$$x^2 = \frac{\hbar}{2m\omega}(a + a^+)^2$$

$$p^2 = -\frac{m\omega\hbar}{2}(a - a^+)^2$$

We can express the Hamiltonian of the harmonic oscillator in terms of the ladder operators using that: $aa^+ = \frac{1}{2\hbar m\omega}(p^2 + (m\omega x)^2) - \frac{i}{2\hbar}[x, p] = -\frac{1}{\hbar\omega}H + \frac{1}{2}.$

Here we recognized the Hamiltonian of the harmonic oscillator and the commentator of x and p which is equal to $i\hbar$.

Now we can express the Hamiltonian H in terms of the ladder operators.

$$H = \hbar\omega(aa^+ - \frac{1}{2}).$$

And it follows that

$$[H, a] = \hbar\omega a[a^+, a] = -\hbar\omega a,$$
$$[H, a^+] = \hbar\omega a^+[a, a^+] = \hbar\omega a^+.$$

The lowering operator will always reduce the energy of the states, since

$$Ha|\psi\rangle = (aH - \hbar\omega a)|\psi\rangle = (E - \hbar\omega)a|\psi\rangle.$$

Similarly the raising operator will always raise the energy of the states, hence the name ladder operator.

$$Ha^+|\psi\rangle = (a^+H + \hbar\omega a^+)|\psi\rangle = (E + \hbar\omega)a^+|\psi\rangle.$$

The ground state of a system is the state with the lowest energy. Since the lowering operator will always reduce the energy of the state; the ground state wave function of the harmonics oscillator $|0\rangle$ must satisfy the equation $a|0\rangle = 0$.

Consequently, the ground state wave function can be determined (see section 2.3 of [3] for more detail).

$$|0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}$$

Using the raising operator the excited states $|n\rangle$ can be calculated (see section 2.3 of [3] for more details). This gives: $|n\rangle = \frac{1}{\sqrt{n!}}(a^+)^n|0\rangle$

$$a^+|n\rangle = \sqrt{n+1}|1+n\rangle,$$
$$a|n\rangle = \sqrt{n}|n-1\rangle.$$

Furthermore the wave functions are orthogonal so $\langle n|m\rangle = \delta_{nm}$

We also know from [3] that energy of the harmonic oscillator is quantized

$$H|n\rangle = E_n|n\rangle = \hbar\omega(n+\frac{1}{2})|n\rangle$$

Using the above two relations for the ladder operator acting on the wave function $|n\rangle$, and Hermite polynomials we can prove that the collection of wave function of the harmonic oscillator from basis for $L^2(\Re)$.

1.2. Coherent states

Before the coherent states will be defined, we introduced the uncertainty principle.

Uncertainty Principle: Consider a system with suitable normalized wave function $|\psi\rangle . e.g |\psi\rangle \in D = \{f(x) : x^n f^{(m)}(x) \in \Re \forall n, m \in \mathrm{N}\}$ and two symmetric operators A and B .then $(\sigma_A)(\sigma_B) \geq \frac{1}{2}|\langle[A,B]\rangle|$. this is called the uncertainty principle. The uncertainty in operator A and B is defined by $\sigma_A = \left(\langle A^2 \rangle - \langle A \rangle^2\right)^{1/2} \geq 0, \sigma_B = \left(\langle B^2 \rangle - \langle B \rangle^2\right)^{1/2} \geq 0$ and the expectation values by $\langle A \rangle = \langle \psi | A | \psi \rangle, \langle B \rangle = \langle \psi | B | \psi \rangle$.

Proof- if $\sigma_B \neq 0$ Define the following operator

$C := A - \langle A \rangle + i\lambda(B - \langle B \rangle)$ Using the operator the fact A and B is symmetric operator and the fact that $|\psi\rangle$ is normalized we obtain the following inequality for every real λ $0 \leq \langle C\psi | C\psi \rangle = \langle \psi | C^{+}C | \psi \rangle = (\sigma_A)^2 + \lambda^2 (\sigma_B)^2 + \lambda \langle i[A,B] \rangle$ the right side of the equation has a minimum for $\lambda = -\frac{1}{2}\langle i[A,B] \rangle / (\sigma_B)^2$ this minimum is equal to

$$(\sigma_A)^2 - \frac{\langle i[A,B] \rangle^2}{4(\sigma_B)^2} \geq 0$$

Rearranging gives us the uncertainty principle $(\sigma_A)(\sigma_B) \geq \frac{1}{2}\left|\langle [A,B] \rangle\right|$ when $\sigma_B = 0 and \sigma_A \neq 0$ we can obtain the uncertainty principle in the same way but with the roles of A and B interchanged .If $\sigma_A = \sigma_B = 0$ then the form of equation follows that $\langle [A,B] \rangle = 0$ because λ can be negative. This result is in according with the uncertainty principle.

Heisenberg discovered this uncertainty relation in 1926. He realized that every pair of physical properties that do not commute result in an uncertainty relation. This implication led the foundation of the contemporary quantum mechanics. In this thesis we will use the uncertainty relation with the physical properties position and momentum.

Definition 2.*4* Wave function that satisfies the Heisenberg uncertainty principle with equality are called the minimum uncertainty wave functions

The ground state wave function $|0\rangle$ of the harmonic oscillator is minimum uncertainty wave function

We need the following expectation values to prove the theorem:

$$\langle 0|(a+a^+)(a+a^+)|0\rangle = \langle 0|aa^+ + a^+a + aa^+ + a^+a^+|0\rangle$$
$$= \langle 0|aa|0\rangle + \langle 0|a^+a|0\rangle + \langle 0|aa^+|0\rangle + \langle 0|aa^+|0\rangle.$$

Because $a|0\rangle = 0 and \langle 0|2\rangle = 0$ there is only one term nonzero

$$\langle 0|aa^+|0\rangle = \langle 0|\left(\frac{H}{\hbar\omega}+\frac{1}{2}\right)|0\rangle = \frac{E_0}{\hbar\omega}+\frac{1}{2} = 1.$$

In similar way it follows that $\langle 0|(a-a^+)(a-a^+)|0\rangle = \langle 0|-aa^+|0\rangle = -1$

Now the expectation values for position and momentum can be calculated. We obtain

$$\langle x\rangle_0 = \langle 0|x|0\rangle = \sqrt{\frac{\hbar}{2m\omega}}\langle 0|(a+a^+)|0\rangle = 0$$

$$\langle p\rangle_0 = \langle 0|p|0\rangle = \frac{1}{i}\sqrt{\frac{\hbar m\omega}{2}}\langle 0|(a-a^+)|0\rangle = 0$$

Where we used that $a|0\rangle = 0$ and that the wave

Functions are orthonormal so $\langle 0|1\rangle = 0$

$$\langle x^2\rangle_0 = \langle 0|x^2|0\rangle = \frac{\hbar}{2m\omega}\langle 0|(a+a^+)^2|0\rangle = \frac{\hbar}{2m\omega}$$

$$\langle p^2\rangle_0 = \langle\ |p^2|\ \rangle = -\frac{\hbar m\omega}{2}\langle 0|(a-a^+)^2|0\rangle = \frac{\hbar\omega m}{2}$$

Now we can calculate the uncertainty in x and p which result

$$(\sigma_x)^2{}_0 = \langle x^2\rangle - \langle x\rangle^2 = \frac{\hbar}{2m\omega}$$

$$(\sigma_p)^2_0 = \langle p^2\rangle - \langle p\rangle^2 = \frac{\hbar m\omega}{2}$$

We obtain the Heisenberg uncertainty principle with equality $(\sigma_x)^2_0(\sigma_p)^2_0 = \frac{\hbar^2}{4}$

So the ground state wave function $|0\rangle$ of the harmonic oscillator is a minimum uncertainty wave function.

Definition 2.5 *The state* $|\alpha\rangle$ *defined by* $a|\alpha\rangle = \alpha|\alpha\rangle$, *with* $\langle\alpha|\alpha\rangle = 1$ *are called the coherent states*

Proof-from

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

It follows that

$$\langle\alpha|a^+ = \langle\alpha|\overline{\alpha}$$

So, $\langle\alpha|a^+a|\alpha\rangle = |\alpha|^2.$

Further more

$$\langle\alpha|a + a^+|\alpha\rangle = \alpha + \overline{\alpha}.$$
$$\langle\alpha|a - a^+|\alpha\rangle = \alpha - \overline{\alpha}.$$
$$\langle\alpha|(a + a^+)^2|\alpha\rangle = \langle\alpha|aa|\alpha\rangle + \langle\alpha|a^+a^+|\alpha\rangle + \langle\alpha|[a, a^+]|\alpha\rangle + 2\langle\alpha|a^+a|\alpha\rangle$$
$$= \alpha^2 + (\overline{\alpha})^2 + 1 + 2\alpha\overline{\alpha} = (\alpha + \overline{\alpha})^2 + 1.$$

In similar way it follows that

$$\langle\alpha|(a - a^+)^2|\alpha\rangle = \alpha^2 + (\overline{\alpha})^2 - 1 - 2\alpha\overline{\alpha} = (\alpha - \overline{\alpha})^2 - 1.$$

Now we can calculate the expectation values for position and momentum.

$$(\sigma_x)_\alpha^2 = \langle x^2 \rangle_\alpha - \langle x \rangle_\alpha^2 = \frac{\hbar}{2m\omega}\left(\langle \alpha | (a + a^+)^2 \right)\alpha\rangle - \langle \alpha | a + a^+ | \alpha \rangle$$

$$= \frac{\hbar}{2m\omega}[(\alpha + \overline{\alpha})^2 + 1 - (\alpha + \overline{\alpha})^2] = \frac{\hbar}{2m\omega},$$

$$(\sigma_p)_\alpha^2 = \langle p^2 \rangle_\alpha - \langle p \rangle_\alpha^2 = -\frac{\hbar m\omega}{2}\left(\langle \alpha | (a - a^+)^2 | \alpha \rangle - \langle \alpha | a - a^+ | \alpha \rangle\right)$$

$$= -\frac{\hbar m\omega}{2}\left[(\alpha - \overline{\alpha}^2) - 1 - (\alpha - \overline{\alpha})^2\right] = \frac{\hbar m\omega}{2}$$

whichimpli es

$$(\sigma_x)_\alpha^2 (\sigma_p)_\alpha^2 = \frac{\hbar^2}{4}.$$

So the coherent state satisfies the minimum uncertainty relation.

1.3. Coherent state in the n-representation

The wave function of the harmonic oscillator from a basis for $L^2 \in (\Re)$ this is the theorem 3.1 Therefore we can express the coherent state in the wave functions. In the $|n\rangle$ basis the coherent state $|\alpha\rangle$ is written as $|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$ multiplying this expression from the left with the bra $\langle m|$

Gives a expression for the coefficient c_m so.

$$\langle m | \alpha \rangle = \sum_{n=0}^{\infty} c_n \langle m | n \rangle = c_m.$$

Here we used the fact that the wave functions of the harmonic oscillator are orthonormal. As a result we obtain the following expression

$$|\alpha\rangle = \sum_{n=0}^{\infty} |n\rangle\langle n | \alpha\rangle$$

Now, the expression for the wave function is $|n\rangle$ then,

$$\langle n | \alpha \rangle = \frac{1}{\sqrt{n!}}\langle 0 | a^n \alpha \rangle = \frac{\alpha^n}{\sqrt{n!}}\langle 0 | \alpha \rangle$$

On combining this with expression, we obtain

$$\left|\alpha\right\rangle = \left\langle 0|\alpha\right\rangle \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \left|n\right\rangle.$$

This constant factor $\left\langle 0|\alpha\right\rangle$ must still be determined which can be done using normalization since the coherent state $\left|\alpha\right\rangle$ has to be normalized so

$$1 = \left\langle \alpha|\alpha\right\rangle = \left(\sum_{m=0}^{\infty}\left|m\right\rangle\left\langle m\right|\right)\left(\sum_{n=0}^{\infty}\left|n\right\rangle\left\langle n|\alpha\right\rangle\right).$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left\langle \alpha|m\right\rangle\left\langle m|n\right\rangle\left\langle n|\alpha\right\rangle = \sum_{n=0}^{\infty} \left\langle \alpha|n\right\rangle\left\langle n|\alpha\right\rangle$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}(\bar{\alpha})^n\alpha^n\left\langle \alpha|0\right\rangle\left\langle 0|\alpha\right\rangle = \sum_{n=0}^{\infty}\frac{1}{n!}\left|\alpha\right|^{2n}\left|\left\langle 0|\alpha\right\rangle\right|^2.$$

Here the exponential function of $\left|\alpha\right|^2$ can be recognized

$$= \left|\left\langle 0|\alpha\right\rangle\right|^2 e^{\left|\alpha\right|^2}.$$

Solving for $\left|\left\langle 0|\alpha\right\rangle\right| = e^{-\frac{1}{2}\left|\alpha\right|^2}$.

Now we know $\left\langle 0|\alpha\right\rangle = e^{-\frac{1}{2}\left|\alpha\right|^2} e^{i\beta}$. Substituting this above equation then we obtain final form.

$$\left|\alpha\right\rangle = e^{-\left|\alpha\right|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \left|n\right\rangle$$

The constant phase factor $e^{i\beta}$ is left out because it does not contribute to the expectation value of the wave function since $\left|e^{i\beta}\right|^2 = 1$ and every multiple of a coherent state by a nonzero constant factor is still a coherent state. We can check that the coherent state $\left|\alpha\right\rangle$ is indeed orthonormal:

$$\left\langle \alpha|\alpha\right\rangle = e^{-\left|\alpha\right|^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\alpha^n(\bar{\alpha})^m}{\sqrt{n!m!}}\left\langle m|n\right\rangle = e^{-\left|\alpha\right|^2}\sum_{n=0}^{\infty}\frac{\left|\alpha\right|^{2n}}{n!} = e^{-\left|\alpha\right|^2}e^{\left|\alpha\right|^2} = 1$$

The coherent state $\left|\alpha\right\rangle$ is not an Eigen function of the harmonic oscillator which can be seen from

$$H|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}H|n\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}E_n|n\rangle \neq \lambda|\alpha\rangle \text{ With } \lambda \in C$$

The coherent state $|\alpha\rangle$ can be expressed in terms of the displacement operator $D(\alpha)$ which is given by $D(\alpha) = e^{\alpha a^+ - \overline{\alpha} a}$ and we know that

$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$. This gives us:

$$= e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}(a^+)^n|0\rangle = e^{-\frac{1}{2}|\alpha|^2}e^{\alpha a^+}|0\rangle$$

Where we recognized the exponential function of αa^+.

To rewrite this expression we need the Baker-Campbell-Hausdorff formula which state that if X and Y are Hilbert space operator that both commute with $[X,Y]$ then $e^{X+Y} = e^{-\frac{1}{2}[X,Y]}e^X e^Y$. We apply this formula on the displacement operator with $X = \alpha a^+$ and $Y = -\overline{\alpha} a$

There commentator is $[\alpha a^+, -\overline{\alpha} a] = -|\alpha|^2[a^+, a] = |\alpha|^2, and [a, a^+] = 1$

So the coherent state $|\alpha\rangle$ is equal to the displacement operator $D(\alpha)$ operating on the ground state of the harmonic oscillator.

1.4. Orthogonally and completeness relations

We can calculate the overlap between two coherent states. Let $|\alpha\rangle and |\beta\rangle$ be two coherent states so $a|\alpha\rangle = a|\alpha\rangle$ and $a|\beta\rangle = \beta|\beta\rangle$ then these two states can be expressed by

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\langle n|, and |\beta\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{m=0}^{\infty}\frac{\beta^m}{\sqrt{m!}}|m\rangle$$

Then the overlap is calculated using

$$\langle\alpha|\beta\rangle=\left(e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{(\overline{\alpha})^n}{\sqrt{n!}}\langle n|\right)\left(e^{-\frac{1}{2}|\beta|^2}\sum_{m=0}^{\infty}\frac{\beta^m}{\sqrt{m!}}|m\rangle\right)$$

$$=e^{-\frac{1}{2}|\alpha|^2}e^{-\frac{1}{2}|\beta|^2}\sum_{n=0}^{\infty}\frac{(\overline{\alpha})^n\beta^n}{n!}=e^{-\frac{1}{2}|\alpha|^2}e^{-\frac{1}{2}|\beta|^2}e^{\overline{\alpha}\beta}$$

And similarly $\langle\beta|\alpha\rangle=e^{-\frac{1}{2}|\alpha|^2}e^{-\frac{1}{2}|\beta|^2}e^{\overline{\beta}\alpha}$

So that the overlap is given by

$$|\langle\alpha|\beta\rangle|^2=\langle\beta|\alpha\rangle\langle\alpha|\beta\rangle=e^{-|\alpha|^2-|\beta|^2+\overline{\alpha}\beta+\alpha\overline{\beta}}=e^{-|\alpha-\beta|^2}.$$

Suppose a system is in quantum state $|\alpha\rangle$, then there is a nonzero chance that the system is in quantum state $|\beta\rangle$ because $|\langle\alpha|\beta\rangle|^2\neq 0 if \alpha\neq\beta$ consequently, since $\langle\alpha|\beta\rangle\neq 0 if \alpha\neq\beta$ the collection of coherent states forms an over complete set. The number of coherent state is grater then the needed number for a basis.

Nevertheless there is closure relation:

$$\int_C d^2\alpha|\alpha\rangle\langle\alpha|=\int_C d^2\alpha e^{-|\alpha|^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(\overline{\alpha})^n\alpha^m}{\sqrt{n!m!}}|m\rangle\langle n|$$

Now writing α in polar form $\alpha=re^{i\phi}$ and $d^2\alpha=rdrd\phi$ gives:

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{|n\rangle\langle m|}{\sqrt{n!m!}}\int_0^{2\pi}e^{i\phi(m-n)}d\phi\int_0^{\infty}re^{-r^2}r^{(n+m)}dr$$

$$=\sum\frac{|n\rangle\langle n|}{n!}2\pi\int_0^{\infty}re^{-r^2}r^{2n}dr$$

Change variable from r to $x=r^2$ then $dx=d(r^2)=2rdr$ and we obtain

$$=\sum_{n=0}^{\infty}\frac{|n\rangle\langle n|}{n!}2\pi\int_0^{\infty}\frac{1}{2}e^{-x}x^n dx=\pi\sum_{n=0}^{\infty}|n\rangle\langle n|=\hat{1}\pi$$

Here, $(n+1) \in \mathrm{N} - \{0\} so \Gamma(n+1) = n!$

We used the closure relation from corollary 3.1 to obtain this result.

We can conclude that the closure relation for coherent state is given by

$$\int_C \frac{d^2\alpha}{\pi} |\alpha\rangle\langle\alpha| = \hat{1}$$

1.5. Time evolution of coherent states

In this section we will investigate the time evolution of a coherent state. It turns out that coherent state remains coherent under time evolution.

The time evolution of a state is given by the Schrodinger equation:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t)|\psi(t)\rangle.$$

Here $H(t)$is the Hamilton operator.

The Schrodinger equation is a first order differential equation when a state $|\psi(t)\rangle$ is known on a time $t = t_0$then the state can be determined for every t.

Definition 2.9.*The* time evolution operator $U(t, t_0)$*gives the time evolution of a state .It has following properties*: $|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle$

$$U(t, t_0) = \hat{1}, \forall_{t1} U(t, t_0) = U(t, t_1)U(t_1, t_0), U^{-1}(t, t_0)$$

According the equation,

$$i\hbar \frac{\partial}{\partial t} U(t, t_0)|\psi(t_0)\rangle = H(t)U(t, t_0)|\psi(t_0)\rangle$$

So that,

$$i\hbar \frac{\partial}{\partial t} U(t, t_0) = H(t)U(t, t_0),$$

With precondition $U(t,t_0)=\hat{1}$.

The Hamiltonian of the harmonic oscillator $H=\frac{1}{2m}[p^2+(m\omega x)^2]$ *is* time independent. So, $\frac{\partial}{\partial t}H(t)=0$

Therefore the differential equation has a direct solution given by:

$$U(t,t_0)=e^{-(t-t_0)H/\hbar}$$

So the time evolution of state is:

$$|\psi(t)\rangle=e^{-(t-t_0)H/\hbar}|\psi(t_0)\rangle$$

This expression can be used to determine the time evolution of a coherent state. We use the expression to define the coherent state at $t_0=0$ as:

$$|\alpha(0)\rangle=e^{-\frac{1}{2}|\alpha(0)|^2}\sum_{n=0}^{\infty}\frac{\alpha(0)^n}{\sqrt{n!}}|n\rangle$$

The coherent state at arbitrary time t is found by applying the equation resulting in:

$$|\alpha(t)\rangle=U(t,t_0)|\alpha(0)\rangle=e^{-\frac{i}{\hbar}Ht}|\alpha(0)\rangle=e^{-\frac{1}{2}|\alpha(0)|^2}\sum e^{-\frac{i}{\hbar}Ht}\frac{\alpha(0)^n}{\sqrt{n!}}|n\rangle.$$

Since the wave function $|n\rangle$ are the eigen state of the Hamiltonian with eigenvalue E_n we obtain:

$$=e^{-\frac{1}{2}|\alpha(0)|^2}\sum e^{-\frac{t}{\hbar}E_n t}\frac{\alpha(0)^n}{\sqrt{n!}}|n\rangle.$$

Here $E_n=\hbar\omega\left(n+\frac{1}{2}\right)$ substituting $|n\rangle$ from result in:

$$= e^{-i\omega t/2} e^{-\frac{1}{2}|\alpha(0)|^2} \sum_{n=0}^{\infty} \frac{[\alpha(0)(a^+)e^{-it\omega}]^n}{n!} |0\rangle$$

$$= e^{-i\omega t/2} e^{-\frac{1}{2}|\alpha(0)|^2} e^{\alpha(0)(a^+)} e^{-i\omega t} |0\rangle$$

We recognized an exponential function; no we rewrite the obtained expression using that: $|e^{i\omega t}|^2 = 1$ Then:

$$= e^{-i\omega t/2} \left(e^{-\frac{1}{2}|\alpha(0)|^2} |e^{i\omega t}|^2 + \alpha(0) e^{-i\omega t} (a^+) |0\rangle \right).$$

Comparing the expression between the parentheses with equation, then we see that this gives a coherent state with the time dependent eigenvalue $e^{-i\omega t}\alpha(0)$

$$= e^{-i\omega t/2} | e^{-i\omega t}\alpha(0)\rangle.$$

We can conclude that a coherent state remain coherent under time evolution.

Furthermore

$$\alpha(t) = e^{-i\omega t}\alpha(0).$$

This implies that $\frac{d}{dt}\alpha(t) = -i\omega\alpha(t).$

This differential equation can be rewritten using the real part of $\alpha(t)$, denoted by $\Re(\alpha(t))$ and the imaginary part $\mathrm{I}(\alpha(t))$. These are defined by:

$$\Re(\alpha(t)) = \frac{1}{2}(\alpha(t) + \alpha(\bar{t})) = \langle \alpha(t) | \frac{a + a^+}{2} | \alpha(t) \rangle$$

$$\mathrm{I}(\alpha(t)) = \frac{1}{2i}(\alpha(t) - \alpha(\bar{t})) = \langle \alpha(t) | \frac{a - a^+}{2i} | \alpha(t) \rangle.$$

Then above third equation follows that:

$$\frac{d}{dt}\Re(\alpha(t)) = \omega \mathrm{I}(\alpha(t)),$$
$$\frac{d}{dt}\mathrm{I}(\alpha(t)) = -\omega \Re(\alpha(t)),$$

Therefore the expectation values for position and momentum are given by:

$$x_c(t) = \langle \alpha(t)|x|\alpha(t)\rangle = \sqrt{\frac{2\hbar}{m\omega}}\Re(\alpha(t)),$$
$$p_c(t) = \langle \alpha(t)|p|\alpha(t)\rangle = \sqrt{2m\omega\hbar}\,\mathrm{I}(\alpha(t)),$$

Here the subscript c stands for classical. Combining these expectation values with expression into the following differential equations:

$$\frac{d}{dt}x_c(t) = \sqrt{\frac{\hbar}{2m\omega}}2\frac{d}{dt}\Re(\alpha(t)) = \sqrt{\frac{h}{2m\omega}}2\omega \mathrm{I}(\alpha(t)) = \frac{p_c(t)}{m},$$
$$\frac{d}{dt}p_c(t) = \frac{1}{i}\sqrt{\frac{\hbar m\omega}{2}}2i\frac{d}{dt}\mathrm{I}(\alpha(t)) = -\sqrt{\frac{\hbar m\omega}{2}}2\omega\Re(\alpha(t)) = m\omega^2 x_c(t).$$

After rewriting and introducing $v_c(t) = \frac{d}{dt}x_c(t)$ we obtain a more familiar from:

$$p_c(t) = m\frac{d}{dt}x_c(t) = mv_c(t).$$
$$\frac{d}{dt}p_c(t) = -m\omega^2 x_c(t).$$

So the equations of motion for the classical harmonic oscillator are valid in terms of the quantum mechanical expectation values for x and p. We could have expected this because of the following theorem:

Chapter 2

Exploring Macroscopic Entanglement with a single photon and coherent states

Entanglement between macroscopically populated states can easily be created by combing a single photon and a bright coherent state on beam splitters motivated by the simplicity of this technique. We report on method using displacement operations in the phase space and basics photon detections to reveal such an entanglement. We demonstrate through preliminary experimental result that this eminently feasible approach provides an attractive way for exploring entanglement at various scales ranging from one to a thousand photons. This offers an instructive view point to gain insight into the reasons that make it hard to observe quantum feature in our macroscopic.

2.1. INTRODUCTION

Why do we not easily observe entanglement between macroscopically populated systems? De-coherence is widely accepted as being responsible [1] . Loss or any other form of interactions with the surroundings more and more rapidly destroys the quantum features of physical systems as their size increase .Technologically demanding experiments involving Rydberg atoms interacting with electromagnetic field of a high- fines cavity [2]or superconducting device cooled down to a few tens of mK [3] have strengthened this idea.

Measurement precision is likely another issue [4] . In a recent experiment [5] a phase covariant cloner has been used to produce ten thousand clones of a single photon belonging initially to an entanglement pair. In the absence of loss this leads to a micro-macro entanglement states [6]. Nobody knows however how the entanglement degrades with loss amplification [7]. This led to a lively debate [8] concerning the presence of entanglement in the experiment reported in Ref. [5] . What is known is that under moderate coarse grained measurement, the micro-macro entanglement resulting from a lossless amplification leads to probability distribution of results that is very close to the one coming from a separates micro-macro state [9]. This suggests that even if micro system could be perfectly isolated from its environment its quantum nature would require very precise measurement to be observed.

Both for practical consideration and from a conceptual point of view it is of great interest to look for ways as simple as possible to generate and measure macro entanglement so that the effect of de-coherence process and the requirements on the measurements can all be studied to get them. In this letter we focus on an approach based on linear optics only where a single photon and a coherent state are combined on a mere beam splitter. The resulting path entanglement state [10] allows one to easily explore entanglement over various photon scales spanning from the micro to the macro domain by simply tuning the intensity of the laser producing the input coherent state. We show that entanglement is more and more sensitive to phase fluctuations between the paths when grows. However it features surprising robustness against loss making it well suited to travel over long distances and to be stored in atomic ensemble. We further present a simple and natural method relying on local displacement operations in the phase space and basics photon detection to reveal this entanglement. Our analysis shows that the precision of the proposed measurement is connected to the limited ability to control the phase of the local oscillator that is used to perform the phase space displacements. We also report on preliminary experimental results demonstrating that entanglement containing more than a thousand photons could be created and measured with currently available technology.

2.2. Creating macro entanglement by combing a single photon with a bright coherent states on a beam splitter

A particular simple way to generate the entanglement we will use a beam splitter. Let a single photon sent through a 50:50 beam splitter. The beam splitter occupied the two output modes A and B with same probability and creates the simple form of entanglement between spatial modes $\frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B - |0\rangle_A|1\rangle_B)$ Known as single photon entanglement[11]. Any product input states of the form $\rho_A \otimes |\beta\rangle\langle\beta|_B \, where |\beta\rangle$ is the coherent states leads to the entanglement after the beam splitter .If and only if ρ_A is non-classical[12],[13] It cannot be written in the form of mixture coherent states[14].Thus a mere beam splitter links two fundamental concepts of quantum physics: non classicality and entanglement. It also gives an attractive way for bringing entanglement to macroscopic level. As explained below.

$$|\psi_{in}\rangle = a^{+}|0\rangle_A \otimes D_b(\sqrt{2}\alpha)|0\rangle_B$$

Let us focus on the beam splitter inputs are a

single photon $|1\rangle$ and a coherent state with $2|\alpha|^2$ photon on Creation and detection of macro entanglement by combining a single photon Fock state $|1\rangle$ and a coherent state $|\sqrt{2}\alpha\rangle$ on a 50:50 beam splitter. Photon on average and $D_b(\sqrt{2}\alpha) = e^{\sqrt{2}(\alpha b^{+} - \alpha^{*} b)}$ Is the displacement operator generating a coherent states $|\sqrt{2}\alpha\rangle$ from vacuum, a and b are Bo sonic annihilation operator associated with modes A and B respectively. A 50:50 beam splitter transforms (a,b) into $((a-b)/\sqrt{2},(a+b)/\sqrt{2}$ .Since a and b commute the output states

$|\psi_{out}\rangle = \frac{1}{\sqrt{2}}(D_a(\alpha)|1\rangle_A|\alpha\rangle_B - |\alpha\rangle_A D_b(\alpha)|1\rangle_B)$ The structure of these states is very simple and follows from displacement of the single photon entanglement. It corresponds to a non -Gaussian states which describes the entanglement of two modes and each mode showing individually a mixture of classical and quantum states. The average number of photons $2|\alpha|^2 + 1$ can easily adjusted by varying the amplitude of the initial coherent states. This allows for the exploration of entanglement at various scales ranging from a single photon to macroscopic photon number.

2.3. Robustness with respect to transmission loss

In general entanglement is seen to be increasingly fragile to transmission loss as it size increases. The coherent states entanglement $|\alpha\rangle_A|\alpha\rangle_B + |-\alpha\rangle_A|-\alpha\rangle_B$ [15]

provides a good example. If the mode B is subjected to loss (modeled by a beam splitter with transmission coefficient η_t) the amount of entanglement measured by the negativity (see $[16,17]$decreases exponentially $N = \frac{1}{2}e^{-2(1-\eta_t)|\alpha|^2}$ with the size of $|\alpha|^2$ and loss $1-\eta_t$ $[18]$.Under the assumption that the mode B undergoes loss. $|\psi_{out}\rangle$ Becomes a statistically mixture of $\frac{1}{\sqrt{1+\eta_t}} D_a(\alpha) D_b(\sqrt{\eta_t}\alpha)(|1\rangle_A \langle 0|_B - \sqrt{\eta_t}|0\rangle_A |1\rangle_B)$ and $D_a(\alpha) D_b(\sqrt{\eta_t}\alpha)|0\rangle_A |0\rangle_B$ with weight $\frac{1+\eta_t}{2}$ *and* $\frac{1-\eta_t}{2}$. After applying the local displacement operator $D_a(-\alpha) and D_b(-\sqrt{\eta_t}\alpha)$ for two modes A and B. And $\mathrm{N} = \frac{\eta_t}{2}$ since the entanglement cannot increase through the local displacement operations this provides a lower bound for the entanglement before the displacements such that between the macroscopically populated states. Therefore the amount of entanglement in $|\psi_{out}\rangle$ decays linearly with independently loss of its size. This robustness may be understood in the light of the intimate link between non-classically and entanglement at bean splitter as mentioned before indeed loss that is modeled by abeam splitter, can be seen as an interaction process that entangles the non-classical states and the environment. However the displacement is classical operation that does not promote the entanglement of a given quantum systems with its environment when it is amplified $(D_b(\alpha) \rightarrow D_b(\sqrt{\eta}\alpha) D_E(\sqrt{1-\eta\alpha}))$.The robustness of the state make it well suited for storage in atomic medium for example. Entanglement between two ensembles containing each a macroscopic number of atoms have been successfully created by mapping a single photon entanglement into two atomic ensembles $[19,20]$. the storage of the displaced single photon entanglement $|\psi_{out}\rangle$would lead to a similar entanglement in terms of the number of ebits but it would contain a macroscopic number of excited atoms.

2.4. Robustness with respect to coupling inefficiency

The starting point in our scheme is the creation of a single photon. It is thus natural to ask how the Resulting macro entanglement degrades when the single photon is subjected to loss .For comparison consider micro-macro entanglement obtained by amplifying one photon of entangled pair [6] with an optimal universal cloner [21]. Such entanglement can be revealed even if the amplification is followed by arbitrary large loss [22]. The states become separable as soon as the overall coupling efficiency η_c before the cloner is lower than $\frac{n}{n+1}$, n being the average number of photons in the macro component [22]. One can show following the lines presented in the previous paragraph that the negativity of the displaced single photon entanglement scales like $\mathrm{N} = -\frac{1}{2}(1-\eta_c - \sqrt{1-2(1-\eta_c)\eta_c}) \approx \eta_c{}^2/4 + Q(\eta^3{}_c) \geq 0$ where η_c stands for the coupling efficiency of the input single photon.

2.5. Robustness with respect to phase instabilities

Another DE coherence process for path entanglement is associated with the relative phase fluctuations due to vibrations and thermal fluctuations. If the two optical path corresponding to A and B acquire a phase difference φ the displaced single photon entanglement becomes $\left|\psi^{\varphi}{}_{out}\right\rangle = \frac{1}{\sqrt{2}}(D_a(\alpha e^{i\varphi})e^{i\varphi}\left|1\right\rangle_A \left|\alpha\right\rangle_B - \left|e^{i\varphi}\alpha\right\rangle_A D_b(\alpha)\left|1\right\rangle_B)$. If φ varies from trial to trial the states $\left|\psi^{\varphi}{}_{out}\right\rangle\left\langle\psi^{\varphi}{}_{out}\right|$ has to be averaged over probability distribution $p(\varphi)$ associated to the phase noise. The question of the sensitivity of the displaced single photon entanglement with respect to phase instability thus reduced to a measure of the entanglement contained in $\overline{\rho} = \int d\varphi p(\varphi)\left|\psi^{\varphi}{}_{out}\right\rangle\left\langle\psi^{\varphi}{}_{out}\right|$. The negativity of this state can easily be obtained numerically by projecting $\overline{\rho_{out}}$ in to a finite dimensional Hilbert space. To derive an analytical lower bound on the negativity of this state, we first notice that for any density matrix ρ and any vector $\left|v\right\rangle$ the following inequality holds $\left\langle v\right|\rho^{\Gamma}\left|v\right\rangle \geq \lambda_{\min} \geq -N_{\rho}$. Where Γ is partial transposition and $\lambda_{\min}$ is the smallest eigen value of ρ^{Γ} . For the states where $p(\varphi) = \delta(0)$, It is easy to verify that vector $|v\rangle$ saturating the inequality is $\frac{1}{\sqrt{2}} D_a(\alpha) D_b(\alpha)(|0\rangle|0\rangle - |1\rangle|1\rangle)$. For a general $\overline{\rho_{out}}$ is not optimal however it provides a lower bound for the estimation of N. We find $N_{\bar{\rho}_{out}}(\delta\varphi, |\alpha|^2) \geq \int d\varphi p(\varphi) \frac{e^{-2\alpha^2(2-\cos\varphi)}}{2} (\alpha^2(1-\cos(2\varphi)) - \cos\varphi$ .For a Gaussian probability distribution $p(\varphi)$ with variance $\delta\varphi$, the lower bound can be approximated by $\frac{2-\delta\varphi}{4(1+2|\alpha|^2 \delta\varphi)^{\frac{3}{2}}}$ reveals what is expected from a macroscopic quantum state: The larger the size $2|\alpha|^2 + 1$ of the state is the more it becomes sensitive to phase noise.

2.6. Revealing displaced single photon entanglement

So we have discussed the properties of the displaced single photon entanglement. We now present a simple way to reveal it. The basic idea is to displace each of the electromagnetic field describing the modes A and B by $-\alpha$. Such a displacement in the phase space can be easily performed by mixing the mode to be displaced with an auxiliary strong coherent field on a highly unbalanced beam splitter in a manner similar to homodyne measurements .Since $D_a(-\alpha) = D_a(\alpha)^{-1}$ the modes A and B ideally end up in the state which can be revealed by tomography using a single photon detectors.

In particular the approach developed in reference (C.W.Chouet.al.Nature (London) 438,828(2005) does not require a full tomography after the local displacements. It gives a lower bound on the entanglement between modes A and B from the estimation of the entanglement contained in the two qubit subspace $\{|00\rangle, |01\rangle, |11\rangle, |10\rangle\}$. More precisely the concurrence C of the detected fields is bounded by $C \geq \max\{0, V(p_{01} + p_{10}) - 2\sqrt{p_{00} p_{11}}\}$. Where V is the visibility of the interference obtained by recombining the modes A and B on a 50:50 beam splitter ,and the coefficient p_{mn} are the probability of detecting m photons in A and n in B.

This tomography approach for characterizing single photon entanglement is attractive in practices and already triggered highly successful experiments.

The statistical fluctuations in the phase of local oscillator that are used to perform the displacement ,limit the precision of the measurement process under the assumption that the two local displacements $D_a(-\alpha), D_b(\alpha)$ are performed with a common local oscillator the measured state is of the form $\int d\overline{\varphi}\,\overline{p}(\overline{\varphi}) D_a(-\alpha e^{i\overline{\varphi}}) D_b(-\alpha e^{i\overline{\varphi}}) \rho_{out} D_a^{+}(-\alpha e^{i\overline{\varphi}}) D_b^{+}(-\alpha e^{i\overline{\varphi}}) where\, \overline{p}(\overline{\varphi})$ stands for the phase noise distribution and $\rho_{out} = |\psi_{out}\rangle\langle\psi_{out}|$ .let $\overline{V}$ be the visibility of the interference that characterizes the phase stability of the local oscillator .One can show that for small imperfections $\varepsilon = (1-\overline{V}) << 1,$ the concurrence is bounded by $C \geq \max\left\{0, 1-10(1-\overline{V})|\alpha|^2\right\}$.The necessary precision of the measurement thus scale as $\frac{1}{\varepsilon} = \frac{1}{10|\alpha|^2}$ .This result strengthens the idea that precise measurements are generally essential for revealing quantum properties of macro systems.

2.7. Proposed experiment

Our main aim was to realization of the single photon source from a pair source based on spontaneous parametric down conversion to detect the single photon heralding the production of its twin. The heralding photon can be made indistinguishable from the one of a coherent states emitting by a laser. Let η_c is a coupling efficiency of the single photon and η_t is the global detection efficiency including the transmission from the 50:50 beam splitter to the detector .If the heralding efficiency is small and parametric process is weakly pumped then the success probability for the emission of single photon pair is small. The concurrence is bounded by $C \geq \max\left\{0, \eta - 2\sqrt{2\eta(1-\eta)}\sqrt{\varepsilon\eta_t|\alpha|^2} - 2(2+3\eta)\varepsilon\eta_t|\alpha|^2\right\}$. Here $\eta = \eta_c\eta_t and \varepsilon = 1-V$ where V is the interferometry visibility that characterized the phase stability of A and B the local oscillator. To determine the value of visibility that we can obtain in practices we built a balanced Mach-Zehnder interferometer and by this we find the value of visibility. Let the coupling efficiency $\eta_c = 50\%$ and detection efficiency $\eta_t = 60\%$ the concurrence remains positive $C \approx 0.01$ for $|\alpha| = 28$. So this translates into entanglement populated by more than $(2|\alpha|^2 + 1) = 1500$ photons.

When we tuning the wave length the phase of the interferometer can be tuned continuously. The quality of the interference can be probed using a probe laser. Because, the wave length of probe laser is fixed. Measure power (in dBm) and the probe detector is the function of the time.

2.8. Conclusion

We have proposed a scheme to create and revealing macroscopic entanglement with a single photon coherent states and linear optical elements. But it give a question that resulting states are macroscopic states. We have shown through experimental results that entangled state that could be obtained with currently available technology would involve a large enough number of photons to be seen with the naked eye. If macroscopic means sensitive to DE coherence and highlight the complexity of possible interaction between a given quantum systems and its surroundings.

Chapter -3

Elementary gates for quantum information with superposed coherent states

3.1. Quantum Bit

The bit is the fundamental concept of classical computation and in formation. Each bit has two possible values: 0 and 1.The elementary carriers in quantum computation and information are the quantum bits, or qubits. In contrast to classical bits qubits can be in every superposition of the state $|0\rangle$ and $|1\rangle$. This means that a vector describing a qubit may be any vector in a two dimensional Hilbert space:

$$|Q\rangle = \mu|0\rangle + \nu|1\rangle = \mu\begin{pmatrix}1\\0\end{pmatrix} + \nu\begin{pmatrix}0\\1\end{pmatrix} \quad (1.1)$$

Where μ and ν are complex numbers and $|\mu|^2 + |\nu|^2 = 1$

And $|0\rangle$ and $|1\rangle$ from an orthonormal basis for this Hilbert space which is referred to as the computational basis.

A geometric representation of a qubit can be done using an unit three dimensional sphere called Bloch sphere. We can write equation (1.1) in the following form

$$|Q\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle \quad (1.2)$$

Where the number θ and ϕ define a point on the Bloch sphere. The Bloch sphere provides a good visualization of the state of a qubit, but be must keep in mind that the use of the Bloch sphere is limited since there is no simple generalization of it for multiple qubits.

In classical computation if we have two bits we would have four possible states given by 00,01,10 and 11.Correspondingly a two qubit system has four computational basis states denoted $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$ A pair of qubits can also exist in super positions of these four states such that the state vector describing the two qubits is:

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

$$= \alpha_{00}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \alpha_{01}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + \alpha_{10}\begin{pmatrix}0\\0\\1\\1\end{pmatrix} + \alpha_{11}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \quad (1.3)$$

After a measurement the states of the qubit is $|x\rangle$ $(x = 00,01,10,11)$ with probability $|\alpha_x|^2$.

A very important two qubit states is the Bell states or EPR Pair.

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad (1.4)$$

This state is a key ingredient in quantum teleportation as we will see in section 1.5 and the prototype for many other interesting quantum states. States like Bell states have been the subjected of intense study since the famous paper by Einstein Podolsky and Rosen [22] in which they pointed out the strange properties of these states .If we measure the first qubit be obtain 0 with probability $1/2$ leaving the post measurement states $|\phi'\rangle = |00\rangle$, and 1 with probability $1/2$ leaving $|\phi'\rangle = |11\rangle$.As result a measurement of the second qubit always gives the same result as the measurement of the qubit. The measurement outcomes are correlated. In 1964 John Bell proved that these measurement correlations are stronger than could ever exist between classical systems [4]

3.2. Qubit Gates

Classical computers operate on a string of input bits and return a string of output bits. The function in between can be described as a logical circuit consisting of wires and logic gates. The wires carry information around the circuit and the logic gates performs simple computational task. A logic gates is a function $f:\{0,1\}^k \to \{0,1\}^l$ from some fixed number k of input bits to some fixed number l of output bits. The circuit model for the quantum computer is actually very similar to the classical circuit model. The input –output function is replaced by a quantum operation taking quantum states into quantum states.

3.2.1. One qubits gate

The operation on one qubit must preserve its norm and are described by a 2×2 unitary matrices. Of these some of the most important are the Pauli's matrices defined by:

$$\sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_y = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(1.5)

Notice that X is the quantum NOT gate. It takes the states $|0\rangle$ and replaces it by the $|1\rangle$ vice -versa. The Z gate leaves $|0\rangle$ unchanged and flip the sign of $|1\rangle$ to give $-|1\rangle$.The Pauli matrices are mutually anti-commuting and the square to the identity.

$$\sigma_k\sigma_l + \sigma_l\sigma_k = 2\delta_{kl}\mathrm{I},$$

Where k and l can be x, y and z.

Another useful single qubit operation is the Hadamard gate is defined by

$$\mathrm{H}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

This turns a $|0\rangle$ into $\frac{(|0\rangle+|1\rangle)}{\sqrt{2}}$ (half way between $|0\rangle$ and $|1\rangle$), and turns $|1\rangle$ into $\frac{(|0\rangle-|1\rangle)}{\sqrt{2}}$ (which is also half way between $|0\rangle$ and $|1\rangle$). Simple algebra shows that $H^2 = \mathrm{I}$, and thus applying H twice to a state does nothing to it.

Three useful operations are created when the Pauli matrices are exponentiated. The rotation operators about the $\hat{x}, \hat{y},$ and $\hat{z}$ axes by an angle θ, are defined by

$$R_x(\theta) = e^{-i\theta X/2} = \cos\left(\frac{\theta}{2}\right)\mathrm{I} - i\sin\left(\frac{\theta}{2}\right), X = \begin{bmatrix} cos\frac{\theta}{2} & -isin\frac{\theta}{2} \\ -isin\frac{\theta}{2} & cos\frac{\theta}{2} \end{bmatrix}$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos\left(\frac{\theta}{2}\right)\mathrm{I} - i\sin\left(\frac{\theta}{2}\right), Y = \begin{bmatrix} cos\frac{\theta}{2} & -sin\frac{\theta}{2} \\ sin\frac{\theta}{2} & cos\frac{\theta}{2} \end{bmatrix}$$

$$R_z(\theta) = e^{-i\theta Z/2} = \cos\left(\frac{\theta}{2}\right)\mathrm{I} - i\sin\left(\frac{\theta}{2}\right), Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

$R_i(\theta)$ Rotates the Bloch sphere vector by an angle θ about the axes i.As we expect these rotation share the property that $R_i(\theta)R_i(\gamma) = R_i(\theta + \gamma)$, where $i = x, y$ or y

Any unitary operation $U(\alpha, \beta, \gamma, \delta)$ on a single qubit can be expressed using four angles α, β, γ and δ:

$$U(\alpha, \beta, \gamma, \delta) = \begin{bmatrix} e^{i(\alpha - \beta/2 - \delta/2)} \cos\frac{\gamma}{2} & -e^{(\alpha - \beta/2 + \delta/2)} \sin\frac{\gamma}{2} \\ e^{i(\alpha + \beta/2 - \delta/2)} \sin\frac{\gamma}{2} & e^{i(\alpha + \beta/2 + \delta/2)} \cos\frac{\gamma}{2} \end{bmatrix}$$

(1.11)

By direct multiplication be can verify that

$$U(\alpha, \beta, \gamma, \delta) = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

(1.12)

The factor $e^{i\alpha}$ has no physical significance. Therefor the set of all R_z and R_y rotations is a universal set of single qubit operation. We can find similar decomposition of $U(\alpha, \beta, \gamma, \delta)$ using R_x and R_y or R_x and R_z

3.2.2. Two Qubit gates

The most useful of two qubit gates is the controlled not (CNOT) gate. This gate has two input qubits, the control qubits and the target qubit. The CNOT applies the X operator to the target qubit flipping it, if the control qubit is in the state $|1\rangle$.When the control is in the $|0\rangle$ state, the target does not change. The circuit representation of CNOT gate is shown following. The CNOT is written as a 4×4 matrix given by:

$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (1.13)$$

Notice that ordering of the computational basis states is $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

Another important two qubit controlled is the controlled sign –flip or C-Z. This gate is applied the Z operator to the target qubit when the control qubit is $|1\rangle$

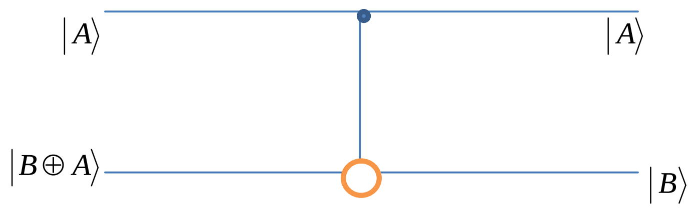

Figure.1: Circuit representation of the CNOT gate.

The "target" and "control "are symmetric foe C-Z .It is represented by the matrix:

$$U_{C-Z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1.14)$$

Note that the Hadamard gate can be used to change the C-Z into the CNOT gate

$HZH = X$ And $HH = 1$

3.2.3. Universal Logic Operation

A set of gates is considered to be universal for quantum computation if any unitary operation may be approximated (too arbitrary) accuracy by a quantum circuit involving only those gates. The CNOT gate and single qubit transformations form a universal set for quantum computation, as was proved in [56].

3.3.4. A Universal set of Quantum gates

A quantum computer must be able to transform any set of input qubits to any vector in the Hilbert space containing the qubits. We described some one –and two qubit gates used to manipulates qubit. Note that requirement give us a universal quantum computer, capable of performing any unitary transformation on qubits. This requirement can be relaxed if we want to design a device to execute a particular algorithm, not a universal quantum computer.

3.2.5. A qubit –specific measurement capability

After being prepared and undergoing some unitary operator evolution through logic gates, the qubits need to be measured. For that we require the ability to measure specific qubits. In an ideal measurement we detect one of the two computational basis states with certainty. Such ideal measurement is said to have 100% efficiency. Real measurement always have less than 100% efficiency, but it is not really necessary for quantum computation .measurement should not occur when not desired, otherwise they can be a decoherence process. In the coherent state quantum computer and many other measurements has an important and expanded role. In this scheme of computation measurements are required to perform many of the logic gates. In some case they signal the success or failure of the gates. In other cases they even tell which logic gate has been applied to the qubit.

3.3. Quantum Teleportation

Quantum teleportation is a technique for moving quantum states from one place to another, even in the absence of a quantum communication channel linking the sender and the recipient of the quantum states.It is very useful tool and it plays a key role in some of the current optical quantum computer proposals. Quantum teleportation was first described by Bennett at all-in 1993[5].

Let us imagine that Alice and Bob met some time ago and generated an EPR pair. Each taking one qubit of the EPR pair when they separated. Now Alice wants to send a single qubit whose state she does not know to Bob. She can use only classical channels and we know that if she tries to perform a measurement on the qubit and call to Bob on the phone or send a letter to convey him the result of her measurement. She would not be able to know the full state of the qubit $|Q\rangle = \mu|0\rangle + \nu|1\rangle$. Fortunately for Alice, quantum teleportation is a way of utilizing the entangled EPR pair in order to send $|Q\rangle$ to Bob, with a small overhead of classical communication. What Alice needs to do first is interact the qubit $|Q\rangle$ with her half of the EPR pair. The three qubit system will be in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\mu|0\rangle\left(|00\rangle + |11\rangle\right) + \nu|1\rangle\left(|00\rangle + |11\rangle\right)\right],$$

Where the first two qubit (on the left) belong to Alice and third qubits belongs to Bob. Alice performs a CNOT on her two qubits using her half of the EPR pair as the target and $|Q\rangle$ as the control. This operation will produced the state

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left[\mu|0\rangle\left(|00\rangle + |11\rangle\right) + \nu|1\rangle\left(|10\rangle\right) + |01\rangle\right]$$

Now Alice sends the first qubit through a Hadamard gate to get

$$|\psi_3\rangle = \frac{1}{2}\left[\mu\left(|0\rangle + |1\rangle\right)\left(|00\rangle + |11\rangle\right) + \nu\left(|0\rangle - |1\rangle\right)\left(|10\rangle + |01\rangle\right)\right]$$

Which can we rewritten as

$$|\psi_3\rangle = \frac{1}{2}\left[|00\rangle\left(\mu|0\rangle + \nu|1\rangle\right) + |01\rangle\left(\mu|1\rangle + \nu|0\rangle\right) + |10\rangle\left(\mu|0\rangle - \nu|1\rangle + |11\rangle\left(\mu|1\rangle - \nu|0\rangle\right)\right)\right]$$

Next, Alice measures the state of her two qubits. From the previous expression .We see that depending on the result of Alice measurement; Bob will have one of the four possible states. To know which state he has Bob needs to know the result of Alice measurement. After performing her measurement Alice should telephone or send an e-mail to Bob to let him know her measurement result. This fact prevents teleportation from being used to transmit information faster than light –Bob dose not gain possession of the qubit until Alice transmits her measurement result which she cannot do faster than the speed of light. Once Bob has learned the measurement outcome, Bob can perform the appropriate X and or Z gates to transforms the state of his qubit into $|Q\rangle$

Chapter-4

Two Coherent state interferometry

Over the last decade coherent states interferometry and two particle interferometry have provided new confirmation of quantum mechanics and greater violation Bell type inequality [1-22]. Two particle interferometry involves entangled microscopic systems. Two coherent state interferometer involves that can be macroscopic while still behaving similarly to microscopic pairs[1-6], and the coherent state superposition refer to as Schrödinger cat states emphasizing the quantum mechanics is used to described macroscopic physical system [7-12]

Almost all of the analysis of quantum optical interferometry have cantered on elements as strictly orthogonal Hilbert space the orthogonally of coherent state is only approximate and is strictly present only in the large average particle number $|\alpha| \rightarrow \infty$. Here in addition to showing evidence for the complementary of one and two coherent state interference visibilities that is in accord with the thing found in two particle interferometry. We found a counter intuitive result for Bell type inequality at odds with the corresponding principle. The corresponding principle demands that as a system gets more macroscopic in the sense of going large number of particle. Its behaviour should become increasingly like that of the corresponding classical mechanic system. Thus one expect that in the microscopic limit quantum effect such as the violation of Bell type inequality will disappear .The violation of a Bell type inequality was to shown to increase as the intensity of the coherent states increase [13]. Here using a bell type inequality increase as the system gets more macroscopic.

4.1. THE TWO-COHERENT STATES INTERFEROMETER

Two coherent state interferometry are the application of technique of coherent state recombination to macroscopic photon system pairs of the general form

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\alpha\rangle_1|\gamma\rangle_2 + i|\delta\rangle_1|\beta\rangle_2\right]$$

Where $|\alpha\rangle_1$ and $|\delta\rangle_1$ are near-orthonormal coherent state vector in the Hilbert space H_1 of system 1,and $|\beta\rangle_2$ and $|\gamma\rangle_2$ are element of H_2 system 2.State of the above equation are entangled i. e they cannot factorized in any way into the form $|\chi\rangle_1|\xi\rangle_2$,where $|\chi\rangle_1 \in H_1$ and $|\xi\rangle_2 \in H_2$,the new phenomena studied here arise when the production of entangled coherent state pairs is combined with interferometry techniques tailored to coherent state pairs is combined with interferometry technique tailored to coherent state. In particular detection probabilities consistent with a complementary between one coherent state and two coherent state visibilities are given and the violation of a Bell type inequality is demonstrated. And two point are following emphasized (i)two coherent state interferometry depends on the preparation of entangled coherent state pairs and (ii) Entangled states like the $|\psi\rangle$ of starting equation could be produced via the nonlinear interaction with Hamiltonian

[7] $\hat{H}_1 = \hbar\chi\left(\hat{a}^+\hat{a}\right)^n$

For n>1 an integer , χ being proportional to the medium's nonlinear susceptibility of order $2n-1$ (iii) The phenomena described here depend on the utilization of a nonlinear version of well-known interferometer such as Mach-Zhender interferometer (iv) quantum effect persist in a macroscopic limit .The general arrangement that we propose for two coherent state interferometry as shown in figure.

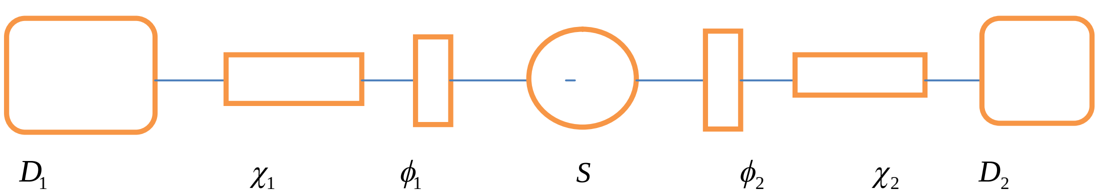

Figure.2: Schematic experimental arrangement for two coherent state interferometry.

The source S creates the entangled state in given equation -2. The output wing 1(2) proceeds to the coherent phase shifters $\phi_1(\phi_2)$ and the nonlinear cell $\chi_1(\chi_2)$and the finally coherent state detector $D_1(D_2)$.

Source simultaneously produced macroscopic photon.

System 1 and 2. In the most interesting case each pair is prepared in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\alpha\rangle_1|-\alpha\rangle_2 + i|-\alpha\rangle_1|\alpha\rangle_2] \tag{3}$$

A coherent superposition of two distinct pairs of correlated coherent state of system 1 and 2 .This state can be created by the injecting coherent state $|\alpha\rangle and |-\alpha\rangle$ into the two input ports of a nonlinear Mach-Zhender interferometer $[11,20-22]$ .In one of these pairs system 1 is in the coherent state $|\alpha\rangle$and undergoes phase shift upon encountering the coherent state phase shifters ϕ_1 [20] on the way to nonlinear cell 1 form which it inters to detector 1 ;similarly for system is in the coherent state $|-\alpha\rangle$ and encounters coherent state phase shifter ϕ_1, proceeds to cell 1 and detector 1,while system 2,in state $|\alpha\rangle$encounter ϕ_2 cell 2 and then enters detector 2. Hence the violation of Bell type inequality in this limit is expected to be maximal if the appropriate measurement can be made.

The transformation operator

$$\hat{K} = e\left[-i\chi\left(\hat{a}^{+}a\right)^{2}\right] \tag{4}$$

$\hat{K}$ is associated with the optical Kerr nonlinearity [24-26]. For the value $\chi = \pi/2$ this nonlinear operator acting on a coherent state creates a so-called Schrodinger cat state, a superposition of the $|\alpha\rangle$ and $|-\alpha\rangle$ coherent state [11]:

$$\hat{K}|\pm\alpha\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\pi/4}|\pm\alpha\rangle + e^{i\pi/4}|\mp\alpha\rangle\right).$$

The unitary operator [20]

$$\hat{\phi}_\alpha = \exp\left(i\phi|\alpha\rangle\langle\alpha|\right)$$

This above equation denotes the evolution due to the coherent phase shifters. This transforms a coherent input state $|\alpha\rangle$ with the result

$$\hat{\phi}_\alpha|\alpha\rangle = e^{i\phi}|\alpha\rangle$$

This transformation is analogous to that for a normal phase shifters transforming a single photon state with the phase term outside the Ket. However due to the non orthogonality of the coherent state $|\alpha\rangle$ and $|-\alpha\rangle$ the coherent phase shifters for $|\alpha\rangle$, $\hat{\phi}_\alpha$ does not leave the $|-\alpha\rangle$ state completely unaltered rather on $\hat{\phi}_\alpha|-\alpha\rangle = |-\alpha\rangle + (e^{i\phi}-1)\langle\alpha|-\alpha\rangle|\alpha\rangle$.

Nonetheless if α large so that $|\alpha| \to \infty$ then $|\langle\alpha|-\alpha\rangle| \to 0$ and the state $|-\alpha\rangle$ will remain effectively unchanged by the coherent phase shifters for the $\hat{\phi}_\alpha$. An exact experimental realization of the unitary operator may not be possible but an approximate realization is possible by exploiting media with higher order nonlinear susceptibility.

4.4. BELL-TYPE INEQUALITY VIOLATION

The amplitude for a coincidence measurement of any combination of the state $|\pm\alpha\rangle_1$ and $|\pm\alpha\rangle_2$ given the state is

$$A(\pm\alpha, \pm\alpha|\phi_1,\phi_2) = N[(1+e^{-4|\alpha|^2})(ie^{i\phi_1} - e^{i\phi_2}) + \gamma\left(\pm 1 + 2ie^{-2|\alpha|^2} \mp e^{-4|\alpha|^2}\right)]$$

$$A(\pm\alpha, \pm\alpha|\phi_1,\phi_2) = N[\pm(1+e^{-4|\alpha|^2})(e^{i\phi_1} - e^{i\phi_2}) + 2e^{-2|\alpha|^2}(ie^{i\phi_1} - e^{i\phi_2}) + i\gamma(1+e^{-4|\alpha|^2})]$$

The probabilities for a coincidence measurement are then calculated by multiplying the relevant amplitude with its complex conjugate. These coincidence measurements of specific combinations of the coherent state can be achieved by applying quadrature phase homodyne measurement [27-30]

The power of $e^{-|\alpha|^2}$ that appears in the equation and above equations are due the non orthogonality of the state $|\alpha\rangle$ and $|-\alpha\rangle$ since for an output state $|-\alpha\rangle$ there is a nonzero probability of measuring this system as $|\alpha\rangle$.One result of this non orthogonality is that in the limit $|\alpha| \to 0$,

$$P(\alpha,\alpha), P(-\alpha,-\alpha), P(\alpha,-\alpha), P(-\alpha,\alpha) \to 1.$$

However in the macroscopic limit where $|\alpha| \to \infty$,

$$0 \le P(\alpha,\alpha), P(-\alpha,-\alpha), P(\alpha,-\alpha), P(-\alpha,\alpha) \le 1/2$$

As $|\alpha| \to \infty$ occurs for two particle interferometry with a pair of particle [1-6] and gives each detection a value: the detection of the $|\alpha\rangle$ state is designed by $D(\alpha)=1$ and the detection of the $|-\alpha\rangle$ state by $D(-\alpha)=-1$Then for a single experiment we have

$$E(\alpha,\phi_1,\phi_2) = P(\alpha,\alpha|\phi_1,\phi_2) + P(-\alpha,-\alpha|\phi_1,\phi_2) - P(\alpha,-\alpha|\phi_1,\phi_2) - P(-\alpha,\alpha|\phi_1,\phi_2)$$

As is usual for the CHSH-type inequalities [31], we construct the function

$$B(\alpha,\phi_1,\phi_2,\phi_1',\phi_2') = E(\alpha,\phi_1,\phi_2) + E(\alpha,\phi_1,\phi_2') + E(\alpha,\phi_1',\phi_2) - E(\alpha,\phi_1',\phi_2')$$

A Bell locality violation would then be indicated by the result $|B|$ ›2.The value of $|B|$ is maximized when $\phi_1 = \frac{3\pi}{4}, \phi_2 = 0, \phi_1' = \frac{\pi}{4}, \phi_2' = \frac{\pi}{2}$

There is violation for sufficiently large values of $|\alpha|$.The larger $|\alpha|$ the larger the Bell locality violation approaching the limit $|B| \to 2\sqrt{2}$ as $|\alpha| \to \infty$.

For $|\alpha| \to \infty$ we find that $e^{-2|\alpha|^2} \to 0$ and $e^{-4|\alpha|^2} \to 0$. In this case the coincidence detection probabilities become assuming perfect detector efficiencies

$$P(\alpha,\alpha\,|\,\phi_1,\phi_2) \to \frac{1}{4}[1+\sin(\phi_1-\phi_2)]\ ,\ P(-\alpha,-\alpha\,|\,\phi_1,\phi_2) \to \frac{1}{4}[1+\sin(\phi_1-\phi_2)]$$

$$P(\alpha,-\alpha\,|\,\phi_1,\phi_2) \to \frac{1}{4}[1-\sin(\phi_1-\phi_2)],\ P(-\alpha,\alpha\,|\,\phi_1,\phi_2) \to \frac{1}{4}[1-\sin(\phi_1-\phi_2)]$$

The above result in equation are analogous to those obtained for two particle interferometry using entangled photon pairs [4,5,32], and a Bell type inequality is violated.

4.3. CONCLUSION

We have shown in this chapter how the interferometry of entangled pairs of quantum coherent states has several characteristics in the common two particle interferometry. These include the complementary between one system and two system interference visibilities in the extreme case of product and maximally entangled quantum states and the violation of Bell type inequality. This quantum behaviour persists even in the limit of macroscopic average particle numbers .Indeed a Bell type inequality is maximally violated in this limit.

The correspondence principle demands that as a system get more macroscopic, its behaviour should become increasingly like that of the corresponding classical mechanics system. Thus the corresponding principle suggest that as coherent state become more macroscopic, the possibility of violation of Bell type inequality should diminish.

5 References

Chapter 1

[1]Bergstrom, L.andH. LectureNotes in Relativistic Quantum Mechanics .Stockholm: Stockholm University,1999 .

[2] Griffiths,D.Introduction to Quantum Mechanics: International edition. Pearson: Prentice Hall, 2005.

[3] Rudin,W. Real and Complex Analysis: International student edition. New York: McGraw-Hill Inc, 1987

Chaptrer-2

[1] W.H.Zurek, Rev.Mod.Phys.75, 715 (2003).

[2] M.Brune et al., Phys.Rev.Lett.77, 4887 (1996).

[3] J.R.Friedman et al., Nature 406,43 (2000).

[4] N.D Mermin, Phys. Rev. D 22,(1980).

[5] F.De Martini et al.,Phys. Rev.Lett.100, 253601 (2008).

[6] F.DeMartini,V.Mussi,and F.Bovino,Opt.Commun.179,581 (2000);C.Simon,G.Weihs,and A.Zeilinger,Phys.Rev.Lett.84,2993 (2000).

[7] In the presence of loss before and after the phase covariant cloner,we do not know when the amplified state becomes separable .

[8] F.De Martini and F.Sciarrino, arXiv: 1202.5518 and references therein.

[9] S.Raeisi,P.Sekatski, and C.Simon,Phys.Rev. Lett.107, 250401 (2011).

[10] A.Windhager et al., Opt.Comm.284,1907 (2011).

[11] S.J.vanEnk,Phys.Rev. A 72,064306 (2005).

[12] J.K.Asboth, J.Calsamiglia, H.Ritsch.Phys.Rev.Lett.94, 173602 (2005)

[13] J.Solomon Ivan, N. Mukunda, and R. Simon, arxiv; 0603255.

[14] R.J.Glauber, Phys.Rev.131, 2766 (1963)

[15] B.C .Sanders,Phys.Rev.A 45,6811(1992)

[16] Negativity N [17] conventionallyquantities entanglement

[17] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod.Phys.81,865 (2009).

Chapter 3

[1]H.Aschauer,W. Dr and H-J. Briegel, Multi particle entanglement purification for two colourable states. Phys. Rev. A, 71:012319 (2005)

[2] A.Auffeves, P. Maioli, T. Meunier,S. Gleyzes, G.Nogues, M.Brune,J.M.Raimond and S. Haroche, Entanglement of a mesoscopicfield with an atom induced by photon graininess in a cavity.Phys.Rev.Lett.,91 :230405 (2003).

[3]S. D. Bartlett, H. de Guise and B. C. Sanders, Quantum encodings in spin Systems and harmonic oscillators. Phys. Rev. A, 65: 052316 (2002).

[4] J. S. Bell, On the einstein-podolsy-rosen paradox. Physics, 1: 195(1964).

[5] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W. K. Wootters, Teleporting an unknown quantum state via dual classical and einsteinpodolsky-rosen channels. Phys. Rev. Lett., 70: 1895 (1993).

Chapter 4

[1] Z.Y.Ou and L. Mandel, Quantum Opt.2, 71 (1990).

[2] C.O.Alley and Y.H.Shih, Phys.Rev.Lett.61, 2921 (1998).

[3] X.Y.Zou,L.J.Wang, and L.Mandel,Phys.Rev.Litt.67,318 (1991).

[4] J.G.Rarity and P.R.Tapster, Phys.Rev.Litt.64, 2495 (1990).

[5] M.A.Horne, A.Shimony, and A.Zeilinger, Phys.Rev.Litt.62, 2209 (1989).

[6] G.Jaeger, A.Shimony, and L.Vaidman, Phys. Rev. A 51,54 (1995)