

# Generalized Lomb-Scargle analysis of $^{36}\text{Cl}$ decay rate measurements at PTB and BNL

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Recently Pomme et al [1] did an analysis of  $^{36}\text{Cl}$  radioactive decay data from measurements at the Physikalisch-Technische Bundesanstalt (PTB), in order to verify the claims by Sturrock and collaborators of an influence on beta-decay rates measured at Brookhaven National Lab (BNL) due to the rotation-induced modulation of the solar neutrino flux. Their analysis excluded any sinusoidal modulations in the frequency range from 0.2-20/year. We carry out an independent analysis of the same PTB and BNL data using the generalized Lomb-Scargle periodogram, to look for any statistically significant peaks in the range from 0 to 14 per year, and by evaluating the significance of every peak using multiple methods. Our results for the PTB data are in agreement with those by Pomme et al. For BNL data, we do find peaks at some of the same frequencies as Sturrock et al, but the significance is much lower. All our analysis codes and datasets have been made publicly available.

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## I. INTRODUCTION

Sturrock and collaborators have argued in a number of works over more than a decade (eg. Refs. [2–4] and references therein) that beta decay rates for a large number of nuclei exhibit variability and show periodicities at multiple frequencies, some of which they have associated with solar rotation as well as other processes in the solar core. They have also found similar peaks at 12.7 per year in the Super-K solar neutrino flux (from the first five years of data) [5], which they have associated with solar rotation. Consequently, they have argued for a correlation between the two sets of findings and argued for an influence of solar neutrinos on beta-decay rates.

However, many other groups have failed to reproduce the periodicities in the beta-decay results, while analyzing the same data as well as decays of the same element from other experiments. A review of some of these claims and rejoinders can be found in Refs. [3, 6, 7]. In our previous works, we have also carried out an independent analysis of some these claims and found evidence of periodicities at some of the same frequencies as found by Sturrock et al, albeit with lower significance [8, 9].

In this work, we focus on addressing the claimed periodicities in the beta decay rates of  $^{36}\text{Cl}$ . Sturrock et al [10] have argued for periodicities with periods at 1/year and 12.7/year (or 28.7 days) in the  $^{36}\text{Cl}$  decay rates of the Brookhaven National Lab [11] counting experiment. They have argued that the peak at 12.7/year is indicative of the synodic rotation rate of the radiative zone of the Sun, since it matches the value of 28.7 days determined using helioseismology [12, 13]. These results were rebutted by Pomme et al [1], who found no evidence for periodicities in the decay rates of  $^{36}\text{Cl}$  using more accurate measurements at Physikalisch-Technische Bundesanstalt Braunschweig (PTB), obtained using the triple-to-double coincidence ratio measurement techniques [6]. Pomme [14, 15] has also raised concerns about the detector stability and control of experimental uncertainties in the BNL measurements, which are now more than three

decades old. Furthermore, the invariability of the decay constants for  $^{36}\text{Cl}$  was also demonstrated using triple-to-double coincidence ratio measurements [16] to refute claims of oscillations ascribed to the changes in Earth-Sun distance.

In this work, we independently try to adjudicate the conflicting results between these two works by doing an independent analysis of the beta decay residual data from both the BNL and PTB measurements (which were kindly provided to us by S. Pomme) using the Generalized (or Floating Mean) Lomb-Scargle periodogram [17–19]. We search the frequency range between 0-14 /year (or upto 26 days), since this encompasses the sweet spot for frequencies associated with solar rotation [10, 20]. We calculate the significance of the peaks using all the available methods provided in the `astropy` [21] library used to calculate the Lomb-Scargle periodogram.

The outline of this paper is as follows. We briefly recap some details of the Lomb-Scargle periodogram and different methods of calculating the FAP in Sect. II. A summary of the results by Sturrock and collaborators and the re-analysis by Pomme et al is discussed in Sect. III. Our analysis of the PTB and BNL datasets is described in Sect. IV. We conclude in Sect. V.

## II. GENERALIZED LOMB-SCARGLE PERIODOGRAM

The Lomb-Scargle (hereafter, L-S) [17, 18] (see Ref. [22, 23] for a recent extensive reviews) periodogram is a widely used technique to look for periodicities in unevenly sampled datasets. The main goal of the L-S periodogram is to determine the frequency ( $f$ ) of a periodic signal in a time-series dataset  $y(t)$  given by:

$$y(t) = a \cos(2\pi ft) + b \sin(2\pi ft). \quad (1)$$

The L-S periodogram calculates the power as a function of frequency, from which one needs to infer the presence of a sinusoidal signal and assess the significance.

For this analysis, we use a modified version of the L-S periodogram proposed by Zechmeister and Kurster [19], which is known in the literature as the generalized L-S periodogram [19, 24] or the floating mean periodogram [22, 25, 26] or the Date-Compensated Discrete Fourier Transform [27]. The main change in this method is that an arbitrary offset is added to the mean values. More details about this method and comparison with the normal L-S periodogram outlined in Refs. [19, 22–24, 28] and references therein.

For any sinusoidal modulations at a given frequency, one would expect a peak in the L-S periodogram. To assess the statistical significance of such a peak, we need to calculate its false alarm probability (FAP, hereafter) or  $p$ -value. A plethora of methods have been developed to estimate the FAP of peaks in L-S periodogram, ranging from analytical methods [18] to Monte-Carlo simulations [29]. We enumerate the different methods used to calculate the FAP for our analysis. All of these can be implemented using the `astropy` package, which we used in this work.

- Method = `baluev`

This method implements the approximation proposed by Baluev [30], which uses extreme value statistics for stochastic process, to compute an upper-bound of the FAP for the alias-free case. Their analytical formula for the FAP can be found in Refs. [22, 30].

- Method = `bootstrap`

This method uses non-parametric bootstrap resampling as described in Ref. [22]. Effectively, it computes many Lomb-Scargle periodograms on simulated data at the same observation times. The bootstrap approach can very accurately determine the false alarm probability, but is very computationally expensive.

- Method = `davies`

This method is related to the `Baluev` method, but loses accuracy at large false alarm probabilities, and is described in Ref. [31].

- Method = `naive` This method is a simplistic method, based on the assumption that well-separated areas in the periodogram are independent. The total number of such independent frequencies depend on the sampling rate and total duration and is explained in Ref. [22].

Once the FAP is calculated, one can convert this FAP to a  $Z$ -score or significance in terms of number of sigmas. This is traditionally estimated from the number of standard deviations that a Gaussian variable would fluctuate in one direction to give the corresponding FAP [32, 33].

### III. RECAP OF RESULTS BY STURROCK ET AL AND POMME ET AL

Here, we briefly summarize the analysis in Sturrock et al [10] (S16 hereafter) and Pomme et al [7] (P17, hereafter). S16 analyzed the  $^{36}\text{Cl}$  decay data from BNL. The data was detrended and normalized to account for the exponential decay. Power spectrum of this detrended data, based on the procedure outlined in Ref. [34] was used to search for periodicities at different frequencies. The maximum power was found at a frequency of 1 /year, corresponding to a  $p$ -value of  $2.7 \times 10^{-7}$ . This  $p$ -value was computed using the Press-Bahcall shuffle test [35].

Since the  $^{36}\text{Cl}$  decay data was found to be non-uniform, the same data was analyzed using spectrograms and phasegrams, in order to look for transient oscillatory cycles. From this, it was discerned that for  $^{36}\text{Cl}$ , annual modulation was conspicuous between 1984 and 1986, but later switched off. They further analyzed the data to look for evidence of solar rotation. The range of frequencies, which they scanned based on observed synodic rotation [36] corresponds to 9-14 per year or periods between 26-41 days. From these spectrograms, S16 found evidence for oscillations with a frequency of 12.7/ year, which is compatible with a source in the solar radiative zone. The  $p$ -value of this peak was also obtained using the shuffle test, and found to be  $1.5 \times 10^{-5}$  in a band of unit width. If you account for the look-elsewhere effect by incorporating the bandwidth of 5 per year, the  $p$ -value was found to be  $7.5 \times 10^{-5}$ . S16 then discuss the correlation between these peaks and peaks near this same frequency found in Super-K solar neutrino data [34].

P17 also applied the generalized (or floating-mean) Lomb-Scargle periodogram[19, 22] to both the BNL and PTB datasets in order to look for periodicities between 0 and 20 per year. They also looked for concurrent peaks in PTB, BNL, and Super-K solar neutrino datasets. They do not find any peak at 9.43/year (seen in the Super-K data) in the BNL or PTB data. Although they found many significant peaks in the BNL data at 11 and 12.7/year, none of these peaks were visible at the same frequencies in the PTB data. P17 then fit the BNL and PTB data to sinusoids at frequencies of 9.43, 11.0, and 12.7 /year. The amplitudes in the PTB data found are  $\mathcal{O}(10^{-3})\%$  and are statistically indistinguishable from zero and were about an order of magnitude lower than the amplitudes reported in S16. Therefore, they disagree with the conclusions in S16.

### IV. ANALYSIS AND RESULTS

We first recap the input datasets used for our analysis, then describe the L-S analysis procedure and finally present our results.

## A. BNL and PTB Datasets

The BNL dataset comprises of 364 measurements formed from countings of  $^{36}\text{Cl}$  and  $^{32}\text{Si}$  decays in gas flow proportional counter at BNL [11], over the period of 1982 to 1990. More details of these measurements can be found in Ref. [11]. For each nuclei, the daily decay rate was obtained by averaging over 20 measurements. The data was detrended and normalized to account for the exponential decay of the experimental uncertainties used in these measurements are discussed in Refs. [14, 15]. The exact total duration of this dataset is 7.83 years and the median sampling interval is 0.00279 years or approximately 1 day. We used an uncertainty of 0.1% for every data point. The raw BNL decay data (after removing the exponential dependence) as a function of time can be found in Fig. 3.

The PTB experiment consists of liquid scintillation vials with  $^{36}\text{Cl}$  in solution, which were prepared in December 2009. The decays were measured 66 times between December 2009 and April 2013 in the custom-built TDCR detector at the PTB. More details of the PTB experiment and the setup used for these measurements can be found in Ref. [16]. The exact total duration of the PTB dataset is 66 days and the median sampling time is equal to 0.0328 years or approximately 12 days. The raw PTB data is shown in Fig. 1. The uncertainty in each data point was 0.009%. Linear correction was applied to the dataset to compensate for increasing colour quenching in the scintillation cocktail with time. For this, `numpy.polyfit` function is used to apply a linear correction (Fig. 1), and the residuals after applying this linear correction are shown in Fig. 2.

## B. Power spectrum analysis

For our analysis, we applied the generalized L-S periodogram as described in Sect. II. We used the `astropy` [21] implementation of the L-S periodogram. These periodograms can be found Figs. 2 and Fig. 3 for BNL and PTB respectively. We normalized the periodogram by the residuals of the data around the constant reference model. With this normalization, the power varies between 0 and 1. The same normalization has been used in P17. However, S16 (and also one of our earlier works [8]) used the normalization proposed by Scargle [18]. The relation between these two normalizations can be found in Ref. [9].

For our analysis, we report results for frequencies from 0 to 14 per year. This frequency range, covers the sweet spot for solar rotation related phenomenon (from 8-14 per year) and is also sensitive to lower frequencies (such as 1/year), which were found to be significant in S16. The corresponding Nyquist frequency for the two data sets is equal to 132/year for BNL and 15/year for PTB. Therefore, the maximum frequency which we have searched for is greater than the Nyquist frequency. We also searched

for peaks at higher frequencies (up to the Nyquist limit). But we do not report these results in this work, as none of them were found to be significant from our analysis (as well as from any previous analysis by Sturrock and collaborators). The frequency resolution used in the periodograms was 0.025 per year and 0.06 per year for BNL and PTB respectively. This resolution is determined from the total duration of the dataset. Our results on the power spectrum for the generalized L-S periodogram for both BNL and PTB can be found in Fig. 4 and Fig. 5 respectively.

While considering these frequencies and corresponding power, we need to identify the significance of each peak. This significance is usually determined from the FAP and Z-score. We calculated the FAP (along with associated Z-score) using all the four methods discussed in Sect. II. A tabular summary of the powers at some of the largest peaks along with FAP and Z-score at each of these frequencies can be found in Tables I and II for BNL and PTB respectively. We now summarize our results.

## C. Results

We now discuss below the results for each of the two datasets:

- **BNL.** The BNL L-S power spectrum is shown in Fig. 4. We find peaks with power  $> 0.05$  at frequencies (less than 14 /year) corresponding to 1.04, 1.93, 1.08, 11.08, 12.65, and 12.82 per year. The powers and FAP of these peaks are tabulated in Table I. These peaks have also been found to be significant in S16. However, the FAPs which we get are about two to three orders of magnitude smaller than in S16. The FAPs calculated for each frequency using the multiple methods are of the same order of magnitude. All Z-scores, which we obtained are less than  $5\sigma$ , (a traditional threshold used for deeming something as a new discovery [37]). The smallest FAP which we get is close to 1 per year with a FAP of  $\mathcal{O}(10^{-4})$ , corresponding to Z-score between  $3.3\sigma$  and  $3.8\sigma$ . This frequency also had the maximum significance in S16. For frequencies in the range from 8 to 14/year, the minimum FAP is at a frequency of 12.65/year, corresponding to a FAP of  $\mathcal{O}(10^{-2})$ , with a significance of  $2.4\sigma$ . This is the closest frequency to 12.7/year, found to be interesting in S16. Therefore, although we do see peaks at some of the same frequencies as seen in S16, the FAPs, which we obtain are about two to three orders of magnitude larger than in S16. Therefore, the significance we obtained for these peaks is marginal and not as large as in S16.
- **PTB** The PTB L-S power as a function of frequency can be found in Fig. 5, and a tabular summary of the powers and significance of the most significant peaks (with power  $> 0.1$ ) can be found

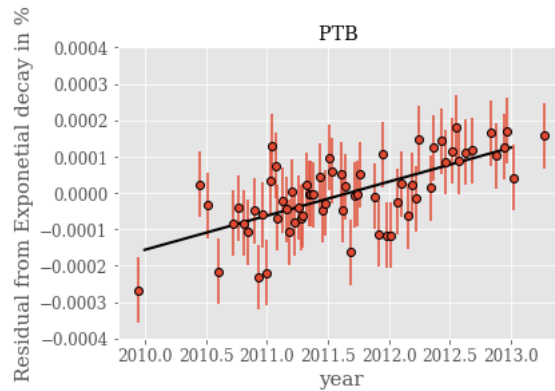


FIG. 1: The original PTB dataset [7] showing the  $^{36}\text{Cl}$  decays. The best-fit line has a slope equal to  $9.36 \times 10^{-5}$  and  $y$ -intercept is equal to 0.81.

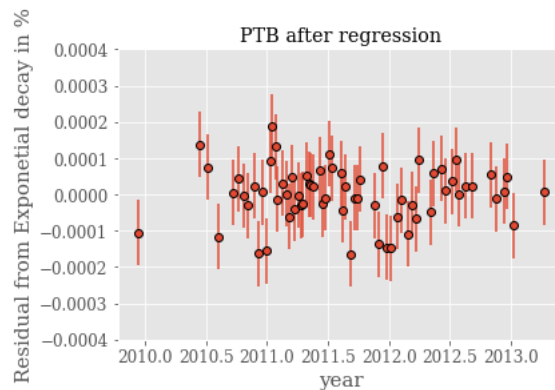


FIG. 2: Residuals in the PTB dataset after applying the linear correction (outlined in Fig. 1). The uncertainty for each data point is 0.009%.

in Table II. We see that none of the FAPs have values less than 0.1, implying that all of them are consistent with statistical fluctuation, and there is no periodicity at any of the frequencies found to be significant in S16. Therefore, we concur with the findings of P17. If the peaks found in S16 were indicative of a solar influence, similar periodicities should have been seen in the PTB data. We do not find any such evidence.

## V. CONCLUSIONS

The aim of this work was to adjudicate the controversy between two groups (S16 and P17) regarding the periodicities in nuclear beta decay rates of  $^{36}\text{Cl}$ , and possible solar influence on these potential periodic decay rates.

For this purpose, we independently re-analyzed both the BNL data, for which S16 found evidence for statistically significant peaks at multiple frequencies as well as the PTB data, for which P17 could not find any corroborative evidence at the same frequencies as in S16.

We have used the generalized or floating-mean L-S periodogram [19] (similar to our previous works, where we analyzed the Super-K solar neutrino [8] and  $^{90}\text{Sr}/^{90}\text{Y}$  decay data [9]), to look for periodicities in the frequency range from 0 to 14 per year, since this is the same frequency range, wherein which S16 found periodicities.

When we analyzed the BNL data, we found peaks in the L-S periodogram at mostly the same frequencies found to be significant in S16. However, the FAP which we found is about 2-3 orders of magnitude larger than S16. Therefore, according to our analysis, none of the peaks found in S16 are statistically significant and indicative of any solar or any other external influence. When we analyzed the PTB data, we do not find any peaks with  $\text{FAP} < 0.1$ , indicating that the  $^{36}\text{Cl}$  PTB decay data contain no periodicities. Therefore, we agree with the conclusions in P17 regarding this dataset.

To promote transparency in data analysis, we have made our analysis codes and data available online and they can be found at <https://github.com/d-a-r-t-h-vader/Beta-decay-analysis-using-Lombscargle-periodogram->. These can be easily applied to look for periodicities in

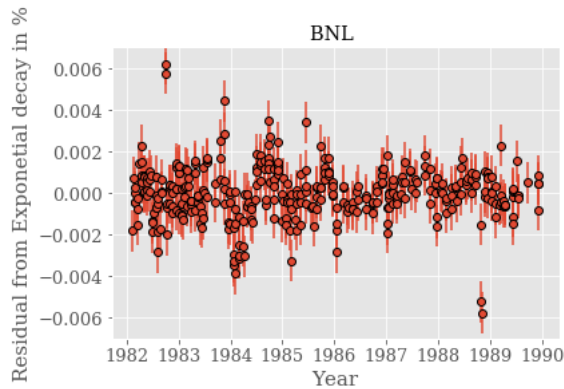


FIG. 3: Relative difference from the mean value of measured  $^{36}\text{Cl}$  at the BNL (applying 0.13% uncertainty on individual data) [11].

Frequency( $a^{-1}$ )	L-S Power	FAP : <b>baluev</b>	FAP : <b>davies</b>	FAP : <b>naive</b>	FAP : <b>bootstrap</b>
1.04	0.09	$3.58 \times 10^{-4}(3.38\sigma)$	$0.85 \times 10^{-4}(3.76\sigma)$	0.001(3.09 $\sigma$ )	$3.58 \times 10^{-4}(3.38\sigma)$
1.93	0.06	0.07 (1.48 $\sigma$ )	0.07 (1.48 $\sigma$ )	0.02 (2.64 $\sigma$ )	0.05 (1.64 $\sigma$ )
11.08	0.06	0.08 (1.41 $\sigma$ )	0.08 (1.41 $\sigma$ )	0.02 (2.64 $\sigma$ )	0.05 (1.64 $\sigma$ )
12.65	0.07	0.01 (2.33 $\sigma$ )	0.01(2.33 $\sigma$ )	0.008 (2.41 $\sigma$ )	0.01 (2.33 $\sigma$ )
12.82	0.06	0.06 (1.55 $\sigma$ )	0.06 (1.55 $\sigma$ )	0.02 (2.64 $\sigma$ )	0.04 (1.73 $\sigma$ )
14.10	$5.23 \times 10^{-2}$	0.16(0.99 $\sigma$ )	0.05(1.64 $\sigma$ )	0.12(1.17 $\sigma$ )	0.17(0.95 $\sigma$ )

TABLE I: L-S powers and FAP for BNL data using multiple methods : **Baluev**, **Davies**, **Naive**, **Bootstrap** methods, for only those frequencies for which the power is greater than 0.05. The numbers in parenthesis represent the Z-score, found using the method prescribed in Ref. [32]. We find peaks at some of same frequencies found to be statistically significant in S16. But our FAPs are about 2-3 orders of magnitude higher than that found in S16.

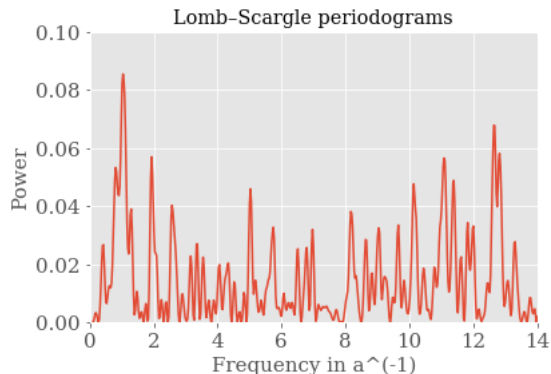


FIG. 4: Weighted LS periodograms of  $^{36}\text{Cl}$  decay rate data measured at the BNL. (Considering frequencies in the range 0 - 14  $a^{-1}$ ).

other datasets.

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Frequency( $a^{-1}$ )	L-S Power	FAP : baluev	FAP : davies	FAP : naive	FAP : bootstrap
0.87	0.12	0.99	4.39	0.93	0.99
2.85	0.12	0.99	5.07	0.96	0.99
6.76	$1.16 \times 10^{-1}$	0.99	5.42	0.97	1
13.07	$1.14 \times 10^{-1}$	0.99	5.68	0.97	1

TABLE II: PTB L-S powers along with FAP for the top four most significant peaks (with power  $> 0.1$ ) along with their FAP and Z-scores (in parenthesis) computed using the same methods as in Table I. We find that none of the FAPs are less than 0.1, and hence all the peaks discussed here are consistent with noise.

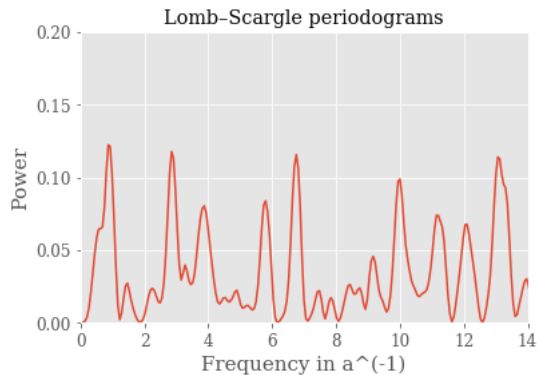


FIG. 5: Weighted LS periodograms of  $^{36}\text{Cl}$  decay rate data measured at the PTB. (Considering frequencies in the range 0 - 14  $a^{-1}$ ).

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