Error Analysis for Decode and Forward Cooperation in Nakagami -m fading for arbitrary m

ANUGU RATHNAKAR

A Thesis Submitted to

Indian Institute of Technology Hyderabad

In Partial Fulfillment of the Requirements for

The Degree of Master of Technology

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Department of Electrical Engineering

July 2014

Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

A. Rathnakar

(Signature)

ANUGU RATHNAKAR

(ANUGU RATHNAKAR)

EE12M1005

(Roll No.)

Approval Sheet

This Thesis entitled Error Analysis for Decode and Forward Cooperation in Nakagami -m fading for arbitrary m by ANUGU RATHNAKAR is approved for the degree of Master of Technology from IIT Hyderabad

(Dr. Sumohana) Examiner
Dept. of Elec Eng
IITH

(Dr. Balasubramaniam Jayaram) Examiner
Dept. of Mathematics
IITH

Acknowledgements

I sincerely thank my advisor Dr. G. V. V. Sharma. I benefitted a lot from discussions with him. His knowledge and approach at problems in the area helped me in understanding the meaning of research. I also thank all my professors from the department and other disciplines who inspired me through their presence.

Dedication

.

Abstract

Bit error rate (BER) expressions for Decode and Forward(DF) cooperative systems, compared to amplify and forward (AF) cooperation, are difficult to evaluate, hence there is considerable interest in finding analytical expressions for the BER for DF cooperative systems. In this thesis we obtain exact BER expression in DF cooperative system for a λ-MRC receiver in Nakagami m fading for multiple relays where m need not be integer. A different approach is proposed for a single relay.

For single relay, BER analysis is done by employing approximate statistics of a gamma conditionally gaussian (CG) random variable (RV) (in the decision rule) obtained through the Loskot-Prony approximation. Numerical results obtained using the analytical BER expressions are shown to closely follow the simulation results, despite the cumulative distribution function (CDF) of the gamma CGRV being a high signal to noise ratio (SNR) approximation.

For multiple relays, the analysis is done through the use of Mellin-Barnes integral representation of special functions. We first show that the exact cumulative distribution function (CDF) for a gamma CGRV can be expessed in terms of the extended Fox-$\hat{H}$ function. This approach is then used for obtaining the exact BER expressions by extrapolating the close relation between the Nakagami-m and gamma distributions. The analytical expressions so obatined are verified through simulations. Previous results were unavailable even for integer values of m.

Further, for a single relay Piecewise Linear (PL) Combiner which is a close approximation of Maxium Likelihood (ML) detector in Nakagami-m fading for integer m, BER analysis is done by evaluating the inverse laplace transform of the Moment Generating Function (MGF) of the decision variable through residue theorem. Previous analysis relied on the CDF and the PDF for BER evaluation.

Contents

Chapter 1

Introduction

1.1 Challenges of Wireless Channel

There has been continous growth in global wireless industry. The fundamental characteristic of wireless channel is fading (random variations of signal strength). Fading degrades the performance of the system when signal components that are received over different propagation paths add destructively. A solution to this is to employ diversity techniques namely Temporal, Frequency and Spatial diversity.

In addition to these traditional diversity techniques, a class of methods called Cooperative diversity enables the mobiles to relay each other's information and the signals from source and relays will provide additonal diversity [1]. The advantages of MIMO (Multiple Input Mulitple Output) systems are well known, and have been incorporated into wireless standards. Although it is beneficial to employ transmit diversity, it may not be practical for certain scenarios where size, cost, hardware are limitaions. The idea in Cooperative Communication is that a single antenna mobile in multi-user scenario can share their antenna to create a virtual MIMO system. Relay technologies are considered in the standardization process of next-generation mobile broadband communication systems such as 3GPP LTE-Advanced, IEEE802.16j, and IEEE802.16m.

1.2 Cooperative Communication

The relay channel was introduced by Vander Meulen [2] and investigated extensively by Cover and El Gamal [3]. Information theoritic model for relay channels were studied in [3]. This work analyzed the capacity of three-node network consisting of a source, a destination and a relay.The problem of creating and exploiting space diversity using a collection of distributed antennas belonging to multiple terminals (which is referred as cooperative diversity) is studied in [4].

1.2.1 Relaying Strategies

- Amplify and Forward (AF) : AF allows the relay node to amplify and forward the received signal from source node and forward it to the destination. Although noise is amplified by relay, the destination receives two independent faded versions of the signal, which are used to improve the performance. This method was proposed and analyzed in [1].

- Decode and Forward (DF) : Here the relay decodes the received signal, reencodes it and then transmits to the destination. Thus there is a possibility of propagating decoding errors which may lead to wrong decision at the destination.

1.3 Contribution

In this thesis, we study the BER analysis of DF cooperative systems. We assume that the fading coefficients are estimated accurately at the receiver. Exact expressions for the BER for certain single relay DF systems in Nakagami-m are available only for integer values of m [5, 6]. We are not aware of such results for noninteger m. Expressions for the case of multiple relays even for integer m are rare.

Reasons for the scarcity of exact BER analysis in DF cooperation can be traced to the lack of sufficient literature on conditionally Gaussian random variables (CGRV) [5, 7, 8]. These variables can be used to express the decision variables at the receivers for some DF cooperative systems experiencing additive white Gaussian noise (AWGN). For integer parameters, the statistics of gamma CGRVs were obtained in [5, 6] which facilitated the evaluation of the corresponding BERs for relay channels in Nakgami-m fading. Such results for noninteger m are unknown. This has been addressed in this thesis.

Chapter 2

Performance Analysis of λ-MRC Decode and Forward Cooperation

2.1 System Model

The classic three node cooperative system in Figure 2.1 is considered, where, without loss of generality, h represents the Nakagami-m channel gain with fading figures m and Ω, E the transmit power at a node, x the transmitted symbol at a node, and the subscripts s and r the source and relay parameters respectively.

2.2 λ MRC Receiver

The decision statistic for the λ-MRC receiver for BPSK modulation, is given by [6, 9]

$$X + \lambda Y \underset{-1}{\overset{1}{\gtrless}} 0, \quad 0 < \lambda \leq 1 \tag{2.1}$$

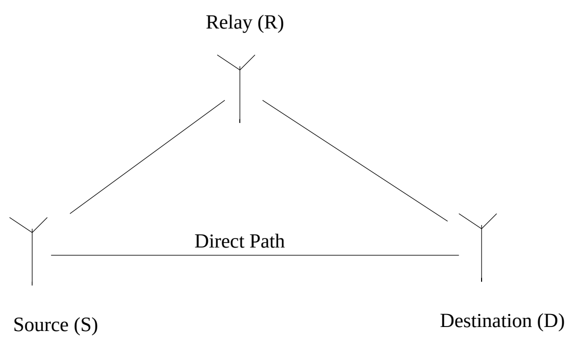

Figure 2.1: Three Node Coperative Diversity System

where $X \sim \mathcal{N}\left(a_s h_s^2, b_s h_s^2\right)$, $Y \sim \mathcal{N}\left(a_r h_r^2, b_r h_r^2\right)$ with $a_i = \frac{4E_i x_i}{N_0}$, $b_i = \frac{8E_i}{N_0}$, $c_i = \frac{m_i}{\Omega_i}$, $i \in \{s, r\}$.

$$p_{h_i^2}(x) = \frac{c_i^{m_i} x^{m_i - 1}}{\Gamma(m_i)} \exp\left(-c_i x\right), \quad x, c_i > 0, m_i \geq 0.5 \tag{2.2}$$

$h_i^2 \sim \mathcal{G}(c_i, m_i)$, where $\mathcal{G}$ denotes the *Gamma* distribution [10].

2.3 Problem Definition

Assuming equal probability of the transmitted symbol $x_s = \{1, -1\}$, from (2.1), the average BER for a λ-MRC cooperative system can be expressed as

$$P_e = \sum_{x_r \in \{1,-1\}} \varepsilon^{\frac{1-x_r}{2}} (1-\varepsilon)^{\frac{1+x_r}{2}} P\left(X + \lambda Y < 0 | x_s = 1, x_r\right). \tag{2.3}$$

where ε is the BER for the S-R link given by [11]

$$\varepsilon = \frac{1}{2\sqrt{\pi}} \frac{\sqrt{\frac{\bar{\gamma}_s}{m_s}}}{\left(1 + \frac{\bar{\gamma}_s}{m_s}\right)^{m_s + \frac{1}{2}}} \frac{\Gamma\left(m_s + \frac{1}{2}\right)}{\Gamma\left(m_s + 1\right)} {}_2F_1\left(\begin{matrix} 1, \ m_s + \frac{1}{2} \\ m_s + 1 \end{matrix}; \frac{1}{1 + \frac{\bar{\gamma}_s}{m_s}}\right) \tag{2.4}$$

From (2.3), we observe that the BER has to be computed separately for the case of correct and incorrect decision at the relay. The probability of error, given a correct decision at the relay, can be expressed as

$$\begin{aligned} P_{e|1} &= P\left(X + \lambda Y < 0 | x_s = 1, x_r = 1\right) \\ &= \int_{-\infty}^{\infty} F_X(-\lambda y) p_Y(y)\, dy \end{aligned} \tag{2.5}$$

To obtain the above, the statistics of X and Y are required. Since X and Y are conditionally Gaussian [5], their statistics are known for integer values of the Nakagami fading parameters m_i [5, 6]. Using this, the BER in (2.3) was obtained in [6]. We have to do this for non integer values of m.

2.4 Statistics of Gamma CGRV

2.4.1 PDF of Gamma CG RV

For arbitrary m_i, the exact PDF of X and Y is given by (2.22) using the approach in [6]. The corresponding CDF, however, is difficult to evaluate. In the following, an approximate expression for the CDF for high SNR is evaluted using the Loskot-Prony approximation [12].

Lemma 2.4.1. The PDF of $X \sim \mathcal{N}\left(a_s h_s^2, b_s h_s^2\right)$ is given by,

$$p_Z(z) = \frac{2c^m e^{\frac{az}{b}}}{\Gamma(m)\sqrt{2\pi b}} \left(\frac{|z|}{\sqrt{a^2+2bc}}\right)^{m-\frac{1}{2}} K_{m-\frac{1}{2}}\left(\frac{|z|}{b}\sqrt{a^2+2bc}\right), \quad a > 0 \tag{2.6}$$

Proof. See Appendix □

2.4.2 CDF of Gamma CG RV

We have two cases i.e. when the relay decodes correctly and when relay decodes incorrectly the information from source and transmits it to the destination.

Case I

First, we try to find the CDF of $X \sim \mathcal{N}(aA, bA)$ for $a > 0$, where $A \sim \mathcal{G}(c, m)$, m not being an integer. The CDF of X is given by

$$\Pr(X < z) = \Pr\left(aA + \sqrt{bA}\eta < z\right) \tag{2.7}$$

where $\eta \sim \mathcal{N}(0, 1)$. The above equation can be expressed as

$$\begin{aligned} F_X(z) &= \Pr\left(-\eta > \frac{aA - z}{\sqrt{bA}}\right) \\ &= \begin{cases} \int_0^\infty Q\left(\frac{ax-z}{\sqrt{bx}}\right) p_A(x)dx, & z \le 0 \\ \int_{\frac{z}{a}}^\infty Q\left(\frac{ax-z}{\sqrt{bx}}\right) p_A(x)dx + \int_0^{\frac{z}{a}} \left\{1 - Q\left(\frac{z-ax}{\sqrt{bx}}\right)\right\} p_A(x)dx, & z > 0 \end{cases} \end{aligned} \tag{2.8}$$

Defining $\epsilon = \frac{a}{c}, \kappa = \frac{b}{a}$, for large ϵ, which corresponds to high average SNR in fading channels, (2.8) can be expressed as In the second equation in (2.8), for $z > 0$,

$$\int_{\frac{z}{a}}^\infty Q\left(\frac{ax-z}{\sqrt{bx}}\right) p_A(x)dx = \frac{c^m}{\Gamma(m)} \int_{\frac{z}{a}}^\infty x^{m-1} Q\left(\frac{ax-z}{\sqrt{bx}}\right) \exp(-cx)\, dx \tag{2.9}$$

Making the substitution $x = \frac{yz}{c}$ in the above,

$$\int_{\frac{z}{a}}^{\infty} Q\left(\frac{ax-z}{\sqrt{bx}}\right) p_A(x)dx = \frac{1}{\Gamma(m)} \int_{\frac{z}{a}}^{\infty} x^{m-1} Q\left(\frac{ax-z}{\sqrt{bx}}\right) \exp\left(-cx\right) dx \tag{2.10}$$

$$= \frac{1}{\Gamma(m)} \int_{\frac{c}{a}}^{\infty} z^m y^{m-1} Q\left(\frac{\frac{a}{c}zy - z}{\sqrt{\frac{b}{c}zy}}\right) \exp\left(-zy\right) dy \tag{2.11}$$

Defining $\epsilon = \frac{a}{c}, \kappa = \frac{b}{a}$, the above integral can be expressed as

$$\int_{\frac{z}{a}}^{\infty} Q\left(\frac{ax-z}{\sqrt{bx}}\right) p_A(x)dx = \frac{1}{\Gamma(m)} \int_{\frac{1}{\epsilon}}^{\infty} z^m y^{m-1} Q\left\{\frac{\sqrt{z}\left(\epsilon y - 1\right)}{\sqrt{\kappa\epsilon y}}\right\} \exp\left(-zy\right) dy \tag{2.12}$$

Similarly, for $z \leq 0$, the first integral in (2.8), on normalizing with c, can be expressed as

$$\int_{0}^{\infty} Q\left(\frac{ax-z}{\sqrt{bx}}\right) p_A(x)dx = \frac{1}{\Gamma(m)} \int_{0}^{\infty} y^{m-1} Q\left(\frac{\epsilon y - z}{\sqrt{\kappa\epsilon y}}\right) \exp\left(-y\right) dy \tag{2.13}$$

Since, for $z > 0$,

$$\int_{0}^{\frac{z}{a}} \left\{1 - Q\left(\frac{z-ax}{\sqrt{bx}}\right)\right\} p_A(x)dx \quad = \quad F_A\left(\frac{z}{\epsilon}\right) \; - \; \frac{1}{\Gamma(m)} \int_{0}^{\frac{1}{\epsilon}} z^m y^{m-1} Q\left\{\frac{\sqrt{z}\left(1 - \epsilon y\right)}{\sqrt{\kappa\epsilon y}}\right\} \exp\left(-zy\right) dy, \tag{2.14}$$

using (2.12), (2.13) and (2.14), we obtain (2.15).

$$F_X(z) \approx \begin{cases} \frac{1}{\Gamma(m)} \int_0^{\infty} x^{m-1} Q\left(\frac{\epsilon x - z}{\sqrt{\kappa\epsilon x}}\right) \exp\left(-x\right) dx & z \leq 0 \\ F_A\left(\frac{z}{\epsilon}\right) + \frac{1}{\Gamma(m)} \int_0^{\infty} z^m x^{m-1} Q\left\{\sqrt{\frac{z}{\kappa\epsilon}}\left(\frac{\epsilon x - 1}{\sqrt{x}}\right)\right\} \exp\left(-zx\right) dx & z > 0 \end{cases} \tag{2.15}$$

Case II

Next, we consider the case when $X \sim \mathcal{N}\left(-aA, bA\right)$ for $a > 0$. The CDF in (2.7) can then expressed as

$$F_X(z) = \Pr\left(\eta < \frac{aA + z}{\sqrt{bA}}\right)$$

$$= \int_0^{\infty} Q\left(\frac{ax+z}{\sqrt{bx}}\right) p_A(x)dx, \quad z \geq 0 \tag{2.16}$$

Letting $z = -t, t > 0$, we have,

$$\begin{aligned} F_X(z) &= F_X(-t) \qquad\qquad t > 0 \\ &= \Pr\left(\eta > \frac{t - aA}{\sqrt{bA}}\right) \\ &= \int_{\frac{z}{a}}^{\infty} \left\{1 - Q\left(\frac{ax - t}{\sqrt{bx}}\right)\right\} p_A(x)dx + \int_0^{\frac{t}{a}} Q\left(\frac{t - ax}{\sqrt{bx}}\right) p_A(x)dx, \\ &\approx \int_{\frac{t}{a}}^{\infty} \left\{1 - Q\left(\frac{ax - t}{\sqrt{bx}}\right)\right\} p_A(x)dx \end{aligned} \tag{2.17}$$

using the approach in Case I. In the following, we obtain the CDF of X for Case I. The corresponding CDF for Case II can be similarly obtained.

2.4.3 Loskot Approximation

From [12, (13d)],

$$Q(x) \approx \sum_{n=1}^{3} a_n e^{-b_n x^2} \tag{2.18}$$

where

$$(a_1, a_2, a_3) = (0.168, 0.144, 0.002), \tag{2.19}$$

$$(b_1, b_2, b_3) = (0.876, 0.525, 0.603) \tag{2.20}$$

For $z \le 0$, using (2.18) in (2.15)

$$\begin{aligned} F_X(z) &\approx \sum_{n=1}^{3} \frac{a_n}{\Gamma(m)} \int_0^{\infty} x^{m-1} e^{-b_n\left(\frac{\epsilon x - z}{\sqrt{\kappa\epsilon x}}\right)^2} \exp(-x)\, dx \\ &= \sum_{n=1}^{3} \frac{a_n}{\Gamma(m)} e^{2b_n z/\kappa} \int_0^{\infty} x^{m-1} e^{-\left(\frac{b_n\epsilon}{\kappa}+1\right)x - \frac{b_n z^2}{\kappa\epsilon x}}\, dx \\ &= \sum_{n=1}^{3} \frac{2a_n(-z)^m e^{2b_n z/\kappa}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2\left(b_n + \frac{\kappa}{\epsilon}\right)}\right)^{m/2} K_m\left(-\frac{2z}{\kappa}\sqrt{b_n\left(b_n + \frac{\kappa}{\epsilon}\right)}\right) \end{aligned} \tag{2.21}$$

using (A.6). Similarly, $F_X(z), z > 0$ can be obtained. The final expression for the approximate CDF using the Loskot approximation, for various cases, is expressed in (2.15). $Z \mid A \sim \mathcal{N}(aA, bA), b > 0$, $A \sim \mathcal{G}(c, m)$, $(a_1, a_2, a_3) = (0.168, 0.144, 0.002)$, $(b_1, b_2, b_3) = (0.876, 0.525, 0.603)$, $\epsilon = \frac{|a|}{c}, \kappa = \frac{b}{|a|}, K_{\cdot}(\cdot)$ is the modified

Bessel function of the second kind [13] and $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function [13].

$$p_Z(z) = \frac{2c^m e^{\frac{az}{b}}}{\Gamma(m)\sqrt{2\pi b}} \left(\frac{|z|}{\sqrt{a^2+2bc}}\right)^{m-\frac{1}{2}} K_{m-\frac{1}{2}}\left(\frac{|z|}{b}\sqrt{a^2+2bc}\right), \quad a > 0 \tag{2.22}$$

$$F_Z(z) \approx \begin{cases} \sum_{n=1}^{3} \frac{2a_n(-z)^m e^{2b_n z/\kappa}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa}{\epsilon}\right)}\right)^{m/2} K_m\left(-\frac{2z}{\kappa}\sqrt{b_n\left(b_n+\frac{\kappa}{\epsilon}\right)}\right) & a > 0, z \le 0 \\ 1 - \frac{\gamma(m,\frac{-z}{\epsilon})}{\Gamma(m)} - \sum_{n=1}^{3} \frac{2(-z)^m a_n e^{\frac{-2b_n z}{\kappa}}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2(b_n+\frac{\kappa}{\epsilon})}\right)^{\frac{m}{2}} K_m\left(\frac{-2z}{k}\sqrt{b_n(b_n+\frac{\kappa}{\epsilon})}\right) & a < 0, z \le 0 \\ \frac{\gamma(m,\frac{z}{\epsilon})}{\Gamma(m)} + \sum_{n=1}^{3} \frac{2a_n z^m e^{2b_n z/\kappa}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa}{\epsilon}\right)}\right)^{m/2} K_m\left(\frac{2z}{\kappa}\sqrt{b_n\left(b_n+\frac{\kappa}{\epsilon}\right)}\right) & a > 0, z > 0 \\ 1 - \sum_{n=1}^{3} \frac{2(z)^m a_n e^{\frac{-2b_n z}{\kappa}}}{\Gamma(m)} \left(\frac{b_n}{\epsilon^2(b_n+\frac{\kappa}{\epsilon})}\right)^{\frac{m}{2}} K_m\left(\frac{2z}{\kappa}\sqrt{b_n(b_n+\frac{\kappa}{\epsilon})}\right) & a < 0, z > 0 \end{cases} \tag{2.15}$$

2.5 BER Analysis for λ-MRC

The BER in (2.3) is a combination of two error probabilities, conditioned on whether the decision at the relay is correct or not. These are $P_{e|1}$, defined in (2.5) and $P_{e|-1}$, that can be similarly defined. Both expressions are computed in the following.

2.5.1 Correct Decision at Relay

(2.5) can be expressed as

$$P_{e|1} = \int_0^\infty F_X(-\lambda y) p_Y(y)\, dy + \int_0^\infty F_X(\lambda y) p_Y(-y)\, dy \tag{2.21}$$

Substituting $F_X, a_s > 0$ from (2.22) and (2.15) in the first integral in (2.21), we have

$$\int_0^\infty F_X(-\lambda y) p_Y(y)\, dy = \sum_{n=1}^{3} \frac{2a_n(\lambda)^{m_s}}{\Gamma(m_s)} \left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s/2} \frac{2c_r^{m_r}}{\Gamma(m_r)\sqrt{2\pi b_r}} \left(\frac{1}{\sqrt{a_r^2+2b_r c}}\right)^{m_r-\frac{1}{2}}$$
$$\times \int_0^\infty y^{m_s+m_r-\frac{1}{2}} e^{\frac{a_r y}{b_r} - \frac{2b_n\lambda y}{\kappa_s}} K_{m_s}\left(\frac{2\lambda y}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right) K_{m_r-\frac{1}{2}}\left(\frac{y}{b_r}\sqrt{a_r^2+2b_r c_r}\right) dy \tag{2.22}$$

The above integral is of the form

$$\mathcal{I}_{m,n}(\alpha,\beta,\delta) = \int_0^\infty y^{m+n} e^{\alpha y} K_m(\beta y) K_n(\delta y)\, dy \qquad \{m,n,\beta,\delta\} > 0. \tag{2.23}$$

This integral does not appear to be tabulated and is difficult to obtain in closed form. However, from (2.21), (2.22),(2.15), it is evident that the integral appears in the final expression for the BER and we will use (2.23) repeatedly in the following to represent integrals of the form in (2.22). The second integral in (2.21) can now be expressed as

$$\int_0^\infty F_X(\lambda y)p_Y(-y)\,dy = \frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s, \frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy + \sum_{n=1}^{3}\frac{2a_n(\lambda)^{m_s}}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s/2}$$
$$\times \frac{2c_r^{m_r}}{\Gamma(m_r)\sqrt{2\pi b_r}}\left(\frac{1}{\sqrt{a_r^2+2b_r c}}\right)^{m_r-\frac{1}{2}} \mathcal{I}_{m_s,m_r-\frac{1}{2}}\left(-\frac{a_r}{b_r}+\frac{2b_n\lambda}{\kappa_s}, \frac{2\lambda}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}, \frac{1}{b_r}\sqrt{a_r^2+2b_rc_r}\right) \quad (2.24)$$

The first integral in (2.24) can be expressed using integration by parts as

$$\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s, \frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy =$$
$$-\frac{1}{\Gamma(m_s)}\left[\left\{\gamma\left(m_s, \frac{\lambda y}{\epsilon_s}\right)F_Y(-y)\right\}_0^\infty + \left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\int_0^\infty y^{m_s-1}e^{-\frac{\lambda y}{\epsilon_s}}F_Y(-y)\,dy\right] \quad (2.25)$$

resulting in

$$\begin{aligned}\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s, \frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy &= \frac{1}{\Gamma(m_s)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\int_0^\infty y^{m_s-1}e^{-\frac{\lambda y}{\epsilon_s}}F_Y(-y)\,dy\\ &= \frac{1}{\Gamma(m_s)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{m_r/2}\\ &\quad\times\int_0^\infty y^{m_s+m_r-1}e^{-\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)y}K_{m_r}\left(\frac{2y}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)dy \end{aligned}\quad (2.26)$$

upon substituting for F_Y, $a_r > 0$ from (2.15). From [14, (6.619.3)]

$$\int_0^\infty x^{\mu-1}e^{-\alpha x}K_\nu(\beta x)\,dx = \frac{\sqrt{\pi}\,(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\,\Gamma(\mu-\nu)}{\Gamma\left(\mu+\frac{1}{2}\right)}{}_2F_1\left(\mu+\nu, \nu+\frac{1}{2}; \mu+\frac{1}{2}; \frac{\alpha-\beta}{\alpha+\beta}\right)$$
$$\mathrm{Re}\,\{\mu\} > |\mathrm{Re}\nu|, \mathrm{Re}\,(\alpha+\beta) > 0. \quad (2.27)$$

Using (2.27) in (2.26), we obtain

$$\frac{1}{\Gamma(m_s)}\int_0^\infty \gamma\left(m_s, \frac{\lambda y}{\epsilon_s}\right)p_Y(-y)\,dy = \frac{\Gamma(m_s+2m_r)}{\Gamma\left(m_s+m_r+\frac{1}{2}\right)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{m_r/2}$$
$$\times\frac{\sqrt{\pi}\left[\frac{4}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_r}}{\left[\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_s+2m_r}}{}_2F_1\left(\begin{matrix}m_s+2m_r,\ m_r+\frac{1}{2}\\ m_s+m_r+\frac{1}{2}\end{matrix}; \frac{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)-\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}\right), \quad (2.28)$$

Thus, from (2.5)-(2.28), we obtain

$$
\begin{aligned}
&P\left(X+\lambda Y<0|x_s=1,x_r=1\right)\\
&\quad=\sum_{n=1}^{3}\frac{2a_n(\lambda)^{m_s}}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s/2}\frac{2c_r^{m_r}}{\Gamma(m_r)\sqrt{2\pi b_r}}\left(\frac{1}{\sqrt{a_r^2+2b_rc}}\right)^{m_r-\frac{1}{2}}\\
&\qquad\times\left\{\mathcal{I}_{m_s,m_r-\frac{1}{2}}\left(\frac{a_r}{b_r}-\frac{2b_n\lambda}{\kappa_s},\frac{2\lambda}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)},\frac{1}{b_r}\sqrt{a_r^2+2b_rc_r}\right)\right.\\
&\qquad\left.+\mathcal{I}_{m_s,m_r-\frac{1}{2}}\left(-\frac{a_r}{b_r}-\frac{2b_n\lambda}{\kappa_s},\frac{2\lambda}{\kappa_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)},\frac{1}{b_r}\sqrt{a_r^2+2b_rc_r}\right)\right\}\\
&\quad+\frac{\Gamma\left(m_s+2m_r\right)}{\Gamma\left(m_s+m_r+\frac{1}{2}\right)}\left(\frac{\lambda}{\epsilon_s}\right)^{m_s}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{m_r/2}\frac{\sqrt{\pi}\left[\frac{4}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_r}}{\left[\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right]^{m_s+2m_r}}\\
&\qquad\times{}_2F_1\left(\begin{matrix}m_s+2m_r,\ m_r+\frac{1}{2}\\ m_s+m_r+\frac{1}{2}\end{matrix};\frac{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)-\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}{\left(\frac{\lambda}{\epsilon_s}+\frac{2b_n}{\kappa_r}\right)+\frac{2}{\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}}\right)
\end{aligned}
\tag{2.29}
$$

2.5.2 Incorrect Decision at Relay

Given that an incorrect decision is made at the relay, the probability of error can be expressed as

$$
\begin{aligned}
P_{e|-1}&=\Pr\left(X+\lambda Y<0|x_s=1,x_r=-1\right)\\
&=\Pr\left(Y<\frac{-X}{\lambda}\middle|x_s=1,x_r=-1\right)\\
&=\int_{-\infty}^{\infty}F_Y\left(\frac{-x}{\lambda}\right)p_X(x)dx\\
&=\int_{0}^{\infty}F_Y\left(\frac{-x}{\lambda}\right)p_X(x)dx+\int_{0}^{\infty}F_Y\left(\frac{x}{\lambda}\right)p_X(-x)dx
\end{aligned}
\tag{2.30}
$$

The first integral in (2.30), upon substitution from (2.15) for $F_Y, a_r<0$ is

$$
\begin{aligned}
\int_0^{\infty}F_Y\left(\frac{-x}{\lambda}\right)p_X(x)dx&=\int_0^{\infty}p_X(x)dx-\frac{1}{\Gamma(m_r)}\int_0^{\infty}\gamma\left(m_r,\frac{x}{\lambda\epsilon_r}\right)p_X(x)dx\\
&-\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n+\frac{k_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}\frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2+2b_sc_s}}\right)^{m_s-\frac{1}{2}}\\
&\times\int_0^{\infty}e^{x\left(\frac{a_s}{b_s}+\frac{2b_n}{\lambda\kappa_r}\right)}x^{m_r+m_s-\frac{1}{2}}K_{m_r}\left(\frac{2x}{\lambda\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)K_{m_s-\frac{1}{2}}\left(\frac{x}{b_s}\sqrt{(a_s^2+2b_sc_s)}\right)dx
\end{aligned}
\tag{2.31}
$$

where the third integral in (2.31) is obtained after subsituting for $p_X, a_s>0$ from (2.22). The second integral in (2.31) can be expressed using the approach in (2.25) as

$$
\frac{1}{\Gamma(m_r)}\int_0^{\infty}\gamma\left(m_r,\frac{x}{\lambda\epsilon_r}\right)p_X(x)dx=1-\frac{1}{\Gamma(m_r)}\int_0^{\infty}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}x^{m_r-1}e^{\frac{-x}{\lambda\epsilon_r}}F_X(x)dx \tag{2.32}
$$

Substituting $F_X, (a_s > 0)$, from (2.15) in (2.32),

$$\frac{1}{\Gamma(m_r)}\int_0^\infty \gamma\left(m_r, \frac{x}{\lambda\epsilon_r}\right)p_X(x)dx = 1 - \frac{1}{\Gamma(m_r)}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\left\{\int_0^\infty \frac{x^{m_r-1}e^{\frac{-x}{\lambda\epsilon_r}}\gamma(m_s, \frac{x}{\epsilon_s})}{\Gamma(m_s)}dx\right.$$
$$\left. + \int_0^\infty \sum_{n=1}^{3}\frac{2a_n x^{m_s}}{\Gamma(m_s)}e^{\frac{2b_n x}{\kappa_s}}\left(\frac{b_n}{\epsilon_s^2\left(b_n + \frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}} K_{m_s}\left(\frac{2x}{k_s}\sqrt{b_n\left(b_n + \frac{k_s}{\epsilon_s}\right)}\right)x^{m_r-1}e^{\frac{-x}{\lambda\epsilon_r}}dx\right\} \quad (2.33)$$

The first integral in (2.33) can be expressed using [15, p. 138, (7)] as

$$\frac{1}{\Gamma(m_r)}\left(\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_r}\frac{1}{\Gamma(m_s)}\int_0^\infty t^{m_r-1}e^{\frac{-t\epsilon_s}{\lambda\epsilon_r}}\gamma(m_s, t)dt$$
$$= \frac{1}{\Gamma(m_r)}\left(\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_r}\frac{1}{\Gamma(m_s)}\frac{\Gamma(m_s+m_r)}{m_s\left(1+\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_s+m_r}}{}_2F_1\left(1, m_s+m_r; m_s+1; \frac{\lambda\epsilon_r}{\lambda\epsilon_r+\epsilon_s}\right) \quad (2.34)$$

The second integral in (2.33) can be expressed using (2.27) as

$$\frac{1}{\Gamma(m_r)}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}}\int_0^\infty x^{m_s+m_r-1}e^{-x\left(\frac{1}{\lambda\epsilon_r}-\frac{2b_n}{k_s}\right)}K_{m_s}\left(\frac{2x}{k_s}\sqrt{b_n\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)$$
$$= \frac{\Gamma(2m_s+m_r)}{\Gamma(m_s+m_r+\frac{1}{2})}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\sum_{n=1}^{3}\frac{a_n}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}} \times \frac{\sqrt{\pi}\left(2\times\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s}}{\left(\frac{1}{\lambda\epsilon_r}+\frac{2b_n}{\kappa_s}+\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{2m_s+m_r}}$$
$$\times {}_2F_1\left(\begin{matrix}2m_s+m_r, & m_s+\frac{1}{2}\\ m_s+m_r+\frac{1}{2}\end{matrix}; \frac{\left(\frac{1}{\lambda\epsilon_r}-\frac{2b_n}{\kappa_s}-\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)}{\left(\frac{1}{\lambda\epsilon_r}-\frac{2b_n}{\kappa_s}+\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)}\right) \quad (2.35)$$

The third integral in (2.31) can be expressed using (2.23) as,

$$\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}\frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2+2b_sc_s}}\right)^{m_s-\frac{1}{2}}$$
$$\times \mathcal{I}_{m_r, m_s-\frac{1}{2}}\left(\frac{a_s}{b_s}+\frac{2b_n}{\lambda\kappa_r}, \frac{2}{\lambda k_r}\sqrt{b_n\left(b_n+\frac{k_r}{\epsilon_r}\right)}, \frac{1}{b_s}\sqrt{\left(a_s^2+2b_sc_s\right)}\right) \quad (2.36)$$

With this, all integrals in (2.31) are evaluated. The second integral in (2.30) can be expressed as

$$\begin{aligned}
&\int_0^\infty F_Y\left(\frac{x}{\lambda}\right)p_X(-x)dx \\
&= \int_0^\infty p_X(-x)dx - \sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n+\frac{k_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}\frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2+2b_sc_s}}\right)^{m_s-\frac{1}{2}} \\
&\quad\times\int_0^\infty e^{x\left(\frac{-a_s}{b_s}+\frac{-2b_n}{\lambda k_r}\right)}x^{m_r+m_s-\frac{1}{2}}K_{m_r}\left(\frac{2x}{\lambda\kappa_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)K_{m_s-\frac{1}{2}}\left(\frac{x}{b_s}\sqrt{(a_s^2+2b_sc_s)}\right)dx \quad (2.37)
\end{aligned}$$

From (2.23), the second integral in (2.37) is obtained as

$$\begin{aligned}
&\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n+\frac{k_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}\frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2+2b_sc_s}}\right)^{m_s-\frac{1}{2}} \\
&\quad\times\mathcal{I}_{m_r,m_s-\frac{1}{2}}\left(-\frac{a_s}{b_s}-\frac{2b_n}{\lambda\kappa_r},\frac{2}{\lambda k_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)},\frac{1}{b_s}\sqrt{(a_s^2+2b_sc_s)}\right) \quad (2.38)
\end{aligned}$$

From (2.30)-(2.38), we obtain

$$\begin{aligned}
&P(X+\lambda Y<0|x_s=1,x_r=-1)= \\
&-\sum_{n=1}^{3}\frac{2a_n}{\Gamma(m_r)}\left(\frac{b_n}{\epsilon_r^2\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)}\right)^{\frac{m_r}{2}}\left(\frac{1}{\lambda}\right)^{m_r}\frac{2c_s^{m_s}}{\Gamma(m_s)\sqrt{2\pi b_s}}\left(\frac{1}{\sqrt{a_s^2+2b_sc_s}}\right)^{m_s-\frac{1}{2}}\times \\
&\left\{\mathcal{I}_{m_r,m_s-\frac{1}{2}}\left(-\frac{a_s}{b_s}-\frac{2b_n}{\lambda\kappa_r},\frac{2}{\lambda k_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)},\frac{1}{b_s}\sqrt{(a_s^2+2b_sc_s)}\right)\right. \\
&\left.+\mathcal{I}_{m_r,m_s-\frac{1}{2}}\left(\frac{a_s}{b_s}+\frac{2b_n}{\lambda\kappa_r},\frac{2}{\lambda k_r}\sqrt{b_n\left(b_n+\frac{\kappa_r}{\epsilon_r}\right)},\frac{1}{b_s}\sqrt{(a_s^2+2b_sc_s)}\right)\right\} \\
&+\frac{1}{\Gamma(m_r)}\left(\frac{1}{\lambda\epsilon_r}\right)^{m_r}\sum_{n=1}^{3}\frac{a_n}{\Gamma(m_s)}\left(\frac{b_n}{\epsilon_s^2\left(b_n+\frac{k_s}{\epsilon_s}\right)}\right)^{\frac{m_s}{2}}\frac{\sqrt{\pi}\left(2\times\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{m_s}}{\left(\frac{1}{\lambda\epsilon_r}+\frac{2b_n}{\kappa_s}+\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_s}{\epsilon_s}\right)}\right)^{2m_s+m_r}} \\
&\times\frac{\Gamma(2m_s+m_r)\Gamma(m_r)}{\Gamma(m_s+m_r+\frac{1}{2})}{}_2F_1\left(\begin{matrix}2m_s+m_r,\ m_s+\frac{1}{2}\\ m_s+m_r+\frac{1}{2}\end{matrix};\frac{\left(\frac{1}{\lambda\epsilon_r}-\frac{2b_n}{\kappa_s}-\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_S}{\epsilon_S}\right)}\right)}{\left(\frac{1}{\lambda\epsilon_r}-\frac{2b_n}{\kappa_s}+\frac{2}{k_s}\sqrt{b_n\left(b_n+\frac{\kappa_S}{\epsilon_S}\right)}\right)}\right) \\
&+\frac{1}{\Gamma(m_r)}\left(\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_r}\frac{1}{\Gamma(m_s)}\frac{\Gamma(m_s+m_r)}{m_s\left(1+\frac{\epsilon_s}{\lambda\epsilon_r}\right)^{m_s+m_r}}{}_2F_1\left(1,m_s+m_r;m_s+1;\frac{1}{1+\frac{\epsilon_s}{\lambda\epsilon_r}}\right) \quad (2.39)
\end{aligned}$$

Substituting (2.29) and (2.39) in (2.3), we obtain the final expression for the BER.

2.6 Results

In Figure 2.2, the analytical and simulated BER are plotted with respect to the average SNR for the S-D link. For convenience, we have chosen $E_r = E_s$, i.e. the source and relay transmit with equal power. (2.29) and

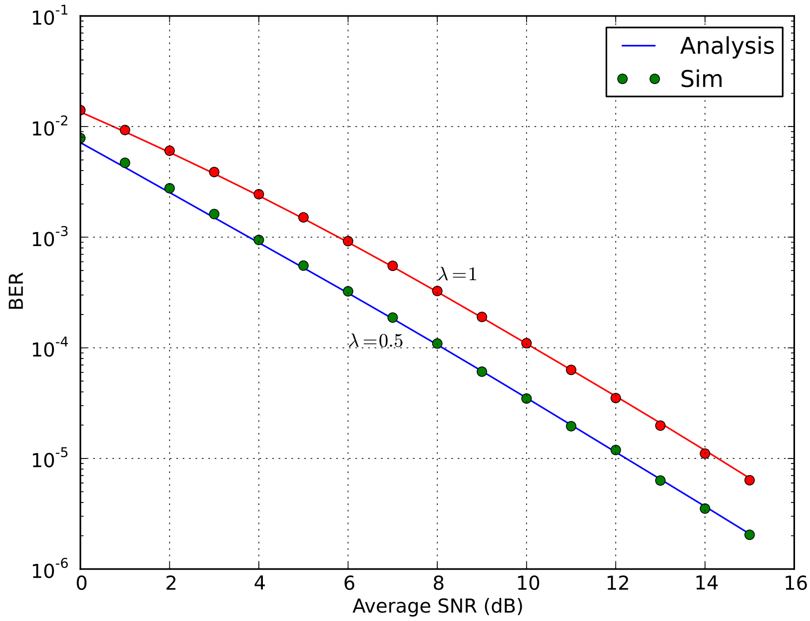

Figure 2.2: Comparison of the simulation and analytical results for $m = 3.7, m_s = 2.6, m_r = 2.6$. Both match perfectly.

(2.39) are used to compute the analytical BER using (2.3) for two cases, $\lambda = 0.5$ and $\lambda = 1$. As we can see, there is an excellent match between the simulation and analytical results, validating the expressions derived in the paper. Note that the Nakagami fading parameters are not integers.

Figure 2.3 provides some interesting insights into the diversity order for λ-MRC cooperation. Firstly, we note that the middle and bottom curves in Figure 2.3 have the same slope at high SNR, indicating the same diversity order. We note that $m_s + m_r = 2.7$ for the middle curve is exactly equal to $m = 2.7$ for the bottom curve. This validates the result in [16] where the diversity order was shown to be $\min(m, m_s + m_r)$ when $\lambda = 1$. Note that the top curve has a diversity order $2 < 2.7$ and its slope is less compared to that of the other two curves, at high SNR.

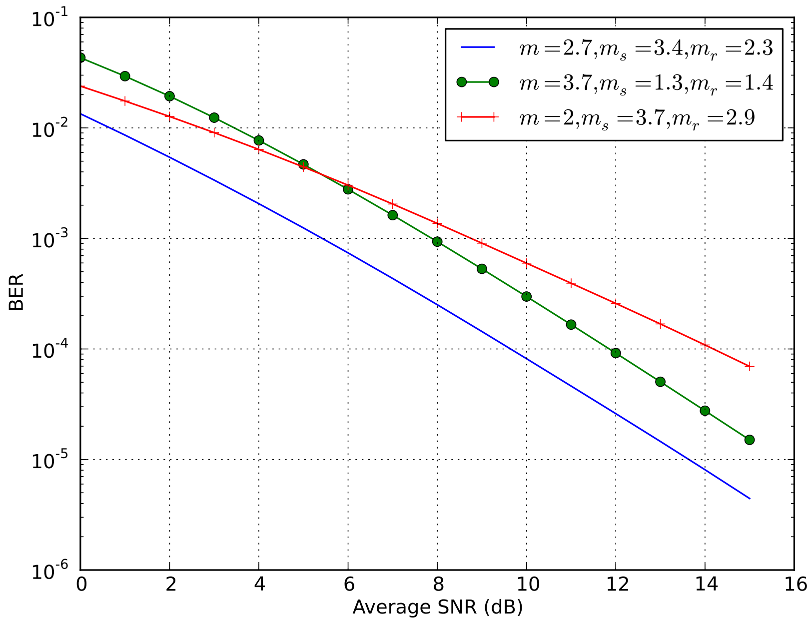

Figure 2.3: Analytical BER plots for $\lambda = 1$. Slopes for the lower two curves almost identical at high SNR indicating a similar diversity order.

Chapter 3

Exact Error Analysis for Decode and Forward λ-MRC Cooperation for N Relays

3.1 System Model and Problem Definition

For N relays the expression for BER is ,

$$P_e = \sum_{x_{ri}\in\{1,-1\}} \prod_{i=1}^{N} \epsilon_i^{\frac{1-x_{ri}}{2}} (1-\epsilon_i)^{\frac{1+x_{ri}}{2}} P\left(X + \sum_{i=1}^{N} \lambda_i Y_i < 0 | x_s = 1, x_{ri}\right) \tag{3.1}$$

where x_s stands for symbol transmitted from source, x_{r1} from relay 1 and so on.

3.2 CDF of Z

The exact cumulative distribution function (CDF) for a gamma CGRV is expessed in terms of the extended Fox-$\hat{H}$ function [17,18]. This approach is then used for obtaining the exact BER expressions by extrapolating the close relation between the Nakagami-m and gamma distributions.Z is gamma CG with parameters $a, b > 0$ if $Z \mid A \sim \mathcal{N}(aA, bA), A \sim \mathcal{G}(c, m)$ being Gamma distributed [10] with scale parameter $c > 0$ and order $m > 0$. The MGF of Z can be expressed as [**?**]

$$M_Z(s) = \left(1 + \frac{4x\bar{\gamma}}{m}s - \frac{4\bar{\gamma}}{m}s^2\right)^{-m} \tag{3.2}$$

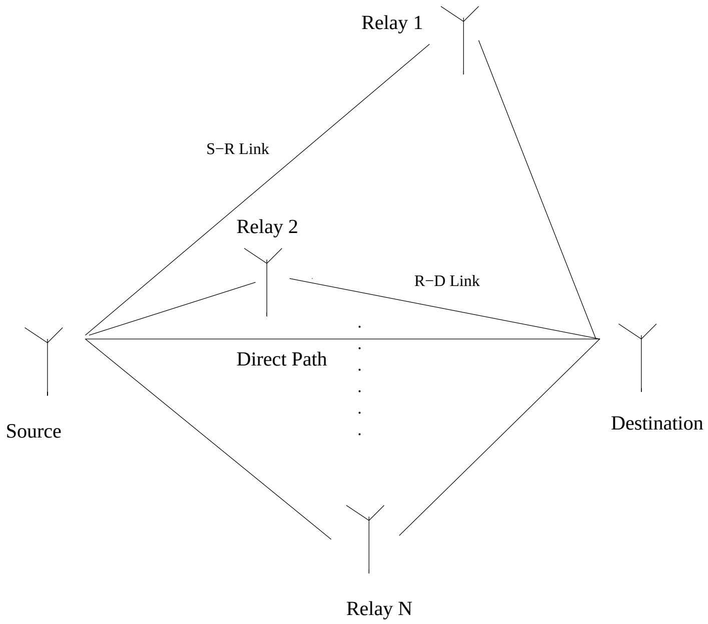

Figure 3.1: System Model

The CDF of Z can be expressed as

$$F_Z(y) = \mathcal{L}^{-1}\left\{\frac{M_Z(s)}{s}\right\} \tag{3.3}$$

From (3.2),

$$\frac{M_Z(s)}{s} = \frac{(\kappa_1\kappa_2)^m}{s\,(\kappa_1 - s)^m\,(\kappa_2 + s)^m} \tag{3.4}$$

where

$$\kappa_1, \kappa_2 = \frac{1}{2}\left(\sqrt{1 + \frac{m}{\gamma}} \pm x\right) \tag{3.5}$$

The statistics of Z for $m = 1$, i.e. Rayleigh fading, are known [7]. The method of finding $F_Z(x)$ using (3.3) for arbitrary m is explained through the case of $m = 1$.

3.2.1 $m = 1$

For $m = 1$,

$$\frac{M_Z(s)}{s} = \frac{\kappa_1\kappa_2}{s\,(\kappa_1 - s)\,(\kappa_2 + s)} \tag{3.6}$$

Using well known results for the inversion of Laplace transforms through contour integrals [19],

$$F_Z(y) = \begin{cases} 1 + \frac{1}{2\pi \jmath} \oint_{C_1} \frac{M_Z(s)}{s} e^{sy}\, ds & y \geq 0 \\ -\frac{1}{2\pi \jmath} \oint_{C_1} \frac{M_Z(-s)}{s} e^{-sy}\, ds & y < 0, \end{cases} \tag{3.7}$$

where C_1 is a suitable contour encompassing all poles in the left half plane. Note that the above form is necessary since the poles of $M_Z(s)$ are on both sides of the imaginary axis, since $\kappa_1, \kappa_2 > 0$. Using residue calculus [19],

$$F_Z(y) = \begin{cases} 1 - \frac{\kappa_1}{\kappa_1+\kappa_2} e^{-\kappa_2 y} & y \geq 0 \\ \frac{\kappa_2}{\kappa_1+\kappa_2} e^{-\kappa_1 y} & y < 0, \end{cases} \tag{3.8}$$

which was obtained in [7].Interestingly, $F_Z(x)$ has an alternative representation in the form of the extended Fox $\hat{H}$ function [17]. This is obtained as follows.

$M_Z(s)$ in (3.6) can be expressed in terms of Gamma functions [13] as

$$\frac{M_Z(s)}{s} = (\kappa_1 \kappa_2) \frac{\Gamma(s)\Gamma(\kappa_1 - s)\Gamma(\kappa_2 + s)}{\Gamma(1+s)\Gamma(1+\kappa_1 - s)\Gamma(1+\kappa_2 + s)} \tag{3.9}$$

Noting the fact that the above expression fits into the Mellin-Barnes integral expression for the Fox $\hat{H}$ function, [18, (T.I.1)], [17],

$$F_Z(y) = \begin{cases} 1 + (\kappa_1 \kappa_2)\, \hat{H}^{1,2}_{3,3}\left[e^{y} \middle| \begin{matrix} \Upsilon^1 \\ \Upsilon^2 \end{matrix} \right] & y \geq 0 \\ (\kappa_1 \kappa_2)\, \hat{H}^{1,2}_{3,3}\left[e^{-y} \middle| \begin{matrix} \Upsilon^1 \\ \Upsilon^2 \end{matrix} \right] & y < 0, \end{cases} \tag{3.10}$$

where

$$\Upsilon^1 = \begin{cases} \{(1,1,1), (1-\kappa_2,1,1), (1+\kappa_1,1,1)\} & y \geq 0 \\ \{(1,1,1), (1-\kappa_1,1,1), (1+\kappa_2,1,1)\} & y < 0 \end{cases} \tag{3.11}$$

$$\Upsilon^2 = \begin{cases} \{(\kappa_1,1,1), (0,1,1), (-\kappa_2,1,1)\} & y \geq 0 \\ \{(\kappa_2,1,1), (0,1,1), (-\kappa_1,1,1)\} & y < 0 \end{cases} \tag{3.12}$$

While the notation in [18] is used above, the Fox $\hat{H}$ function was originally defined in [17] where several interesting properties are listed. It is reiterated that (3.8) and (3.10) are equivalent.

3.2.2 Arbitrary m

(3.4) can be expressed in terms of Gamma functions as

$$\frac{M_Z(s)}{s} = \left(\frac{m}{4\bar{\gamma}}\right)^m \frac{\Gamma(s)\Gamma^m(\kappa_1 - s)\,\Gamma^m(\kappa_2 + s)}{\Gamma(1+s)\Gamma^m(1+\kappa_1 - s)\,\Gamma^m(1+\kappa_2+s)} \tag{3.13}$$

Then, from [18, (T.I.1)], [17],

$$F_Z(y) = \begin{cases} 1 + \left(\frac{m}{4\bar{\gamma}}\right)^m \hat{H}^{1,2}_{3,3}\left[e^{y}\,\middle|\,\begin{matrix}\Upsilon^1\\ \Upsilon^2\end{matrix}\right] & y \geq 0 \\ \left(\frac{m}{4\bar{\gamma}}\right)^m \hat{H}^{1,2}_{3,3}\left[e^{-y}\,\middle|\,\begin{matrix}\Upsilon^1\\ \Upsilon^2\end{matrix}\right] & y < 0, \end{cases} \tag{3.14}$$

where C is a suitable contour and

$$\Upsilon^1 = \begin{cases} \{(1,1,1)\,,(1-\kappa_2,1,m)\,,(1+\kappa_1,1,m)\} & y \geq 0 \\ \{(1,1,1)\,,(1-\kappa_1,1,m)\,,(1+\kappa_2,1,m)\} & y < 0 \end{cases} \tag{3.15}$$

$$\Upsilon^2 = \begin{cases} \{(0,1,1)\,,(\kappa_1,1,m)\,,(-\kappa_2,1,m)\} & y \geq 0 \\ \{(0,1,1)\,,(\kappa_2,1,m)\,,(-\kappa_1,1,m)\} & y < 0 \end{cases} \tag{3.16}$$

Now that general closed form expression for the CDF of a gamma CGRV has been obtained ,the approach for evaluating this CDF is now used for BER analysis.

3.3 BER Analysis for N Relays

Consider3.1for the general case where we have N Relays,then expression for BER is as follows,

$$P\left(X + \sum_{i=1}^{N} \lambda_i Y_i < 0 | x_s = 1, x_{r1}, x_{r2}, \ldots, x_{rN}\right) = \frac{1}{2\pi \jmath} \oint_C \frac{M_X(s) M_{Y1}(\lambda_1 s) \ldots M_{YN}(\lambda_N s)}{s}\, ds \tag{3.17}$$

Using 3.2

$$= \left(\frac{m_s}{4\bar{\gamma}_s}\right)^{m_s} \frac{1}{2\pi\jmath} \oint_C \frac{\Gamma(s)}{\Gamma(1+s)} \frac{\Gamma^{m_s}(\kappa_1 - s)\Gamma^{m_s}(\kappa_2 + s)}{\Gamma^{m_s}(1+\kappa_1 - s)\Gamma^{m_s}(1+\kappa_2+s)}$$

$$\times \prod_{i=1}^{N} \left(\frac{m_{ri}}{4\lambda_i^2\bar{\gamma}_{ri}}\right)^{m_{ri}} \frac{\Gamma^{m_{ri}}(\eta_{1ri} - s)\Gamma^{m_{ri}}(\eta_{2ri} + s)}{\Gamma^{m_{ri}}(1+\eta_{1ri} - s)\Gamma^{m_{ri}}(1+\eta_{2ri}+s)} ds$$

$$= \left(\frac{m_s}{4\bar{\gamma}_s}\right)^{m_s} \prod_{i=1}^{N} \left(\frac{m_{ri}}{4\lambda_i^2\bar{\gamma}_{ri}}\right)^{m_{ri}} \hat{H}^{N+1,N+2}_{2N+3,2N+3}\left[1 \middle| \begin{matrix}\Upsilon^1 \\ \Upsilon^2\end{matrix}\right] \quad (3.18)$$

$$\Upsilon^1 = \{(1,1,1),(1-\kappa_2,1,m_s),(1+\kappa_1,1,m_s),\ (1-\eta_{2r1},1,m_{r1}),(1+\eta_{1r1},1,m_{r1}),$$
$$(1-\eta_{2r2},1,m_{r2}),(1+\eta_{2r2},1,m_{r2})\ldots(1-\eta_{2rN},1,m_{rN}),(1+\eta_{2rN},1,m_{rN})\}$$
$$\Upsilon^2 = \{(0,1,1),(\kappa_1,1,m_s),(-\kappa_2,1,m_s),(\eta_{1r1},1,m_{r1}),\ (-\eta_{2r1},1,m_{r1}),(\eta_{1r2},1,m_{r2}),$$
$$(-\eta_{2r2},1,m_{r2}),\ldots(\eta_{1rN},1,m_{rN}),(-\eta_{2rN},1,m_{rN})\} \quad (3.19)$$

$$\kappa_1,\kappa_2 = \frac{1}{2}\left(\sqrt{1+\frac{m_s}{\gamma_s}} \pm 1\right) \quad (3.20)$$

$$\eta_{1ri},\eta_{2ri} = \frac{1}{2\lambda_i}\left(\sqrt{1+\frac{m_{ri}}{\gamma_{ri}}} \pm x_{ri}\right) \quad (3.21)$$

3.4 BER Analysis for Single Relay

The expression in (2.3) consists of two error probabilities obtained with a) correct ($x_1 = 1$) and b) incorrect ($x_1 = -1$) decision at the relay. Both these probabilities are separately computed below.

3.4.1 Correct Decision at the Relay

The probability of error, given that a correct decision was made at the relay can be expressed using (3.3) as

$$P(X+\lambda Y<0|x_0=1,x_1=1) = \frac{1}{2\pi\jmath}\oint_C \frac{M_X(s)M_Y(\lambda s)}{s} ds \quad (3.22)$$

where C is a suitable contour. Using the subscripts 0 and 1 for the source and relay parameters in M_X and M_Y respectively, and substituting $x_0 = x_1 = 1$, the integrand in (3.22) is

$$\frac{M_X(s)M_Y(\lambda s)}{s} = \left(\frac{m_0}{4\bar{\gamma}_0}\right)^{m_0}\left(\frac{m_1}{4\lambda^2\bar{\gamma}_1}\right)^{m_1}\frac{\Gamma(s)}{\Gamma(1+s)}\frac{\Gamma^{m_0}(\kappa_1 - s)\,\Gamma^{m_0}(\kappa_2 + s)}{\Gamma^{m_0}(1+\kappa_1 - s)\,\Gamma^{m_0}(1+\kappa_2+s)} \times \frac{\Gamma^{m_1}(\eta_1 - s)\,\Gamma^{m_1}(\eta_2+s)}{\Gamma^{m_1}(1+\eta_1-s)\,\Gamma^{m_1}(1+\eta_2+s)} \tag{3.23}$$

where

$$\kappa_1, \kappa_2 = \frac{1}{2}\left(\sqrt{1+\frac{m_0}{\gamma_0}} \pm 1\right) \tag{3.24}$$

$$\eta_1, \eta_2 = \frac{1}{2\lambda}\left(\sqrt{1+\frac{m_1}{\gamma_1}} \pm 1\right) \tag{3.25}$$

Using the second contour in (3.7) and subsequent approach to obtain (3.14),

$$P(X + \lambda Y < 0|x_0 = 1, x_1 = 1) = \hat{H}^{2,3}_{5,5}\left[1\left|\begin{matrix}\Upsilon^1\\ \Upsilon^2\end{matrix}\right.\right] \tag{3.26}$$

with

$$\Upsilon^1 = \{(1,1,1), (1-\kappa_2, 1, m_0), (1-\eta_2, 1, m_1),\ (1+\kappa_1, 1, m_0), (1+\eta_1, 1, m_1)\} \tag{3.27}$$

$$\Upsilon^2 = \{(\kappa_1, 1, m_0), (\eta_1, 1, m_1), (0,1,1), (-\kappa_2, 1, m_0),\ (-\eta_2, 1, m_1)\} \tag{3.28}$$

3.4.2 Incorrect Decision at the Relay

The probability of error, given that an incorrect decision was made at the relay can be expressed using (3.26) as So,

$$P(X + \lambda Y < 0|x_0 = 1, x_1 = -1) = \hat{H}^{2,3}_{5,5}\left[1\left|\begin{matrix}\Upsilon^1\\ \Upsilon^2\end{matrix}\right.\right] \tag{3.29}$$

with

$$\Upsilon^1 = \{(1,1,1), (1-\kappa_2, 1, m_0), (1-\eta_2, 1, m_1),\ (1+\kappa_1, 1, m_0), (1+\eta_1, 1, m_1)\} \tag{3.30}$$

$$\Upsilon^2 = \{(\kappa_1, 1, m_0), (\eta_1, 1, m_1), (0,1,1), (-\kappa_2, 1, m_0),\ (-\eta_2, 1, m_1)\} \tag{3.31}$$

Table 3.1: Various parameters required for computing the conditional probabilites for two relays

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$	κ_1, κ_2	η_1, η_2	α_1, α_2
$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$	$\frac{1}{2}\left(\sqrt{1+\frac{m_0}{\gamma_0}} \pm 1\right)$	$\frac{1}{2\lambda_1}\left(\sqrt{1+\frac{m_1}{\gamma_1}} \mp 1\right)$	$\frac{1}{2\lambda_2}\left(\sqrt{1+\frac{m_2}{\gamma_2}} \mp 1\right)$
$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	$\frac{1}{2}\left(\sqrt{1+\frac{m_0}{\gamma_0}} \pm 1\right)$	$\frac{1}{2\lambda_1}\left(\sqrt{1+\frac{m_1}{\gamma_1}} \mp 1\right)$	$\frac{1}{2\lambda_2}\left(\sqrt{1+\frac{m_2}{\gamma_2}} \pm 1\right)$
$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\frac{1}{2}\left(\sqrt{1+\frac{m_0}{\gamma_0}} \pm 1\right)$	$\frac{1}{2\lambda_1}\left(\sqrt{1+\frac{m_1}{\gamma_1}} \pm 1\right)$	$\frac{1}{2\lambda_2}\left(\sqrt{1+\frac{m_2}{\gamma_2}} \mp 1\right)$
$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\frac{1}{2}\left(\sqrt{1+\frac{m_0}{\gamma_0}} \pm 1\right)$	$\frac{1}{2\lambda_1}\left(\sqrt{1+\frac{m_1}{\gamma_1}} \pm 1\right)$	$\frac{1}{2\lambda_2}\left(\sqrt{1+\frac{m_2}{\gamma_2}} \pm 1\right)$

where

$$\eta_1, \eta_2 = \frac{1}{2\lambda}\left(\sqrt{1+\frac{m_1}{\gamma_1}} \mp 1\right) \tag{3.32}$$

Note that $x_1 = -1$ results in different values for η above and distinguishes (3.30) from (3.26). Substituting (3.26) and (3.30) in (2.3) we obtain the final expression for BER.

3.5 Two Relays

In this case, the expression for the BER is given by

$$P_e = \sum_{x_1 \in \{1,-1\}} \sum_{x_2 \in \{1,-1\}} \varepsilon_1^{\frac{1-x_1}{2}} (1-\varepsilon_1)^{\frac{1+x_1}{2}} \varepsilon_2^{\frac{1-x_2}{2}} (1-\varepsilon_2)^{\frac{1+x_2}{2}} P(X + \lambda_1 Y_1 + \lambda_2 Y_2 < 0 | x_0 = 1, x_1, x_2)$$

Similar to (3.22), we have the MGF

$$\begin{aligned} \frac{M_X(s) M_{Y1}(\lambda_1 s) M_{Y2}(\lambda_2 s)}{s} &= \left(\frac{m_0}{4\bar{\gamma}_0}\right)^{m_0} \left(\frac{m_1}{4\lambda_1^2 \bar{\gamma}_1}\right)^{m_1} \left(\frac{m_2}{4\lambda_2^2 \bar{\gamma}_2}\right)^{m_2} \times \frac{\Gamma^{m_0}(\kappa_1 - s)\Gamma^{m_0}(\kappa_2 + s)}{\Gamma^{m_0}(1+\kappa_1 - s)\Gamma^{m_0}(1+\kappa_2+s)} \\ &\times \frac{\Gamma^{m_1}(\eta_1 - s)\Gamma^{m_1}(\eta_2 + s)}{\Gamma^{m_1}(1+\eta_1 - s)\Gamma^{m_1}(1+\eta_2+s)} \frac{\Gamma^{m_2}(\alpha_1 - s)\Gamma^{m_2}(\alpha_2 + s)}{\Gamma^{m_2}(1+\alpha_1 - s)\Gamma^{m_2}(1+\alpha_2+s)} \frac{\Gamma(s)}{\Gamma(1+s)} \end{aligned} \tag{3.33}$$

where more terms appear due to an extra relay between the source and destination. Using the approach the previous section, the conditional probability in (3.33) can be expressed as

$$P(X + \lambda_1 Y_1 + \lambda_2 Y_2 < 0 |\ x_0 = 1, x_1, x_2) = \hat{H}_{7,7}^{3,4}\left[1 \middle| \begin{matrix} \Upsilon^1 \\ \Upsilon^2 \end{matrix}\right] \tag{3.34}$$

$$\Upsilon^1 = \{(1,1,1),(1-\kappa_2,1,m_0),(1-\eta_2,1,m_1),\ (1+\kappa_1,1,m_0),(1+\eta_1,1,m_1),$$
$$(1-\alpha_2,1,m_2),(1+\alpha_1,1,m_2)\}$$

$$\Upsilon^2 = \{(\kappa_1,1,m_0),(\eta_1,1,m_1),(0,1,1),(-\kappa_2,1,m_0),\ (-\eta_2,1,m_1),(\alpha_1,1,m_2),(-\alpha_2,1,m_2)\} \quad (3.35)$$

The above expression is different for different values of the vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and results in four different values depending upon a combination of κ, η and α. These are tabulated in Table 3.1. These values for the conditional probability in (3.33) are then used to obtain a closed form expression for the BER.

3.6 Results and Discussion

In Figure 3.2 the simulation and analytical results for various combinations of m across different links are provided for a single relay. The expressions in (3.26), (3.30) and (2.4) are used in (2.3) to evalute the exact BER. We have assumed $E_s = E_r$ for generating the results. The simulations perfectly follow the analysis, validating the expressions obtained for single relay.

Figure 3.3 and 3.4 validate concepts related to the diversity order. The diversity order is the same for a) different λ but constant m and b) constant λ but same value of $m + \min(m_s, m_r)$ [16]. For the single relay system, Figures 3.3, 3.4 provide a practical verification of the these results.

The expressions in (3.33) and the following equations are used to generate the plots in Figure 3.5 for two relays. In Figure 3.5 for choosen fading parameters simulations follow analysis verifying our expression for two relays,here we have assumed $E_s = E_{r1} = E_{r2}$. Cooperation with multiple relays is supposed to improve the system performance. This is now established for λ-MRC in Figure 3.6. The additional relay between the source and desitnation brings down the BER curve, leading to relay diversity. Such results for multiple relays based on exact analysis are rare.

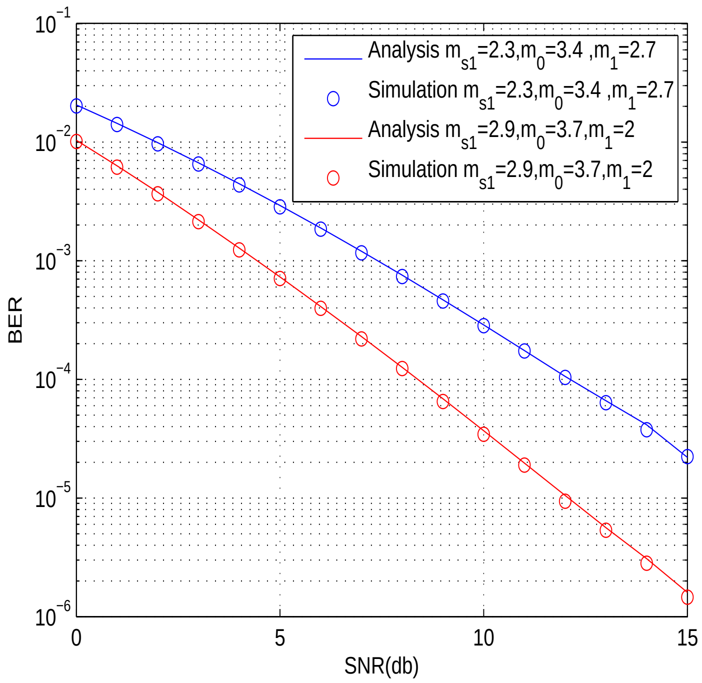

Figure 3.2: Analysis and Simulation for Single Relay for $\lambda = 1$

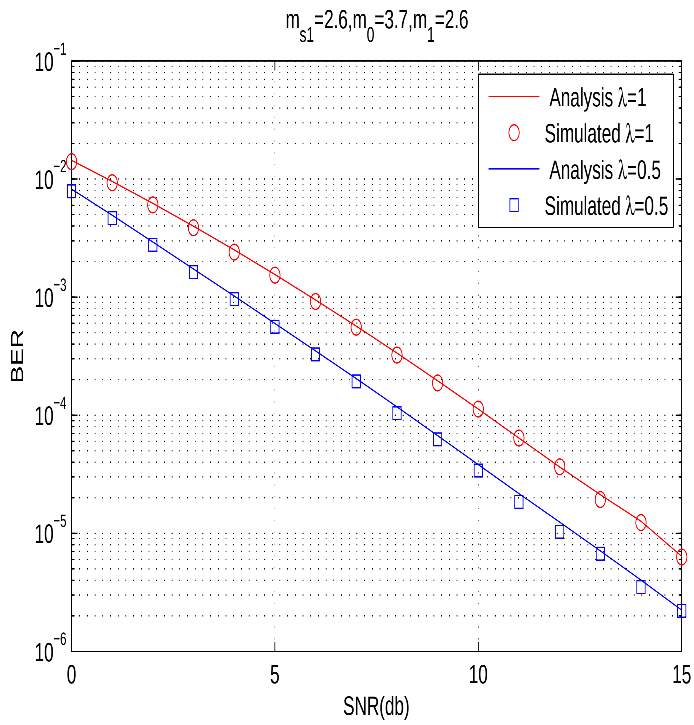

Figure 3.3: Analysis and Simulation BER Plot for Single Relay. Slopes for two curves almost identical at high SNR indicating a similar diversity order.

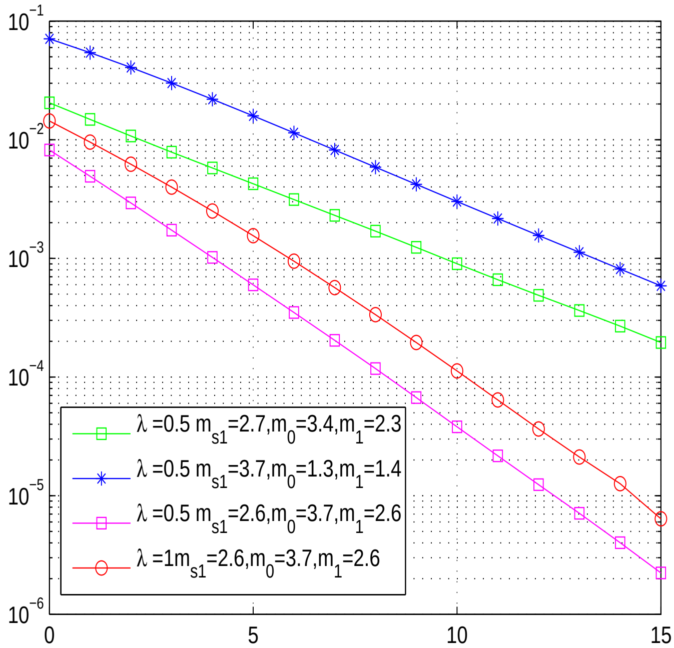

Figure 3.4: Verification of Diversity Order

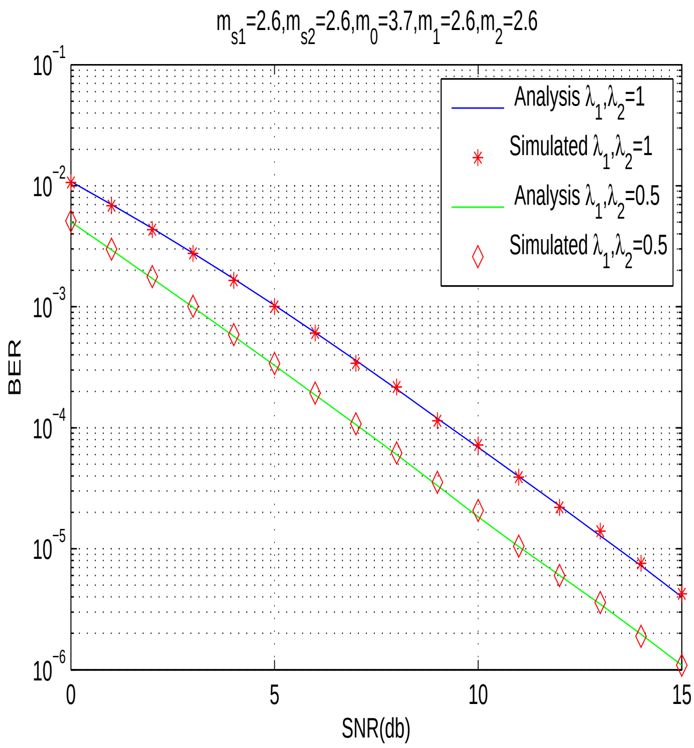

Figure 3.5: BER plots for $\lambda = 1, \lambda = 0.5$ for two relays.They match at all snr values

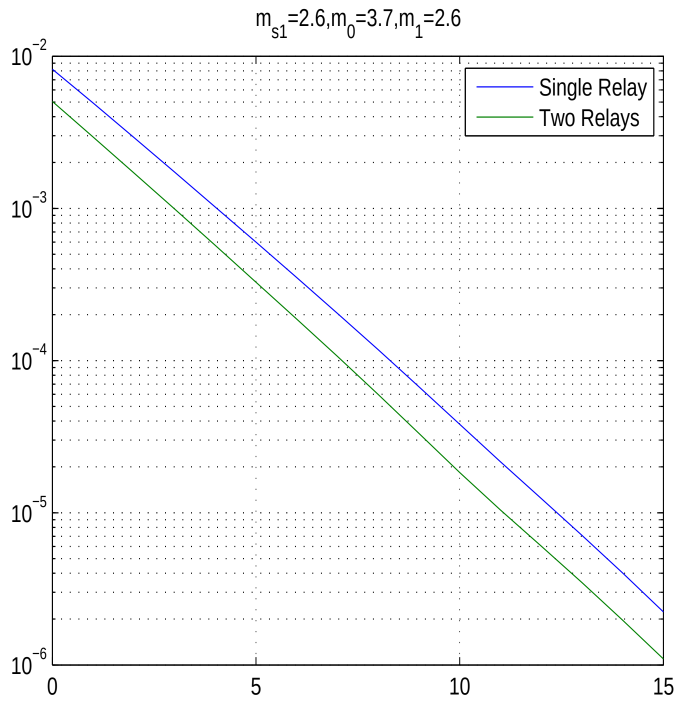

Figure 3.6: λ = 0.5 i.e constant , and all relays have same fading parameter m

Chapter 4

Error Analysis for PL combiner in Nakagami *m* fading

4.1 Introduction

The ML decision rule for DF cooperative systems was presented in [20], followed by a detailed derivation in [21]. Further, the PL combiner was suggested as useful practical alternative for the ML detector in [21]. The BER in a Nakagami-m fading channel for integer values of m for PL combiner are obtained in [6]. The approach used in the above methods is direct and results in complicated analysis. Here we employ MGF approach.

4.2 ML Decision

The ML decision criterion at the destination is obtained from [7, 22] as

$$X + f(Y) \underset{-1}{\overset{1}{\gtrless}} 0, \tag{4.1}$$

$$\tag{4.2}$$

4.3 PL combiner

The PL combiner for ML-DF cooperative systems is obtained by using the following piecewise linear approximation for $f(t)$ [22] in [16]

$$f(t) \approx \begin{cases} \ln\frac{1}{\delta} & t \geq \ln\frac{1}{\delta} \\ t & \ln\delta < t < \ln\frac{1}{\delta} \\ \ln\delta & t < \ln\delta \end{cases} . \tag{4.3}$$

4.4 Statistics of X,Y and f(Y)

Pdf and MGF of X and Y are already defined in 3.4 .MGF of f(y)

$$\phi_{f(y)}(s) = E\left[e^{sf(y)}\right] \tag{4.4}$$

$$= \int_{-\infty}^{\infty} e^{sf(y)} p_Y(y)dy \tag{4.5}$$

$$= \int_{-\infty}^{ln\delta} e^{sln(\delta)} p_Y(y)dy + \int_{ln\delta}^{0} e^{sy} p_Y(y)\,dy \quad + \int_{0}^{ln\delta} e^{sy} p_Y(y)\,dy + \int_{ln(\frac{1}{\delta})}^{\infty} e^{sln\frac{1}{\delta}} p_Y(y)\,dy$$

$$= \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3 + \mathcal{I}_4 \tag{4.6}$$

$$\mathcal{I}_1 = \delta^s \int_{-\infty}^{ln\delta} \frac{2c^m e^{\frac{ay}{b}}}{\Gamma(m)\sqrt{2\pi b}} \left(\frac{|x|}{\sqrt{a^2+2bc}}\right)^{m-\frac{1}{2}} K_{m-\frac{1}{2}}\left(\frac{|x|}{b}\sqrt{a^2+2bc}\right) dx \tag{4.7}$$

Using

$$K_{m-\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^{m-1} \frac{(m-1+k)!}{k!\,(m-1-k)!\,(2z)^k} \tag{4.8}$$

$$\mathcal{I}_1 = \frac{\delta^s c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m} \sum_{k=0}^{m-1} \frac{(m-1+k)!}{k!\,(m-1-k)!} \left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k \int_{-\infty}^{ln\delta} e^{\frac{ax}{b} + \frac{-|x|\sqrt{a^2+2bc}}{b}} |x|^{m-1-k}\,dx$$

$$= \frac{\delta^s c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m} \sum_{k=0}^{m-1} \frac{(m-1+k)!}{k!\,(m-1-k)!} \left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k$$

$$\times \int_{-\infty}^{ln\delta} e^{x\left(\frac{a}{b} + \frac{\sqrt{a^2+2bc}}{b}\right)} (-x)^{m-1-k}\,dx \tag{4.9}$$

which yields

$$\mathcal{I}_1 = \frac{c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m}\sum_{k=0}^{m-1}(-1)^{m-1-k}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k$$
$$\times\left\{\frac{(ln\delta)^{m-1-k}\delta^{\alpha+s}}{\alpha}+\sum_{j=1}^{m-1-k}\frac{(-1)^j(ln\delta)^{m-k-j-1}\delta^{\alpha+s}}{\alpha^{j+1}}\frac{(m-1-k)!}{(m-k-j-1)!}\right\} \quad (4.10)$$

similarly

$$\mathcal{I}_2 = \frac{c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m}\sum_{k=0}^{m-1}\frac{(-1)^{m-1-k}(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k\left\{\frac{-(ln\delta)^{m-1-k}\delta^{\alpha+s}}{\alpha+s}\right.$$
$$\left.+\sum_{j=1}^{m-1-k}\frac{(-1)^{j+1}(ln\delta)^{m-k-j-1}\delta^{\alpha+s}}{(\alpha+s)^{j+1}}\frac{(m-1-k)!}{(m-k-j-1)!}+(-1)^{m-k-1}\frac{(m-1-k)!}{(\alpha+s)^{m-k}}\right\} \quad (4.11)$$

$$\mathcal{I}_3 = \frac{c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m}\sum_{k=0}^{m-1}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k\left\{-\frac{(ln(\frac{1}{\delta}))^{m-1-k}\left(\frac{1}{\delta}\right)^{(s-\beta)}}{s-\beta}\right.$$
$$\left.-\sum_{j=1}^{m-1-k}\frac{\left(ln(\frac{1}{\delta})\right)^{m-k-j-1}}{(s-\beta)^{j+1}}\left(\frac{1}{\delta}\right)^{(s-\beta)}\frac{(m-1-k)!}{(m-k-j-1)!}+\frac{(m-1-k)!}{(s-\beta)^{m-k}}\right\} \quad (4.12)$$

$$\mathcal{I}_4 = \frac{c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m}\sum_{k=0}^{m-1}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k\left\{+\frac{\left(ln\left(\frac{1}{\delta}\right)\right)^{m-1-k}\left(\frac{1}{\delta}\right)^{(s-\beta)}}{\beta}\right.$$
$$\left.+\sum_{j=1}^{m-1-k}\frac{\left(ln\left(\frac{1}{\delta}\right)\right)^{m-k-j-1}}{\beta^{j+1}}\left(\frac{1}{\delta}\right)^{(s-\beta)}\frac{(m-1-k)!}{(m-k-j-1)!}\right\} \quad (4.13)$$

where

$$\alpha = \frac{\sqrt{a^2+2bc}+a}{b} \quad (4.14)$$
$$\beta = \frac{\sqrt{a^2+2bc}-a}{b} \quad (4.15)$$

Finally characteristic function of f(Y) is given by

$$\phi_{f(Y)}(s) = \frac{c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m}\sum_{k=0}^{m-1}(-1)^{m-1-k}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k \times \Bigg\{(ln\delta)^{m-1-k}\delta^{\alpha+s}$$
$$\times\frac{s}{(\alpha)(\alpha+s)} + \sum_{j=1}^{m-1-k}\frac{(-1)^j(m-1-k)!}{(m-k-j-1)!}\times\left(\frac{(ln\delta)^{m-k-j-1}\delta^{\alpha+s}}{\alpha^{j+1}} - \frac{(ln\delta)^{m-k-j-1}\delta^{\alpha+s}}{(\alpha+s)^{j+1}}\right) + (-1)^{m-k-1}$$
$$\times\frac{(m-1-k)!}{(\alpha+s)^{m-k}}\Bigg\} + \frac{c^m}{\Gamma(m)\left(\sqrt{a^2+2bc}\right)^m}\sum_{k=0}^{m-1}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k\Bigg\{(ln(\frac{1}{\delta}))^{m-1-k}\left(\frac{1}{\delta}\right)^{(s-\beta)}$$
$$\times\frac{s}{\beta(s-\beta)} - \sum_{j=1}^{m-1-k}\left(ln(\frac{1}{\delta})\right)^{m-k-j-1}\left(\frac{1}{\delta}\right)^{(s-\beta)}\frac{(m-1-k)!}{(m-k-j-1)!}\left(\frac{1}{(s-\beta)^{j+1}} - \frac{1}{\beta^{j+1}}\right) + \frac{(m-1-k)!}{(s-\beta)^{m-k}}\Bigg\} \quad (4.16)$$

4.5 BER Analysis

The conditional probability in (2.3) can be expressed as

$$P(X+f(Y)<0|x_s=1,x_r) = \frac{1}{2\pi\mathrm{J}}\oint_c\frac{\phi_X(s)\phi_{f(Y)}(s)}{s}\,ds \quad (4.17)$$

$$P(X+f(Y)<0|x_s=1,x_r) = \frac{1}{2\pi\mathrm{J}}\frac{c_1^m}{\Gamma(m)\left(\sqrt{a_1^2+2b_1c_1}\right)^m}\sum_{k=0}^{m-1}(-1)^{m-1-k}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b_1}{2\sqrt{a_1^2+2b_1c_1}}\right)^k$$
$$\times\Bigg\{\oint_c(ln\delta)^{m-1-k}\delta^{\alpha_1-s}\frac{1}{(\alpha_1)(\alpha_1+s)(\alpha+s)^m(s-\beta)^m}\,ds + \sum_{j=1}^{m-1-k}\frac{(-1)^j(m-1-k)!}{(m-k-j-1)!}\times\Bigg(\oint_c\frac{(ln\delta)^{m-k-j-1}\delta^{\alpha_1+s}}{\alpha_1^{j+1}s(\alpha+s)^m(s-\beta)^m}\,ds$$
$$-\oint_c\frac{(ln\delta)^{m-k-j-1}\delta^{\alpha_1+s}}{(\alpha_1+s)^{j+1}s(\alpha+s)^m(s-\beta)^m}\,ds\Bigg) + \oint_c(-1)^{m-k-1}\frac{(m-1-k)!}{(\alpha_1+s)^{m-k}s(\alpha+s)^m(s-\beta)^m}\,ds\Bigg\}$$
$$+\frac{1}{2\pi\mathrm{J}}\frac{c_1^m}{\Gamma(m)\left(\sqrt{a_1^2+2b_1c_1}\right)^m}\sum_{k=0}^{m-1}\frac{(m-1+k)!}{k!\,(m-1-k)!}\left(\frac{b_1}{2\sqrt{a_1^2+2b_1c_1}}\right)^k$$
$$\times\Bigg\{\oint_c(ln(\frac{1}{\delta}))^{m-1-k}\left(\frac{1}{\delta}\right)^{(s-\beta_1)}\frac{1}{\beta_1(s-\beta_1)(\alpha+s)^m(s-\beta)^m}\,ds - \sum_{j=1}^{m-1-k}\left(ln(\frac{1}{\delta})\right)^{m-k-j-1}\left(\frac{1}{\delta}\right)^{(s-\beta_1)}\frac{(m-1-k)!}{(m-k-j-1)!}$$
$$\times\left(\oint_c\frac{1}{(s-\beta_1)^{j+1}s(\alpha+s)^m(s-\beta)^m}\,ds - \oint_c\frac{1}{\beta_1^{j+1}s(\alpha+s)^m(s-\beta)^m}\right)ds + \oint_c\frac{(m-1-k)!}{(s-\beta_1)^{m-k}s(\alpha+s)^m(s-\beta)^m}\,ds\Bigg\}$$
$$(4.18)$$

For the first four integrals(T1) the contour comprises the semicircle on the right half of the s plane as $\delta^{(s)} < 0$ in the first and fourth quadrants [16].Therefore all of them have pole of order m at $s = \beta$. For the last four integrals(T2) the countour is semicircle on left half of s plane for the same reason and they all have poles at $s = \alpha$.

$$\begin{aligned} T_1 &= \frac{|\delta^{\alpha_1+z}|}{|(\alpha_1+z)(\alpha+z)^m(z-\beta)^m|} \\ &= \frac{\delta^{\alpha_1+Rcos\theta+jsin\theta}}{\left(\sqrt{R^2+\alpha_1^2+2\alpha_1 zcos\theta}\right)\left(\sqrt{R^2+\alpha^2+2\alpha Rcos\theta}\right)^m\left(\sqrt{R^2+\beta^2-2\beta Rcos\theta}\right)^m} \end{aligned} \tag{4.19}$$

$$\begin{aligned} T_2 &= \frac{|\left(\frac{1}{\delta}\right)^{z-\beta_1}|}{|(s-\beta_1)(\alpha+z)^m(z-\beta)^m|} \\ &= \frac{\left(\frac{1}{\delta}\right)^{Rcos\theta+jsin\theta-\beta_1}}{\sqrt{R^2+\beta_1^2-2\beta_1 zcos\theta}\left(\sqrt{R^2+\alpha^2+2\alpha Rcos\theta}\right)^m\left(\sqrt{R^2+\beta^2-2\beta Rcos\theta}\right)^m} \end{aligned} \tag{4.20}$$

Applying the residue theorem to 4.18 and using Leibnitz rule for differentiation , where we have pole of order m

$$(f.g.h)^{(n)} = \sum_{i=0}^{n}\binom{n}{i}f^{(n-i)}\left\{\sum_{j=0}^{i}\binom{i}{j}g^{(i-j)}h^{(j)}\right\} \tag{4.21}$$

We get,

$$
\begin{aligned}
P(X+f(Y)<0|x_s=1,x_r) &= \left(\frac{-2c}{b}\right)^m \frac{c^m}{\Gamma m\sqrt{a^2+2bc}} \sum_{k=0}^{m-1} \frac{(m-1+k)!}{k!(m-1-k)!} \left(\frac{b}{2\sqrt{a^2+2bc}}\right)^k \\
&\times \left\{\left\{\left\{-\frac{ln(\delta)^{m-1-k}}{\alpha_1} \times \frac{\delta^{\alpha_1+\beta}}{(m-1)!} \sum_{r=0}^{m-1} \binom{m-1}{r} \frac{(-1)^m (m-1-r)!(m+r-1)!}{(\beta+\alpha_1)^{m-r}(\beta+\alpha)^{m+r}(m-1)!}\right\}\right.\right. \\
&+ \left\{\sum_{j=1}^{m-1-k} \frac{(-1)^{j+1}\delta^{\alpha_1+\beta} ln(\delta)^{m-1-k-j}(m-k-1)!}{(m-1-k-j)!\alpha_1^{j+1}} \left\{\frac{1}{(m-1)!}\sum_{r=0}^{m-1}\binom{m-1}{r}\left\{\frac{(-1)^m(m-1-r)!(m+r-1)!}{\beta^{m-r}(\beta+\alpha)^{m-r}(m-1)!}\right.\right.\right. \\
&- \sum_{r=0}^{m-1} \frac{(-1)^{m-1-r}(m-1-r)!}{\beta^{m-r}} \sum_{p=0}^{r} \frac{(-1)^r(j+r-p)!}{j!(\beta+\alpha_1)^{j+1+r-p}} \times \left.\left.\left.\frac{(m+p-1)!}{(\beta+\alpha)^{m+p}(m-1)!}\right\}\right\}\right\} + \left\{\frac{(-1)^k(m-1-k)!}{2\alpha_1^{m-k}(\alpha\beta)^m}\right\} \\
&- \left\{(-1)^{m-1-k}(m-1-k)! \sum_{r=0}^{m-1}\binom{m-1}{r}\frac{(-1)^{m-1-r}(m-1-r)!}{(m-1)!\beta^{m-r}} \sum_{p=0}^{r}\binom{r}{p}\frac{(-1)^r(m-k-1+r-p)!}{(m-k-1)!(\beta+\alpha_1)^{m-k+r-p}}\right. \\
&\times \left.\left.\frac{(m+p-1)!}{(\beta+\alpha)^{m+p}(m-1)!}\right\}\right\} \times (-1)^{m-1-k} + \left\{\left\{\left\{\frac{ln(\frac{1}{\delta})^{m-1-k}\left(\frac{1}{\delta}\right)^{-\alpha-\beta_1}}{\beta_1} \frac{1}{(m-1)!}\sum_{r=0}^{m-1}\binom{m-1}{r}\right.\right.\right. \\
&\times \left.\frac{(-1)^{m-1}(m-1-r)!(m+r-1)!}{(-\alpha-\beta_1)^{m-r}(-\alpha-\beta)^{m+r}(m-1)!}\right\} + \left\{\sum_{j=1}^{m-1-k}\frac{\left(\frac{1}{\delta}\right)^{\alpha_1+\beta} ln\left(\frac{1}{\delta}\right)^{m-1-k-j}(m-k-1)!}{(m-1-k-j)!}\right. \\
&\times \left\{\frac{1}{(m-1)!\beta_1^{j+1}}\sum_{r=0}^{m-1}\binom{m-1}{r}\left\{\frac{(-1)^{m-1}(m-1-r)!(m+r-1)!}{(-\alpha)^{m-r}(-\alpha-\beta)^{m+r}(m-1)!} + (-1)^j \sum_{r=0}^{m-1}\frac{(-1)^{m-1-r}(m-1-r)!}{(-\alpha)^{m-r}(m-1)!}\right.\right. \\
&\times \sum_{p=0}^{r}\frac{(-1)^r(j+r-p)!(m+p-1)!}{j!(-\alpha-\beta_1)^{j+1+r-p}} \times \left.\left.\left.\frac{(m+p-1)!}{(-\alpha-\beta)^{p+m}(m-1)!}\right\}\right\}\right\} + \left\{\frac{(-1)^m(m-1-k)!}{2\beta_1^{m-k}(\alpha\beta)^m}\right\} \\
&+ \left\{(-1)^{m-k}(m-1-k)!\sum_{r=0}^{m-1}\binom{m-1}{r}\frac{(-1)^{m-1-r}(m-1-r)!}{(m-1)!(-\alpha)^{m-r}}\sum_{p=0}^{r}\binom{r}{p}\frac{(-1)^r(m-k-1+r-p)!}{(m-k-1)!(-\alpha-\beta_1)^{m-k+r-p}}\right. \\
&\times \left.\left.\left.\frac{(m+p-1)!}{(-\alpha-\beta)^{m+p}(m-1)!}\right\}\right\}\right\} \qquad (4.22)
\end{aligned}
$$

4.6 Results

The exact BER expression of DF cooperative systems for BPSK signals in Nakagami-m fading channels with PL combiner at the receiver are obtained. 4.1 shows the analysis and simulation results for $m = 3$ and $l = .3$ (l stands for relay location). The simulation perfectly follows analysis validating the expressions.

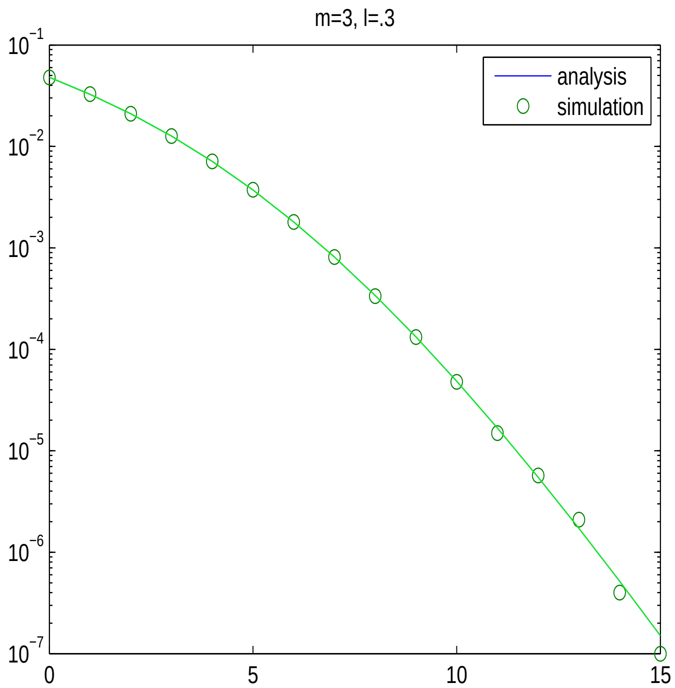

Figure 4.1: Analysis and Simulation BER plot

Chapter 5

Conclusion and Future work

We have obtained a close but approximate expression for the BER for the λ-MRC-DF cooperative system for single relay. The final expression contains only one integral in terms of simple, well defined functions. Numerical results obtained using this expression match exactly with the actual simulation results, indicating the usefulness of this work.

Exact expressions for the BER for λ-MRC DF cooperation in Nakagami-m fading have been obtained for upto N relays. These results were obtained for arbitrary values of m, which, to the best of our knowledge, has not been addressed in the available literature. In the process, we obtained an exact expression for the CDF of gamma CGRV for arbitrary m. A relatively new approach, using the Mellin-Barnes integral representation of the extended Fox-$\hat{H}$ function, has been employed for this. We also found the exact expressions for the BER for a PL-DF cooperative diversity for Nakagami m fading for integer m. Here the contour is choosen such that it facilitates the application of Residue theorem. The same concept can be applied to obtain BER for multiple relays.

Chapter 6

Publications

- A. Rathnakar and G. V. V. Sharma, ”Performance Analysis of λ-MRC Decode and Forward Cooperation in Nakagami-m Fading for Arbitrary Parameters,” Malaysia International Conference on Communicaitons (MICC), Nov 2013.
- Sachin Kumar, A. Rathnakar and G. V. V. Sharma ”On the Linear Combination of Gamma Conditionally Gaussian Distributions,” under preparation.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, December 2004.

[2] E. C. van der Meulen, “Three-terminal communication channels,” *Adv. Appl. Prob*, vol. 3, pp. 120 – 154, 1971.

[3] T. Cover and A. Gamal, “Capacity theorems for the relay channel,” *Information Theory, IEEE Transactions on*, vol. 25, pp. 572–584, 1979.

[4] J. Laneman, D. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *Information Theory, IEEE Transactions on*, vol. 50, pp. 3062–3080, 2004.

[5] G. V. V. Sharma, U. B. Desai, and S. N. Merchant, “Conditionally Gaussian Distributions and their Application in the Performance of Maximum Likelihood Decode and Forward Cooperative Systems,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, pp. 27 –32, january 2011.

[6] Y. G. Kim and N. C. Beaulieu, “Exact Closed-Form Solutions for the BEP of Decode-and-Forward Cooperative Systems in Nakagami-m Fading Channels,” *Communications, IEEE Transactions on*, vol. 59, no. 9, pp. 2355 –2361, september 2011.

[7] G. V. V. Sharma, V. Ganwani, U. B. Desai, and S. N. Merchant, “Performance Analysis of Maximum Likelihood Decode and Forward CooperativeSystems in Rayleigh Fading,” *Proc. IEEE International Conference on Communications (ICC)*, June 2009.

[8] ——, “Performance Analysis of Maximum Likelihood Detection for Decode and Forward MIMO Relay Channels in Rayleigh Fading,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 9, pp. 2880 –2889, September 2010.

[9] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity, part ii: implementation aspects and performance analysis,” *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, November 2003.

[10] W. Feller, *An introduction to Probability Theory and Its Applications, Vol II*, 2nd ed. John Wiley and Sons, 1970.

[11] M. K. Simon and M.-S. Alouini, *Digital Communications over Fading Channels*, 2^{nd} ed., ser. Wiley Series in Telecommunication and Signal Processing, J. G. Proakis, Ed. John Wiley and Sons, 2005.

[12] P. Loskot and N. C. Beaulieu, “Prony and Polynomial Approximations for Evaluation of the Average Probability of Error Over Slow-Fading Channels,” *IEEE Trans. Veh. Technol.*, vol. 58, no. 3, pp. 1269–1280, March 2009.

[13] G. E. Andrews, R. Askey, and R. Roy, *Special Functions*, 1st ed. Cambridge University Press, 1999.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 5th ed. Academic Press, 1994.

[15] H. Bateman, *Higher Transcendental Functions*, A. Erdelyi, Ed. McGraw-Hill Book Company Inc., 1953, vol. 2.

[16] G. V. V. Sharma, “Exact error analysis for decode and forward cooperation with maximal ratio combining,” in *National Conference on Communications (NCC)*, January 2011.

[17] A. K. Rathie, “A New Generalization of Generalized Hypergeometric Functions,” *LE MATEMATICHE*, vol. LII, pp. 297–310, 1997.

[18] I. S. Ansari, F. Yilmaz, M.-S. Alouini, and O. Kucur, “New Results on the Sum of Gamma Random Variates With Application to the Performance of Wireless Communication Systems over Nakagami-*m* Fading Channels.”

[19] J. E. Marsden and M. J. Hoffman, *Basic Complex Analysis*, 3rd ed. W H Freeman and Company, 1999.

[20] J. Laneman and G. Wornell, “Energy-efficient antenna sharing and relaying for wireless networks,” in *Wireless Communications and Networking Confernce, 2000. WCNC. 2000 IEEE*, 2000.

[21] D. Chen and J. Laneman, “Cooperative diversity for wireless fading channels without channel state information,” in *Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on*, 2004.

[22] D. Chen and J. N. Laneman, “Modulation and demodulation for cooperative diversity in wireless systems,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1785–1794, July 2006.

Appendix A

PDF of X (CGRV)

The PDF of $X \sim \mathcal{N}(aA, bA)$ can be expressed as [6]

$$
\begin{aligned}
p_X(z) &= \int_{-\infty}^{\infty} p_{X|A}(x) p_A(x)\, dx && \text{(A.1)}\\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{x}} e^{-\frac{(z-ax)^2}{2bx}} p_A(x)\, dx && \text{(A.2)}\\
&= \frac{c^m}{\Gamma(m)\sqrt{2\pi}} \int_0^{\infty} x^{m-\frac{3}{2}} \exp\left\{-\frac{(z-ax)^2}{2bx} - cx\right\} dx && \text{(A.3)}\\
&= \frac{c^m e^{\frac{az}{b}}}{\Gamma(m)\sqrt{2\pi}} \int_0^{\infty} x^{m-\frac{3}{2}} \exp\left\{-\frac{|z|^2}{2bx} - \left(\frac{a^2}{2b} + c\right)x\right\} dx && \text{(A.4)}\\
&= \frac{2c^m e^{\frac{az}{b}}}{\Gamma(m)\sqrt{2\pi}} \left(\frac{|z|}{\sqrt{a^2+2bc}}\right)^{\frac{m}{2}-\frac{1}{4}} \times K_{m-\frac{1}{2}}\left(\frac{|z|}{b}\sqrt{a^2+2bc}\right) && \text{(A.5)}
\end{aligned}
$$

resulting in (2.22) using

$$
\int_0^{\infty} x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\, dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right) \qquad [\operatorname{Re}\beta > 0, \operatorname{Re}\gamma > 0] \tag{A.6}
$$

in [14, 3.471.9].