Exact Error Analysis For Decode And Forward Multirelay Cooperative Systems : An Algorithmic Approach

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Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

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Approval Sheet

This Thesis entitled Exact Error Analysis For Decode and Forward Multirelay Cooperation Systems : An Algorithmic Approach by G.V.S.S.Praneeth Varma is approved for the degree of Master of Technology from IIT Hyderabad

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Dedication

In memory of my grand parents

Abstract

The bit error rate (BER) performance analysis of maximum-likelihood (ML) based decode and forward (DF) cooperative diversity systems has been a subject of considerable interest. Exact analysis of ML-DF transmission has been considered a challenging problem due to the nonlinear characteristic of the ML detector. In this thesis, we provide exact expressions for the BER of ML-DF cooperative systems employing a single relay. This is done by using a novel theory of conditionally Gaussian random variables and special hypergeometric functions. By expressing the ML decision variable in terms of functions of conditionally Gaussian variables, exact expressions for the BER of the ML-DF system are obtained. Through simulation results, we verify the validity of the derived analytical expressions.

We observe the PL combiner performs well with that of the ML detector.It is difficult to work with the special functions because of the convergence of their infinite series. Hence we extend our thesis to the multiple relays with the PL combiner at the receiver. Using Gil-Pelaez inversion formula, BER analysis is done with the concepts of contour integral approach.As there is recursive relation existing for multiple relays [3],we propose an algorithm which evaluate the analytical results for arbitrary relays.We verify the validity of the analytical results from the algorithm with the simulations. We also derive the complexity of the algorithm.

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Chapter 1

Introduction

1.1 Challenges of Wireless Channel

Wireless Communication has been the rapidly growing mostly because of the possibility of mobility.The wireless channel is the free space with resources and also suffering from attenuation, noise, interference etc. The performance of wireless system is very much degraded by fading than the noise. To deal with multipath fading, diversity techniques are employed in wireless communications.In the thesis we work on the techniques that overcome noise and fading problems.

In wireless networks, transmission of information directly over large distance is expensive because of the need for more transmitted power. This results in fast dicharge of battery resulting in shorter network life. Also increased transmitted power will introduce interference to the local radios nearby. Fading and noise characteristics of all the wireless channels are independent.Each channel experience different fading characteristics. Using this fact, Diversity techniques are employed to improve the reliability of the signal. Using multiple antennas at the transmitter and receiver helps in improving the performance of the system. But multiple antennas at the transmitter may lead to interference between them.

For efficient utilization of power and bandwidth resources and to achieve broader coverage, relays are employed. The receiver combines the signals from the relays and the source which are multiple transmissions of the same signal to reduce the effect of fading. We refer to this technique as spatial diversity. The main aim is to design a receiver that overcome the uncertainity in relay decision and still exploit spatial diversity. Coperative Diversity acheives this antenna diversity with the cooperation of the antennas located between the transmitter and receiver.

1.2 Diversity Techniques for Fading Channels

When the channel is in deep fade, there is more probable error at the receiver. When the receiver has several replicas of the transmitted signal over independent fading channels then the probability of that all the signals fade simultaneously is reduced.So the providing receiver with multiple independently fading replicas can be done in several ways.

One such way is transmitting the same information over multiple carrier,i.e on two frequency bands with separation more than coherence bandwidth.This is called frequency diversity.Other method is to transmit the same signal in different time slots with separation more thamn coherence time.This is time diversity.Most commonly used technique is employing multiple antennas,where a single transmitting antennas send the information while the multiple antennas at receiver receivesignal from different fading channels.This is space diversity.There is no need of coherent time and coherent bandwidth like time and frequency diversity.

1.3 Cooperative System

Cooperative Diversity acheives antenna diversity by combining the ideas of intermediate relays and multiple antennas. The problem of multiple antennas at the node in MIMO is avoided in this technique. Cooperative Diversity is a technique where multiple antennas relay the information from transmitter to receiver exploiting the diversity by allowing the antennas to relay in parallel and combining the signals from source and the relays at the receiver.Relaying can be done in different ways, the relays can decode or amplify or compress the signal from the transmitter and then forward it to the receiver.

In this thesis,we assume that the fading coefficients are estimated accurately at the receiver. All the transmissions are over orthogonal channels and hence the bandwidth efficiency decreases with the number of relays. We focus on Decode and Forward(DF) signalling at the relay, find the Exact Close-form expressions for the probability of error. Also extends this to multiple relays with a novel algorithmic approach. And also verify there is an increase in diversity order for multiple relays.

1.4 Standards of Relay Technologies

1.4.1 IEEE 802.16j

This is a change to the standard IEEE 802.16e mobile WiMAX standards supporting the relay functionalities to improve throughput of cell users and extend coverage to interiors and with in mobile transportation vehicles.Relays are categorized as transparent and non transparent based on whether mobile stations(MS) are aware of relay existance.Non transparent relays are used when the MS are at he end of the coverage area by forming virtual base station(BS).As the transparent relays to enhance throughput of MS, they communicate using the same frequency on different links.While nontransparent relays use different frequency bands

1.4.2 Long Term Evolution(LTE)-Advanced

The main aim in adding relay technologies to LTE is to extend coverage of high data rates.Here the relays are classified as inband and outband based on whether the communication between the BS and MS are over same frequency band.A type1 realay is inband relaywhich controls its own cell, thus have its own synchronization,scheduling information,physical cell ID and it appears as base station to the MS.

Chapter 2

Performance Analyis for Decode and Forward Cooperative System

2.1 Exact BER Analysis for ML-DF cooperative system with one relay

2.1.1 System Model

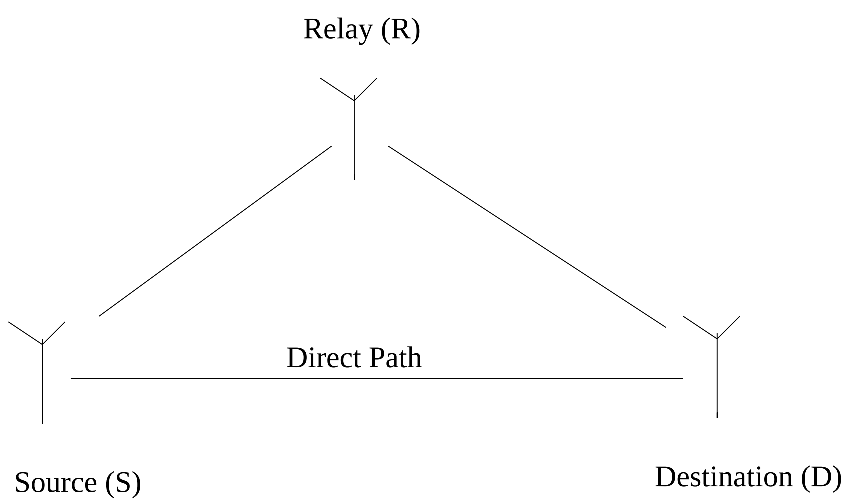


Figure 2.1: Three node cooperative diversity system.

Consider the model as shown in Fig.1 with single relay (R) between the source (S) and destination (D). We assume that all the transmissions are on orthogonal channels and the modulation is BPSK. Let the transmission bits at the source and relays be $x_s \in \{1,\text{-}1\}$ and $x_r \in\{1,\text{-}1\}$ respectively with powers E_s and E_r. The received symbols on S-D, S-R, and R-D links are

$$\begin{aligned} y_{d,s} &= \sqrt{E_s} h_{d,s} x_s + z_{d,s} \\ y_{r,s} &= \sqrt{E_s} h_{r,s} x_s + z_{r,s} \\ y_{d,r} &= \sqrt{E_r} h_{d,r} x_s + z_{d,r} \end{aligned} \tag{2.1}$$

And $z_{d,s}, z_{r,s}, z_{d,r} \sim CN(0, N_0)$ represent additive white Gaussian noise at the relay and destination.The fading coefficients $h_{d,s} \sim CN(0, \Omega_{d,s})$, $h_{r,s} \sim CN(0, \Omega_{r,s})$ and $h_{d,r} \sim CN(0, \Omega_{d,r})$ are due to Rayleigh fading channel.

2.1.2 ML decision

We assume the BPDK symbols are equi-probable,hence the optimum Maximum A Posteriori decision is same as the Maximum Likelihood decision. The decision rule for BPSK modulation at the destination can be obtained from [4] as

$$X + f(Y_r) \underset{-1}{\overset{1}{\gtrless}} 0 \tag{2.2}$$

where

$$X = \frac{4\sqrt{E_s} Re\{h^*_{d,s} y_{d,s}\}}{N_0}, Y_r = \frac{4\sqrt{E_s} Re\{h^*_{d,r} y_{d,r}\}}{N_0} \tag{2.3}$$

$$f(t) = ln \frac{\delta + e^t}{1 + \delta e^t}, \qquad 0 < \delta < 1 \tag{2.4}$$

The parameter δ in (2.4) is defined as $\delta = \frac{\epsilon}{1-\epsilon}$ for the relay, where ϵ is average probability of error for S-R link. For the analysis, we consider the suboptimal ML scheme proposed in [9] which results in $\epsilon = \frac{1}{2}\left[1 - \left(1 + \frac{\Omega_{r,s} E_s}{N_0}\right)^{-\frac{1}{2}}\right]$.

2.1.3 Problem Definition

We see that the Λ in [1, 12] is evaluated by PL combiner [1, 5] and close form approximations are obtained for BER.Problem is to find the exact closed form expressions for BER .So the problem reduces to evaluating the integral

$$\Lambda(p, q) = \frac{1}{p} \int_0^{\ln \frac{1}{\delta}} \left(\frac{e^{-t} - \delta}{1 - \delta e^{-t}} \right)^p e^{-qx} dx \tag{2.5}$$

2.1.4 Derivation of Exact Expressions for ML Detector

Approach to this problem, is possible with the knowledge of special functions Appell,Horn etc. and convergence of their infinite series.

$$\Lambda(p,q)=\frac{1}{p}\int_0^{\ln\frac{1}{\delta}}\left(\frac{e^{-t}-\delta}{1-\delta e^{-t}}\right)^p e^{-qx}dx \tag{2.6}$$

can be expressed as

$$\Lambda(p,q)=\frac{1}{p}\int_0^{1}\left(\frac{t-\delta}{1-\delta t}\right)^p t^{q-1}dt-\frac{1}{p}\int_0^{\delta}\left(\frac{t-\delta}{1-\delta t}\right)^p t^{q-1}dt \tag{2.7}$$

From [10, 3.211], we have

$$\begin{aligned}&\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-ux)^{-\rho}(1-vx)^{-\sigma}dx\\ &\qquad=B(\mu,\lambda)F_1(\lambda,\rho,\sigma,\lambda+\mu;u,v)\,,\quad \Re\{\lambda,\mu\}>0\end{aligned} \tag{2.8}$$

Substituting (2.8) in (2.7), we get

$$\begin{aligned}\Lambda(p,q)\ &=\tfrac{(\frac{-1}{\delta})^{-p}}{p}[B(1,q)F_1(q,p,-p,1+q,\delta,\delta^{-1})\\ &\quad-\delta^q B(q,p+1)_2F_1(q,p,p+q+1,\delta^2)]\end{aligned} \tag{2.9}$$

where the functions F_1 and ${}_2F_1$ are defined in [10, 9.180] as

$$F_1(\alpha,\beta,\beta',\gamma,x,y)\ =\ \sum_{m=0}^{\infty}\sum_{m=0}^{\infty}\frac{(\alpha)_{m+n}(\beta)_m(\beta')_n}{m!n!(\gamma)_{m+n}}x^m y^n \tag{2.10}$$

$${}_2F_1(\alpha,\beta,\gamma,x)\ =\ \sum_{m=0}^{\infty}\frac{(\alpha)_m(\beta)_m}{(\gamma)_m}\frac{x^m}{m!} \tag{2.11}$$

It can be seen from above that the function in (2.10) is absolutely convergent if and only if $\mid x\mid<1\mid y\mid<1$ and the function in (2.11) is convergent if $\mid x\mid<1$ and $\gamma\geq 0$ Therefore, F_1 is not convergent hence we transform the variables (x,y) to (u,v)=(x/y,1/y) as stated in [11, 22] resulting

$$F_1(\alpha,\beta,\beta',\gamma,x,y) = \frac{\Gamma(\gamma)\Gamma(\beta'-\alpha)}{\Gamma(\beta')\Gamma(\gamma-\alpha)}(-y)^{-\alpha}F_1(\alpha,\beta,1+\alpha-\gamma,\alpha-\beta'+1,\frac{x}{y},\frac{1}{y}) + \frac{\Gamma(\gamma)\Gamma(\alpha-\beta')}{\Gamma(\alpha)\Gamma(\gamma-\beta_2)}(-y)^{-\beta'}G_2(\beta,\beta',1+\beta'-\gamma,\alpha-\beta'+1,-x,\frac{-1}{y}) \quad (2.12)$$

where Horn G_2 function is defined in [15] as

$$G_2(\alpha,\beta,\beta',\gamma,x,y) = \sum_{m=0}^{\infty}\sum_{m=0}^{\infty}\frac{(\alpha)_m(\beta)_n(\beta')_{n-m}(\gamma)_{m-n}}{m!n!}x^m y^n \quad (2.13)$$

In the convergence of the F_1 function, the transformation results in complex numbers with small residues in the imaginary part which can be neglected.

2.1.5 Simulation Results

The total power used by the system is one unit, therefore $E_s + E_r = 1$, and L_r is distance between S-R while the distance of S-D is 1 with fading power $\Omega_{r,s} \propto \frac{1}{L_r^4}$. The simulation results are compared for the relay placed at location $L_r = 0.3$.

From 2.2, Also we can infer PL-Combiner gives very good approximation to the ML Detector. We observed from the figure 2.3 for PL combiner at different locations, at high SNR , BER performance is best when relay is closer to the source.

ML Detector is very difficult to implement when compared to PL combiner,so our further analysis consider the PL combiner for multiple relays.All the simulation further use the same fading power, BPSK modulation and signal power.

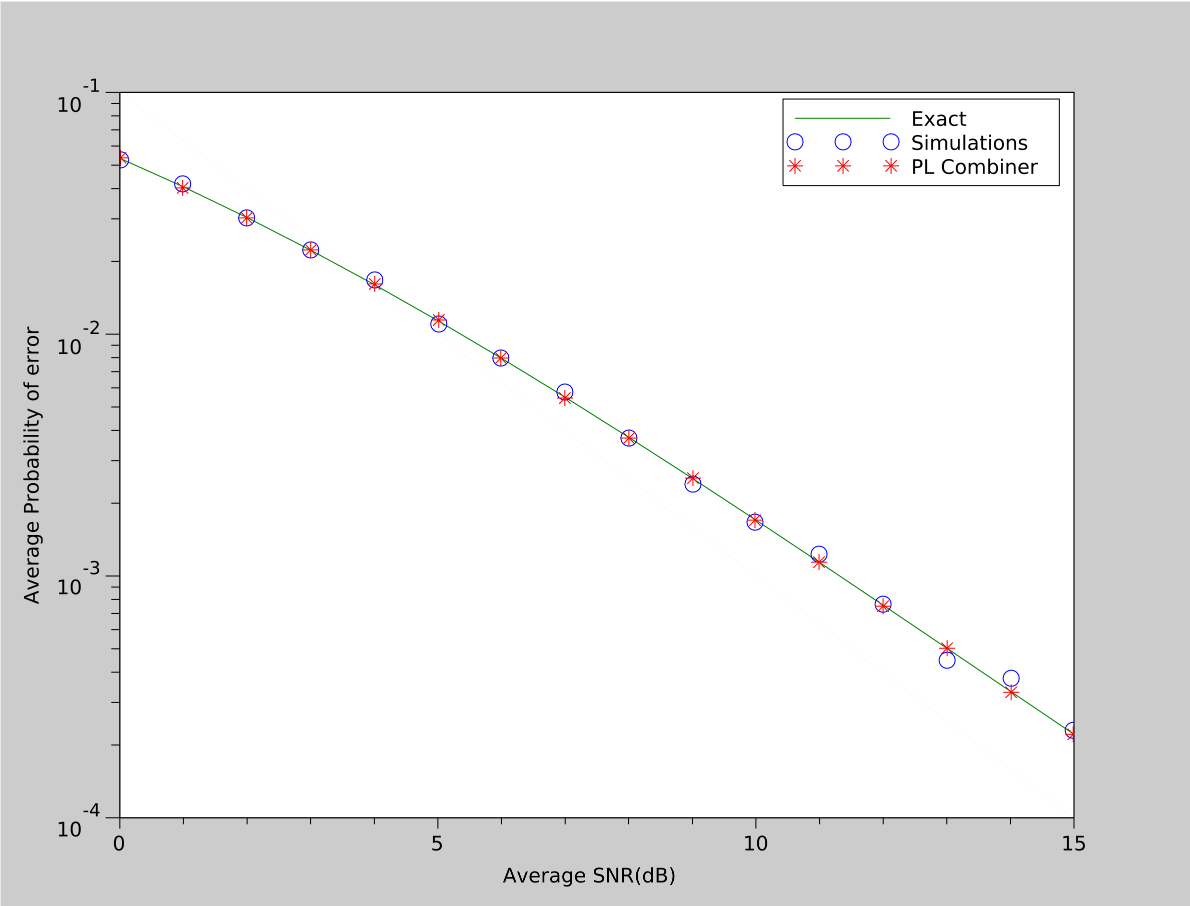


Figure 2.2: Comparison of Exact, PL Analytical and Simulations for L = 0.3

2.2 Algorithmic Approach for PL-DF cooperative system for multiple relays

2.2.1 Algorithm

From the analysis in [2], [3], we find it very difficult to obtain expression for BER analytically for three relays, hence an algorithmic approach is much feasible for multiple relays. The algorithm is similar to the analysis provided in [2], [3], using contour integral and residue calculus.
Using Gil-Pelaez theorem,

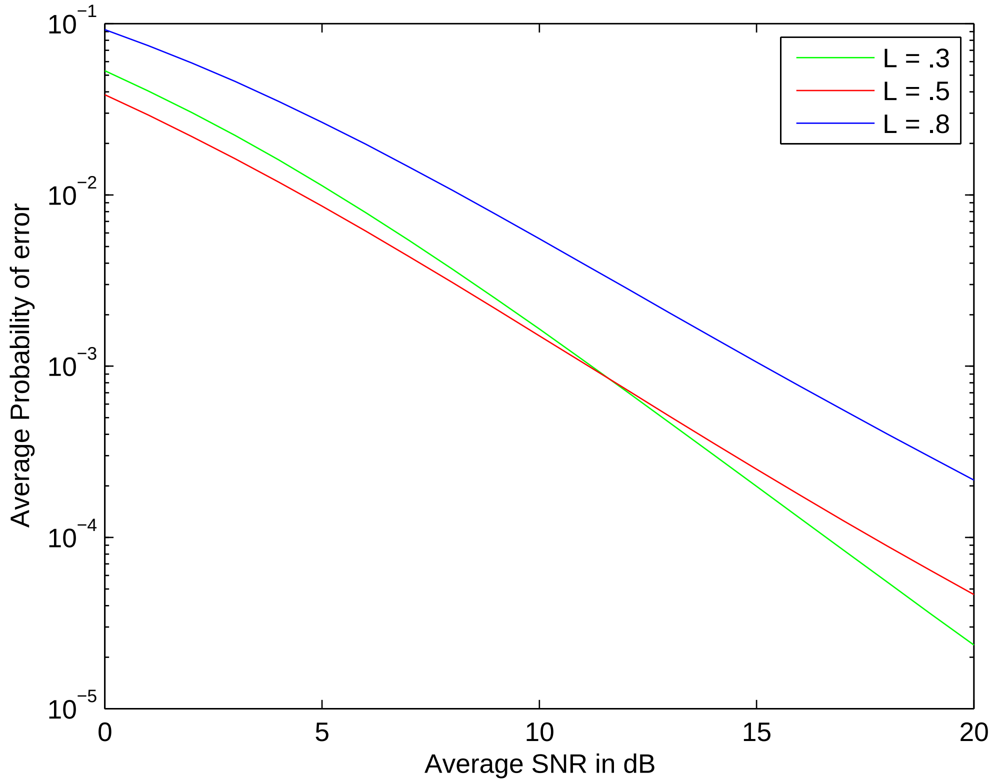


Figure 2.3: Comparison of BER for BPSK with relay located at L = 0.3,0.5 and 0.8

$$P\left(X + \sum_{r=1}^{N} f(Y_r) < 0 | x_s = 1, \mathbf{x}\right) = \frac{1}{2} - \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{\Phi_X(t)}{t} \Pi_{r=1}^{N} \Phi_{V_r}(t) dt$$
$$= A(\Phi_{V_1}, ..., \Phi_{V_N}) \tag{2.14}$$

where $\phi_X(t)$ and $\phi_{f(Y_r)}(t)$ are the characteristic functions of X and $V_r = f(Y_r)$ and V_r=$f(Y_r)$ given by

$$\phi_X(jw) = \frac{g(\alpha + \beta)}{(\beta - jw)(\alpha + jw)} \tag{2.15}$$
$$\Phi_{V_r}(jw) = [1 - g\phi(\alpha, jw) - g\phi(\beta, -jw)] \tag{2.16}$$
$$\phi(x, t) = \frac{t(1 - \delta^{x+t})}{x(x + t)}, \tag{2.17}$$

$$\alpha_i = \frac{1}{2}\left(\sqrt{1+\frac{1}{\gamma_{d,i}}}+1\right),$$
$$\beta_i = \frac{1}{2}\left(\sqrt{1+\frac{1}{\gamma_{d,i}}}-1\right)$$
$$g = \frac{\alpha\beta}{\alpha+\beta} \tag{2.18}$$

BER analysis can be done using contour integration.

$\frac{\phi(.,.)}{w}$ function has removable singularity and does not contribute any poles.

$$\begin{aligned} |\phi(\alpha_1, jw)| &= \frac{|w||1-\delta^{\alpha_1+jw}|}{\alpha_1|\alpha_1+jw|} \\ &= \frac{R|1-\delta^{\alpha_1-Rsin\theta+jRcos\theta}|}{\alpha_1\sqrt{R^2-\alpha_1^2-2\alpha_1 R}} \end{aligned} \tag{2.19}$$

This becomes infinite when $R \to \infty$ on the contour $w = Re^{j\theta}, 0 < \theta < \pi$ and is finite for $w = Re^{j\theta}, -\pi < \theta < 0$. Therefore the integrals are evaluated based on the pole location and the contour.This is the main principle in the implementation of the algorithm.

There is a recursive relation existing for the conditional BER for systems with N relays [3] which can be expressed as

$$\begin{aligned} A(\Phi_{V_1},....,\Phi_{V_N}) = (-1)^{N-1}[\frac{1}{2} - \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\Phi_X(jw)}{w}dw + \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\Phi_X(jw)\varepsilon_N(\phi)}{w}dw \\ + \sum_{r=1}^{N-1}\sum_{l_1<...<l_r}\sum_{l_1,..,l_r=1}^{N}(-1)^r A(\Phi_{l_1},..,\Phi_{l_r})] \end{aligned} \tag{2.20}$$

where

$$\varepsilon_r(\phi) = \Pi_{l=1}^{r} g_l \sum_{p=1}^{2^r} \Pi_{i\in I_k, j\in J_k}\phi(\alpha_i, jw)\phi(\beta_j, -jw)$$
$$I_k \cup J_k = 1,2,..,r, I_k \cap J_k = \{\}$$

For every additional relay, we need to evaluate the integral $\varepsilon_r(\phi)$.

Converting this recursive into iterative relation,we have

$$A(\Phi_{V_1},....,\Phi_{V_N}) = \sum_{r=1}^{N-1} A(\Phi_{V_r}) - (N-1)\left(\frac{1}{2} - \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\Phi_X(jw)}{w}dw\right) + \sum_{r=2}^{N}\sum_{k=1}^{{}^{N}C_r}(-1)^{r-1} in_r^k \tag{2.21}$$

where

$$\begin{aligned} in_r^k &= \frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\Phi_X(jw)\varepsilon_r^k(\phi)}{w}dw \\ &= \sum_{p=1}^{2^r}\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{\Phi_X(jw)W_p}{w}dw \end{aligned}$$

$$\begin{aligned} \varepsilon_r^k(\phi) &= \Pi_{l=1}^{r} g_l \sum_{p=1}^{2^r} \Pi_{i\in I_k, j\in J_k}\phi(\alpha_i, jw)\phi(\beta_j, -jw) \\ &= \sum_{p=1}^{2^r} W_p \\ I_k \cup J_k &= Z_k, I_k \cap J_k = \{\}. \end{aligned}$$

The algorithm for N multiple relays for different SNR's proceeds as:

$E_s = 0.5,\ E_r = \frac{(1-E_s)}{M}$

for i=1:snr

Step 1: Calculate α_r, β_r, g_r as in (2.18)

sum1=0;

for r=1:N

Step 2: Go to step 5 and evaluate A_r

sum1=sum1+A_r

end

sum3=0;

for r=2:N

sum2=0;

Step 3: Form ${}^{N}C_r$ combinations of relays

$$eg.:N=3,r=2$$
$$Z_k=\{\{1,2\},\{1,3\},\{2,3\}\}$$

for k=1:${}^{N}C_r$

Step 4: Go to step 6 and evaluate in_r^k

sum2=sum2+in_r^k

end

sum3=sum3+$(-1)^{(r-1)}*$sum2

end

final_ sum=sum1+sum3-(N-1)$\frac{g}{\alpha}$

end

Step 5: $A_r=\frac{g}{\alpha}+gg_r\left[\frac{\phi(\alpha_r,\beta)}{\beta}-\frac{\phi(\beta_r,\alpha)}{\alpha}\right]$.

Step 6: To evaluate the in_r^k,

i. Find poles in the integrand as described

$\Phi_X(jw)$ in (2.15) has two poles, one in upper and the other in lower half of S-plane.

Similarly, $\phi(\alpha_i,\beta)$ and $\phi(\beta_i,\alpha)$ in (2.17) has a pole in lower and upper half of S-plane respectively.

$$eg.:N=3,r=2,k=2;p=2$$
$$integrand=$$
$$\frac{g'}{jw}\frac{(\alpha+\beta)}{(\alpha+jw)(\beta-jw)}\frac{jw\left(1-\delta_1^{\alpha_1+jw}\right)}{\alpha_1(\alpha_1+jw)}\frac{-jw\left(1-\delta_3^{\beta_3-jw}\right)}{\beta_3(\beta_3-jw)}$$

Here the poles are $j\alpha,-j\beta,j\alpha_1,-j\beta_3$

ii. Expand the integrand as a polynomial of δ as power of 'jw'.

$$eg.: N=3, r=2, k=2; p=2$$
$$integrand =$$
$$\frac{-g'(\alpha+\beta)(jw)\left(1-\delta_1^{\alpha_1}\delta_1^{jw}-\delta_3^{\beta_3}(\frac{1}{\delta_3})^{jw}+\delta_1^{\alpha_1}\delta_3^{\beta_3}(\frac{\delta_1}{\delta_3})^{jw}\right)}{\alpha_1\beta_3(\alpha+jw)(\beta-jw)(\alpha_1+jw)(\beta_3-jw)}$$

iii. In each term, consider a^{jw},

Separate the integrand into two functions **a** $\underset{f_{upper}}{\overset{f_{lower}}{\lesseqgtr}}$ **1**

This is needed for identifying the contour and considering the pole location the residues are evaluated.

Without loss of generality consider $\delta_1 < \delta_3$

$$eg.: f_{lower} = \frac{-g'(\alpha+\beta)(jw)\left(1-\delta_1^{\alpha_1}\delta_1^{jw}+\delta_1^{\alpha_1}\delta_3^{\beta_3}(\frac{\delta_1}{\delta_3})^{jw}\right)}{\alpha_1\beta_3(\alpha+jw)(\beta-jw)(\alpha_1+jw)(\beta_3-jw)}$$
$$f_{upper} = \frac{g'(\alpha+\beta)(jw)\left(\delta_3^{\beta_3}(\frac{1}{\delta_3})^{jw}\right)}{\alpha_1\beta_3(\alpha+jw)(\beta-jw)(\alpha_1+jw)(\beta_3-jw)}$$

iv. Find residues of these functions.

For the $f_{lower}(f_{upper})$ function use poles in lower(upper) half of s-plane to find the residues

$$eg.: Res\left[f_{lower}; -j\beta, -j\beta_3\right]$$
$$Res\left[f_{upper}; j\alpha, j\alpha_1\right]$$

v.Using residue theorem, $2\pi j\times$[sum of residues] is the value of integral in_r^k

2.2.2 Computaional complexity

Eg: $integrand =$

$$g'(\alpha+\beta)z^2 \quad \frac{1}{\beta_1\alpha_2\alpha_3} \quad \frac{[sumterms]}{(\beta-z)(\beta_1-z)(\alpha+z)(\alpha_2+z)(\alpha_3+z)}$$

$$commonfactor \qquad combinationfactor \qquad \frac{sumterm}{sumfactor}$$

Derivation of Complexity :

For each pole :

Computations for sum terms

$$\begin{aligned} \leq \qquad & r2^{r-1} + 2^r - 1 \ \ Additions \\ & 2^{r-1}(r-2) + 1 \ \ Multiplications \end{aligned}$$

Computations for sum factors

$$r + 1 \ \ Additions$$
$$r + 1 \ \ Multiplications$$

For each pole combination :

The above has to be evaluated for r+2 poles in each combination.

Computations for Combination factors

$$r \ \ Multiplications$$

For r relay

There are 2^r such pole combinations

Computation for common factor

$$1 \ \ Addition$$
$$2(r+1) \ \ Multiplications$$

Plugging this into (2.21) results in complexity of algorithm as

$$5^{N-2}\left(8N^2 + 42N + 50\right) + 3^{N-2}\left(4N^2 + 14N + 9\right) - 16N - 4 \quad Additions$$
$$5^{N-2}\left(8N^2 + 2N - 50\right) + 3^{N-2}\left(4N^2 + 38N + 63\right) + 2^N\left(N + 2\right) - 17N \quad Multiplications$$

2.2.3 Simulation Results

Figure 2.4 shows the analytical results for the arbitrary relays located at the L=0.5, and also the algorithm is validated with the simulation results indicates as circles.Also we can say that there is an increase in diversity order with the number of relays. Figure 2.5 gives the comparison of central limit theorem approximation for multiple relays with the algorithmic approach.It is observed that at high SNR the algorithmic approach gives upper bound over CLT approximation.

2.2.4 Diversity Order

Antenna gain is the gain in signal to noise ratio.It tells how the signal strength is improved with multiple antennas. Diversity order tells how many independent copies are available.Diversity gains tells the improvement in the power for certain BER.

Here for the system with N multiple relays the diversity order should be N+1 ,for PL Combiner acheives half the diversity i.e $\frac{N+1}{2}$.

Diversity order D is defined as

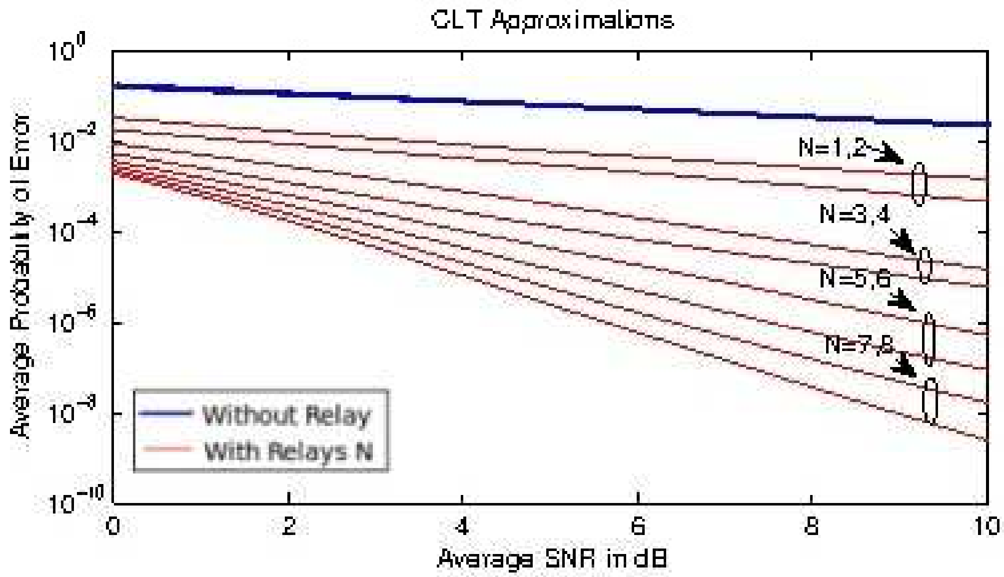
$$P_e \propto \frac{1}{SNR^D}$$

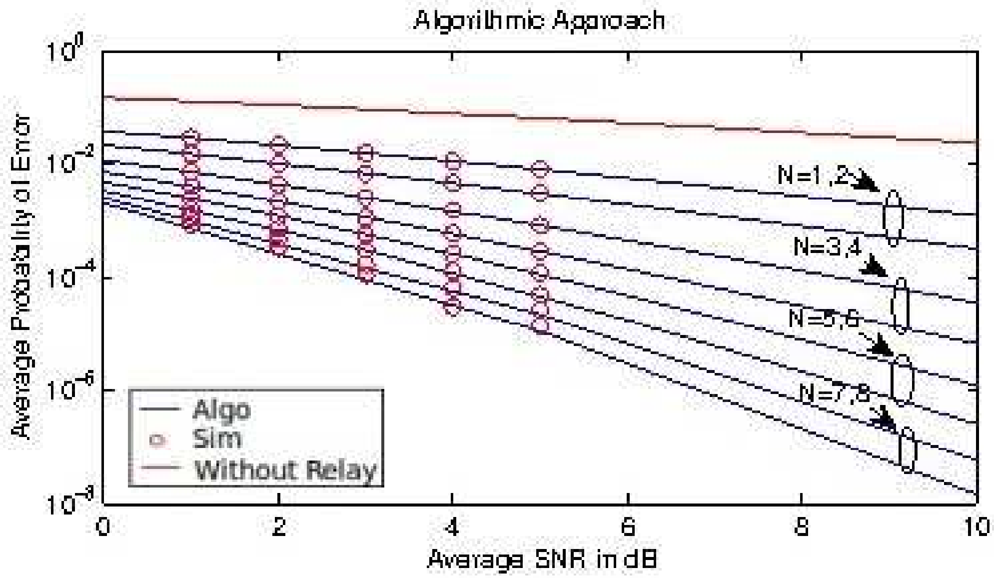
or

$$D = \lim_{SNR \to \infty} -\frac{log(P_e)}{log(SNR)}$$

As D is real,we expect the log-log plot of BER vs SNR is linear function in diversity system. Therefore, D is the slope of the BER curve vs SNR in log-log plot.

From the figure 2.5, the diversity order is found graphically, and indicated as (relays,diversity order).we have (1,1.48), (2,1.59), (3,1.91), (4,2.17), (5,2.55), (6,2.96),(7,3.3) and (8,3.8).We can observe from the CLT approximation that every two relays have almost same slope at high SNR. We can infer that the diversity order is $\frac{N}{2}$ for large values of N.For small values of N, where the diversity order is observed to be same, then the use of number of relays depends on the coding gain.


CLT Approximations
Average Probability of Error
Average SNR in dB
N=1,2
N=3,4
N=5,6
N=7,8
Without Relay
With Relays N


Algorithmic Approach
Average Probability of Error
Average SNR in dB
N=1,2
N=3,4
N=5,6
N=7,8
Algo
Sim
Without Relay

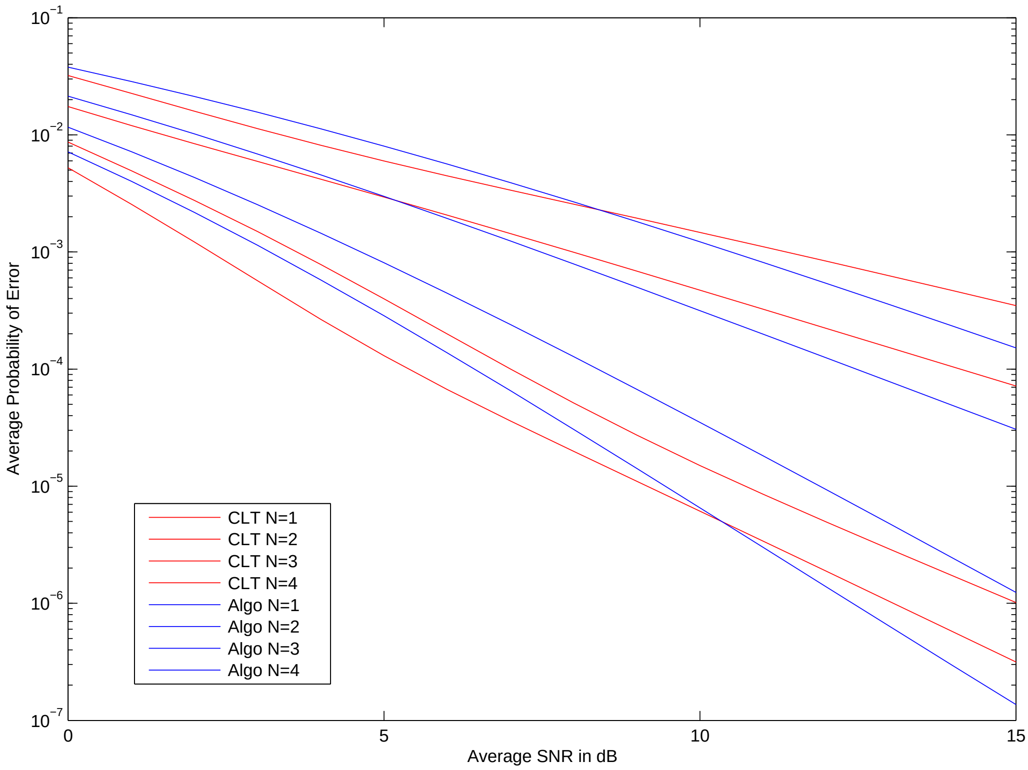


Figure 2.5: Comparison of BER for multiple relays located at L=0.5

Chapter 3

Conclusion

We found the exact closed form expressions for BER for ML-DF cooperative system.From the simulations we observe that PL combiner gives good approximation to the ML detector.An algorithm is implemented to evaluate the BER for multiple relays analytically. We also provide the complexity of the algorithm. From the analytical results, we observe that there is an increase in diversity order with the number of relays.At high SNR,we observe that our algorithmic approach when compared with CLT approximation [1] gives upper bound on diversity order.

This work is extended to optimization of algorithm in terms of fast execution. For this we implement the algorithm in OpenCL which is heterogeneous programming which execute the applications across devices like CPU and GPU .It is optimized by identifying different forms of parallelism present in the algorithm.Other factor that helps in reduction of time is load balancing on the devices.

In future, Channel state information at transmitter helps in increasing the bandwidth efficiency.With this, only one relay can be selected from multile relays based on the new contributing factor and all the relays convey the decision at that relay to destination. This also should perform comparably well with the existing scenario.

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