Phase Noise Reduction in an Oscillator Using Harmonic Mixing Technique

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Department of Electrical Engineering

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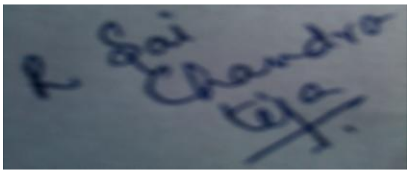

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Last but not least thank god for making things possible.

Dedicated to

My parents, Sister& God

Abstract

This thesis presents a new technique of injection of multi-harmonics into an oscillator, study the behavior and response of the oscillator. First general locking equations are derived using the feedback model of an oscillator. Second, using the Laplace domain modeled phase noise analysis giving insight into the phase noise model of an oscillator under multiple harmonics are analytically studied and experimentally validated with good agreement. From the locking process and phase noise analysis we can observe when compared to single tone injection there is a substantial improvement of phase noise under multi-harmonic injection, enhancement in the locking range. We present Injection Locked Frequency Divider (ILFD) circuit under multi-harmonic injection (2^{nd} and 3^{rd} harmonics injected) with simulation, theoretical validation. The proposed circuit is designed using the UMC library by using Op-Amp based feedback current reference circuit. The amplitude dependency of the injection signals on the phase noise is observed as increase in the injection levels decreases the phase noise.

With the above mentioned performances and advantages, it is believed that the studied Injection Locked Frequency Divider (ILFD) under multi-harmonic injection is attractive for further development toward practical applications in the modern microwave and millimeter wave wireless communications systems.

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Chapter 1

Introduction

1.1 Introduction

In order to obtain a reference signal with a low phase noise, high frequency stability, and low cost for the microwave/millimeter-wave applications, the injection-locked oscillators [1] have been widely used in the wireless, optics, and local network systems. Signal generation and division are very important to provide a reference frequency signal for the communication systems. The reference signal based on a voltage-controlled oscillator or a phase-locked loop must be included in the system to offer a desired frequency for the channel selection or the modulation/demodulation through up/down conversion. To reach the microwave and millimeter-wave operation band, the design of the oscillator also has more challenges due to the limitation about the signal transmission effect in such band, high-frequency performance of the transistors, and cost issues. No matter what kinds of the reference signal are generated, low phase noise, frequency tuning range, and the carrier frequency stability always have to pay attention. For this reason, the injection locking mechanism has been proposed to improve the performance of an oscillator [1-3]. Since the injection-locked technique has the characteristics of the super-/sub-harmonically synchronization and the phase tunable function, the injection-locked oscillator has been widely used to synchronize the frequency and phase for some applications, such as the active/phase array antenna with the phase tuning function to control the beam angle . The super-harmonic ILO can to be an injection-locked frequency divider (ILFD), and the sub-harmonic ILO can synchronize an injected signal with a frequency multiple function[4].

1.2 Aim and Motivation

Research on frequency division can be tracked back to 1920's, indicating a long history topic. Researches focused on frequency divider circuits primarily involve

- Wide operation range
- High operating frequency
- High division ratio
- Low power consumption
- Low cost

Generally wide operating range is achieved by flip-flop based digital dividers [].for analog dividers, ring oscillator based divider feature wide locking range due to low-Q characteristic. On the other hand LC-oscillator based dividers works at high frequency with less power dissipation. Advances in devices motivate the evolvement of circuits, components and systems toward compactness, high operating frequency and low cost. Frequency dividers with increase in division ratios improve the phase noise, narrowing down locking range with increase in power and area. Since there is a need to achieve higher phase noise improvement with wide locking range. In present work we propose a low-cost IL–LC-VCO with Multiple Harmonic injection technique (i.e. injection of multiple harmonics like $2\boldsymbol{\omega_o}$ and $3\boldsymbol{\omega_o}$ at different nodes of oscillator) to meet above mentioned needs, the phase noise constraints of modern RF transceivers.

1.3 Literature Survey

The injection-locking of an electrical oscillator was first described by Van Der Pol in 1927, and the first locking bandwidth equation for electrically injection-locked oscillators was developed by Adler in 1946 [1]. A vacuum tube transistor was used in Adler's oscillator model. Razavi [8], Abidi [9] etc have extended this analysis. They arrived to a conclusion that the increase in division ratio will narrow down the locking range. Over the years, various attempts for analysis of phase noise have resulted in various improved expressions for Leeson's classic formula [12]. Leeson's model of phase noise was based on viewing an oscillator as a time-invariant system. It was heuristic derivation without formal proof. Rael and Abidi [13] were successful in deriving the unspecified noise factor in Leeson's heuristic derivation as a function of circuit parameters for a popularly current steered LC-tank topology. Based on the definition by Rael and Abidi, Hegazi [14] proposed a filtering technique to remove high frequency noise thereby reducing the phase noise considerably. However, Rael's description of phase noise did not completely describe the physical phenomena of noise to phase noise conversion, since they neglected the time-variant nature of oscillator. More recent developments are reported by Hajimiri and Lee [15] [16], have explicitly proven the importance of incorporating the time varying nature of oscillator's phase noise. Key idea in this linear time-variant approach is the introduction of sensitivity function called Impulse Sensitivity Function (ISF), ISF quantitatively represents the impact of stationary and cyclo-stationary noise sources on phase noise conversion varies across the oscillation period. Rategh and Lee [5] using ISF derived the phase noise of an ILFD. From the analysis a divide-by N ILFD network shows a phase noise improvement of $20\log N$. By using the analogy between first-order PLL and ILO, the phase noise of an Injection Locked Oscillator is calculated in terms of noise filtering bandwidth.K_{ILO} from Paciorek's equation and Laplace domain modeling.

1.4 Contribution of the Thesis

This work focuses on the design and analysis of an oscillator under multi-harmonic injection. The phase noise model using Laplace domain modeling for multi-harmonic injection locked oscillator by extending the single tone injection analysis. The main contributions of this research work are as follows:

- General locking expressions of an Injection locked oscillator under multiple injections using feedback model of an oscillator.
- Phase noise model using Laplace domain modeling for multi-harmonic injection.
- Multi-harmonic injection locked oscillator design and theoretical validation using the proposed model.
- Ideas for exploring the possibility of multi-harmonic injection in other oscillator topologies.

1.5 Thesis Organization.

- **Chapter 1:** is the introduction describing the motivation behind the work, literature survey, objectives and contributions of the present work.
- **Chapter 2:** analysis of the oscillator response under multi-harmonic injection, theoretical locking expressions using feedback model of an oscillator, Extending the Laplace domain phase noise model for single tone injection to Multi-harmonic injection.
- **Chapter 3:** describes existing Injection locked frequency divider circuit topologies for even, odd division ratios.

- **Chapter 4:** gives insight into Multi-harmonic ILFD design, analysis of the circuit and theoretical validation of the proposed design based on proposed phase noise model.
- **Chapter 5:** presents the conclusion to the thesis as well as future directions of this work

Chapter 2
Theoretical Analysis of Multi-Harmonic Injection Locking

2.1 Introduction

Injection of a periodic signal into an oscillator leads to interesting locking or pulling phenomena. The injection-locking of an electrical oscillator was first described by Van Der Pol in 1927, and the first locking bandwidth equation for electrically injection-locked oscillators was developed by Adler in 1946 [1]. A vacuum tube transistor was used in Adler's oscillator model. Injection locking becomes useful in a number of applications, including frequency division, quadrature generation, and oscillators with finer phase separations. Injection pulling, on the other hand, typically proves undesirable. Depending upon the ratio of the incident frequency to the oscillation frequency, three classes of injection-locked oscillators (ILO's) may be defined: first-harmonic, sub harmonic, and super harmonic ILO's. In a first-harmonic ILO, the oscillation frequency is the same as the fundamental frequency of the incident signal, while in a sub harmonic ILO, the incident frequency is a sub harmonic of the oscillation frequency. Likewise, in a super harmonic ILO, the incident frequency is a harmonic of the oscillation frequency.

In order to maintain the phase and frequency stability of the reference signal for establishing a coherent carrier at each active T/R module, thus, the sub-harmonic injection-locked oscillators are used to establish a coherent carrier and to synchronize the frequency of reference local oscillators through indirect. Because figures of merit, such as the locking range and the FM noise degradation of the sub harmonically injection locked LO, have a great effect on the frequency stability and the noise behavior of the modulated carrier, it is important to analyze the sub-harmonic injection locking process quantitatively.

2.2 Theoretical Formulation

2.2.1 Model of an Injection Locked Oscillator

In general, an injection locked oscillator can be modeled as a non-linear block f(e) with a linear sensitivity block H(ω) in a positive feedback loop; and an external signal(i.e., injection signal) is injected into the oscillator. Analysis and study on single source injection locked oscillator with locking phenomena is done by Jia-Lin Li, Shi-Wei Qu and Quan Xue.[11] We extend the analysis to Multi-Harmonic injection (i.e. Multiple sources are injected) in an oscillator as shown in Figure (2.1)

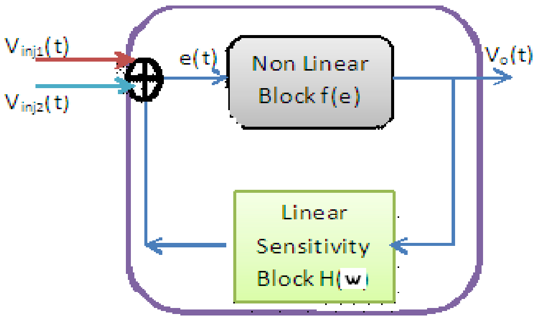


Figure (2.1): Model of a multi-harmonic injection locked oscillator.

Let the oscillator be described by []

$v_{i1}(t) = V_{i1}\cos(\omega_{i1}t + \varphi_1)$

$v_{i2}(t) = V_{i2}\cos(\omega_{i2}t + \varphi_2)$

$u(t) = H(\omega).v_o(t) = U\cos(\omega_o t)$

$v_o(t) = f(e(t)) = f(u(t) + v_{i1}(t) + v_{i2}(t))$

$$H(\omega) = \frac{H_o}{1 + j2Q\dfrac{\Delta\omega}{\omega_o}}$$

Where $v_{i1}(t)$ and $v_{i2}(t)$ are the injection signals, $u(t)$ is the positive feedback signal, $v_o(t)$ is the output signal of the oscillator,φ_1, φ_2 is the phase difference between $v_{i1}(t), v_{i2}(t)$ and $u(t)$. ω_{i1}, ω_{i2} Injection frequencies. $\omega_o, \Delta\omega$ and Q are the resonant frequency, frequency deviation from the resonant frequency, and quality factor of a single-tuned resonant circuit, respectively. The output of the nonlinear block $v_o(t)$ may contain harmonic and intermodulation terms of $u(t)$ and $v_{i1}(t), v_{i2}(t)$. For a small injection signal $v_{i1}(t), v_{i2}(t)$, function $f(u(t)+v_{i1}(t)+v_{i2}(t))$ can be expanded into a Taylor series given by

$$v_o(t) = f(e(t)) = f\big(u(t)+v_{i1}(t)+v_{i2}(t)\big)$$
$$= \sum_{n=0}^{\infty} f^n(U\cos(\omega_o t))\frac{(v_{i1}(t)+v_{i2}(t))^n}{n!}$$

Where function $f^n(.)$ denotes the nth-order derivative. It can be seen that $f^n(.)$ is a periodic function with periodicity 2π, and hence, one can further expand $f^n(.)$ in terms of the complete orthogonal basis set of exponential Fourier series, resulting in

$$v_o(t) = f(e(t)) = f\big(u(t)+v_{i1}(t)+v_{i2}(t)\big) = \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty} A_{m,n}(U)e^{jm\omega_o t}\frac{(v_{i1}(t)+v_{i2}(t))^n}{n!}$$

Where

$$A_{m,n}(U) = A_{-m,n}(U) = \frac{1}{2\pi}\int f^n(U\cos\alpha)\cos(m\alpha)\,d\alpha$$

Based on trigonometric identities of $\cos(m \pm 1)\alpha$, the following relationships are valid for the coefficients $A_{m,n}(U)$ are

$$A_{m-1,n}(U) - A_{m+1,n}(U) = 2\frac{m}{U}A_{m,n-1}(U)$$
$$A_{m-1,n}(U) + A_{m+1,n}(U) = 2\frac{dA_{m,n-1}(U)}{dU}$$

Now, to obtain general locking equations, we expand $v_o(t)$ as

$$v_o(t) = f\big(u(t)+v_{i1}(t)+v_{i2}(t)\big)$$
$$= \sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty} A_{m,n}(U)e^{jm\omega_o t}\frac{1}{2^n n!}\Big[v_{i1}^n e^{j(n\omega_{i1}t+n\varphi_1)} + v_{i1}^n e^{-j(n\omega_{i1}t+n\varphi_1)}$$
$$+ v_{i2}^n e^{j(n\omega_{i2}t+n\varphi_2)} + v_{i2}^n e^{-j(n\omega_{i2}t+n\varphi_2)} + rest\ of\ polynomial\ terms\Big]$$

In the proposed design we inject multiple harmonics (2nd harmonic and 3rd harmonic) into oscillator. We simplify the above expression by considering

$$\omega_{i1} = 2\omega_o, \omega_{i2} = 3\omega_o$$

$$v_o(t) = f\big(u(t) + v_{i1}(t) + v_{i2}(t)\big)$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} A_{m,n}(U) e^{jm\omega_o t} \frac{1}{2^n n!} \big[v_{i1}^n e^{j(2n\omega_o t + n\varphi_1)} + v_{i1}^n e^{-j(2n\omega_o t + n\varphi_1)}$$

$$+ v_{i2}^n e^{j(3n\omega_o t + n\varphi_2)} + v_{i2}^n e^{-j(3n\omega_o t + n\varphi_2)} + rest\ of\ polynomial\ terms \big]$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} A_{m,n}(U) \frac{1}{2^n n!} \big[v_{i1}^n e^{j((2n+m)\omega_o t + n\varphi_1)} + v_{i1}^n e^{-j((2n-m)\omega_o t + n\varphi_1)}$$

$$+ v_{i2}^n e^{j((3n+m)\omega_o t + n\varphi_2)} + v_{i2}^n e^{-j((3n-m)\omega_o t + n\varphi_2)} + rest\ of\ polynomial\ terms \big]$$

From the above expression the fundamental oscillation frequency is set by considering $2n + m = \pm 1\ , 2n - m = \pm 1, 3n + m = \pm 1, 3n - m = \pm 1$

$$terms\ correspomding\ to\quad m + 2n = \pm 1$$

$$\longmapsto\ A_{1,0} e^{j\omega_o t} + A_{-1,0} e^{-(j\omega_o t)} + \frac{A_{-2n+1,n}}{2^n n!} v_{i1}^n e^{j(\omega_o t + n\varphi_1)}$$

$$+ \frac{A_{-2n-1,n}}{2^n n!} v_{i1}^n e^{j(-(\omega_o t) + n\varphi_1)} + higher\ order\ terms$$

$$terms\ correspomding\ to\quad 2n - m = \pm 1$$

$$\longmapsto\ A_{1,0} e^{j\omega_o t} + A_{-1,0} e^{-(j\omega_o t)} + \frac{A_{2n+1,n}}{2^n n!} v_{i1}^n e^{-j(-(\omega_o t) + n\varphi_1)}$$

$$+ \frac{A_{-2n-1,n}}{2^n n!} v_{i1}^n e^{-j(\omega_o t + n\varphi_1)} + higher\ order\ terms$$

$$terms\ correspomding\ to\quad 3n + m = \pm 1$$

$$\longmapsto\ A_{1,0} e^{j\omega_o t} + A_{-1,0} e^{-(j\omega_o t)} + \frac{A_{-3n+1,n}}{2^n n!} v_{i2}^n e^{j(\omega_o t + n\varphi_2)}$$

$$+ \frac{A_{-3n-1,n}}{2^n n!} v_{i2}^n e^{j(-(\omega_o t) + n\varphi_2)} + higher\ order\ terms$$

$$terms\ correspomding\ to\quad 3n - m = \pm 1$$

$$\longmapsto\ A_{1,0} e^{j\omega_o t} + A_{-1,0} e^{-(j\omega_o t)} + \frac{A_{3n+1,n}}{2^n n!} v_{i2}^n e^{-j(-(\omega_o t) + n\varphi_2)}$$

$$+ \frac{A_{-3n-1,n}}{2^n n!} v_{i2}^n e^{-j(\omega_o t + n\varphi_2)} + higher\ order\ terms$$

The output signal at ω_o of the oscillator is simplified by truncating the higher order terms

$$v_{\omega_o}(t) = B e^{j\omega_o t} + B^* e^{-(j\omega_o t)}$$

Where the asterisk denotes the complex conjugate, and

$$B = \left\{4A_{1,0} + \frac{v_{i1}^n}{2^{n-1}n!}\frac{dA_{2n,n-1}}{dU}\cos(n\varphi_1) + \frac{v_{i2}^n}{2^{n-1}n!}\frac{dA_{3n,n-1}}{dU}\cos(n\varphi_2)\right\} + j\left\{\frac{2n}{U}\frac{v_{i1}^n}{2^{n-1}n!}A_{2n,n-1}\sin(n\varphi_1) + \frac{3n}{U}\frac{v_{i2}^n}{2^{n-1}n!}A_{3n,n-1}\sin(n\varphi_2)\right\}$$

Using the linear transfer function $H(\omega)$ and oscillation condition the output signal at ω_o is expressed by

$$u(t) = H(\omega).v_o(t) = U\cos(\omega_o t)$$

The real and imaginary parts are separated as

$$U = 2H_o\left\{4A_{1,0} + \frac{v_{i1}^n}{2^{n-1}n!}\frac{dA_{2n,n-1}}{dU}\cos(n\varphi_1) + \frac{v_{i2}^n}{2^{n-1}n!}\frac{dA_{3n,n-1}}{dU}\cos(n\varphi_2)\right\}$$

$$2Q\frac{\Delta\omega}{\omega_o} = \frac{2H_o}{U}\left\{\frac{2n}{U}\frac{v_{i1}^n}{2^{n-1}n!}A_{2n,n-1}\sin(n\varphi_1) + \frac{3n}{U}\frac{v_{i2}^n}{2^{n-1}n!}A_{3n,n-1}\sin(n\varphi_2)\right\}$$

The above expressions give the fundamental and frequency locking of Multi-harmonic injection locked oscillator.

2.2.2 Locking range

The frequency deviation from the above expression considering the steady state response of the system to the single tone injection signal being replaced by the instantaneous frequency gives an insight into locking process. For multi-harmonic injection the frequency deviation derived is similar in the form to the one studied by Alder [1], Razavi [8] and Abidi [9].

$$\Delta\omega = \frac{\omega_o}{2Q}\frac{2H_o}{U}\left\{\frac{2n}{U}\frac{v_{i1}^n}{2^{n-1}n!}A_{2n,n-1}\sin(n\varphi_1) + \frac{3n}{U}\frac{v_{i2}^n}{2^{n-1}n!}A_{3n,n-1}\sin(n\varphi_2)\right\}$$

$$\Delta\omega = \frac{\omega_o}{2Q}\frac{\frac{2n}{U}\frac{v_{i1}^n}{2^{n-1}n!}A_{2n,n-1}\sin(n\varphi_1) + \frac{3n}{U}\frac{v_{i2}^n}{2^{n-1}n!}A_{3n,n-1}\sin(n\varphi_2)}{4A_{1,0} + \frac{v_{i1}^n}{2^{n-1}n!}\frac{dA_{2n,n-1}}{dU}\cos(n\varphi_1) + \frac{v_{i2}^n}{2^{n-1}n!}\frac{dA_{3n,n-1}}{dU}\cos(n\varphi_2)}$$

From the above expression when compared to the single injection we have excess terms in the numerator and the denominator the locking range entirely depends on the non-linear function of the oscillator.

2.3 **Phase Noise Analysis**

Any practical Oscillator has variations in its amplitude and frequency. Short term Frequency fluctuations of a standalone oscillator are mainly due to the noise and interference. The one sided spectrum of an ideal oscillator which is an impulse function at the frequency of operation exhibits skirts around it due to these short term frequency variations. These skirts are characterized as phase noise side bands [13]. Any reference frequency signal in a wireless communication system has stringent phase noise specifications. In order to obtain a reference frequency signal with a low phase noise, high frequency stability and low cost, the injection-locked oscillators have been widely used in wireless, optics, and local networks. In the frequency domain viewpoint, an oscillator's short term instabilities are usually characterized in terms of the single sideband noise spectral density as shown in Figure (2.2). It is conventionally given the units of decibels below the carrier per Hertz (dBc/Hz) and is defined as:

$$L_{total}\,\{\Delta\omega\} = 10.log\left[\frac{P_{sideband}(\omega_0 + \Delta\omega, 1Hz)}{P_{carrier}}\right]$$

Where$P_{sideband}(\omega_0 + \Delta\omega, 1Hz)$, represents the single side band power at a frequency offset $\Delta\omega$ from the carrier in a measurement bandwidth of 1Hz and $P_{carrier}$ is the total power under the power spectrum.

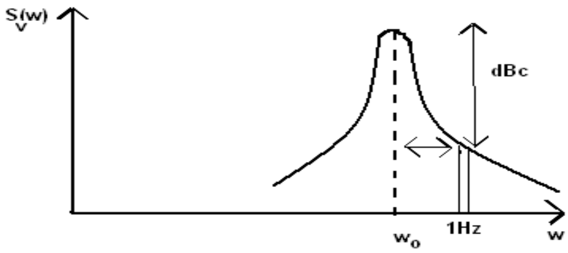


Figure (2.2): Phase Noise per unit bandwidth.

Its disadvantage is that it shows the sum of both amplitude and phase variations; it does not show them separately. It is often important to know the amplitude and phase noise separately because they behave differently in a circuit. For instance, the effect of amplitude noise can be reduced by amplitude limiting, while the phase noise cannot be reduced in an analogous manner. Therefore, in most practical oscillators, $L_{total}\,\{\Delta\omega\}$ is dominated by its phase portion.

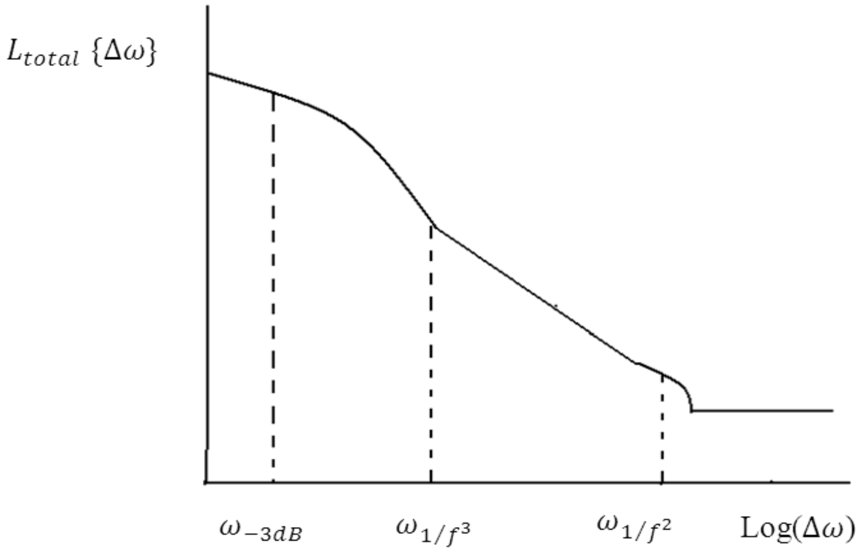


Figure (2.3): Phase noise in an oscillator

If one plots $L_{total}\{\Delta\omega\}$for a free running oscillator as a function of $\Delta\omega$ on logarithmic scales, region of different slopes may be observed as shown in Figure (2.3) .At large offset frequencies; there is a flat noise floor. At small offsets, one may identify regions with a slope of $1/f^2$ and $1/f^3$ .where the corner is called ω_{1/f^3}. An early phase noise model was proposed by Leeson [12] in 1966 considering an oscillator as Time-Invariant system. Oscillator properties such as signal power, resonator Q, Noise factor, which don't vary with time, are used to obtain an estimation of phase noise.

$$L(\Delta\omega) = 10.\log\left[\frac{2FkT}{P_s}.\left[1+\left(\frac{\omega_o}{2Q_l\Delta\omega}\right)^2\right].\left[1+\frac{\omega_{1/f^3}}{\Delta\omega}\right]\right]$$

Where F is the empirical noise factor parameter, k is Boltzmann's constant; T is the absolute temperature, P_s is the average power dissipated in the positive resistive part of the tank, ω_o is the oscillation frequency, Q_l is the effective quality factor of the tank with all loadings included, $\Delta\omega$ is the offset frequency from the carrier, and ω_{1/f^3} is the frequency of the corner between the $1/f^3$ and $1/f^2$ regions as shown in Figure ().

The main drawback of the Leeson's model is the lack of knowledge about the constant of proportionality F, which Leeson leaves as an unspecified noise factor that strongly depends on the oscillator topology. Over the years large number of attempts for analysis of phase noise has resulted in various improved expressions for the Leeson's classic formula. Rael and Abidi [13] were successful in deriving the unspecified noise factor in Leeson's heuristic derivation as a function of circuit parameters for a popularly current steered LC-tank topology. Based on the definition by Rael and Abidi , Hegazi [14] proposed a filtering technique to remove high frequency noise thereby reducing the phase noise considerably. However, Rael's description of phase noise did not completely describe the physical phenomena of noise to phase noise conversion, since they neglected the time-variant nature of oscillator.

More recent developments are reported by Hajimiri and Lee [15] [16], have explicitly proven the importance of incorporating the time varying nature of oscillator's phase noise. Key idea in this linear time-variant approach is the introduction of sensitivity function called Impulse Sensitivity Function (ISF), ISF quantitatively represents the impact of stationary and cyclo-stationary noise sources on phase noise conversion varies across the oscillation period.

Rategh and Lee [5] using Impulse sensitivity function (ISF) derived the phase noise of an Injection Locked Frequency Divider (ILFD). From the analysis a divide-by N ILFD network shows a phase noise improvement of $20\log N$. This approximation is valid under the assumption the contribution of the phase noise to ILFD by input injection sources and internal noises are independent and uncorrelated.

Phase noise of a Divide-by-N ILFD is given by

$$L_{\varphi,total} = L_{osc}\frac{\Delta\omega^2}{\omega_p{}^2+\Delta\omega^2}+L_{source}\frac{\left(\omega_p/N\right)^2}{\omega_p{}^2+\Delta\omega^2}$$

Where $L_{\varphi,total}$ is the phase noise of an ILFD and L_{osc}, L_{source} are the phase noise of the free running oscillator, injection source. $\Delta\omega$ is the offset frequency, ω_p is the pole frequency where ILFD phase noise follows the oscillator phase noise. Just as in the first order PLL, the internal free-running phase noise of an ILFD is filtered with a high pass filter while the noise from the injection source is filtered with a low pass filter. The pole frequency is analogous to the loop-bandwidth frequency of conventional PLL's. Phase noise of the output is dominated by the phase noise from the input which the output tracks with a scale factor of $1/_{N^2}$.

Using the phase noise spectrum of an ILFD for a single source injection extending it to Multi-harmonic injection sources under the assumption the contribution of phase noise to ILFD by input injection sources and internal oscillator noise are independent and uncorrelated it can be concluded from Figure (2.4) that there will be phase noise improvement of $20\log N_1 + 20\log N_2$ under injection of two harmonics where N_1 and N_2 being the harmonics injected (like 2nd and 3rd harmonic).

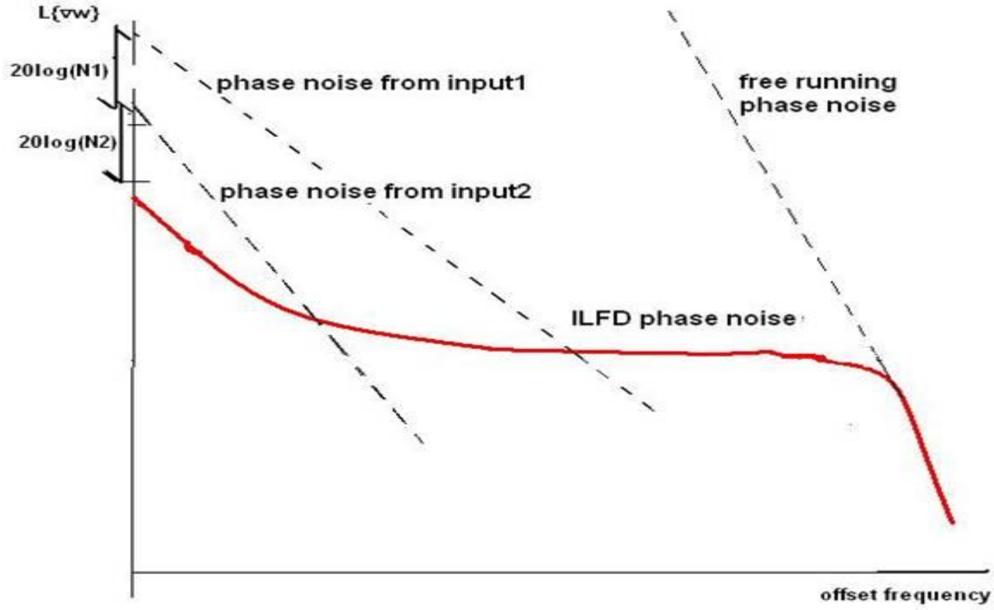


Figure (2.4): Phase Noise Spectrum of an ILFD under Multiple injections.

From [4] using the analogy between the first order Phase Locked Loop (PLL) and Injection Locked Oscillator (ILO), the phase noise of an injection locked oscillator is calculated in terms of the noise filtering bandwidth K_{ILO} analogous to the pole frequency as derived by Rategh and Lee [5]. From the Laplace domain modeling and based on the Paciorek's equation the phase noise of an ILO is given by

$$L_{ILO}(\Delta\omega) = \frac{{N_1}^2[K_{ILO}(\varphi_{ss})]^2 * L_{src}(\Delta\omega) + \Delta\omega^2 * L_{osc}(\Delta\omega)}{\Delta\omega^2 + {K_{ILO}}^2}$$

Where $K_{ILO}(\varphi_{ss})$ is the noise filtering bandwidth given by $K_{ILO}(\varphi_{ss}) = \frac{\epsilon^2 + \epsilon\cos(\varphi_{ss})}{[1+\epsilon\cos(\varphi_{ss})]^2}\frac{\omega_{free}}{2Q}$

φ_{ss} is the fixed phase difference between the injection output and the oscillator output, Relative injection level $\epsilon = \frac{I_{inj}}{I_{osc}}$, N1 is the ratio of injection locked frequency to the injection frequency(>1:Multiplier , <1:Divider).

Till date all the phase noise models proposed were inherently taken into account a single injection source and analysis was performed. Considering the Laplace domain modeled phase noise analysis where its dependency on the quantities being easily derivable by designers we emphasize on it and extend the analysis to multi-harmonic injection. Using the superposition theorem under the assumption contribution to phase noise by the injection sources and the internal noise are independent and uncorrelated the phase noise of a multi-harmonic injection locked oscillator intuitively is proposed as:

$$L_{ILO}(\Delta\omega) = \frac{{N_1}^2[K_{ILO1}(\varphi_{ss})]^2 * L_{src1}(\Delta\omega) + {N_2}^2[K_{ILO2}(\varphi_{ss})]^2 * L_{src2}(\Delta\omega) + \Delta\omega^2 * L_{osc}(\Delta\omega)}{\Delta\omega^2 + [K_{ILO1}(\varphi_{ss})]^2 + [K_{ILO2}(\varphi_{ss})]^2}$$

Where$K_{ILO1}(\varphi_{ss}) = \frac{{\epsilon_1}^2+\epsilon_1 \cos(\varphi_{ss})}{[1+\epsilon_1 \cos(\varphi_{ss})]^2}\frac{\omega_{free}}{2Q}$,φ_{ss} is the fixed steady state phase difference between the injection output and the oscillator output, $K_{ILO2}(\varphi_{ss}) = \frac{{\epsilon_2}^2+\epsilon_2 \cos(\varphi_{ss})}{[1+\epsilon_2 \cos(\varphi_{ss})]^2}\frac{\omega_{free}}{2Q}$. Relative injection levels$\epsilon_1 = \frac{I_{inj1}}{I_{osc}}$, $\epsilon_2 = \frac{I_{inj2}}{I_{osc}}$ of the multiple injection sources, N1 and N2 are the division ratio.

2.4 **Discussions**

Understanding the injection locking based on the feedback model of an oscillator and the phase noise phenomena in an injection locked frequency divider under single tone injection. We tried exploring the possibility of multiple harmonic injections into an oscillator; understand the response and behavior of the oscillator. Based on the feedback model the locking equation of Multi-harmonic injection locked oscillator is derived. The expression derived is similar to that of one by Alder, Razavi. By analyzing the phase noise models derived for single tone injection we emphasize on the Laplace domain model analysis for understanding the phase noise of a Multi-harmonic injection locked oscillator.

Chapter 3

Study of Injection Locked Frequency Divider (ILFD)

3.1 **Introduction**

Modern Wireless and wire-line communications consists of many sub-circuits each performing a special function of frequency modulation demodulation, power amplification, frequency up-down conversion, etc. such systems are treated as frequency conversion networks and among them, a key component is the frequency divider. Generally the different microwave frequency dividers can be classified into two categories: digital and analog.

The fundamental element of a digital frequency divider is a flip-flop, typically consisting of two level-sensitive latches in a negative feedback loop to form an edge triggered master-slave flipflop. The advantage of digital frequency divider over analog one is the broadband operation from dc to a maximum division frequency as long as the slew rate is high enough. The disadvantage is the relatively low maximum division frequency and complicated circuit configurations.

All the analog frequency dividers can be classified into three sub-types: regenerative, parametric, and injection- locked frequency dividers [17]. Regenerative frequency divider consists of a modulator or mixer, a frequency selectivity network H(ω) and a power amplification unit limited by small division ratios and complicated circuit topologies. Parametric divider can be described by a negative conductance generator and input/output isolation networks .The major advantage of this kind of frequency divider is high operation frequency with relatively inexpensive circuit. However, the division ratio higher than two will result in a narrow operation band and poor noise performance.

The injection-locked frequency divider is generally an oscillator-based divider. In this way, an oscillator is required to design before the final implementation of this kind of frequency divider. There are two kinds of oscillators: relaxation and harmonic types. The relaxation oscillator is characterized by an extremely distorted waveform, an inherent lack of frequency stability, and a high degree of susceptibility to synchronization even at very high-order super-harmonic components. The harmonic oscillator, on the contrary, features a relatively high degree of frequency stability, and a correspondingly smaller susceptibility to synchronization the harmonic oscillator can be further classified as two sub-types: ring and LC oscillators. The ring oscillator has low quality factor Q, and a wide operation range for frequency divider applications. However, its relatively complicated circuit topology leads to higher dc power consumptions LC oscillator generally characterizes high Q, which leads to the relatively narrow operation bandwidth for frequency divider applications, and this becomes the only disadvantage. However, low power dissipation as well as high speed (high-frequency operation) makes it very attractive for application to the microwave and millimeter-wave frequency-division circuits.

3.2 **Designed ILFD**

3.2.1 Divide-by-2 ILFD

The injection method for popular LC ILFD's can be tail or direct injection. A classical topology suitable for divide-by 2 ILFD operation is a differential LC oscillator with injection of the signal at the tail node is shown in Figure (3.1). The classical CMOS injection locked oscillator topology [17] has a limitation on locking range due to inefficient injecting path through the tail transistor and it is not suitable for the high frequency operation because of the large parasitic capacitance at drain of injected tail transistor and not a good option for low voltage applications. Direct injection is used for high frequency operation

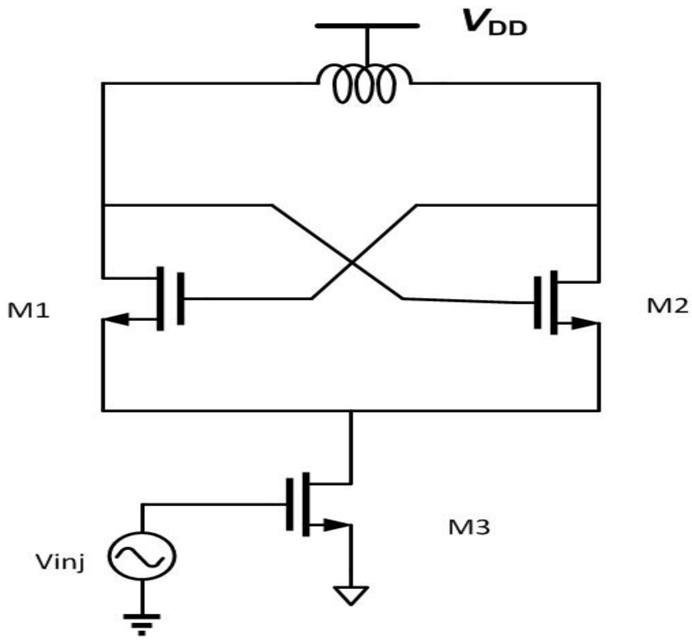


Figure (3.1): Classic ILO Topology

A differential LC oscillator has become a popular choice for ILFDs [4]. When there is no input signal, i.e., the oscillator is free-running, there exists a signal at node A (the drain of the tail transistor, Mtail), which is at the second harmonic of the free-running oscillation frequency. To avoid this problem the versatile structure is shown in Figure (3.2), where the signal is injected parallel to the tank of the oscillator i.e. equivalent to injection at the tail node with a factor of mixer conversion gain. Differential pair M1and M2 shown in figure produces an odd non-linear function mixed with injection signal forces the oscillator to lock to even division ratios [18]. There is a phase noise improvement of 6dBc/Hz for divide-by 2 network.

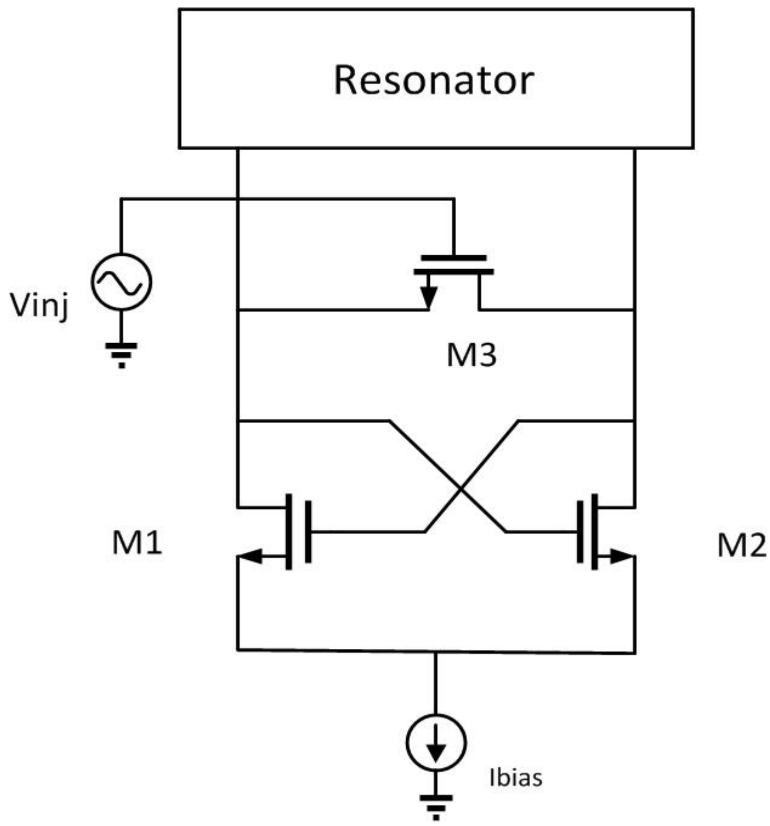


Figure (3.2): Divide-by-2 circuit

3.2.2 Divide-by 3 ILFD

In case of divide-by odd [19] number operation needs a configuration of even-order non-linearity with differential topology. Most suitable architecture for divide-by 3 is shown in Figure (3.3). Here differential pair of M3 and M4 is added with the cross coupled source node of M1 and M2 forming a differential cascode topology. As a result M3 and M4 mixes the differential injection signals with even order non linearity function forcing the oscillator to lock to odd division ratios [19]. There is a phase noise improvement of 9.5dBc/Hz for divide-by 3.

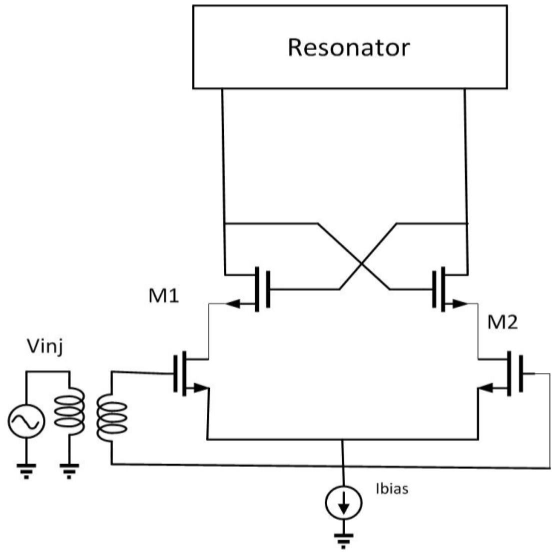


Figure (3.3): Divide-by-3 circuit

3.3 Results and Discussion

Simulations are carried out using Cadence tool Spectre-RF on 0.35μm UMC CMOS Technology, we observe a phase noise improvement of 6dBc/Hz for divide-by 2 circuit, 9dBc/Hz for divide-by 3 circuit. The dependence of amplitude of injection signal on the phase noise has been examined and the phase noise of an oscillator decreases with increase in the injection levels. For a divide-by 2 circuit for injection levels varied from 10mv to 2v the phase noise variations are shown in Figure (3.4) below.

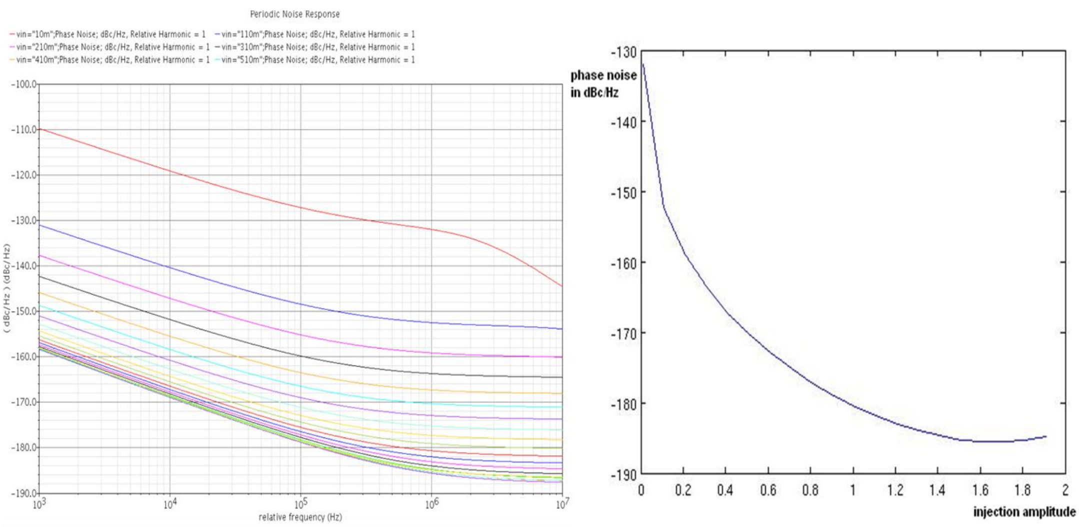


Figure (3.4): Phase noise variation with input injection levels.

Chapter 4

Proposed Multi-harmonic Injection Locked Frequency Divider (ILFD)

4.1 Introduction

Injecting signal into an oscillator is done either by direct injection or tail injection. The drawback with the direct injection being the channel resistance of MOS can degrade the quality factor of the resonator, high power consumption, whereas tail injection uses stack MOSFET's which is not a good option for low voltage application. Hence there being a possibility of injection of multiple signals into oscillator. We propose Multi-harmonic injection locked frequency divider which intuitively has more advantages over single tone injection.

4.2 Proposed Design for Multi-Harmonic ILFD

Practically to achieve this phase noise improvement through multiple injection, in our proposed circuit we have combined the differential cascode topology for the odd division ratios [19] with the differential nature of even division ratio [18] as shown in Figure (4.1). Designed ILFD is composed of double cross-coupled VCO with LC resonator, with 2^{nd} harmonic injected through M3 parallel to the tank and 3^{rd} harmonic differential signal is injected into M4, M5 through Balun (converts the single ended input signal to a differential signal). By preserving the differential nature we confine signals of different harmonics locally by circuit topology and filtering.

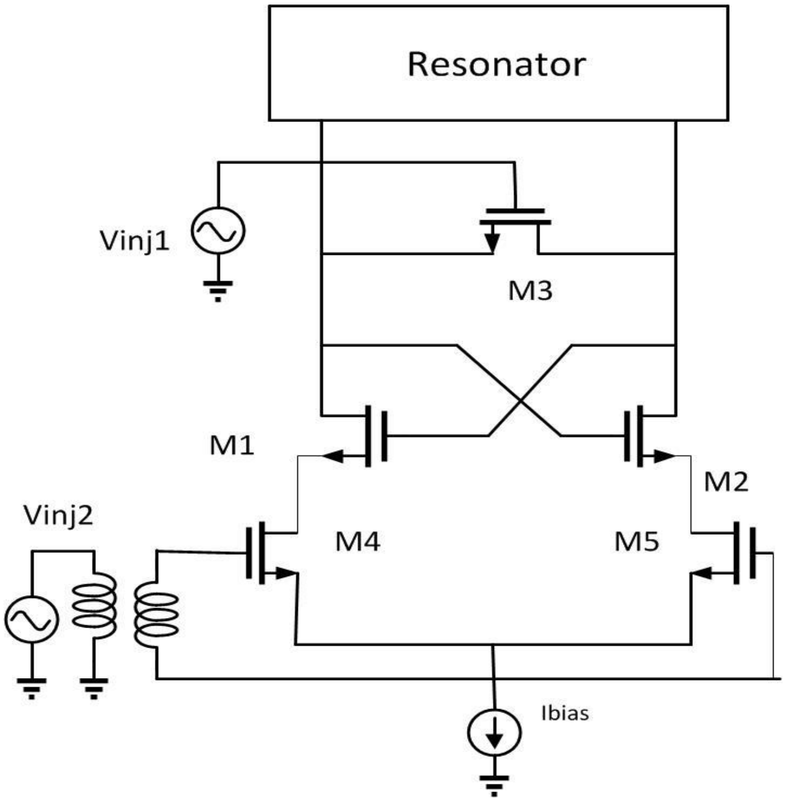


Figure (4.1): Proposed Multi-Harmonic ILFD.

We have designed the proposed circuit using UMC library components. The bias current is generated using the op-amp based feedback current reference circuit as shown in Figure (4.2).

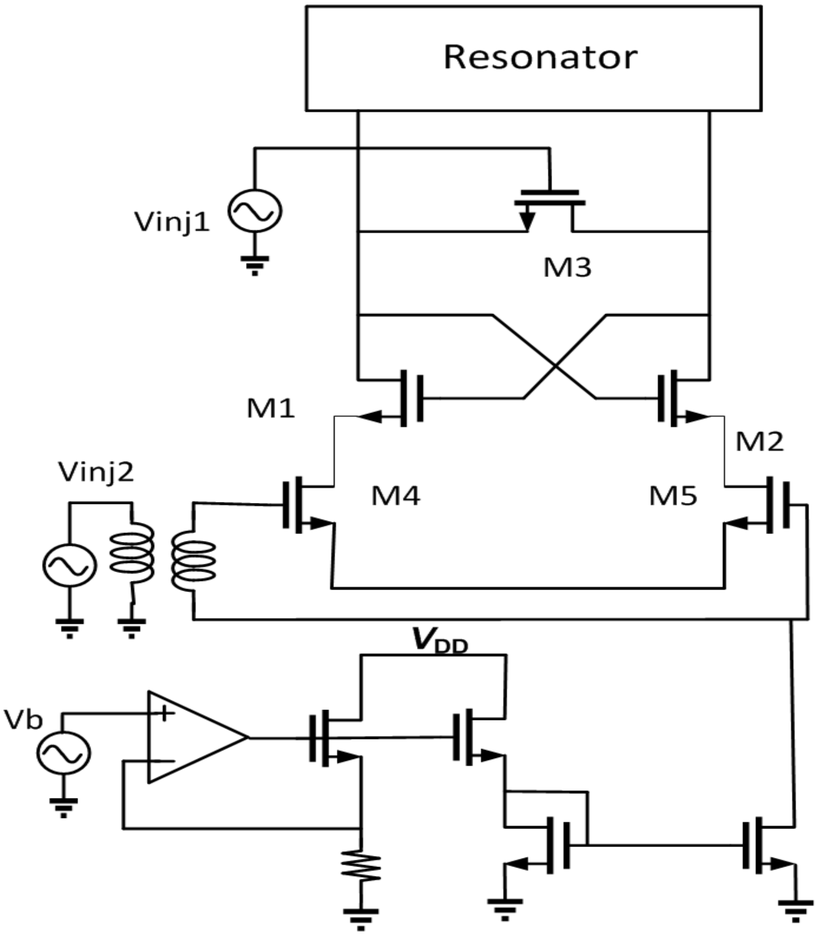


Figure (4.2): Proposed Circuit Using Op-Amp based Current reference.

4.3 Theoretical validation of the proposed design

For small injection levels using the proposed Phase Noise model for phase noise analysis of the proposed ILFD the measurements from the simulated phase noise values were observed to be in close agreement with the theoretical predicted values as shown in Figure (4.3) validating the predicted model for multi-harmonics injection locked oscillator.

Parameters used are $\varphi_{ss} = {}^{\pi}/_{100}$, $\epsilon_1 = \epsilon_2 = 0.007$, $\omega_{free} = 2\pi * 2.7G$, Q=10, N_1=2, N_2=3.

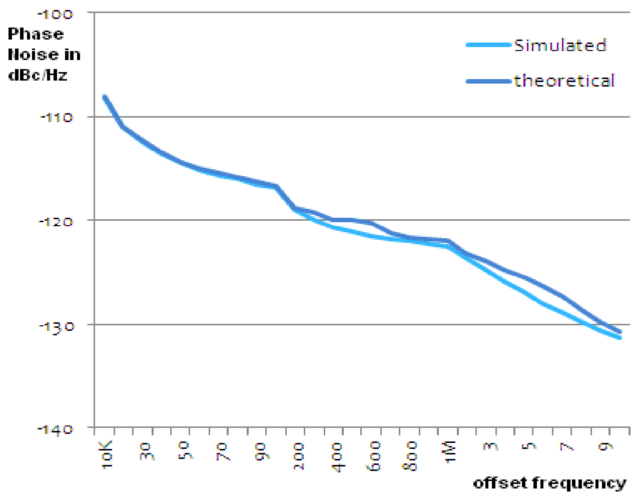


Figure (4.3): theoretical and simulated phase noise values

4.4 Results and Discussion

Proposed ILFD oscillates at free running frequency of 2.72GHz and phase noise at 1.04MHz offset frequency is -108.5dBc/Hz. Under the injection of 2nd and 3rd harmonic signal, phase noise of the locked output at 1MHz frequency offset is -123.24dBc/Hz i.e. phase noise of the locked output is improved by 14.72dBc/Hz at 1MHz offset frequency compared to free-running one as shown in Figure (4.4).

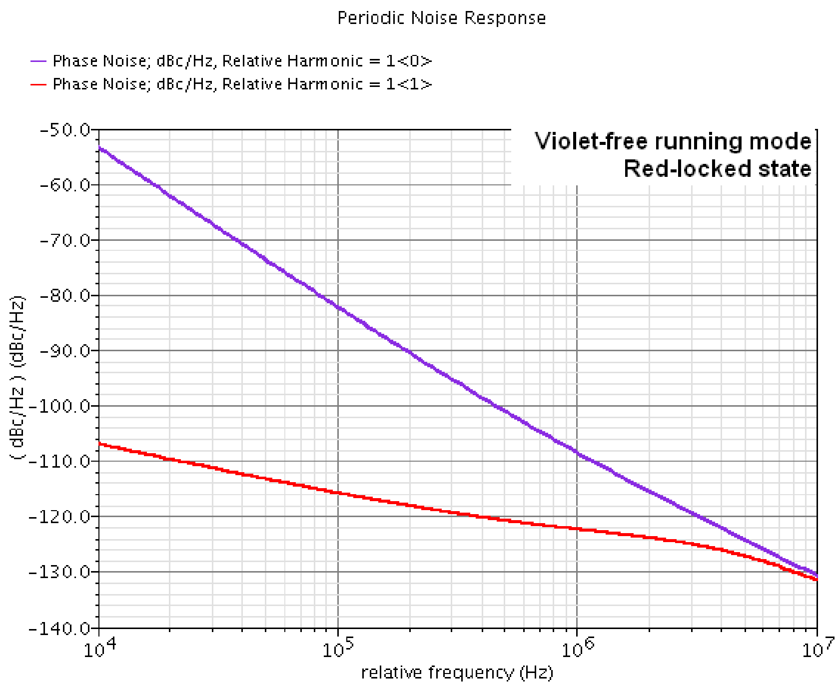


Figure (4.4): Simulated phase noise plots of free running and locked state

Frequency response of proposed ILFD for injection frequencies 5GHz to 5.8GHz is shown in Figure (4.5).We observe for frequencies 5.1GHz to 5.7GHz at fundamental tones of the oscillator the oscillator enters the locking region and at 2nd harmonic it gets perfectly locked as observed at frequency 5.4GHz and produces beats at frequencies outside locking range.

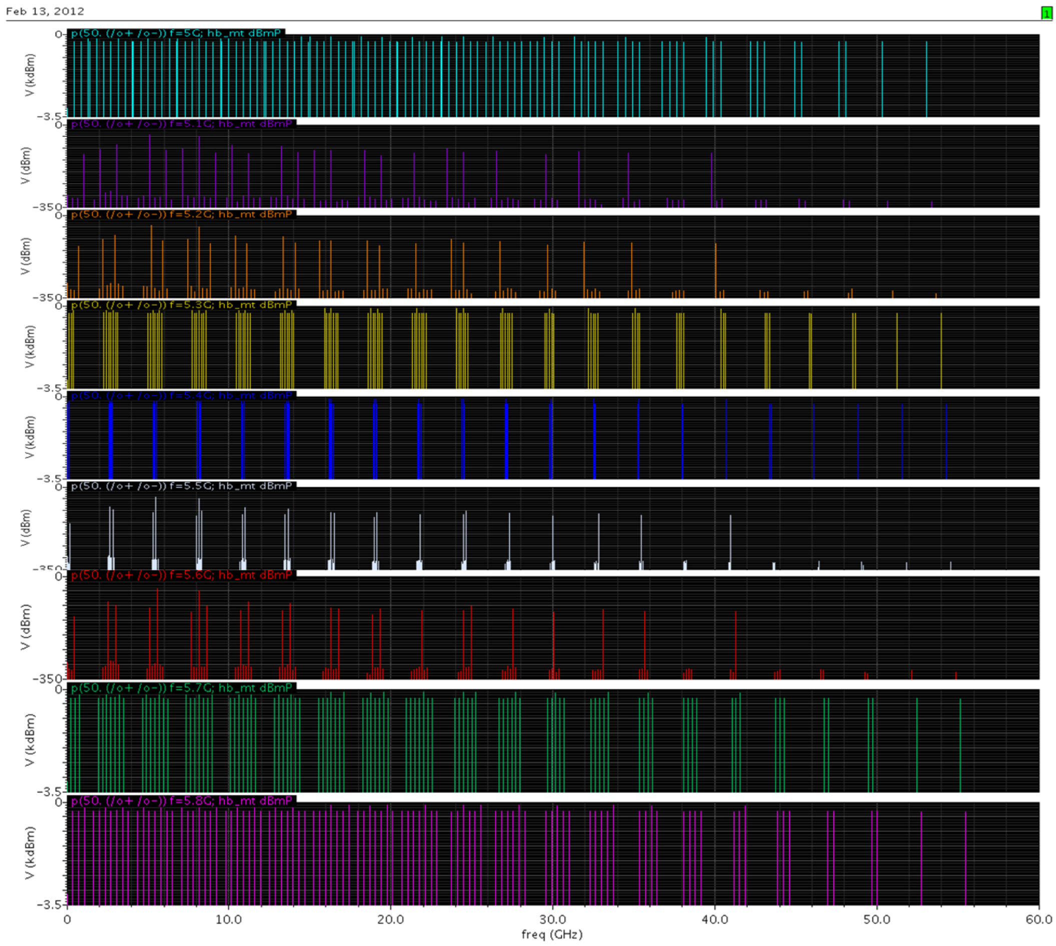


Figure (4.5): Simulated power in dBm at different injection frequencies

It is substantiated from the DFT magnitude plot shown in Figure (4.6). For the injection frequencies within the locking range the DFT magnitude is observed to be maximum, In comparison to the [18] divide-by 2 ILFD the proposed circuit shows a wide locking range. The performance summary is shown in Table I.

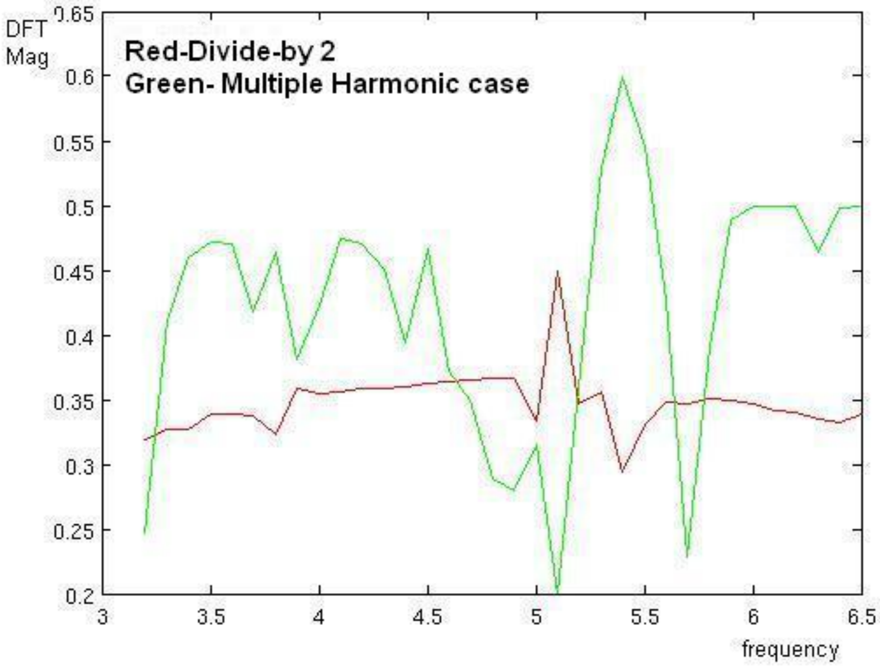


Figure (4.6): DFT Magnitude vs. Injected Frequency plot

Multiple-harmonic injection improves the locking range of VCO, as 5.1- 5.7GHz compare to 5.1-5.3 GHz in divide by 2 networks.

TABLE I: Performance Summary

	Unit	***Performance***
Technology		UMC 0.35μm
Supply Voltage	V	2.4
Power Consumption	mW	6.4
Free-running	GHz	2.72
Locking range @V_{ctrl}=1.6v	GHz	5.1-5.7
Phase Noise: @1MHz offset Frequency	dBc/Hz	Free running: -108.5
		Locked: -123.24

The phase noise dependency on the 2nd harmonic injection source injection levels and on the 3rd harmonic injection source injection levels as shown in Figures (4.7), (4.8).

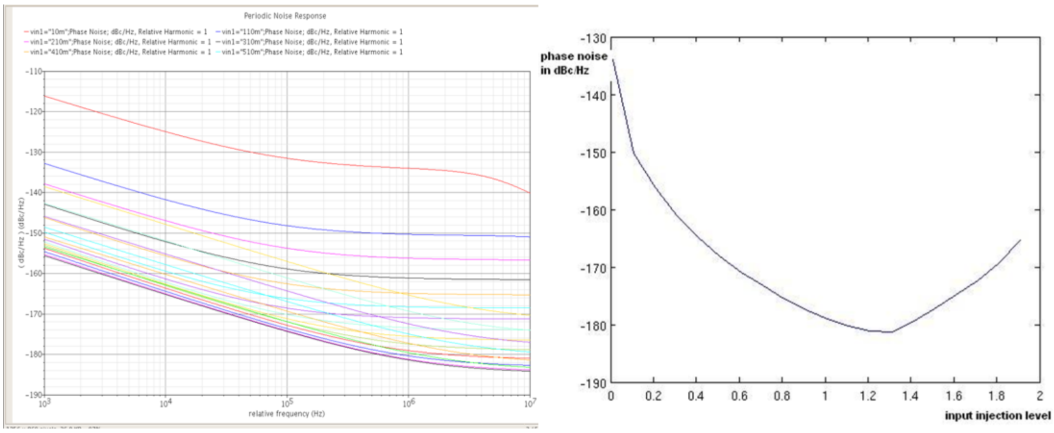


Figure (4.7): variation of phase noise on 2nd harmonic injection source amplitude

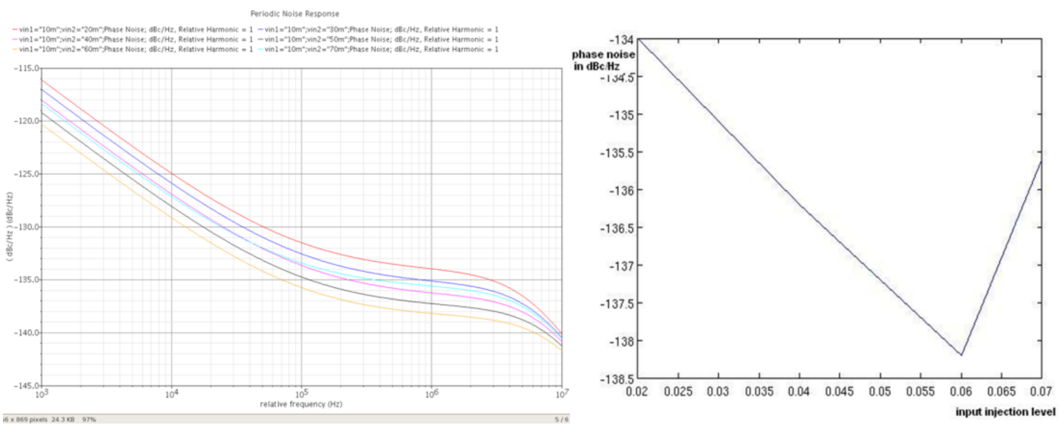


Figure (4.8): variation of phase noise on 3rd harmonic injection source amplitude

The phase noise variation with input injection sources amplitudes both simultaneously varied is shown in Figure (4.9)

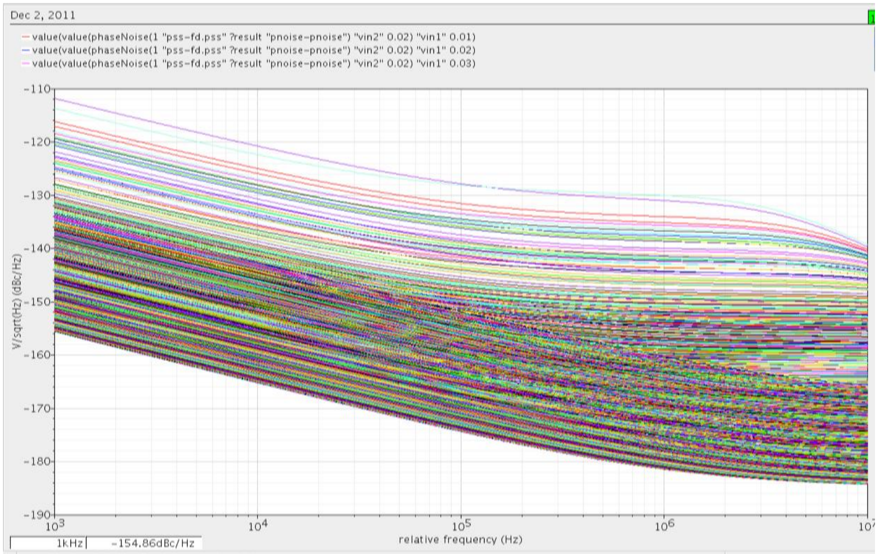


Figure (4.9): phase noise variation with both injection sources amplitude variation simultaneously

Performance comparison of the proposed circuit with recently published papers is shown in table II below:

TABLE: Performance Comparison of Injection Locked Frequency Dividers

	Unit	**[18]**	**[19]**	**[20]**	**[21]**	**[22]**	***Our Work***
Technology		AMS 0.35µm	0.18µm	UMC 0.18µm	TSMC 0.35µm	TSMC 0.35µm	UMC 0.35µm
Supply Voltage	V	2.5	1.8	1.8	2.0	3.0	2.4
Power Consumption	mW	3.43	-	12.51	8	4.32	5.75(6.4)
Division Ratio		2	3	3	3	3	2 and 3
Phase Noise Improvement @1MHz offset frequency	dBc/Hz	6	9	8.2	7.7	9.5	14.2(14.72)

Chapter 5

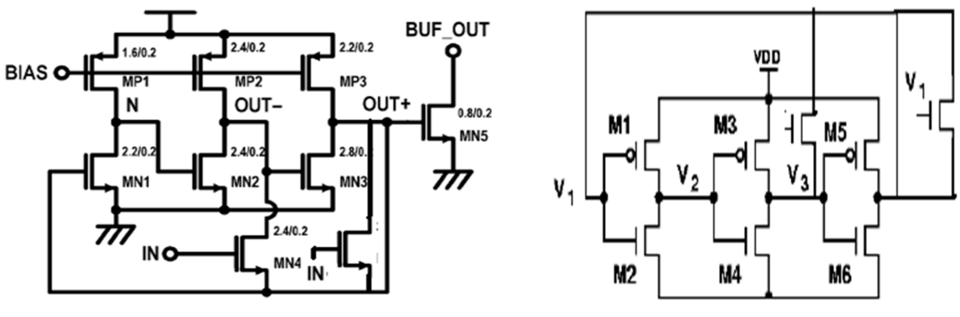
Conclusion and Future work

5.1 Conclusion

Multi-harmonic injection locking technique has been proposed. Using the feedback model of an oscillator general locking expressions under multiple injections is derived, the expressions look similar to that of derived by Alder, Razavi. From the frequency deviation expression the locking range of an oscillator under multi-harmonic injection improves when compared to the single tone injection. Insight into the phase noise model of an oscillator under multi-harmonic injection by extending the Laplace domain derived phase noise model with single source injection, validate the predicted model with the simulated phase noise of the proposed design.

5.2 Future Work

Ongoing work includes exploring the possibility of multi-harmonic injection in other oscillator topologies like Colpitts, Ring oscillator. Designed Multiple injection Ring oscillators based on Quasi-Differential locking divider [24] and inverter based Ring oscillator [25].



(a) (b)

Figure (5.1): Proposed multi-injection (a) QDL (b) Ring oscillator

For the above proposed designs as shown in Figure (5.1) there isn't much substantial improvement in phase noise. Exploring the multi-harmonic with different topology and theoretical formulation of the phase noise for ring oscillator.

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