M.Tech Project Report

MIMO DETECTION

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Declaration

I declare that this written submission represents my ideas in my own words, and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

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Approval Sheet

This thesis entitled MIMO Detection by B.Raja Kullaya Reddy is approved for the degree of Master of Technology from IIT Hyderabad.

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I would like to thank Dr. Asudeb datta, who advised me opt for IIT-Hyderabad at my admission. I also extend my thanks to Mr. Srinivas Goud, my senior who told me the importance of IIT at my admission. Also I would like thank IIT-Hyderabad for its support.

I would like to thank my batchmates whom make time at IIT enjoyable.

Above all I would like to thank my parents and all my family members for constantly supporting me in building my career.

Abstract

The use of digital wireless communication systems has become more and more common during recent years. A multiple-input-multiple-output (MIMO) system techniques can be implemented to enhance the capacity of a wireless link. We have investigated the performances of MIMO detectors : Linear detectors(ZF detector, MMSE detector), SIC(Successive Interference Cancellation) signal detectors, Maximum Likelihood detector, Sphere decoding. In SIC signal detection we use MMSE weight matrix.

The optimal decoder is based on the maximum likelihood principle. But as the number of the antennas in the system and the data rates increase, the maximum likelihood decoder becomes too complex to use. Less complex decoding techniques are zero-forcing and MMSE. at the price of reduced performance at the receiver. We have investigated the performance of the sphere decoding algorithm. As it has shown in the computer simulations, the decoder based on the sphere decoding algorithm has almost the same performance of a maximum likelihood decoder with much lower complexity. Further simulations of the sphere decoding algorithms has shown, the decoder with the sphere decoding algorithm has the same performance as in a ML decoder without increase the decoding complexity.

Contents

Chapter 1

Introduction

1.1 MIMO

In order to meet the demands on higher data rates, better quality and availability from an ever increasing number of wireless subscribers, new techniques in signal processing and coding need to be developed and implemented. To achieve high data rate communication, the system has to overcome problems such as additive noise and channel fading. One way is to make several replicas of the signal available to the receiver with the hope that at least some of them are not severely attenuated. This technique is called diversity [5]. Examples of diversity techniques include time diversity, frequency diversity and space diversity. As the available bandwidth is finite, space diversity schemes seem promising, since they do not involve any loss of bandwidth. The use of multiple antennas at both ends of a wireless link is illustrated in Figure 1.1. Such a multiple-input-multiple-output (MIMO) system promises significant improvements in terms of spectral efficiency, link reliability and also improves the system capacity compared to conventional systems [2].

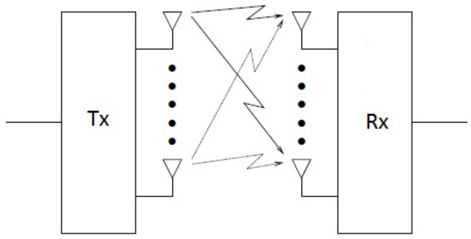

Figure 1.1: A multiple input-multiple output system

1.1.1 Benefits of MIMO

The benefits of MIMO technology that help achieve such significant performance gains are array gain, spatial diversity gain, spatial multiplexing gain and interference reduction [3][4]. These gains are described in brief below.

Array gain

Array gain is the increase in receive SNR that results from a coherent combining effect of the wireless signals at a receiver. The coherent combining may be realized through spatial processing at the receive antenna array and/or spatial pre-processing at the transmit antenna array. Array gain improves resistance to noise, thereby improving the coverage and the range of a wireless network [7].

Spatial diversity gain

As mentioned earlier, the signal level at a receiver in a wireless system fluctuates or fades. Spatial diversity gain mitigates fading and is realized by providing the receiver with multiple (ideally independent) copies of the transmitted signal in space, frequency or time. With an increasing number of independent copies (the number of copies is often referred to as the diversity order), the probability that at least one of the copies is not experiencing a deep fade increases, thereby improving the quality and reliability of reception. A MIMO channel with M_T transmit antennas and M_R receive antennas potentially offers $M_T M_R$ independently fading links, and hence a spatial diversity order of $M_T M_R$ [8].

Spatial multiplexing gain

MIMO systems offer a linear increase in data rate through spatial multiplexing i.e., transmitting multiple, independent data streams within the bandwidth of operation. Under suitable channel conditions, such as rich scattering in the environment, the receiver can separate the data streams. Furthermore, each data stream experiences at least the same channel quality that would be experienced by a single-input single-output system, effectively enhancing the capacity by a multiplicative factor equal to the number of streams. In general, the number of data streams that can be reliably supported by a MIMO channel equals the minimum of the number of transmit antennas and the number of receive antennas, i.e., $\min\{M_T, M_R\}$. The spatial multiplexing gain increases the capacity of a wireless network.

Interference reduction and avoidance

Interference in wireless networks results from multiple users sharing time and frequency resources. Interference may be mitigated in MIMO systems by exploiting the spatial dimension to increase the separation between users. For instance, in the presence of interference, array gain increases the tolerance to noise as well as the interference power, hence improving the signal-to-noise-plus-interference ratio (SINR). Additionally, the spatial dimension may be leveraged for the purposes of interference avoidance, i.e., directing signal energy towards the intended user and minimizing interference to other users. Interference reduction and avoidance improve the coverage and range of a wireless network.

In general, it may not be possible to exploit simultaneously all the benefits described above due to conflicting demands on the spatial degrees of freedom. However, using some combination of the benefits across a wireless network will result in improved capacity, coverage and reliability [6].

1.1.2 System Model

MIMO system model is shown in Figure 1.2. MIMO system typically consists of M_T transmit and M_R receive antennas. By using the same channel, every antenna receives not only the directed components intended for it, but also the indirect components intended for the other antennas. The direct connection from antenna 1 to 1 is specified with h_{11}, direct connection from antenna 2 to 2 is specified with h_{22} etc., while the indirect connection from antenna 1 to 2 is identified as cross component h_{21} , indirect connection from 2 to 1 is identified as h_{12} etc. [5]. From this the channel transmission matrix H with the dimensions $M_R \times M_T$ can be obtained as

$$\begin{bmatrix} h_{11} & h_{12} & h_{1.} & h_{1M_T} \\ h_{21} & h_{22} & h_{2.} & h_{2M_T} \\ h_{.1} & h_{.2} & h_{..} & h_{.M_T} \\ h_{M_R1} & h_{M_R2} & h_{M_R.} & h_{M_RM_T} \end{bmatrix}$$

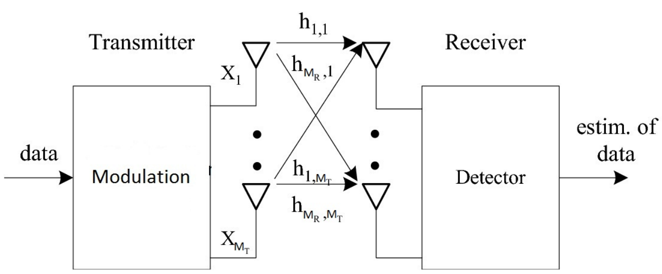

Figure 1.2: MIMO System model

The following formula is the transmission formula results from receive vector y, transmit vector x, and noise n.

$$y = Hx + n \tag{1.1}$$

1.1.3 MIMO Detection

In MIMO detection we have Linear detection methods, Successive Interference Cancellation(SIC) detection methods, Maximum Lkelihood detection, Sphere decoding.

Linear detection methods

In this channel matrix is inverted using a zero-forcing(ZF) or minimum mean squared error(MMSE) criterion. The received vectors are then multiplied by the channel inverse, and then demodulate.

Successive Interference Cancellation

Instead of jointly detecting signals from all the antennas, the strongest signal is detected first and its interference is cancelled from each received signal in the SIC receiver. Then the second strongest signal is detected and cancelled from the remaining signals and so on. The detection method is called successive nulling and interference cancellation.The strongest signal is determined from the weight matrix based on ZF, MMSE.

ML detection

Maximum Likelihood detection solves

$$\hat{x} = \underset{x \in O^{M_T}}{\arg} min\|y - Hx\|^2 \tag{1.2}$$

A straightforward approach to solve above equation is an exhaustive search. The search space increases with the number of antennas increasing.

Sphere decoding

The Sphere decoder reduces the search space. It only searches the lattice points only inside the sphere of certain radius.

Chapter 2

Linear Signal Detection

Linear signal detection method treats all transmitted signals as interferences except for the desired stream from the target transmit antenna. Therefore, interference signals from other transmit antennas are minimized or nullified in the course of detecting the desired signal from the target transmit antenna. To facilitate the detection of desired signals from each antenna, the effect of the channel is inverted by a weight matrix W such that

$$\hat{x} = [\hat{x}_1 \hat{x}_2 \dots \hat{x}_{M_T}]^T = Wy \tag{2.1}$$

that is, detection of each symbol is given by a linear combination of the received signals. The standard linear detection methods include the zero-forcing (ZF) technique and the minimum mean square error (MMSE) technique [6].

2.1 ZF Signal Detection

The zero-forcing (ZF) technique reduces the interference by the following weight matrix:

$$W_{ZF} = \left(H^H H\right)^{-1} H^H \tag{2.2}$$

where $(.)^H$ denotes the Hermitian transpose. We detect the received signal y by multiplying it with the weight matrix W_{ZF} to get the estimated signal $\hat{x}_{ZF}$ [9].

$$\begin{aligned}
\hat{x}_{ZF} &= W_{ZF} y \\
&= x + \left(H^H H\right)^{-1} H^H n \\
&= x + \hat{n}_{ZF}
\end{aligned}$$

where $\hat{n}_{ZF} = W_{ZF}n = \left(H^H H\right)^{-1} H^H n$. Error performance is directly connected to the power of $\hat{n}_{ZF}$(i.e., $\|\hat{n}_{ZF}\|_2^2$).

By demodulating the $\hat{x}_{ZF}$ we will get the estimated transmitted vector $\hat{x}$.

2.2 MMSE Signal Detection

The MMSE technique reduces the interference by the following weight matrix:

$$W_{MMSE} = \left(H^H H + \sigma_n^2 I\right)^{-1} H^H \tag{2.3}$$

where σ_n^2 is the statistical information of the noise. We detect the received signal y by multiplying it with the weight matrix W_{MMSE} to get the estimated signal $\hat{x}_{MMSE}$ [10].

Using the MMSE weight matrix in Equation (2.3), we obtain the following relationship:

$$\begin{aligned} \hat{x}_{MMSE} &= W_{MMSE}y \\ &= \left(H^H H + \sigma_n^2 I\right)^{-1} H^H y \\ &= \hat{x} + \left(H^H H + \sigma_n^2 I\right)^{-1} H^H n \\ &= \hat{x} + \hat{n}_{MMSE} \end{aligned}$$

where $\hat{n}_{MMSE} = W_{MMSE}n = \left(H^H H \sigma_n^2\right)^{-1} H^H n$. Error performance is directly connected to the power of $\hat{n}_{MMSE}$(i.e., $\|\hat{n}_{MMSE}\|_2^2$).

By demodulating the $\hat{x}_{MMSE}$ we will get the estimated transmitted vector $\hat{x}$.

2.3 Results

2.3.1 ZF detection

Figure 2.1 is the graph resulting the BER performance of the ZF detector for a 2×2 MIMO system with QPSK modulation.

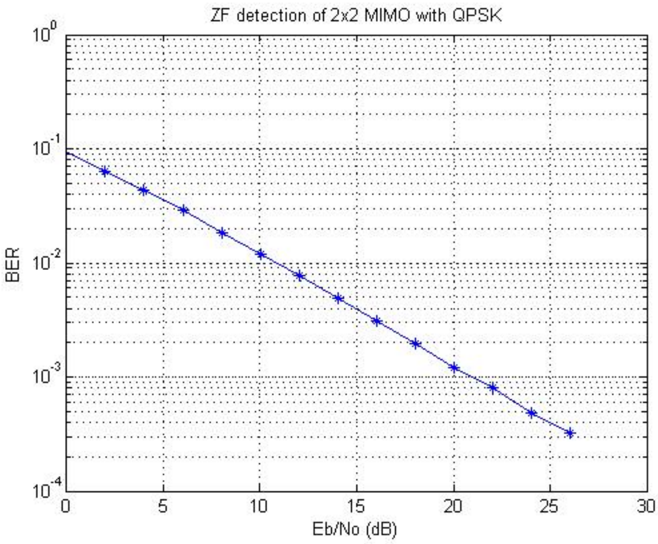

Figure 2.1: BER performance of the ZF detector for a MIMO system with QPSK modulation.

2.3.2 MMSE detection

Figure 2.2 is the graph resulting the BER performance of the MMSE detector for a 2×2 MIMO system with QPSK modulation.

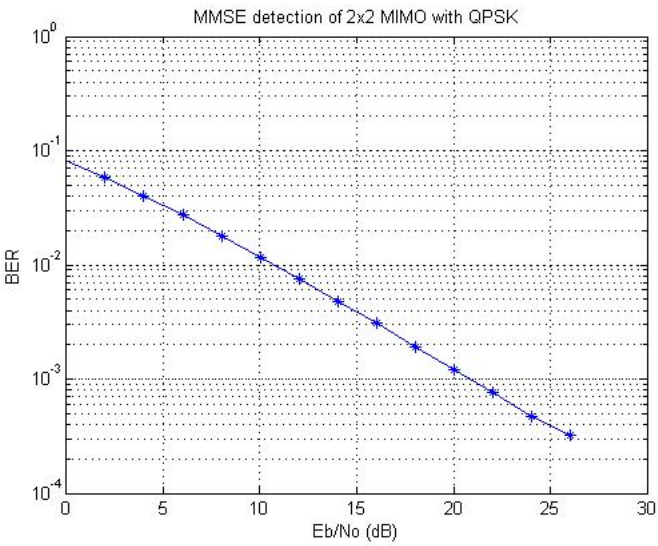

Figure 2.2: BER performance of the MMSE detector for a MIMO system with QPSK modulation.

Figure 2.3 is the comparison of BER performance of the detectors ZF and MMSE. By observing the Figure 2.3 MMSE detector is giving better BER performance than ZF detector.

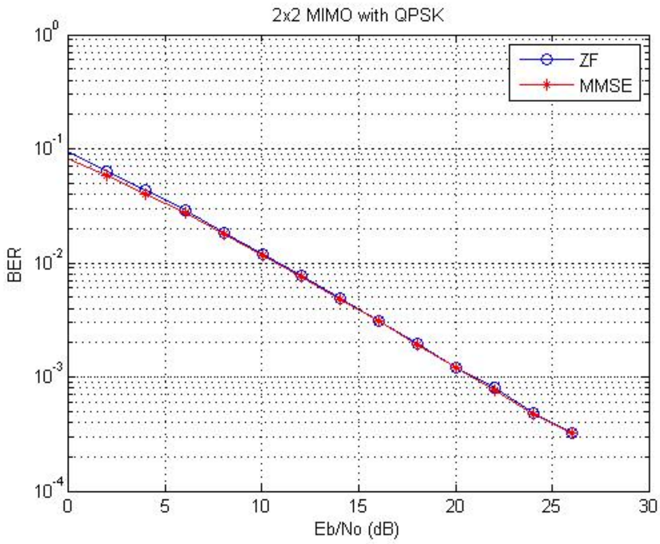

Figure 2.3: Comparison of BER performance of the ZF and MMSE detectors for a MIMO system with QPSK modulation.

Chapter 3

Successive Interference Cancellation

Instead of jointly detecting signals from all the antennas, the strongest signal is detected first and its interference is cancelled from each received signal in the SIC receiver. Then the second strongest signal is detected and its interference cancelled from the remaining signals and so on. The detection method is called successive nulling and interference cancellation (SIC) [1].

SIC is also called as ordered successive interference cancellation (OSIC) method. It is a bank of linear receivers, each of which detects one of the parallel data streams, with the detected signal components successively cancelled from the received signal at each stage. More specifically the detected signal in each stage is subtracted from the received signal so that the remaining signal with the reduced interference can be used in the subsequent stage.

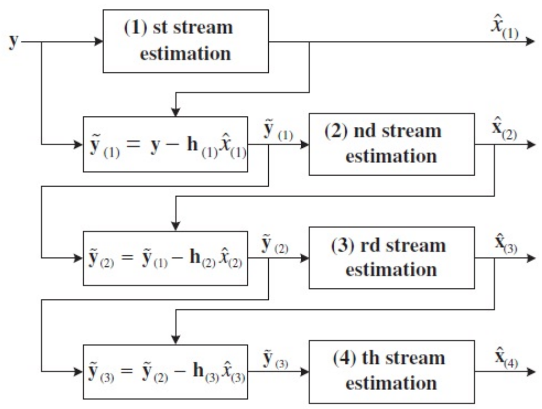

Figure 3.1: SIC signal estimation

The Figure 3.1 illustrates the OSIC signal detection process for four spatial streams. Let $x_{(i)}$ denote the symbol to be detected in the ith order, which may be different from the transmited signal at the ith antenna, since $x_{(i)}$ depends on the order of detection. Let $\hat{x}_{(i)}$denote a sliced value of $x_{(i)}$ [6].

3.1 MMSE based SIC detection

MMSE method is used for symbol estimation. The MMSE weight matrix is given by

$$W_{MMSE} = \left(H^H H + \sigma_n^2 I\right)^{-1} H^H \tag{3.1}$$

Where σ_n^2 is the noise variance.

The (1)st stream is estimated with the (1)st row vector of the MMSE weight matrix. After estimation and slicing to produce $\hat{x}_{(1)}$, the remaining signal in the first stage is formed by subtracting it from the received signal, that is,

$$\begin{aligned} \hat{y}_{(1)} &= y - h_{(1)}\hat{x}_{(1)} \\ &= h_{(1)}\left(x_{(1)} - \hat{x}_{(1)}\right) + h_{(2)}x_{(2)} + \ldots + h_{(M_T)}x_{(M_T)} + n \end{aligned}$$

If $x_{(1)} = \hat{x}_{(1)}$, then the interference is successfully canceled in the course of estimating $x_{(2)}$; however, if $x_{(1)} \neq \hat{x}_{(1)}$, then error propagation is incurred because the MMSE weight that has been designed under the condition of $x_{(1)} = \hat{x}_{(1)}$ is used for estimating $x_{(2)}$.

Due to the error propagation caused by erroneous decision in the previous stages, the order of detection has significant influence on the overall performance of OSIC detection. So we use the post-detection signal-to interference-noise-ratio (SINR) for order.

Signals with a higher post-detection SINR are detected first [12]. Consider the linear MMSE detection with the following post-detection SINR:

$$SINR_i = \frac{E_x |W_{i,MMSE} h_i|^2}{E_x \sum_{l \neq i} |W_{i,MMSE} h_l| + \sigma_n^2 \|W_{i,MMSE}\|^2}, i = 1, 2, \ldots, M_T \tag{3.2}$$

Where E_x is the energy of the transmitted signals, h_i is the ith column vector of the channel matrix H.

Once M_T SINR values are calculated by using the MMSE weight matrix, we choose the corresponding layer with the highest SINR. In the course of choosing the second-detected symbol, the interference due to the first detected symbol is cancelled from the received signals. Suppose that (1)=l (i.e., the lth symbol has been cancelled first). Then, the channel matrix is modified by deleting the channel gain vector corresponding to the lth symbol as follows:

$$H^{(1)} = [h_1 h_2 \ldots h_{l-1} h_{l+1} \ldots h_N] \tag{3.3}$$

Using the modified channel matrix $H^{(1)}$ in place of H the MMSE weight matrix is recalculated. Now (M_T-1) SINR values, $[SINR_i]_{i=1,i\neq l}^{M_T}$, are calculated to choose the symbol with the highest SINR. The same process is repeated with the remaining after cancelling the next symbol with the highest SINR.

3.2 ZF based SIC dtection

In this ZF weight matrix is used for symbol estimation, that is given by

$$W_{ZF} = \left(H^H H\right)^{-1} H^H \tag{3.4}$$

The (1)st stream is estimated with the (1)st row vector of the ZF weight matrix. After estimation and slicing to produce $\hat{x}_{(1)}$, the remaining signal in the first stage is formed by subtracting it from the received signal, that is,

$$\begin{aligned} \hat{y}_{(1)} &= y - h_{(1)}\hat{x}_{(1)} \\ &= h_{(1)}\left(x_{(1)} - \hat{x}_{(1)}\right) + h_{(2)}x_{(2)} + \ldots + h_{(M_T)}x_{(M_T)} + n \end{aligned}$$

If $x_{(1)} = \hat{x}_{(1)}$, then the interference is successfully canceled in the course of estimating $x_{(2)}$; however, if $x_{(1)} \neq \hat{x}_{(1)}$, then error propagation is incurred because the ZF weight that has been designed under the condition of $x_{(1)} = \hat{x}_{(1)}$ is used for estimating $x_{(2)}$.

Due to the error propagation caused by erroneous decision in the previous stages, the order of detection has significant influence on the overall performance of OSIC detection. So we use the post-detection signal-to interference-noise-ratio (SINR) for order.

Signals with a higher post-detection SINR are detected first. Once M_T SINR values are calculated by using the ZF weight matrix, we choose the corresponding layer with the highest SINR. In the course of choosing the second-detected symbol, the interference due to the first detected symbol is cancelled from the received signals. Suppose that (1)=l (i.e., the lth symbol has been cancelled first). Then, the channel matrix is modified by deleting the channel gain vector corresponding to the lth symbol as follows:

$$H^{(1)} = [h_1 h_2 \dots h_{l-1} h_{l+1} \dots h_N] \tag{3.5}$$

Using the modified channel matrix $H^{(1)}$ in place of H the ZF weight matrix is recalculated. Now (M_T-1) SINR values, $[SINR_i]_{i=1,i\neq l}^{M_T}$, are calculated to choose the symbol with the highest SINR. The same process is repeated with the remaining after cancelling the next symbol with the highest SINR [11].

3.3 Results

3.3.1 ZF-SIC

Figure 3.2 is the BER performance of the SIC detector for a 2×2 MIMO system with QPSK modulation.

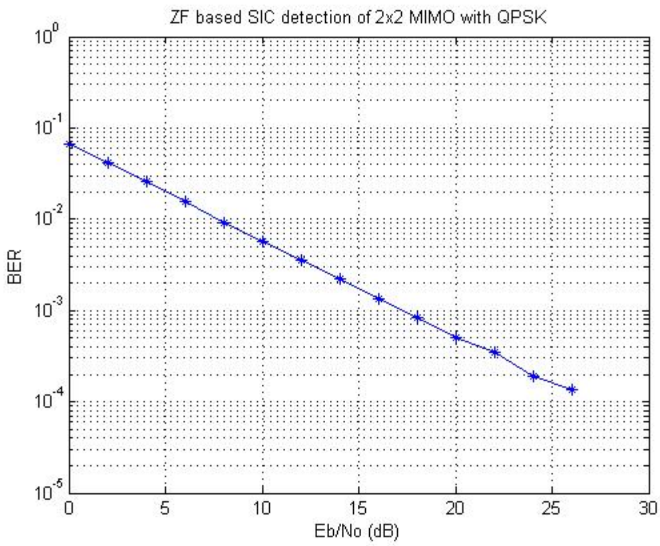

Figure 3.2: BER performance of the ZF based SIC detector for a MIMO system with QPSK modulation.

3.3.2 MMSE-SIC

Figure 3.3 is the BER performance of the SIC detector for a 2×2 MIMO system with QPSK modulation.

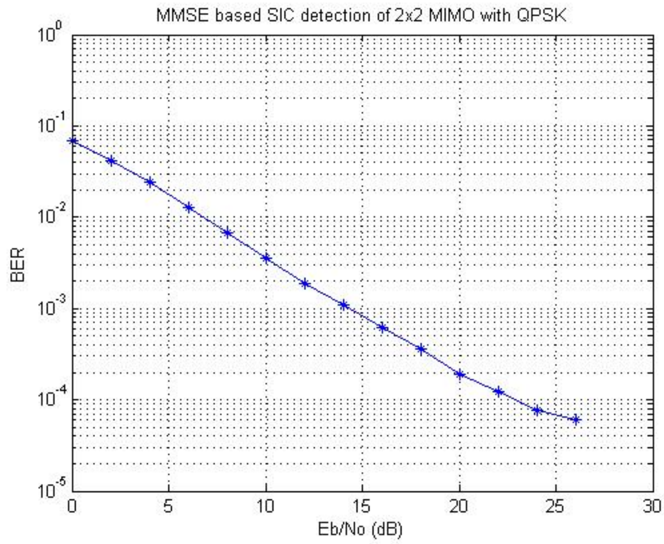

Figure 3.3: BER performance of the MMSE based SIC detector for a MIMO system with QPSK modulation.

By observing Figure 3.4, we can say that SIC is giving better BER performance than the Linear signal detection methods. In SIC detection, MMSE based SIC is giving good BER performance.

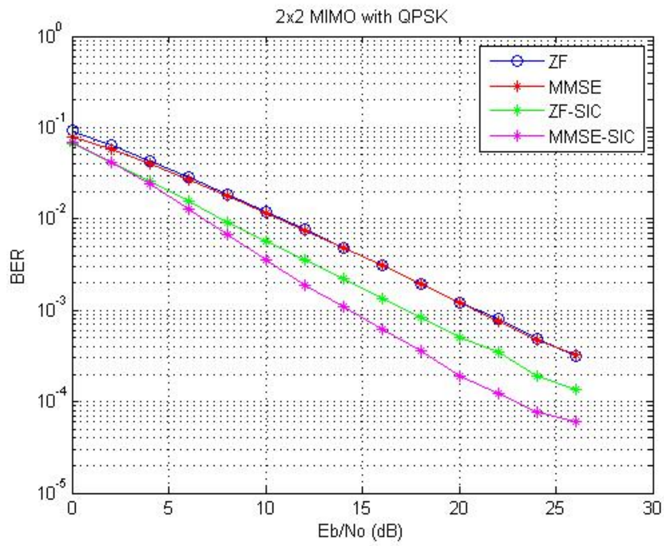

Figure 3.4: Comparison of BER performance of SIC detectors and Linear detectors for a MIMO system with QPSK modulation.

Chapter 4

ML Detection and Sphere Decoding

4.1 ML Detection

The ML detection method minimizes the average error Probability and it is the optimal method for finding the closest lattice point. The ML detector calculates the Euclidean distances (EDs) between the received signal vector and lattice points Hx, and returns the vector with the smallest distance, i.e., it minimizes

$$\hat{x}_{ML} = \underset{x \in X^{M_T}}{\arg} min\|y - Hx\|^2 \tag{4.1}$$

Out of all M_T^Q (Q is the order of modulation type) Euclidean distances between the received vector and the candidate symbol vectors, the symbol with the minimum Euclidean distance is the estimated transmitted vector $\hat{x}_{ML}$.Thus, the ML detector chooses the message $\hat{x}_{ML}$ which yields the smallest distance between the received vector y, and lattice point $H\hat{x}_{ML}$ [13].

As the order of MIMO system and order of modulation increases, the search space of ML detector increases, i.e. it has to calculate many Euclidean distances of search space. So, it becomes difficult to detect as the search space increases and it becomes complex. We can reduce this complxity with sphere decoding method, which reduces the search space [17].

4.2 Sphere decoding

Maximum-likelihood decoding requires an exhaustive search over all the possible codewords, and so the computational complexity of the decoding scheme is exponential in the length of the codeword. Sphere decoding algorithm [14] is proposed to lower the computational complexity. The principle of the sphere decoding algorithm is to search the closest lattice point to the received signal within a sphere radius, where each codeword is represented by a lattice point in a lattice field [15][16].

The ML detection problem of Equation(4.1), i.e.

$$\hat{x}_{ML} = \underset{x \in X^{M_T}}{\arg} min \|y - Hx\|^2 \tag{4.2}$$

By performing QR decomposition to H, we will get $H = QR$. By putting $H = QR$ into Equation(4.2),

$$\hat{x}_{ML} = \underset{x \in X^{M_T}}{\arg} min \|y - QRx\|^2 \tag{4.3}$$

Where Q is an Orthogonal matrix and R is an Upper triangular matrix. So, Equation(4.3) becomes

$$\hat{x}_{ML} = \underset{x \in X^{M_T}}{\arg} min \|y' - Rx\|^2 \tag{4.4}$$

Where $y' = Q^H y$

The idea behind the sphere decoder is to solve Equation(4.4) by enumerating all points which belong to a hypersphere of radius r around the received point y'. That is, all $x \in X^{M_T}$ which satisfy a criterion:

$$\|y' - Rx\|^2 \leq r^2 \tag{4.5}$$

Choosing suitable r is the main thing in Sphere decoding method. If we choose very small r no lattice point(candidate symbol vector) may be present inside the sphere. If we choose very large r many lattice points lie inside the hyper sphere, i.e. search space increases then complexity increases. So, optimal r should be choosed. We can take $r = y - H^+ y$, where $H^+ = \left(H^H H\right)^{-1} H^H$

If we build a tree such that the leaves at the bottom correspond to all possible vector symbols and the possible values of the entry x_{M_T} define its top level, we can uniquely describe each node at level i ($i = 1, 2, \ldots, M_T$) by the partial vector symbols $x^{(i)} = [x_i \; x_{i+1} \; \ldots \; x_{M_T}]^T$, as illustrated in Figure 4.1 for a 4×4 MIMO system with BPSK modulation.

We start at level $i = M_T$ and set $T_{M_T+1}\left(x^{(M_T+1)}\right) = 0$. The partial (squared) Euclidean distances (PEDs) $T_i\left(x^{(i)}\right)$ are then given by

$$T_i\left(x^{(i)}\right) = T_{i+1}\left(x^{(i+1)}\right) + |e_i\left(x^{(i)}\right)|^2 \tag{4.6}$$

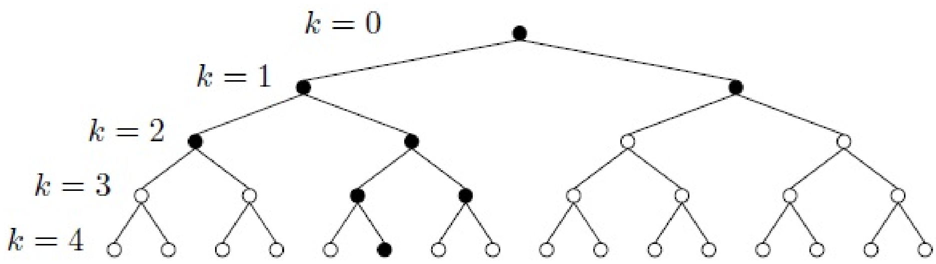

Figure 4.1: Tree search of the Sphere decoder for a 4×4 MIMO system with BPSK modulation.

with $i = M_T, M_{T-1}, \ldots, 1$, where the distance increments $|e_i\left(x^{(i)}\right)|^2$ can be obtained as

$$|e_i\left(x^{(i)}\right)|^2 = |y_i' - \sum_{j=i}^{M_T} R_{ij}x_j|^2 \tag{4.7}$$

We can make the influence of x_i more explicit by writing

$$|e_i\left(x^{(i)}\right)|^2 = |b_{i+1}\left(x^{(i+1)}\right) - R_{ii}x_i|^2 \quad \text{with} \tag{4.8}$$

$$b_{i+1}\left(x^{(i+1)}\right) = y_i' - \sum_{j=i+1}^{M_T} R_{ij}x_j. \tag{4.9}$$

Since the distance increments $|e_i\left(x^{(i)}\right)|^2$ are nonnegative, it follows immediately that whenever the PED of a node violates the (partial) SC(Sphere Constraint) given by

$$T_i\left(x^{(i)}\right) < r^2 \tag{4.10}$$

then the PEDs of all of its children will also violate the SC. Consequently, the tree can be pruned above this node. This approach effectively reduces the number of transmit vector symbols (i.e., leaves of the tree) to be checked.When the tree traversal is finished, the leaf with the lowest $T_1\left(x\right)$ corresponds to the ML solution [18].

4.3 Results

Figure 4.2 is the BER performance of the ML detection for a 2×2 MIMO system with QPSK modulation. ML detection is giving good BER performance.Figure 4.3 is the BER performance of the Sphere decoder for a

2×2 MIMO system with QPSK modulation. BER curve of Sphere decoder is matching with that of ML detection.

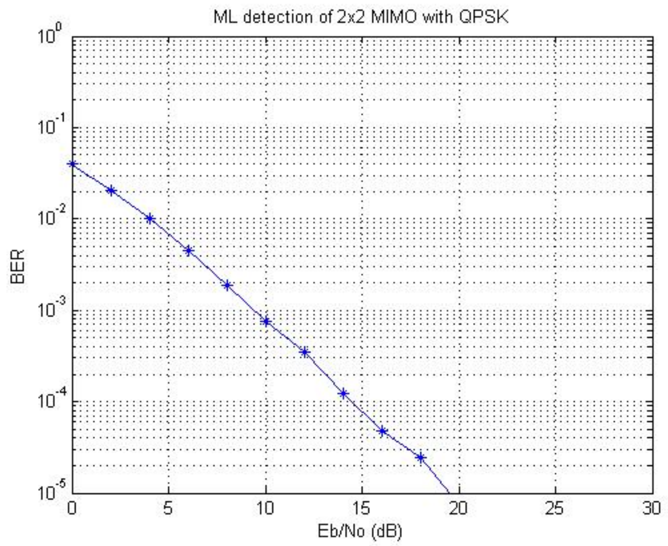

Figure 4.2: BER performance of the ML detector for a MIMO system with QPSK modulation.

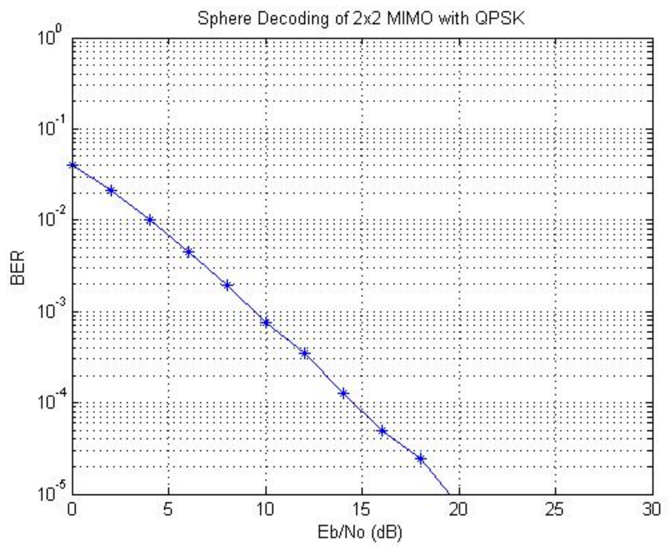

Figure 4.3: BER performance of the Sphere decoder for a MIMO system with QPSK modulation.

By observing the Figure 4.4 we can say that ML detection and Sphere decoder are having the best BER performance.

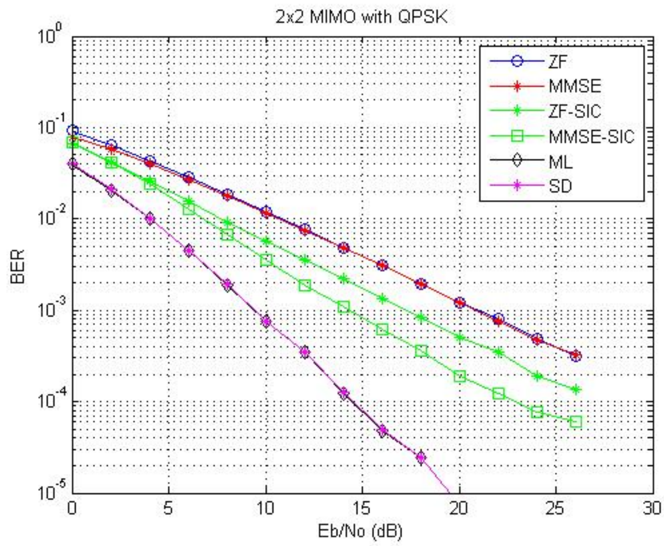

Figure 4.4: BER performances of ZF, MMSE, SIC, ML and Sphere detectors for a MIMO system with QPSK modulation.

Chapter 5

Complexity Comparison of detection schemes

Before determining the complexity of the MIMO SM algorithms, a number of general rules will be introduced, namely, the complexity of a matrix multiplication, the conversion from complex complexity figures to real complexity figures, the complexity of a slicer, and the complexity of finding a minimum value from a set of values.

The complexity of a matrix product is determined as follows. Suppose two matrices A and B (real or complex) with dimensions $C \times E$ and $D \times E$ are multiplied, then the $(i, j)^{th}$ element of the resulting matrix is given by

$$a^i b_j = \sum_{k=1}^{D} a_{ik} b_{kj} \tag{5.1}$$

where a^i represents the $i^t h$ row of matrix A , b_j denotes the j^{th} column of B and a_{ik} and b_{kj} stand for the k^{th} element of this row and column, respectively. Thus, in order to obtain one element of the resulting matrix, $D-1$ additions and D multiplications need to be performed. The resulting matrix is $C \times E$ dimensional and, therefore, a total of $C\,(D-1)\,E$ additions and CDE multiplications are needed to multiply the two A and B.

To write complex additions and complex multiplications in terms of real additions and real multiplications, it is easily verified that one complex addition consists of two real additions; the real and the imaginary part of the two complex numbers are added. Furthermore, a complex multiplication can be rewritten in the following two ways:

$$(a + jb)\,(c + jd) = (ac - bd) + j\,(bc + ad) \tag{5.2}$$

$$(a + jb)(c + jd) = (ac - bd) + j((a + b)(c + d) - ac - bd) \qquad (5.3)$$

The first option consists of 4 real multiplications, ac , bd , bc and ad , and 2 real additions, $ac - bd$ and $bc - ad$. A subtraction is counted as an addition and the addition before the j does not count because the real and imaginary parts are stored separately. The second option has only three real multiplications $(ac, bd, (a + b)(c + d))$, plus five real additions. Compared with the first case, the total operations count is higher by two, but in a number of hardware implementations, a multiplication is a more complex operation. However, the first option will be used.

The complexity of a slicer is minimal in terms of additions and/or multiplications. For an M-PSK constellation scheme, the phase range $[-\pi, \pi]$ is divided in M equal parts. In such a regular structure, a recursive search is done in which half of the (remaining) range the phase of the estimated symbol best fits. This results in a complexity equivalent to $log_2(M)$ comparisons. For an M-QAM constellation diagram, the real and imaginary parts are split. Each of these parts is regularly divided in $\sqrt{M}$ slicing ranges. Also in this case, a recursive search is achieved in which half of the (remaining) range the real or imaginary part of the estimated symbol best fits, and the complexity is equal to $log_2\left(\sqrt{M}\right)$ comparisons for the real and for the imaginary part, or $2log_2\left(\sqrt{M}\right)$ comparisons in total. It is reasonable to assume that a comparison is as complex as a real addition and, therefore, the slicing of the N_t-dimensional vector x_{ext} requires at most $N_t log_2(M)$ R-adds.

In order to find the minimum of N numbers in hardware, the easiest thing to do is start with the first two elements, subtract the second number from the first, and compare the result with zero. If the result is larger than zero, the second number is the smallest; otherwise the first number is the smallest, etc. Obviously, finding the minimum between two real numbers has the complexity of one real addition. As a result, determining the minimum of N values has a complexity of $N - 1$ real additions [19].

5.1 Complexity of Detection schemes

Consider a $N_t \times N_r$ MIMO system of N_t transmitting antennas and N_r receiving antennas. Let M-PSK modulation is employed at the transmitting side.

Complexity of ZF

The complexity of the ZF algorithm per transmitted vector x in terms of real operations equals

$$C_{ZF}\left(flops\right) = 7N_t^3 + 7N_t^2 N_r - 2N_t + 4N_t N_r + \frac{1}{2} N_r log_2\left(M\right) \tag{5.4}$$

Complexity of MMSE

The complexity of the MMSE algorithm per transmitted vector x in terms of real operations equals

$$C_{MMSE}\left(flops\right) = 7N_t^3 + 7N_t^2 N_r - N_t + 4N_t N_r + \frac{1}{2} N_r log_2\left(M\right) \tag{5.5}$$

Complexity of ZF-SIC

The complexity of the ZF-SIC algorithm per transmitted vector x in terms of real operations equals

$$C_{ZF-SIC}\left(flops\right) = N_t^4 + \frac{5}{3}N_t^3 + \frac{8}{3}N_t^3 N_r + \frac{3}{4}N_t^2 + \frac{7}{2}N_t^2 N_r + \frac{55}{6}N_t N_r - \frac{17}{12}N_t + \frac{1}{2}N_t log_2\left(M\right) \tag{5.6}$$

Complexity of MMSE-SIC

The complexity of the MMSE-SIC algorithm per transmitted vector x in terms of real operations equals

$$C_{MMSE-SIC}\left(flops\right) = N_t^4 + \frac{5}{3}N_t^3 + \frac{7}{3}N_t^3 N_r + N_t^2 + \frac{7}{2}N_t^2 N_r + \frac{7}{6}N_t N_r - \frac{1}{6}N_t + \frac{1}{2}N_t log_2\left(M\right) \tag{5.7}$$

Complexity of ML

The complexity of the ML detection[20] per transmitted vector x in terms of real operations equals

$$C_{ML} = 4N_t^2\left(M+1\right) + 4N_t N_r T\left(M+1\right) + 6N_t\left(2M+1\right) \tag{5.8}$$

Complexity of Sphere Decoder

The complexity of the Sphere decoding[16][21] per transmitted vector x in terms of real operations equals

$$C_{SD} = \frac{1}{6}\left(2N_t^3 + 3N_t^2 - 5N_t\right) + \frac{1}{2}\left(N_t^2 + 12N_t - 7\right)\left(\left(2\left\lfloor\sqrt{r^2 t}\right\rfloor + 1\right)\binom{\lfloor 4r^2 t\rfloor + N_t - 1}{\lfloor 4r^2 t\rfloor} + 1\right) \tag{5.9}$$

Where [.] denotes rounding to the nearest smaller value.

Chapter 6

Conclusion

We have discussed and simulated the Linear detection methods, SIC detection methods, ML detection and Sphere decoding for MIMO systems. In Linear detection schemes we just multiplied the weight matrix(ZF and MMSE) with the received signal vector and then demodulated that signal to get the estimated transmitted vector. Linear signal detection scheme is a simple detection scheme. It is easy to implement this scheme. BER performance is poor in Linear detection methods.

In SIC detection we detect the strogest signal first and it's interference is cancelled from each received signal, then the second strongest signal is detected and it's interference cancelled from the remaining signals and so on. SIC is also a simple detection scheme but it is slight complex to implement than Linear detection methods.

ML detection is the optimal detection scheme of MIMO systems. It's BER performance is better than other detection scheme. But it is complex to implement since it is doing an exaustive search over the entire lattice space. Sphere decoding scheme is the ML solution with reduced search space. It searches the closest lattice point to the received signal within a sphere radius. Sphere decoder reduces the search space by tree pruning. Since it need not search the whole lattice space it is less complex than ML decoder.

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