

Probing CP violation in the neutrino sector with magic baseline experiments

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Abstract

We investigate the effect of CP violation in the leptonic sector. Due to the tiny neutrino masses its value is predicted to be very small and it is far beyond the experimental reach of the current experiments. Recently, the magic baseline experiment from CERN to INO (Indian Neutrino Observatory) with $L = 7152$ km has been proposed to get a sensitive limit on $\sin\theta_{13}$. We show that due to such magic baseline neutrino beam it is possible to observe CP violation in the neutrino sector upto several percent for the beam energy between (1-10) GeV.

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It is now well established by the recent neutrino oscillation experiments [1–8] that neutrinos do have a tiny but finite nonzero mass. Because of the non-zero mass, the flavor eigenstates of the neutrinos are no longer be the corresponding mass eigenstates and these two are related by some unitary transformation. Thus, due to the mixing between the flavor and mass eigenstates of neutrinos, it is expected that there could also be CP violation in the neutrino sector analogous to that of the quark sector. CP violation so far has been observed only in the quark sector of the standard model i.e., in the K and B meson systems, the origin of which is basically attributed to the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [9, 10]. Its discovery in the leptonic sector should shed additional light on the understanding of the origin of CP violation in nature. The study of CP violation in the lepton sector though less examined than that of the quark sector, it is indispensable, since neutrinos are allowed to be massive and the corresponding mixing matrix is complex. It seems necessary for us to examine whether there is a chance to observe CP violation in the leptonic sector in the long baseline experiments. In this paper we explore such a possibility.

Let us briefly review the CP violation phenomenon in neutrino oscillation experiments to clarify our notation. Within the framework of three lepton families, the three flavor eigenstates of neutrinos (ν_e, ν_μ, ν_τ) are related to the corresponding mass eigenstates (ν_1, ν_2, ν_3) by the unitary transformation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (1)$$

where U is the 3×3 unitary matrix known as PMNS matrix [11, 12], which contains three mixing angles and three CP violating phases (one Dirac type and two Majorana type). The unitary matrix U can be represented in the standard parametrization [13] as

$$\begin{aligned} U &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \quad (2) \end{aligned}$$

with $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and θ_{12} , θ_{23} and θ_{13} the three neutrino mixing angles, δ is the Dirac type CP violating phase and α and β are Majorana phases. The presence of the leptonic mixing, analogous to that of quark mixing, has opened up the possibility that CP violation could also be there in the lepton sector as it exists in the quark sector. Although the absolute masses of the neutrinos are not yet known, the recent experiments like SNO, KamLand, K2K and MINOS [1–8, 14–17] provide information on the two mass square differences Δm_{21}^2 and Δm_{31}^2 and on the two mixing angles θ_{12} and θ_{23} . The third mixing angle θ_{13} is not yet determined but from the null result of CHOOZ [18] experiment, its value is expected to be quite small. The current best fit values with 1σ errors for three flavour neutrino oscillation parameters from global fit [19] are given as

$$\begin{aligned} \Delta m_{21}^2 &= (7.65_{-0.20}^{+0.23}) \times 10^{-5} \text{ eV}^2, & \sin^2 \theta_{12} &= 0.304_{-0.016}^{+0.022}, \\ |\Delta m_{31}^2| &= (2.40_{-0.11}^{+0.12}) \times 10^{-3} \text{ eV}^2, & \sin^2 \theta_{23} &= 0.50_{-0.06}^{+0.07}, \\ \sin^2 \theta_{13} &= 0.01_{-0.011}^{+0.016}, & (\sin^2 \theta_{13} < 0.04 \text{ (} 2\sigma \text{ bound)}), & \delta \in [0, 2\pi], \end{aligned} \quad (3)$$

while the sign of Δm_{31}^2 is unconstrained. The Majorana phases α and β are currently completely unconstrained.

Let us take a closer look at the discovery reach for CP violation. For this purpose we will first consider the neutrino oscillation phenomenon in vacuum. From eq. (1), one can write the evolution equation for the flavour eigenstates as

$$\begin{aligned} i \frac{d}{dx} \nu_\alpha &= - (U \text{diag}(p_1, p_2, p_3) U^\dagger) \nu_\alpha \\ &\simeq \left(-p_1 + \frac{1}{2E} U \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) U^\dagger \right) \nu_\alpha \\ &\simeq \frac{1}{2E} (U \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) U^\dagger) \nu_\alpha, \end{aligned} \quad (4)$$

where p_i 's are the momenta of the i 'th-type mass eigenstates, E is the energy and $\Delta m_{ij}^2 = (m_i^2 - m_j^2)$ denote the neutrino mass square differences. A term proportional to the unit matrix like p_1 in eq. (4) has been dropped because it is irrelevant to the transition probability. The solution of (4) is given as

$$\nu_\alpha(x) = U \exp \left(-i \frac{x}{2E} \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) \right) U^\dagger \nu_\alpha(0). \quad (5)$$

Thus, one can obtain the conversion probability for $\nu_\alpha \rightarrow \nu_\beta$ process at a distance L as

$$\begin{aligned}
P(\nu_\alpha \rightarrow \nu_\beta; L) &= \left| \sum_{i,j} U_{\beta i} \left[\exp \left(-i \frac{L}{2E} \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) \right) \right]_{ij} U_{\alpha j}^* \right|^2 \\
&= \sum_{i,j} U_{\beta i} U_{\beta j}^* U_{\alpha i}^* U_{\alpha j} \exp \left(-i \Delta m_{ij}^2 (L/2E) \right). \tag{6}
\end{aligned}$$

The simplest measure of CP violation, which is equivalent to T violation if CPT is conserved, would be the difference of oscillation probabilities between neutrinos and antineutrinos, i.e., $P(\nu_\alpha \rightarrow \nu_\beta)$ and $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$, which is represented as

$$\Delta P \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta). \tag{7}$$

The transition probability for the corresponding CP conjugate process $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ can be obtained by replacing the PMNS matrix elements $U_{\alpha i}$ by $U_{\alpha i}^*$. Thus, one can obtain the CP or T violation parameter for the neutrino oscillation case as

$$\begin{aligned}
\Delta P &\equiv P(\nu_\alpha \rightarrow \nu_\beta; L) - P(\nu_\beta \rightarrow \nu_\alpha; L) \\
&= -4 \text{Im}(U_{\beta 1} U_{\beta 2}^* U_{\alpha 1}^* U_{\alpha 2}) (\sin 2\Delta_{21} + \sin 2\Delta_{32} L + \sin 2\Delta_{13} L) \\
&= 4Jf \tag{8}
\end{aligned}$$

where $\Delta_{ij} = \Delta m_{ij}^2 L / 4E$, and L is the distance between the neutrino source and the detector. J , the leptonic analog of Jarlskog Invariant and f are defined by

$$\begin{aligned}
J &= -\text{Im}(U_{\beta 1} U_{\beta 2}^* U_{\alpha 1}^* U_{\alpha 2}) \\
f &= \sin 2\Delta_{21} + \sin 2\Delta_{32} + \sin 2\Delta_{13} \\
&= 4 \sin \Delta_{21} \sin \Delta_{32} \sin \Delta_{13} \tag{9}
\end{aligned}$$

The size of ΔP is proportional to J times the product of the sine of three mass differences. The effect is proportional to E^{-3} for small Δ_{ij} . Therefore, there is a hope that this effect will be visible in long baseline neutrino oscillation experiment provided the Jarlskog invariant factor J is not too small.

In the standard parametrization of the mixing matrix [13], the Jarlskog invariant J can be written as

$$J = \text{Im} (U_{\mu 3} U_{\tau 3}^* U_{\mu 2}^* U_{\tau 2}) = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin \delta \tag{10}$$

where θ_{12} is the mixing angle that directly comes from solar neutrino oscillation, θ_{23} is that for the atmospheric neutrino oscillation and θ_{13} is directly constrained by the $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ oscillation experiment. Now using the data from Eq. (3), one can obtain the maximum value of J is given by

$$J \leq 0.04 \sin \delta. \quad (11)$$

Thus it is found that the value of J in the lepton sector is significantly larger than that of the quark sector ($J_{quark} \sim \mathcal{O}(10^{-5})$), provided δ is not too small.

The CP violation search will require pure neutrino beams with the highest possible intensities. Beta-beams is a new concept for the production of neutrino beams that is based on the beta-decay of boosted radioactive ions, as first proposed by Zucchelli [20]. By exploiting the high ion intensities foreseen in the future, this method can produce intense neutrino beams, pure in flavour and with well known fluxes. The beta-beam concept has several important advantages. The neutrino beams are pure in flavour since only electron neutrinos or anti-neutrinos can be produced, depending on the ion that decays through β^+ or β^- . This means that there is no beam related background. The neutrino intensity and energy spectrum is precisely known, since the number of ions is perfectly controlled.

In the standard beta-beam scenario [20], the beta-beam facility is hosted at CERN. The search for CP violation effects can be performed through the comparison of $\nu_e \rightarrow \nu_\mu$ versus $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ oscillations. If such a beam is allowed to be detected at the INO detector, then the beam has to travel a distance of 7152 km [21], which is very close to the magic baseline length $L_{magic} = (7300-7600)$ km [22, 23]. At such a distance the $\nu_e \rightarrow \nu_\mu$ survival probability has no dependence on δ and it allows to measure the neutrino hierarchy without any degenerate solution. The INO facility is expected to come up at PUSHEP situated close to Bangalore at Southern India. It will have an Iron calorimeter (ICAL) detector, which is expected to detect the charged muon with energies of few GeV.

Now let us consider the matter effect in the CP violating parameter. When the neutrino beam is allowed to travel a long distance, the electron neutrinos could have interaction with the matter fields consisting of electrons, protons and neutrons on their path. Hence the CP violation parameters will be modified due to such matter effect as such interactions are not invariant under CP transformation. The general discussion of matter effect in the long baseline experiments was given by Kuo and Pantaleone [24]. The T violation effects in the Earth were also studied numerically by Krastev and Petcov [25]. The data in the

long baseline experiments include the background matter effect which is not CP invariant. Therefore, it is very important to investigate the matter effect in order to estimate the CP violating effect originating from the neutrino mixing matrix. The CP violation effect in long baseline experiments are well studied in the literature [26–28]. Due to matter effect the evolution equation becomes [28]

$$i\frac{d\nu}{dx} = \mathcal{H}\nu \quad (12)$$

where

$$\mathcal{H} \equiv \frac{1}{2E} (U_m \text{diag}(\mu_1^2, \mu_2^2, \mu_3^2) U_m^\dagger). \quad (13)$$

The matrix U_m and the masses μ_i 's are determined by

$$U_m \begin{pmatrix} \mu_1^2 & 0 & 0 \\ 0 & \mu_2^2 & 0 \\ 0 & 0 & \mu_3^2 \end{pmatrix} U_m^\dagger = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (14)$$

where $A = 2\sqrt{2}G_F N_e E = 7.56 \times 10^{-5} \text{ eV}^2 \frac{\rho}{\text{g cm}^{-3}} \frac{E}{\text{GeV}}$ with N_e is the electron density and ρ is the matter density. The solution of the above equation is given as

$$\nu(x) = S(x)\nu(0)$$

with

$$S = T e^{\int_0^x ds \mathcal{H}(s)},$$

giving the oscillation probability for $\nu_\alpha \rightarrow \nu_\beta$, ($\alpha, \beta = e, \mu, \tau$) at distance L as

$$P(\nu_\alpha \rightarrow \nu_\beta; L) = |S_{\beta\alpha}(L)|^2. \quad (15)$$

Thus, one can obtain a simple approximative result for the appearance probability $P(\nu_e \rightarrow \nu_\mu)$ as [29, 30]

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &\approx \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(\hat{A} - 1)\Delta_{31}]}{(1 - \hat{A})^2} \\ &+ \alpha \sin \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \Delta_{31} \frac{\sin(\hat{A}\Delta_{31})}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta_{31}]}{(1 - \hat{A})} \\ &+ \alpha \cos \delta \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \Delta_{31} \frac{\sin(\hat{A}\Delta_{31})}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta_{31}]}{(1 - \hat{A})} \\ &+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta_{31})}{\hat{A}^2}, \end{aligned} \quad (16)$$

where $\alpha = \Delta m_{21}^2/\Delta m_{31}^2$, $\hat{A} = A/\Delta m_{31}^2$, and $\Delta_{31} = \Delta m_{31}^2 L/4E$. The first and last terms in eq (16) correspond to the atmospheric and solar probabilities while the terms proportional to α are the interference between the solar and atmospheric contributions.

A particularly interesting situation occurs for the case when

$$\sin(\hat{A}\Delta_{31}) = 0, \Rightarrow \hat{A}\Delta_{31} = \pi, \quad (17)$$

for which the δ dependence disappears in the transition probability $P(\nu_e \rightarrow \nu_\mu)$ as seen from Eq. (16). This condition can also be translated as

$$\sqrt{2}G_F N_e L = 2\pi, \quad (18)$$

which is independent of the energy E . The baseline for which condition (18) is satisfied is known as magic baseline, which is basically found to be

$$\left(\frac{\rho}{\text{g/cc}}\right) \left(\frac{L}{\text{km}}\right) \simeq 32725. \quad (19)$$

This magic baseline is found to be

$$L_{magic} = 7690, \quad (20)$$

according to Preliminary Reference Earth Model (PREM) [31] density profile of the earth. The implications of such magic baseline is studied for the clean determination of θ_{13} and $sgn(\Delta m_{31}^2)$ [21]. However, here we are interested to see whether CP violation could be observed in such magic baseline experiments.

Since the transition probability is independent of the CP violating phase δ for L_{magic} , it is naively expected that CP violation would also vanish for such experiments, however in actual practice it is not the case. The intrinsic CP violation due to the complex phase in the PMNS matrix which is proportional to $\sin \delta$ vanishes whereas significant CP violation due to matter effect could be possible.

The transition probability for $\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta$ can be obtained from (16) by replacing $\hat{A} \rightarrow -\hat{A}$ and $\delta \rightarrow -\delta$. Thus the CP violating parameter in the presence of matter can be given as

$$\Delta P(\nu_e \rightarrow \nu_\mu) \equiv P(\nu_e \rightarrow \nu_\mu; L) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu; L). \quad (21)$$

After obtaining the relevant expressions for CP violation, we now proceed to estimate its value both in the vacuum oscillation case (8) and including the matter effect contributions

(21). For numerical estimation, we use the central values of the mixing angles and mass square differences as given in (3) and the baseline length as $L = 7152$ km. Since the Dirac CP violating phase δ is unconstrained, we vary its value between $(10 - 90)^\circ$. With these inputs, in Figure-1 we show the variation of CP violation parameter (in vacuum) with beam energy. From the figure it can be seen that CP violating effect of few percent could be possible for such a long baseline experiment and its dependence on the CP violating phase δ is quite significant. In this case we get the same behavior for the CP violating observable both in the normal as well as inverted hierarchy cases of neutrino masses.

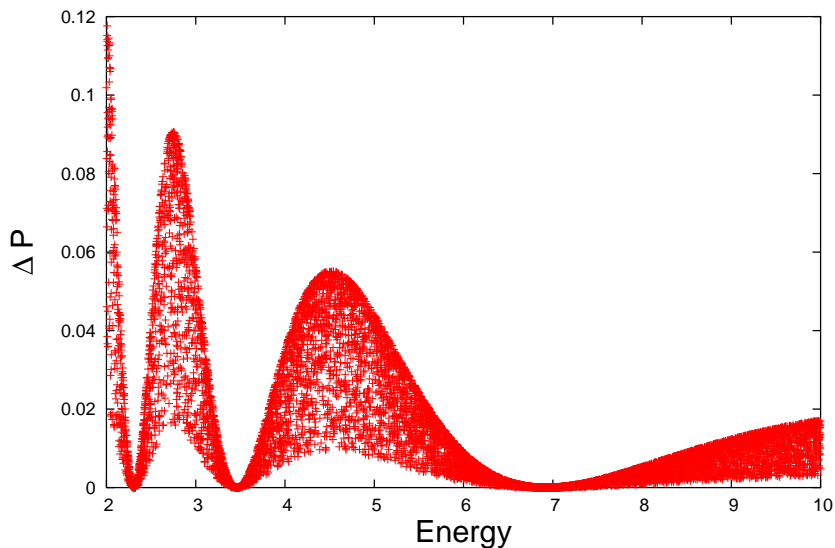


FIG. 1: The variation of CP violating parameter (8) with beam energy (in GeV), where we have varied the CP violating phase between $(10 - 90)^\circ$.

The corresponding variation, including the matter effect (21) is shown in Figure-2 both for normal hierarchy (red region) and inverted hierarchy (blue region), where we have used the same input parameters as figure-1 and vary the CP violating phase δ between $(10 - 90)^\circ$. From the figure it can be also be noted that the dependence δ is almost negligible for such a baseline length. So the measurement of CP violation in such experiment will also provide additional information regarding the hierarchical nature of neutrino masses.

In figure-3 we have shown the CP violation effect (with normal hierarchy) for two representative baseline lengths : $L = 2500$ km and $L = 5000$ km. In this case the δ dependence is not completely negligible. For inverted hierarchy case the CP violation effect will be

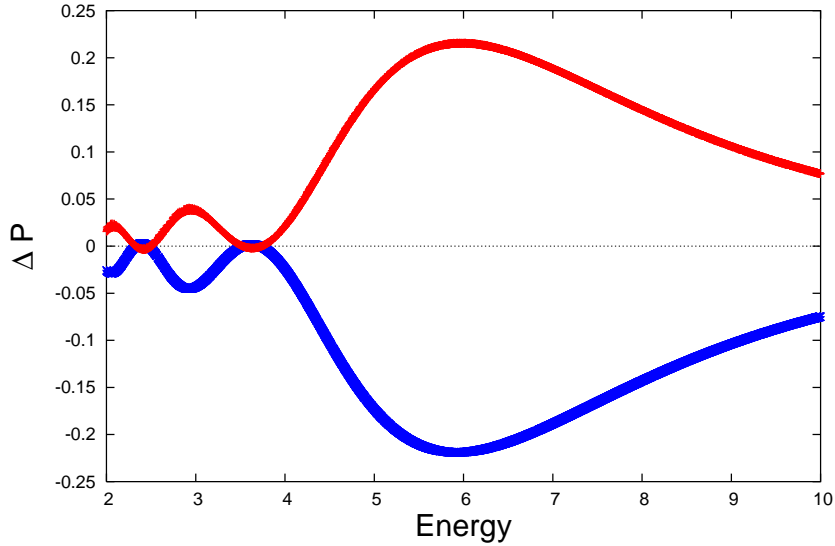


FIG. 2: The variation of CP violating parameter including matter effect (21) with beam energy (in GeV), where the red (blue) plots correspond to normal (inverted) hierarchical behavior of neutrino masses.

opposite to that of normal case.

To summarize, in this paper we have examined the possibility of observing CP violation in the lepton sector in the proposed INO experiment, using the beta beam from CERN. In the lepton sector also CP violation is expected unless neutrinos are exactly massless. In particular CP violation in neutrino flavour oscillation is an important phenomenon because it is directly related to the CP violating phase parameter in the mixing matrix. Unfortunately this CP violating effect is suppressed in the short baseline accelerator experiments if the neutrinos have hierarchical mass spectrum. However the suppression is avoidable in the long baseline accelerator experiments, which are expected to operate in the near future. So there is probability that one can observe CP violating effect in those experiments. We found that CP violating effect of few percent could be observable at the INO detector using the beta beam from CERN. We have also investigated the matter effect on the CP violation parameter and found that it has significant contribution for such base line length. We have shown that CP violation effect as large as $\sim 20\%$ could be possible in such experiment. Furthermore of CP violation in this experiment can also provide us the evidence whether the neutrino masses are normal or inverted hierarchical in nature. It is therefore strongly argued to look for leptonic CP violation effect at INO.

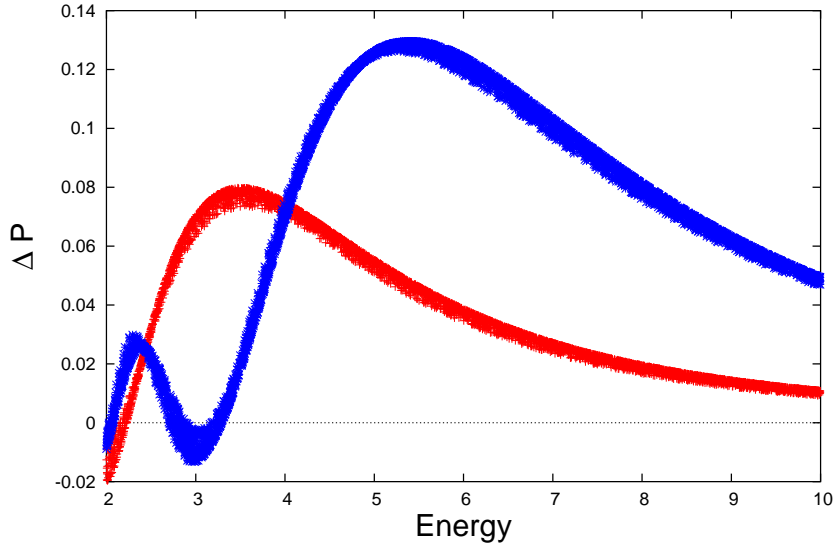


FIG. 3: Same as Figure-2 with normal hierarchy for two different baseline lengths, where the red (blue) regions are for $L = 2500$ (5000) km.

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