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Meshless Natural Neighbor Galerkin method for the analysis of composite plates using higher order shear deformation theories

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Abstract

In the present work a meshless natural neighbor Galerkin method for the bending and vibration analysis of plates and laminates is presented. The method has distinct advantages of geometric flexibility of meshless method. The compact support and the connectivity between the nodes forming the compact support are performed dynamically at the run time using the natural neighbor concept. By this method the nodal connectivity is imposed through nodal sets with reduced size, reducing significantly the computational effort in construction of the shape functions. Smooth non-polynomial type interpolation functions are used for the approximation of inplane and out of plane primary variables. The use of non-polynomial type interpolants has distinct advantage that the order of interpolation can be easily elevated through a degree elevation algorithm, thereby making them suitable also for higher order shear deformation theories. The evaluation of the integrals is made by use of Gaussian quadrature defined on background integration cells. The plate formulation is based on first order shear deformation plate theory. The application of natural neighbor Galerkin method formulation has been made for the bending and free vibration analysis of plates and laminates. Numerical examples are presented to demonstrate the efficacy of the present numerical method in calculating deflections, stresses and natural frequencies in comparison to the Finite element method, analytical methods and other meshless methods available in the literature.

1. Introduction

Laminated composite plates are widely used in Civil, Mechanical and Aerospace engineering industries due to their high strength to weight ratio and flexibility in design. Accurate prediction of structural response characteristic is a challenging problem in the analysis of laminated composites due to the orthotropic structural behavior, the presence of various types of couplings and due to the reduced thickness of the structural elements made of composites. The finite element method (FEM) has been widely used in the analysis of laminated composite structures. Finite element modeling of thin structures with large deformation, material discontinuities and fracture related studies such as crack initiation and propagation requires adaptive mesh generation and or re-meshing that are computationally expensive. This also leads to errors in representation of geometries. Natural Element Method (NEM) possess advantageous properties of both meshless and finite element method. The NEM was developed by Sambridge et al. [1] where the interpolation function is based on Sibson [2] natural neighbor coordinates. NEM was successfully applied to solid mechanics problems by Sukumar et al. [3].

The analysis of laminated composite plates by various two dimensional equivalent single layer theories is available in literature. The basic equivalent single layer plate theory is classical laminated plate theory and it is based on Kirchhoff assumption for the analysis of thin plates. Because of this assumption, the CLPT predicts deflection and natural frequencies accurately for thin plates but for thick plates it under predicts deflection and over predicts natural frequency as well as buckling loads. The transverse shear strains are zero in CLPT, hence is not useful for the analysis of thick plates which require accounting for shear deformation. To this end the shear deformation plate theories were introduced for analysis of thick plates. In the first order shear deformation theory a linear displacement through the thickness results in an uniform transverse shear strain over the thickness of the laminate. However, the actual variation of transverse shear strain over the laminate thickness is parabolic. To account for this an arbitrary shear correction factor is introduced in FSDT.

2. Laminate Plate Formulation using FSDT

The displacement field of the first order shear deformation plate theory is written as

$$u(x,y,z) = u_\circ(x,y) - z\theta_x\,(x,y)$$
$$v(x,y,z) = v_\circ(x,y) - z\theta_y\,(x,y)$$
$$w(x,y,z\,) = w_\circ(x,y)$$

where $u_\circ, v_\circ$ and w$_\circ$ are displacements of a point on the plane z = 0. The rotations of a transverse normal about y and x axis represented as θ_x and θ_y respectively. In the first order shear deformation plate theory, the displacement field is expressed till the first order of z term. The strain-displacement relation can be written as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \varepsilon_{xx}^{(0)} + zk_{xx}^{(1)}$$
$$\varepsilon_{yy} = \frac{\partial v}{\partial x} = \varepsilon_{yy}^{(0)} + zk_{yy}^{(1)}$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^{(0)} + zk_{xy}^{(1)}$$

The shear strain terms are given by

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \gamma_{xz}^{(0)}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} = \gamma_{yz}^{(0)}$$

3. Natural neighbor interpolation

The natural element method allows an element-free spatial discretization which is entirely node based. It ensures quadratic precision of the interpolant over convex and non-convex boundaries. A more detailed description on the implementation of the method for solid mechanics problems can be found in [3] and [4]. The C^0-continuous natural neighbor interpolant N_I^0 about the point x, is defined as

$$N_I^0(x) \;=\; \frac{A(V(p_I) \cap \tilde{V}(x))}{A(\tilde{V}(x))}$$

where A(o) denotes the d-dimensional volume measure. The Sibson interpolants posses properties such as non negativity, $0 \le N_I^0 \le 1$, partition of unity, $\sum_I N_I^0 = 1$, interpolation of data, $N_I^0(x_J) = \delta_{IJ}$, allowing the exact imposition of essential boundary conditions, and linear completeness, $\sum_I p_I N_I^0\,(x) = x$. These interpolants also posses smoothness characteristics such as $C\infty$-continuous everywhere except at the nodes and at the boundaries of the support where they are C^0- and C^1-continuous, respectively. The derivatives of the natural neighbour coordinates are obtained by differentiating above equation and is given by

$$N^0_{(I,J)}(x) = \frac{A_{I,J}(V(p_I) \cap \tilde{V}(x)) - N^0_I(x) A_{,J}(\tilde{V}(x))}{A(\tilde{V}(x))}$$

4. Governing equations

The equations of equilibrium for the analysis of plate are obtained using principle of virtual work. In analytical form it can be written as

$$\delta \int_{t_1}^{t_2} (U + W - E)\, dt = 0$$

Where U is total strain energy due to deformation, W is the potential of external loads and E is the kinetic energy.

$$= \frac{1}{2} \int_{-h/2}^{h/2} \int_A \left(\sigma_{xx} \delta\varepsilon_{xx} + \sigma_{yy} \delta\varepsilon_{yy} + \sigma_{zz} \delta\varepsilon_{zz} + \tau_{xy} \delta\gamma_{xy} + \tau_{yz} \delta\gamma_{yz} + \tau_{xz} \delta\gamma_{xz} \right) dAd$$

$$- \int_A (q \delta w) dA - \frac{\delta}{2} \int_{-h/2}^{h/2} \int_A \rho\, (\dot{u}^2 + v^2 + \dot{w}^2)\ dAdz = 0$$

The independent field variables are $u_{0,} v_{0,} w_{0,} \theta_x$ and θ_y and the detail study about plate mathematical formulation based on FSDT has been adopted from [5]. Simply supported (SS) boundary conditions are $u_{0,} v_{0,} w_{0,} \theta_x = 0$ for x = 0 to a, and $u_{0,} v_{0,} w_{0,} \theta_y = 0$ or y = 0 to b are considered for analysis.

7.Results

Here, simply supported thick orthotropic plate and laminated composite plates subjected to uniformly distributed load (UDL) q_o is considered for bending and modal analysis. The transverse shear strains are constant along thickness in FSDT but these strains vary atleast quadratically through the thickness in 3D elasticity theory. To avoid this discrepancy the shear correction factor 5/6 is considered in FSDT and a, b, and h are plate length, width and height respectively. The natural neighbour Galerkin method results of the plate bending and modal analysis problems are validated with analytical and FEM results taken from literature. The laminated composite plate made from stacking of equal thickness of the laminate are considered for the analysis.

The orthotropic material properties considered are $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$,$G_{23} = 0.2E_2$, υ_{12}= 0.25. The non-dimensional deflection and in-plane stresses are given as

$$\bar{w}(a/2, b/2, 0) = 100 w_c\ E_2 h^3 / a^4 q_o$$
$$\bar{\sigma}_{xx}(a/2, b/2, h/2) = \sigma_{xx}\, h^2 / a^2 q_o$$
$$\bar{\sigma}_{yy}(a/2, b/2, h/4) = \sigma_{yy}\, h^2 / a^2 q_o$$
$$\bar{\tau}_{xy}(a/2, b/2, -h/2) = \tau_{xy}\, h^2 / a^2 q_o$$

A converegence study has been first performed. The deflection and in-plane normal stress of an orthotropic plate obtained from the present analysis are compared and validated with those obtained from literature. The results are presented in Table 1. Six Gauss points are considered in each background triangle cell. The deflection and in-plane normal stress obtained with 21x21 and above nodes are well agreement with FSDT analytical results [5]. It is understood from the results in Table 1 that the developed plate formulation and the natural neighbour Galerkin method code for analysis of plates is working very well.

The Delaunay triangles are used as a background cells for integration purpose. The influence of Gauss quadrature in each background triangle cell and the influence of number of nodes with change in Gauss quadrature on deflection and stresses are shown in Table 2. It is seen in Table 2, for a particular set of nodes in domain the increase in Gauss quadrature from 1x1 to 6x6 the results are approaching reference results. The results with Gauss quadrature 12x12 is not better compare to 6x6 because the stiffness matrix is becoming more stiffer with higher number of Gauss points, so there is a drop in deflection and stress values from 6x6 to 12x12. However, it is observed that the stresses predicted by the present method together with the natural neighbour shape functions and with a Gauss quadrature 6x6 and with a nodal distribtuion of 21x21 nodes are higher than the analytical stress values given in [5].

The deflection and in-plane stresses of orthotropic and four layer symmetric ($0^o/90^o/90^o/0^o$) laminated composite obtained with present method are compared with Reddy [5] analytical FSDT results for different plate aspect ratios in Table 3 and 4 respectively. The lamina in the laminated composite plate is an orthotropic material and same orthotropic material properties considered. It can be observed from Table 3 and Table 4 that the stresses calculated by present natural neighbour Galerkin method are in good agreement with reference results. Here, the present natural neighbour Galerkin method results for isotropic plate are compared with 3D elasticity solution and 2D higher order shear deformation plate theory (HSDT) results. The results are compared with the litreature [7]. It is observed that the proposed method is comparable with the results using natural radial element method (NREM) and NREM with non centered integration (NREM-NC). The material properties of isotropic material are $E_1 = E_2 = 10920, G_{12} = G_{13} = G_{23}, G_{23} = E_2/2.5$, $\upsilon_{12} = 0.25$. The deflection and in-plane normal stress of an isotropic plate are compared with exact 3D solutions and HSDT results in Table 5.

Grid	$\bar{w}$	$\bar{\sigma}_{xx}$
11x11	0.9519	0.7694
15x15	0.9519	0.7669
21x21	0.9521	0.7728
31x31	0.9521	0.7755
Analytical[5]	0.9519	0.7706

Table 1: Convergence study of non-dimensional central deflection and in-plane normal stress of a simply supported orthotropic plate with a/b=1, a/h=10 under uniformly distributed loading.

Nodes 11x11	a/h=10		a/h=20	
	$\bar{w}$	$\bar{\sigma}_{xx}$	$\bar{w}$	$\bar{\sigma}_{xx}$
1x1	0.9532	0.7378	0.7165	0.7650
3x3	0.9447	0.7633	0.6917	0.7415
6x6	0.9519	0.7694	0.7074	0.7583
12x12	0.9492	0.7650	0.7027	0.7523
Nodes 21x21				
1x1	0.9531	0.7631	0.7249	0.7639
3x3	0.9504	0.7819	0.7179	0.7826
6x6	0.9521	0.7828	0.7217	0.7864
12x12	0.9514	0.7783	0.7205	0.7825
Nodes 31x31				
1x1	0.9527	0.7695	0.7260	0.7769
3x3	0.9513	0.7754	0.7226	0.7834
6x6	0.9521	0.7755	0.7242	0.7848
12x12	0.9517	0.7736	0.7237	0.7829
Analytical[5]	0.9519	0.7706	0.7262	0.7828

Table 2: Influence of Gauss quadrature with number of nodes on non-dimensional central deflection and in-plane normal stress of a simply supported orthotropic plate with a/b=1, a/h=10 under uniformly distributed loading.

	a/h=10				a/h=20			
	Present	Analytical[5]	Belinha et al [7]		Present	Analytical[5]	Belinha et al [7]	
			NREM	NREM-NC			NREM	NREM-NC
$\bar{w}$	0.9518	0.9519	0.9433	0.9506	0.7217	0.7262	0.7053	0.7246
$\bar{\sigma}_{xx}$	0.7828	0.7706	0.7609	0.7703	0.7864	0.7828	0.7579	0.7830
$\bar{\sigma}_{yy}$	0.0358	0.0352	0.0352	0.0354	0.0282	0.0272	0.0274	0.0274
$\bar{\tau}_{xy}$	0.0527	0.0539	0.0519	0.0527	0.0468	0.0487	0.0456	0.0473

Table 3: Non-dimensional central deflection and in-plane stresses of a simply supported orthotropic plate with a/b = 1 under uniformly distributed loading with $\bar{\sigma}_{yy}$is calculated at (x,y,z) = (a/2, b/2, h/2).

	a/h=10				a/h=20			
	Present	Analytical[5]	Belinha et al [7]		Present	Analytical[5]	Belinha et al [7]	
			NREM	NREM-NC			NREM	NREM-NC
$\bar{w}$	1.0245	1.0250	1.0141	1.0235	0.7626	0.7694	0.7442	0.7678
$\bar{\sigma}_{xx}$	0.7691	0.7577	0.7464	0.7573	0.8065	0.8045	0.7747	0.8048
$\bar{\sigma}_{yy}$	0.5037	0.5006	0.4943	0.5009	0.3973	0.3968	0.3838	0.3970
$\bar{\tau}_{xy}$	0.0461	0.0470	0.0456	0.0467	0.409	0.0420	0.0400	0.0414

Table 4: Non-dimensional central deflection and in-plane stresses of a simply supported symmetric ($0^o/90^o/90^o/0^o$) laminated plate with a/b = 1 under uniformly distributed loading with $\bar{\sigma}_{yy}$ is calculated at (x,y,z) = (a/2, b/2, h/2).

		Present	Reddy[5]	Exact[8]	Ferreira et. al. [8]
a/h=10	$\bar{w}$	4.7687	4.770	4.791	4.7866
	$\bar{\sigma}_{xx}$	0.2750	0.2899	0.2762	0.2777
a/h=20	$\bar{\sigma}_{yy}$	4.5034	4.570	4.625	4.6132
	$\bar{\tau}_{xy}$	0.2708	0.2683	0.2762	0.2761

Table 5: Non-dimensional central deflection and in-plane normal stress of a simply supported isotropic plate with a/b = 1 under uniformly distributed loading.

6.Summary

The consistency and convergence of solutions for deflection and stresses have been studied by varying order of Gauss quadrature and the number of nodes. For various aspect ratios of isotropic and laminated plates, the deflections, and stresses obtained from the present analysis are validated with the analytical solutions and with the results obtained from literature. It has been observed that good convergence of results are obtained with increase in the quadrature points and nodes. For various aspect ratios of isotropic and laminated plates, the deflections and stresses frequencies obtained from the present analysis are validated with the analytical solutions and with the results obtained from literature. It is observed that the natural neighbour galerkin method FSDT solutions are closer to the FSDT analytical solutions and slightly higher than solutions that are obtained using FSDT finite element method. This is largely attributed to the smooth interpolation functions used in the natural neighbour Galerkin method. The use of non polynomial type approximations has distinct advantage that the order of interpolation can be degree elevated and thereby making them suitable for use with other higher order shear deformation theories[6].

7. References

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