Phenomenology of Right-Handed Neutrino in Beyond Standard Model

Master's Dissertation

presented by

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Abstract

In this work we describe the possible phenomenologies of right-handed neutrinos in two simple extensions of Standard Model. In this respect we briefly summarise the gauge theory and Standard Model as $SU(3)_C \times SU(2)_L \times U(1)_Y$. In particular we describe the two-body decay and Drell-Yan scattering for leptons via weak interaction. Given this set up, we first discuss Type-I seesaw mechanism with the Standard Model gauge singlet right-handed neutrinos. The prompt and displaced decays are also scrutinized with respect to the parameter space. The production mechanism at the LHC are mentioned. Secondly we focus on the Type-III extension of Standard Model and derive the charged and neutral lepton mixings. The phenomenologies at the LHC are also described in this context.

2 Introduction

In this article we attempt to extended Standard Model(SM) by a right - handed neutrinos (RHNs). For this purpose we start with the basic tools of abelian and non-abelian gauge theories. In the process we describe briefly Quantum Electro Dynamics (QED). Next we describe SM as $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory. SM describes the interactions beautifully as we derive the vertices with the gauge bosons and fermions. Nevertheless, SM leave all the particle mass less. Giving ad hoc mass term would break the SM gauge symmetry. Therefore, the symmetry cannot be broken in the Lagrangian but can be broken by the ground state of a scalar field, which is known as Higgs mechanism. This is achieved by the introduction of a $SU(2)_L$ doublet scalar with non-zero hyper-charge, which has four degrees of freedom (DOF). After taking the vacuum expectation value (vev), only one DOF is shown as physical state known as Higgs boson and discovered recently at the Large Hadron Collider (LHC)[1, 2].

Prepared with the setup we the describe the two-body decay formula and Drell-Yan cross-section formulas the charged leptonic sectors are derived within SM. Though SM with Higgs mechanism is very beautiful theory but it fails in many other aspect like it cannot explain the fermion mass hierarchy, smallness of neutrino mass and does not have a cold dark matter candidate. In this article we are going to address one of these aspect, namely neutrino mass generation and its smallness.

This can be achieved by adding a right-handed neutrino which can be SM gauge singlet via Type-I seesaw mechanism [3]. The smallness of neutrino mass can be explained due to existence of heavy neutrinos. We discuss the decay and collider phenomenolgies of such right-handed neutrinos.

Similar results can also be achieved by the introduction of a $SU(2)_L$ triplet fermion. In this case the charged leptons along with the neutrinos mix with charged and neutral right-handed neutrinos [4].

We organise the report as follows. In section 3 we discuss the gauge theory for both abelian and non-abelian cases, where we address the global and local gauge invariance of Lagrangian. In abelian gauge theory as an application of local gauge invariance we discussed Quantum Electrodynamics. Vector current and axial vector current is also discussed in chiral basis in this section. In non-abelian gauge theory portion we basically discuss the transformation of co-variant derivative and the field strength tensor, finally we reach to the Yang-Mills Lagrangian. In section 4 we discuss the Standard model as $SU(2)_L \times U(1)_Y$ gauge theory. In this section we also discuss the electro-weak symmetry breaking and Higgs mechanism and mention how the gauge bosons, quarks and charged leptons get mass. Done with the vertex calculation we then move to the Drell-Yan cross-section in section 5 and two-body decay widths (general case) in section 6. The issues of SM as discussed above prompt us to go beyond the Standard Model scenario in section 7, where we introduce the right-handed neutrino as Majorana fermion. In section 8 we extend the Standard Model by a SM singlet RHN, which generates neutrino mass via type-I seesaw mechanism, where we calculate its decay to different modes. In that context we also discuss the production of right-handed neutrino at collider. Followed by in section 9 we extend Standard Model by a RHN that is a $SU(2)_L$ triplet (type-III seesaw) and the corresponding decay phenomenology is also discussed. In section 10 we discuss and conclude. We attached some of the Mathematica codes in **Appendix A**.

3 Gauge theory in a nutshell

A gauge theory is a theory where the action is invariant under a continuous group symmetry. The continuous symmetry is called a gauge group and this transformation is called a gauge transformation. The invariance of physical system under symmetry transformation implies a set of conservation laws. Invariance under translations, time displacements, rotations leads to the conservation of linear momentum, energy and angular momentum respectively. Similarly in the field theory we consider some internal continuous symmetry transformation which also leads us to some conserved quantity, they imply conserved current. The 1^{st} consequence of symmetry transformation is given by Noether's theorem, which states that for any invariance of the action under a continuous global transformation of the fields there exists conserved currents. But the kinetic part of Lagrangian of free field fails to maintain the symmetry under local gauge transformation. To get back the symmetry under local transformation we have to introduce the interaction of photon with the free field.

Though the Dirac fermions describes the spin half particles nicely in most of the cases but in the context of gauge theory it is more natural to use chiral basis, i.e. left- and right-handed fermions. We will see that QED does not distinguish between left- and right-handed fermions and gives rise to same currents. However, for the transformation of $e^{iQ\gamma^5\alpha}$ we get two different types of currents. Unlike $e^{iQ\alpha}$ rotation, this is not a symmetry for massive fermions and exists only in the massless case [5, 6, 7, 8].

In a nutshell, Gauge theory is the mathematical description of the system which is followed by Nature. Apart from some internal symmetry, there are many more other symmetries to explore, but the question is that whether these symmetries are exact? Do Nature exactly follows symmetry? To explain some more fundamental concept like mass these symmetries must be broken. Gauge theory is the theory of massless particles, and we know that in Standard Model except for gluons and photons all other the particles are massive so there should be some way out which solves this problem.

3.1 Abelian Gauge Theory

We consider an arbitrary complex field $\chi(x)$ which transforms as,

$$\chi(x) \longrightarrow \chi'(x) = e^{iQ\alpha}\chi(x) \tag{1}$$

The result of doing one gauge transformation α_1 followed by another gauge transformation α_2 is always a third gauge transformation parameterised by the function $\alpha_1 + \alpha_2$,

$$\chi \longrightarrow e^{iQ\alpha_2}(e^{iQ\alpha_1}\chi) = e^{iQ(\alpha_1+\alpha_2)}\chi \tag{2}$$

Mathematically these gauge transformations are an example of a continuous group which satisfies the commutative property. The group is represented on a complex field by complex 1×1 matrices $U_Q(\alpha) = e^{iQ\alpha}$. Hence it is Abelian Lie Group $U(1)$.

3.1.1 Global gauge invariance in complex spinor field

Let us consider an arbitrary complex spinor field $\psi(x)$ which transforms under a constant phase transformation as,

$$\psi(x) \longrightarrow \psi'(x) = e^{iQ\alpha}\psi(x) \tag{3}$$

$$\psi^\dagger(x) \longrightarrow \psi'^\dagger(x) = \psi^\dagger(x)e^{-iQ\alpha}. \tag{4}$$

Global symmetry is a symmetry which does not depend on space-time. So here, the gauge parameter α is independent on space-time.

The Lagrangian density of $\psi(x)$ has the form

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \tag{5}$$

Here we use $\bar{\psi}$ instead of $\psi^\dagger$ to make the Lagrangian Lorentz invariant. $\bar{\psi}$ transforms as

$$\begin{aligned}\bar{\psi}' &= \psi'^\dagger\gamma^0 \\ &= \psi^\dagger e^{-iQ\alpha}\gamma^0 \\ &= \psi^\dagger\gamma^0 e^{-iQ\alpha} \\ &= \bar{\psi}e^{-iQ\alpha}\end{aligned}$$

So, under global gauge transformation spinor Lagrangian density transforms as,

$$\begin{aligned}\mathcal{L}' &= i\bar{\psi}e^{-iQ\alpha}\gamma^\mu\partial_\mu e^{iQ\alpha}\psi - m\bar{\psi}e^{-iQ\alpha}e^{iQ\alpha}\psi \\ &= i\bar{\psi}\gamma^\mu e^{-iQ\alpha}e^{iQ\alpha}\partial_\mu\psi - m\bar{\psi}\psi \\ &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi\end{aligned} \tag{6}$$

so, Lagrangian density is invariant under global gauge transformation.

3.1.2 Local gauge transformation of complex spinor field

The global gauge invariance is not the most general invariance because the gauge parameter does not depend on space-time. In case of local gauge invariance we will make the transformation as

$$\psi(x) \longrightarrow \psi'(x) = e^{iQ\alpha(x)}\psi(x) \tag{7}$$

$$\bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{-iQ\alpha(x)} \tag{8}$$

Here the gauge parameter α is a function of space-time.

So,

$$\partial_\mu\psi(x) = e^{iQ\alpha(x)}\partial_\mu\psi(x) + iQ(\partial_\mu\alpha(x))e^{iQ\alpha(x)}\psi(x) \tag{9}$$

So under local gauge transformation the spinor Lagrangian becomes:

$$\mathcal{L}' = i\bar{\psi}e^{-iQ\alpha(x)}\gamma^\mu(e^{iQ\alpha(x)}\partial_\mu\psi(x)+iQ(\partial_\mu\alpha(x))e^{-iQ\alpha(x)}\psi(x))-m\bar{\psi}(x)e^{-iQ\alpha(x)}e^{iQ\alpha(x)}\psi(x)$$

$$= (i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi) - Q\bar{\psi}\gamma^\mu(\partial_\mu\alpha(x))\psi(x) \tag{10}$$

The term $\partial_\mu\alpha(x)$ breaks the invariance of Lagrangian.

3.1.3 Quantum Electrodynamics (QED)

To make the Lagrangian locally gauge invariance we introduce a modified derivative (covariant derivative), D_μ that transforms covariantly under gauge transformation just like ψ.

$$D_\mu\psi \longrightarrow D'_\mu\psi' = e^{iQ\alpha(x)}D_\mu\psi$$

To form the covariant derivative we have to introduce a vector field A_μ and the transformation of this field will be such that, it will cancel the extra term in equation (10) and will make the Lagrangian locally invariant.

- The construction of covariant derivative is: $D_\mu \equiv \partial_\mu - iQA_\mu$

- Vector field transforms as: $A_\mu \longrightarrow A'_\mu = A_\mu + \frac{1}{Q}\partial_\mu\alpha(x)$

Now, if we replace ∂_μ in the Lagrangian by D_μ then it takes the form:

$$\begin{aligned}\mathcal{L} &= i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi \\ &= i\bar{\psi}\gamma^\mu(\partial_\mu - iQA_\mu)\psi - m\bar{\psi}\psi \\ &= (i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi) + Q\bar{\psi}\gamma^\mu A_\mu\psi \end{aligned} \tag{11}$$

So, to make the Lagrangian locally gauge invariant we are introducing a gauge field which interacts with Dirac fermions of charge $-Q$ and the interaction term is $Q\bar{\psi}\gamma^\mu A_\mu\psi$. This field interacts exactly the same way as the photon field.

If we consider the new field as the physical photon field then we have to add a term to the Lagrangian corresponding to it's kinetic energy and this term must be invariant under gauge transformation [5, 6, 7, 8].

So, we introduce the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ in the Lagrangian in the form $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. (The coefficient $-\frac{1}{4}$ is used to get the correct form of Maxwell's equation from here.)
Still here the mass term of photon field is missing. It would have the form

$$\mathcal{L}_\gamma = \frac{1}{2}m^2 A^\mu A_\mu$$

such a mass term would spoil the local gauge invariance. It can be seen that

$$A^\mu A_\mu \longrightarrow A'^\mu A'_\mu = (A^\mu + \frac{1}{Q}\partial^\mu\alpha)(A_\mu + \frac{1}{Q}\partial_\mu\alpha) \neq A^\mu A_\mu$$

Rather it gives an extra term in Lagrangian as

$$\delta\mathcal{L} = \frac{1}{Q} A^\mu \partial_\mu \alpha \quad \text{(upto } 1^{st} \text{ order of } \alpha\text{)}$$

The reason is that a photon field with finite mass does not have infinite range and hence can not compensate local phase transformation in the entire space. So, as a result Local Gauge Invariance requires a massless photon field.

So, the QED Lagrangian becomes:

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu \psi - m)\psi + Q\bar{\psi}\gamma^\mu A_\mu \psi - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}} \tag{12}$$

In covariant derivative notation the QED Lagrangian is

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{13}$$

We already have noticed that the 1^{st} term is invariant under local gauge transformation because we defined the co-variant derivative accordingly.
Now we know, $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
So,

$$\begin{aligned} F'_{\mu\nu} &= \partial_\mu (A_\nu + \frac{1}{Q}\partial_\nu \alpha) - \partial_\nu (A_\mu + \frac{1}{Q}\partial_\mu \alpha) \\ &= \partial_\mu A_\nu + \frac{1}{Q}\partial_\mu \partial_\nu \alpha - \partial_\nu A_\mu - \frac{1}{Q}\partial_\nu \partial_\mu \alpha \\ &= \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{(Hence invariant)} \end{aligned}$$

So, it is verified that the QED Lagrangian is invariant under local gauge transformation.

3.1.4 Vector Current

We have already seen that the Dirac Lagrangian is invariant under global gauge transformation as ($\psi \longrightarrow e^{iQ\alpha}\psi$). This gives rise to the current $j_v^\mu = -e\bar{\psi}\gamma^\mu\psi$. (if the charge is $-e$.) This current is called vector current.

If projection operator operates on spinor ψ then we can write:

$$\psi = \psi_L + \psi_R \quad \text{and} \quad \bar{\psi} = \bar{\psi}_L + \bar{\psi}_R$$

Now, $\bar{\psi_L} = \psi_L^\dagger \gamma^0 = \psi^\dagger \frac{1}{2}(1-\gamma^5)\gamma^0 = \psi^\dagger \gamma^0 \frac{1}{2}(1+\gamma^5) = \bar{\psi}\frac{1}{2}(1+\gamma^5)$

Similarly, $\bar{\psi_R} = \bar{\psi}\frac{1}{2}(1-\gamma^5)$

So, the vector current,

$$\begin{aligned} j_v^\mu &= -e[(\bar{\psi_L}+\bar{\psi_R})\gamma^\mu(\psi_L+\psi_R)] \\ &= -e[\bar{\psi_L}\gamma^\mu\psi_L + \bar{\psi_L}\gamma^\mu\psi_R + \bar{\psi_R}\gamma^\mu\psi_L + \bar{\psi_R}\gamma^\mu\psi_R] \end{aligned} \tag{14}$$

Now,

$$\begin{aligned} \bar{\psi_L}\gamma^\mu\psi_R &= \bar{\psi}\frac{1}{2}(1+\gamma^5)\gamma^\mu\frac{1}{2}(1+\gamma^5) \\ &= \frac{1}{4}\bar{\psi}[(1+\gamma^5)\gamma^\mu(1+\gamma^5)]\psi \\ &= \frac{1}{4}\bar{\psi}[(1+\gamma^5)(1-\gamma^5)\gamma^\mu]\psi \\ &= \frac{1}{4}\underbrace{[1-(\gamma^5)^2)]}_{=0}\gamma^\mu\psi \\ &= 0 \end{aligned}$$

Similarly, we get $\bar{\psi_R}\gamma^\mu\psi_L = 0$ So, we can write,

$$\begin{aligned} j_v^\mu &= (-e\bar{\psi_L}\gamma^\mu\bar{\psi_L}) + (-e\bar{\psi_R}\gamma^\mu\bar{\psi_R}) \\ &= j_{v_L}^\mu + j_{v_R}^\mu \end{aligned} \tag{15}$$

Hence it is seen that the vector current can be written in two types of currents separately. (Same for the axial vector current also in massless condition. We will see later.)

The fact is that the left and right-handed components ψ_L and ψ_R transform in the same way under this symmetry as,

$$\psi_L \longrightarrow e^{iQ\alpha}\psi_L \quad \text{and} \quad \psi_R \longrightarrow e^{iQ\alpha}\psi_R$$

$$\psi_L^\dagger \longrightarrow \psi_L^\dagger e^{-iQ\alpha} \quad \text{and} \quad \psi_R^\dagger \longrightarrow \psi_R^\dagger e^{-iQ\alpha}$$

Now, we can write the Dirac Fermion ψ as: $\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$ (Appendix (**??**)). But, $\bar{\psi} = \psi^\dagger \gamma^0$ and in Weyl or Chiral basis $\gamma^0 = \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix}$ So,

$$\begin{aligned} \bar{\psi} &= \begin{bmatrix} \psi_L^\dagger & \psi_R^\dagger \end{bmatrix} \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \psi_R^\dagger & \psi_L^\dagger \end{bmatrix} \end{aligned}$$

Now if we put all the things in Dirac Lagrangian and try to write the Dirac Lagrangian in two component form of ψ_L and ψ_R then it becomes,

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$= \begin{bmatrix} \psi_R^\dagger & \psi_L^\dagger \end{bmatrix} \left(\begin{bmatrix} 0 & i\sigma^\mu \partial_\mu \\ i\bar{\sigma}^\mu \partial_\mu & 0 \end{bmatrix} - \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \right) \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$$

$$\left[Where, \gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma^\mu} & 0 \end{bmatrix} \text{ and, } \sigma^\mu = (I_2, \sigma^i) \text{ and, } \bar{\sigma^\mu} = (I_2, -\sigma^i) \right]$$

$$= \begin{bmatrix} \psi_R^\dagger & \psi_L^\dagger \end{bmatrix} \begin{bmatrix} -m & i\sigma^\mu \partial_\mu \\ i\bar{\sigma}^\mu \partial_\mu & -m \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$$

$$= \begin{bmatrix} \psi_R^\dagger & \psi_L^\dagger \end{bmatrix} \begin{bmatrix} -m\psi_L + i\sigma^\mu \partial_\mu \psi_R \\ i\bar{\sigma}^\mu \partial_\mu \psi_L - m\psi_R \end{bmatrix}$$

$$= i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R - m\psi_R^\dagger \psi_L + i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L - m\psi_L^\dagger \psi_R$$

$$= i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R + i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L - m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R)$$

- Under $e^{iQ\alpha}$ rotation the mass term transforms as:

$$\begin{aligned} m({\psi'}_R^\dagger {\psi'}_L + {\psi'}_L^\dagger {\psi'}_R) &= m[(\psi_R^\dagger e^{-iQ\alpha} e^{iQ\alpha} \psi_L) + (\psi_L^\dagger e^{-iQ\alpha} e^{iQ\alpha} \psi_R)] \\ &= m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R) \end{aligned}$$

  Hence, the mass term is invariant.

- Under $e^{iQ\alpha}$ rotation the kinetic term transforms as:

$$\begin{aligned}
i\psi'^{\dagger}_{R}\sigma^{\mu}\partial_{\mu}\psi'_{R} + i\psi'^{\dagger}_{L}\bar{\sigma}^{\mu}\partial_{\mu}\psi'_{L} &= i\psi^{\dagger}_{R}e^{-iQ\alpha}\sigma^{\mu}\partial_{\mu}(e^{iQ\alpha}\psi_{R}) + i\psi^{\dagger}_{L}e^{-iQ\alpha}\bar{\sigma}^{\mu}\partial_{\mu}(e^{iQ\alpha}\psi_{L}) \\
&= i\psi^{\dagger}_{R}\sigma^{\mu}e^{-iQ\alpha}e^{iQ\alpha}\partial_{\mu}\psi_{R} - Q\psi^{\dagger}_{R}\sigma^{\mu}(\partial_{\mu}\alpha)\psi_{R} \\
&+ i\psi^{\dagger}_{L}\bar{\sigma}^{\mu}e^{-iQ\alpha}e^{iQ\alpha}\partial_{\mu}\psi_{L} - Q\psi^{\dagger}_{L}\bar{\sigma}^{\mu}(\partial_{\mu}\alpha)\psi_{L} \\
&= i\psi^{\dagger}_{R}\sigma^{\mu}\partial_{\mu}\psi_{R} + i\psi^{\dagger}_{L}\bar{\sigma}^{\mu}\partial_{\mu}\psi_{L} \\
&\underbrace{-Q(\psi^{\dagger}_{R}\sigma^{\mu}(\partial_{\mu}\alpha)\psi_{R} + \psi^{\dagger}_{L}\bar{\sigma}^{\mu}(\partial_{\mu}\alpha)\psi_{L})}
\end{aligned}$$

  Hence, the kinetic term is not invariant due to existence of the extra terms involving the derivative of phase α.

Using Noether's theorem for this continuous symmetry we can get the current as:

$$j^{\mu}_{v_L} = -e\bar{\psi}_{L}\sigma^{\mu}\bar{\psi}_{L} \tag{16}$$

$$j^{\mu}_{v_R} = -e\bar{\psi}_{R}\bar{\sigma}^{\mu}\bar{\psi}_{R} \tag{17}$$

Thus we get both the left and right-handed currents with the same coupling (-e) for QED interaction and it does not distinguish between different chiral vector currents, QED is blind about the handedness of the electrons [5, 6, 7, 8].

3.1.5 Axial Vector Current

If we consider a gauge transformation is like,

$$\begin{aligned}
\psi &\longrightarrow \psi' = e^{iQ\alpha\gamma^5}\psi \\
\text{then,}\bar{\psi} &\longrightarrow \bar{\psi}' = \psi^{\dagger}e^{-iQ\alpha\gamma^5}\gamma^0 \\
&\qquad\;\; = \bar{\psi}e^{iQ\alpha\gamma^5} \qquad (\gamma^5\text{anti-commutes with } \gamma^0)
\end{aligned}$$

Under this transformation Dirac Lagrangian becomes,

$$\begin{aligned}
\mathcal{L} &= i\bar{\psi}'\gamma^{\mu}\partial_{\mu}\psi' - m\bar{\psi}'\psi' \\
&= i\bar{\psi}e^{iQ\alpha\gamma^5}\gamma^{\mu}\partial_{\mu}(e^{iQ\alpha\gamma^5}\psi) - m\bar{\psi}e^{iQ\alpha\gamma^5}e^{iQ\alpha\gamma^5}\psi \\
&= i\bar{\psi}\gamma^{\mu}e^{-iQ\alpha\gamma^5}e^{iQ\alpha\gamma^5}\partial_{\mu}\psi - m\bar{\psi}e^{2iQ\alpha\gamma^5}\psi \\
&= i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \boxed{m\bar{\psi}e^{2iQ\alpha\gamma^5}\psi} \qquad \text{(Not invariant for mass term)}
\end{aligned} \tag{18}$$

So,it is clear that for massless fermion Dirac Lagrangian is invariant under this transformation and admits an extra internal symmetry.

As discussed in (section (3.1.4)) we have Dirac Lagrangian in two component notation as

$$\mathcal{L} = i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R - m\psi_R^\dagger \psi_L + i\psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L - m\psi_L^\dagger \psi_R$$

If we put this Lagrangian in Euler-Lagrangian equation then we get the equation of motion for both ψ_L and ψ_R.
So,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \psi_R} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi_R)} \right) &= 0 \\ -m\psi_L^\dagger - i\partial_\mu \psi_R^\dagger \sigma^\mu &= 0 \\ i\partial_\mu \psi_R^\dagger \sigma^\mu &= -m\psi_L^\dagger \end{aligned}$$

Taking complex conjugate:

$$i\bar{\sigma^\mu} \partial_\mu \psi_R = m\psi_L$$

Taking derivative with respect to ψ_L, we can reach at:

$$i\sigma^\mu \partial_\mu \psi_L = m\psi_R$$

So, for massive fermions both right-handed and left-handed chiral states are present in each equation of motion. To avoid these types of chirarility flipping we will consider $m = 0$.
In that case we will get two separate Dirac equation,

$$i\bar{\sigma}^\mu \partial_\mu \psi_R = 0 \quad \text{(Corresponding Lagrangian } \mathcal{L}_R = i\psi_R^\dagger \bar{\sigma}^\mu \partial_\mu \psi_R) \tag{19}$$
$$i\sigma^\mu \partial_\mu \psi_L = 0 \quad \text{(Corresponding Lagrangian } \mathcal{L}_L = i\psi_L^\dagger \sigma^\mu \partial_\mu \psi_L) \tag{20}$$

Since, $\gamma^5 = \begin{bmatrix} -I_2 & 0 \\ 0 & I_2 \end{bmatrix}$ and $\begin{bmatrix} -I_2 & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} -\psi_L \\ \psi_R \end{bmatrix}$ we get the transformation of two component ψ_L and ψ_R as:

$$\begin{aligned} \psi_L &\longrightarrow e^{-iQ\alpha I_2} \psi_L \quad &\text{and} \quad \psi_L^\dagger &\longrightarrow \psi_L^\dagger e^{iQ\alpha I_2} \\ \psi_R &\longrightarrow e^{iQ\alpha I_2} \psi_R \quad &\text{and} \quad \psi_R^\dagger &\longrightarrow \psi_R^\dagger e^{-iQ\alpha I_2} \end{aligned}$$

So, making the gauge parameter α temporarily space-time dependent we can reach to the axial vector current.

- Chiral Current Associated with ψ_R:

$$\begin{aligned}\mathcal{L}'_{\mathcal{R}} &= i\psi_R^\dagger e^{-iQ\alpha I_2}\bar{\sigma}^\mu \partial_\mu (e^{iQ\alpha I_2}\psi_R) \\ &= i\psi_R^\dagger \bar{\sigma}^\mu e^{-iQ\alpha I_2}\left(e^{iQ\alpha I_2}\partial_\mu \psi_R + iQI_2(\partial_\mu \alpha)e^{iQ\alpha I_2}\psi_R\right) \\ &= i\psi_R^\dagger \bar{\sigma}^\mu \partial_\mu \psi_R - Q\psi_R^\dagger \bar{\sigma}^\mu I_2(\partial_\mu \alpha)\psi_R\end{aligned}$$

So, from the invariance of the Lagrangian we can identify $\partial_\mu \alpha$ as the gauge filed A_μ as shown for the vector currents and get the the right-chiral current as

$$j^\mu_{A_R} = -Q\psi_R^\dagger \bar{\sigma}^\mu I_2 \psi_R. \tag{21}$$

- Chiral Current Associated with ψ_L:

$$\begin{aligned}\mathcal{L}'_{\mathcal{L}} &= i\psi_L^\dagger e^{iQ\alpha I_2}\sigma^\mu \partial_\mu (e^{-iQ\alpha I_2}\psi_L) \\ &= i\psi_L^\dagger \sigma^\mu e^{iQ\alpha I_2}\left(e^{-iQ\alpha I_2}\partial_\mu \psi_L - iQI_2(\partial_\mu \alpha)e^{-iQ\alpha I_2}\psi_L\right) \\ &= i\psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + Q\psi_L^\dagger \sigma^\mu I_2(\partial_\mu \alpha)\psi_L\end{aligned}$$

Similar to the right-chiral current we can get the left-chiral current as

$$j^\mu_{A_L} = Q\psi_L^\dagger \sigma^\mu I_2 \psi_L. \tag{22}$$

So, the two types of currents are different.
Now if we consider the 4-component representation of ψ_L and ψ_R:

$$\psi_L = \begin{bmatrix}\psi_L \\ 0 \\ 0\end{bmatrix} \quad \text{and,} \quad \psi_R = \begin{bmatrix}0 \\ 0 \\ \psi_R\end{bmatrix} \quad \text{and,} \quad \gamma^\mu = \begin{bmatrix}0 & \sigma^\mu \\ \bar{\sigma^\mu} & 0\end{bmatrix}$$

Then the transformation will look like,

$$\begin{aligned}\psi_L \longrightarrow e^{-iQ\alpha\gamma^5}\psi_L \quad &and \quad \bar{\psi_L} \longrightarrow \bar{\psi_L} e^{-iQ\alpha\gamma^5} \\ \psi_R \longrightarrow e^{iQ\alpha\gamma^5}\psi_R \quad &and \quad \bar{\psi_R} \longrightarrow \bar{\psi_R} e^{iQ\alpha\gamma^5}\end{aligned}$$

In massless condition the separate Dirac Lagrangian for ψ_L and ψ_R can be written as:

$$\mathcal{L}_L = i\bar{\psi_L}\gamma^\mu\partial_\mu\psi_L \quad \text{and,} \quad \mathcal{L}_R = i\bar{\psi_R}\gamma^\mu\partial_\mu\psi_R$$

Treating similarly as the two component notation and considering the anti-commutation relation between γ^μ and γ^5 we can reach to the two types of current as:

$$j^\mu_{A_R} = -Q\bar{\psi_R}\gamma^\mu\gamma^5\psi_R \tag{23}$$

$$j^\mu_{A_L} = Q\bar{\psi_L}\gamma^\mu\gamma^5\psi_L \tag{24}$$

These are called axial vector current.
From here also we can conclude that in massless condition the chiral rotation differs between two types of current corresponding to left and right-handed Dirac fermions [5, 6, 7, 8].

3.2 Non-Abelian Gauge Theory

The field theory associated with a non-committing local symmetry is termed as Non-abelian gauge theory [5, 6, 7, 8]. We extend our gauge theory calculations to non-abelian cases because we want to understand the dynamics of strong and weak interactions.

Lets consider a fermionic field ψ which transforms under abstract $3-D$ rotation as,

$$\psi \longrightarrow e^{i\alpha^i\frac{\sigma^i}{2}}, \tag{25}$$

where, σ^i are the Pauli-spin matrices.

This (25) can be promoted to a local symmetry by requiring that the Lagrangian be invariant under this transformation for α^i be an arbitrary function of x.

Let ψ be a doublet of Dirac fields,

$$\psi = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

and the transformation can be written as:

$$\psi'(x) = V(X)\psi(x), \qquad \text{where,} \;\; V(x) = exp\left(i\alpha^i(x)\frac{\sigma^i}{2}\right)$$

To construct a Lagrangian that is invariant under this new group of transformation, we must again define a covariant derivative. In local symmetry, we can define the covariant derivative as:

$$n^\mu D_\mu \psi = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left[\psi(x+\epsilon n) - U(x+\epsilon n, x)\psi(x) \right], \tag{26}$$

where, $U(y,x) = U(x+\epsilon n, x)$ is a scalar quantity called the comparator, that compares for the difference in phase transformations from one point to other. It depends on two points and has the transformation law:

$$U(y,x) \longrightarrow V(y)U(y,x)V^\dagger(x) \tag{27}$$

Here, since ψ is a two component object, $U(y,x)$ is a 2×2 matrix.

$$U(y,y) = 1.$$

At points $(x \neq y)$, $U(y,x)$ can be restricted to be an unitary matrix.

Thus any such matrix can be expanded in terms of Hermitian generator of $SU(2)$,

$$U(x+\epsilon n, x) = 1 + ig\epsilon n^\mu A^i_\mu \frac{\sigma^i}{2} + \mathcal{O}(\epsilon^2), \tag{28}$$

where g is a constant and $A - \mu^i$s are vector fields. Inserting this expansion into the definition (26), we get the expression for covariant derivative associated with local $SU(2)$ symmetry.

$$D_\mu = \partial_\mu - igA^i_\mu \frac{\sigma^i}{2} \tag{29}$$

Thus, 3 vector fields are required, one for each generator of the transformation group. The transformation law for A^i_μ can be obtained by inserting the expansion (28) into the transformation law (26),

$$1 + ig\epsilon n^\mu A^i_\mu \frac{\sigma^i}{2} \longrightarrow V(x+\epsilon n)(1 + ig\epsilon n^\mu A^i_\mu \frac{\sigma^i}{2})V^\dagger(x), \tag{30}$$

and from here we can get,

$$A^i_\mu(x)\frac{\sigma^i}{2} \longrightarrow V(x)\left(A^i_\mu(x)\frac{\sigma^i}{2} = \frac{i}{g}\partial_\mu\right)V^\dagger(x)$$

For infinitesimal transformation, expanding $V(x)$ to 1^{st} order of α, we obtain

$$A^i_\mu(x)\frac{\sigma^i}{2} \longrightarrow A^i_\mu(x)\frac{\sigma^i}{2} + \frac{1}{g}(\partial_\mu \alpha^i)\frac{\sigma^i}{2} + i\left[\alpha^i\frac{\sigma^i}{2}, A^i_\mu\frac{\sigma^i}{2}\right] + \ldots \tag{31}$$

Here, the 3^{rd} term is new and is arising from the non-commutativity of the local transformation.

Combining this relation with the infinitesimal form of the fermion transformation,

$$\psi \longrightarrow \left(1 + i\alpha^i \frac{\sigma^i}{2}\right) \psi + \ldots,$$

we can see that the infinitesimal transformation of the covariant derivative,

$$D_\mu \psi \longrightarrow \left(\partial_\mu - igA^i_\mu \frac{\sigma^i}{2} - i(\partial_\mu \alpha^i)\frac{\sigma^i}{2} + g\left[\alpha^i \frac{\sigma^i}{2}, A^i_\mu \frac{\sigma^i}{2}\right]\right)\left(1 + i\alpha^k \frac{\sigma^k}{2}\right)\psi$$

$$\Longrightarrow D_\mu \psi = \left(1 + i\alpha^i \frac{\sigma^i}{2}\right) D_\mu \psi$$

To write a complete Lagrangian, we must also find a gauge invariant term that depends only on A^i_μ. The transformation law of covariant derivative implies:

$$[D_\mu, D_\nu]\,\psi(x) \longrightarrow V(x)\,[D_\mu, D_\nu]\,\psi(x) \tag{32}$$

Using (29) in the expansion of the commutator, we find,

$$[D_\mu, D_\nu] = -igF^i_{\mu\nu}\frac{\sigma^i}{2}$$

where,

$$F^i_{\mu\nu}\frac{\sigma^i}{2} = \partial_\mu A^i_\nu \frac{\sigma^i}{2} - \partial_\nu A^i_\mu \frac{\sigma^i}{2} - ig\left[A^i_\mu \frac{\sigma^i}{2}, A^j_\nu \frac{\sigma^j}{2}\right]$$

This can be simplified using

$$\left[\frac{\sigma^i}{2}, \frac{\sigma^j}{2}\right] = i\epsilon^{ijk}\frac{\sigma^k}{2},$$

and we get,

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk}A^j_\mu A^k_\nu \tag{33}$$

The transformation law for the field strength tensor is,

$$F^i_{\mu\nu}\frac{\sigma^i}{2} \longrightarrow V(x)F^i_{\mu\nu}\frac{\sigma^i}{2}V^\dagger(x) \tag{34}$$

and the infinitesimal form is

$$F^i_{\mu\nu}\frac{\sigma^i}{2} \longrightarrow F^i_{\mu\nu}\frac{\sigma^i}{2} + \left[i\alpha^i \frac{\sigma^i}{2}, F_{\mu\nu^i}\frac{\sigma^j}{2}\right]$$

It is to be noticed that the field strength tensor is not a gauge invariant quantity, since there are now 3 field strength, each associated with a given direction of rotation in the abstract space.

But we can form gauge invariant combinations of the field strength, for example,

$$\mathcal{L} = -\frac{1}{2} Tr \left[\left(F^i_{\mu\nu} \frac{\sigma^i}{2} \right)^2 \right] = -\frac{1}{4} (F^i_{\mu\nu})^2 \tag{35}$$

To construct a theory of such vector fields, interacting with fermions we add the gauge field Lagrangian (35) to the Dirac Lagrangian with the ordinary derivatives replaced by covariant derivatives.

$$\boxed{\mathcal{L} = \bar{\psi}(i\not{D})\psi - \tfrac{1}{4}(F^i_{\mu\nu})^2 - m\psi\bar{\psi}} \tag{36}$$

This is the famous Yang-Mill's Lagrangian.

4 The Standard Model

The Standard Model is a theory of fundamental particles and it discusses about their interactions. This model is based on gauge theories which explain the strong, weak and electromagnetic interactions. The strong interaction is interactions described by $SU(3)$ gauge group and to explain weak interaction we need $SU(2)_L \times U(1)_Y$ gauge groups. The electromagnetic interactions can be explained by $U(1)_{EM}$ which is a sub-group of $SU(2)_L \times U(1)_Y$ [5, 6, 7, 8]. However, after electro-weak symmetry breaking only $U(1)_{EM}$ stays as symmetry of the Lagrangian leaving the gauge boson viz, photon as a massless gauge boson.

This symmetry breaking also introduces much required massive W and Z bosons which were later discovered at LEP experiment. This symmetry breaking requires as electro-weak scalar and in Standard Model it is in spin half representation of $SU(2)_L$. After symmetry breaking only one spin zero excitation is predicted as Higgs boson which was discovered in 2012 at CERN.

4.1 Standard Model as $SU(2)_L \times U(1)_Y$ gauge theory for weak interaction

The electro-weak theory is an unified theory of electromagnetic and weak interactions. Electromagnetic force occurs between two electrically charged particles through the exchange of a massless spin-1 boson (photon). However, the weak interaction is mediated by three massive spin-1 vector bosons namely, $W^{\pm}, Z$ bosons and and it is a short range force. Coupling strength of electromagnetic interaction is $\sim 10^2$ times larger than that of weak interaction. The Standard Model unifies these two forces.

From observation we know that there are three generation of leptons and three generations of quarks. The left- and right-handed fields behave differently under $SU(2)_L \times U(1)_Y$ gauge groups. The left-handed ones are in spin-$\frac{1}{2}$ (doublet) and right-handed ones are singlet representation of $SU(2)_L$. Both of them carry different hypercharges.

The three generations of lepton fields in Standard Model are given below :

$$L \equiv \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \qquad \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}, \qquad \begin{pmatrix} \nu_\tau \\ \tau_L, \end{pmatrix}$$

,

$$R \equiv e_R, \qquad \mu_R, \qquad \tau_R.$$

Similarly, we can write three generations of quark fields as given below:

$$Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \qquad \begin{pmatrix} t_L \\ b_L, \end{pmatrix}$$

$$U_R \equiv u_R, \qquad c_R, \qquad t_R,$$

$$D_R \equiv d_R, \qquad s_R, \qquad b_R.$$

4.1.1 $SU(2)_L \times U(1)_Y$ interactions

In this subsection we derive the interaction vertices of the leptonics fields under $SU(2)_L \times U(1)_Y$ gauge group. From $SU(2)_L \times U(1)_Y$ we also derive the rotation angle which give rise to photon and Z boson.

The $SU(2)_L$ is a non-abelian gauge group which has three generators represented by Pauli spin matrices

$$T^a = \tfrac{\sigma^a}{2}.$$

The corresponding vector gauge bosons are written as real W_μ^a, [where a=1, 2, 3]. On the other hand the weak hypercharge sub-group $U(1)_Y$ has identity as generator and the real component field is given by the vector boson B_μ.

The transformation of left- and right-handed $SU(2)_L \times U(1)_Y$ transformations are given as:

$$L \longrightarrow L' = e^{ig_2 T.\alpha(x) + ig_1 \frac{Y_L}{2}\beta(x)} L \tag{37}$$

$$R \longrightarrow R' = e^{ig_1 \frac{Y_R}{2}\beta(x)} R, \tag{38}$$

where Y_L and Y_R are the respective hypercharges for the left- and right-handed leptons. According to Gell-Mann-Nishijima formula $Q = T_3 + \frac{Y}{2}$, we have $Y_L = -1$ and $Y_R = -2$. The weak hypercharge is different for left-handed and right-handed fermions, but it is same for the doublets. The covariant derivatives acting on the lepton fields are given below:

$$\begin{aligned} D_\mu \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} &= \left(\partial_\mu + ig_1 B_\mu \frac{Y_L}{2} + ig_2 W_\mu^a T^a\right) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \\ &= \partial_\mu \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} + i\left(g_1 \frac{Y_L}{2} \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} + \frac{g_2}{2} \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}\right) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \end{aligned} \tag{39}$$

and

$$D_\mu e_R = \left(\partial_\mu + ig_1 B_\mu \frac{Y_R}{2}\right) e_R. \tag{40}$$

The transformation laws for the left- and right-handed charged leptons and neutrinos are obtained as:

$$D_\mu \nu_e = \partial_\mu \nu_e + i\Big(g_1 \frac{Y_L}{2} B_\mu + \frac{g_2}{2} W_\mu^3\Big)\nu_e + i\frac{g_2}{2}\Big(W_\mu^1 - iW_\mu^2\Big)e_L, \tag{41}$$

$$D_\mu e_L = \partial_\mu e_L + i\Big(g_1 \frac{Y_L}{2} B_\mu - \frac{g_2}{2} W_\mu^3\Big)e_L + i\frac{g_2}{2}\Big(W_\mu^1 + iW_\mu^2\Big)\nu_e, \tag{42}$$

$$D_\mu e_R = \partial_\mu e_R - ig_1 \frac{Y_R}{2} B_\mu e_R. \tag{43}$$

In order to conserve charge, the covariant derivative of a field must carry the same electric charge as the field itself. So, $W_\mu^1 - iW_\mu^2$ must carry electric charge $+1$ and $W_\mu^1 + iW_\mu^2$ carry electric charge -1. We define,

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \tag{44}$$

The interaction part of the Lagrangian becomes

$$
\begin{aligned}
& g_2\bar{L}\gamma^\mu\left(\frac{\sigma^a}{2}\right)LW_\mu^a + g_1\frac{Y_L}{2}(\bar{L}\gamma^\mu L)B_\mu + g_1\frac{Y_R}{2}(\bar{e_R}\gamma^\mu e_R)B_\mu \\
= \; & \frac{g_2}{2}\left(\left(\bar{\nu_L}\gamma^\mu\nu_L\right)W_\mu^3 + \left(\bar{\nu_L}\gamma^\mu e_L\right)W_\mu^+ + \left(\bar{e_L}\gamma^\mu\nu_L\right)W_\mu^- - \left(\bar{e_L}\gamma^\mu e_L\right)W_\mu^3\right) \\
& + \frac{g_1}{2}Y_L\left(\bar{\nu_L}\gamma^\mu\nu_L\right)B_\mu + \frac{g_1}{2}Y_L\left(\bar{e_L}\gamma^\mu e_L\right)B_\mu + \frac{g_1}{2}Y_R\left(\bar{e_R}\gamma^\mu e_R\right)B_\mu \qquad (45)
\end{aligned}
$$

The vector bosons B_μ and W_μ^3 are both electrically neutral. As a result of spontaneous symmetry breaking $\Big(SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}\Big)$ they will mix. Here the well defined mass eigen state is not B_μ and W_μ^3 but their orthogonal linear combination (one is massless photon field A_μ and other is massive Z boson vector field Z_μ). Hence, the relation between gauge eigen state and mass eigen states is obtained by the rotation matrix defined by the Weinberg angle (θ_W) as given below:

$$
\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}
$$

$$
\begin{aligned}
A_\mu &= B_\mu\cos\theta_W + W_\mu^3\sin\theta_W & \qquad B_\mu &= A_\mu\cos\theta_W - Z_\mu\sin\theta_W \\
Z_\mu &= -B_\mu\sin\theta_W + W_\mu^3\cos\theta_W & \qquad W_\mu^3 &= A_\mu\sin\theta_W - Z_\mu\cos\theta_W
\end{aligned}
$$

The electroweak neutral current becomes:

$$
\begin{aligned}
& \left[\left(\frac{g_2}{2}\sin\theta_W - \frac{g_1}{2}\cos\theta_W\right)(\bar{\nu_\mu}\gamma^\mu\nu_\mu) - \left(\frac{g_2}{2}\sin\theta_W + \frac{g_1}{2}\cos\theta_W\right)(\bar{e_L}\gamma_\mu e_L) - (g_1\cos\theta_W)(\bar{e_R}\gamma_\mu e_R)\right] A_\mu \\
+ & \left[\left(\frac{g_2}{2}\cos\theta_W + \frac{g_1}{2}\sin\theta_W\right)(\bar{\nu_\mu}\gamma^\mu\nu_\mu) - \left(\frac{g_2}{2}\cos\theta_W - \frac{g_1}{2}\sin\theta_W\right)(\bar{e_L}\gamma_\mu e_L) + (g_1\sin\theta_W)(\bar{e_R}\gamma_\mu e_R)\right] Z_\mu
\end{aligned}
$$

The 1^{st} term is electromagnetic interaction. So, the coefficient of neutrino current is zero and the coefficient of electric current is electric charge e.
So,

$$
\begin{aligned}
& \frac{g_2}{2}\sin\theta_W - \frac{g_1}{2}\cos\theta_W = 0 \quad and \quad \frac{g_2}{2}\sin\theta_W + \frac{g_1}{2}\cos\theta_W = e \\
& \Longrightarrow g_2\sin\theta_W = g_1\cos\theta_W = e \\
& \Longrightarrow \sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}} \quad and \quad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}}
\end{aligned}
$$

So, the mixing angle is given by the ratio of the two independent group coupling constant, $\tan\theta_W = \frac{g_1}{g_2}$

4.1.2 Electro-weak symmetry breaking and Higgs Mechanism in SM

The central question of electroweak physics is :"Why are the W and Z boson masses non-zero?" The measured values, $M_W = 80GeV$ and $M_Z = 91GeV$, are far from zero and cannot be considered as small effects. The answer is inside the electro-weak symmetry breaking and Higgs mechanism. In the case of Higgs phenomenon, the vacuum expectation value (vev) of the Higgs field plays the role of mass generator of weak force carriers. Quarks and leptons gain mass through the Yukawa term after EWSB. By Higgs mechanism all the fundamental particles get mass but photons and gluons remain massless as $U(1)$ and $SU(3)_c$ remain as the symmetry of vacuum.

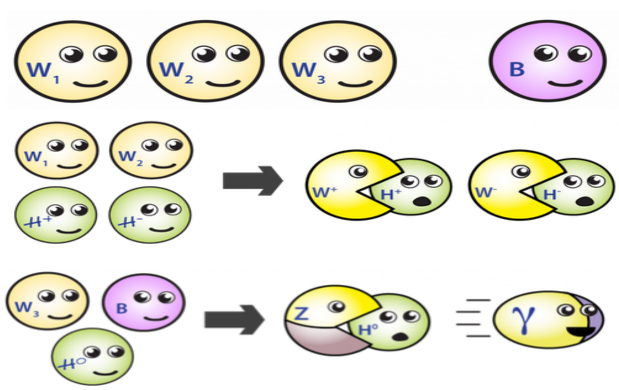


Figure 1:

The Lagrangian of the complex scalar field i.e. Higgs field is given by:

$$\mathcal{L} = (D^\mu \phi)^\dagger (D_\mu \phi) - \mu^2 |\phi^\dagger \phi| - \lambda (|\phi^\dagger \phi|)^2 \tag{46}$$

Here, ϕ is $SU(2)$ doublet, $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

It couples to the gauge boson in a gauge invariant way, and the covariant derivative involving the gauge-fields is given by:

$$D_\mu = \partial_\mu - ig_1 B_\mu \frac{Y_L}{2} - ig_2 W^a_\mu \sigma^a \tag{47}$$

Here, Y is the hypercharge, σ^a are the Pauli matrices (generators of $SU(2)_L$), W^a_μ are the gauge field corresponding to $SU(2)_L$) and B_μ is the gauge field corresponding to $U(1)_Y$, Higgs field gets non-zero vev at the minimum of the potential,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}, \qquad \text{where,} < v >= \sqrt{-\frac{\mu^2}{2\lambda}}$$

After EWSB the potential becomes a Mexican hat potential where the minima of the field is given by a circle in the complex plane and acquire a non-zero vev. If we choose a point in the minima and look around the potential is no more symmetric. Now we look for fluctuation around the minima i.e. we expand the potential around the minima. The field ϕ here is written in Unitary gauge, where we remove the Goldstone bosons i.e the charged part ϕ^+ gives the pair of charged Glodstones $W^\pm$ and the complex part of the neutral component becomes the neutral Glodstone [5, 6, 7, 8].

From the kinetic part of the scalar Lagrangian we get,

$$(D^\mu\phi)^\dagger(D_\mu\phi) = \frac{1}{2}\partial^\mu h\partial_h + \frac{(v+h)^2}{4}\left[g_2^2 W_\mu^+ W^{\mu-} + \frac{1}{2}\left(g_2 W_\mu^3 - g_1 B_\mu\right)\left(g_2 W^{\mu 3} + g_1 B^\mu\right)\right] \tag{48}$$

From here we can write the contribution of mass in the matrix form (in the basis of W_1, W_2, W_3 and B) as:

$$M_\phi^2 = \frac{v_\phi^2}{4} = \begin{pmatrix} g_2^2 & 0 & 0 & 0 \\ 0 & g_2^2 & 0 & 0 \\ 0 & 0 & g_2^2 & -g_1 g_2 \\ 0 & 0 & -g_1 g_2 & g_1^2 \end{pmatrix}$$

This 4×4 mixing matrix is already diagonalised in the above 2×2 block and the lower 2×2 block is non-diagonal, which tells there is a mixing between W_μ^3 and B_μ. They are diagonalised by weak mixing angle (Weinberg angle) as:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ \sin\theta_W & -\cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

Where, $\sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}$ and $\cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}}$ was discussed above.

This diagonalisation gives the Z boson and massless photon eigenstate. In terms of W^+, W^- and Z we can write the kinetic part as:

$$(D^\mu\phi)^\dagger(D_\mu\phi) = \frac{1}{2}\partial^\mu h\partial_h + \frac{(v+h)^2}{4}\left[g_2^2 W_\mu^+ W^{\mu-} + \frac{1}{2}\left(g_2^2 + g_1^2\right) Z^\mu Z_\mu\right] \tag{49}$$

The masses of $W^{\pm}$ and Z boson are given as:

$$M_Z^2 = \frac{(g_2^2 + g_1^2)\, v^2}{4} \tag{50}$$

$$M_W^2 = \frac{g_2^2 v^2}{4} \tag{51}$$

$$\rho = \frac{M_W^2}{\cos^2\theta_W M_Z^2} \tag{52}$$

The electro-weak parameter ρ is precisely measured experimentally and is:

$$\rho = 1,0004 \pm 0.00024$$

The potential part of the scalar field is

$$V(\phi^\dagger\phi) = \mu^2|\phi^\dagger\phi| + \lambda(|\phi^\dagger\phi|)^2 \qquad \text{Before, EWSB} \tag{53}$$

$$V(\phi^\dagger\phi) = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4}h^2 \qquad \text{After, EWSB} \tag{54}$$

The 1^{st} term is the mass term for the Higgs field, It is given by $M_h^2 = 2\lambda v^2$. This is the standard model Higgs boson discovered at the LHC around 125 GeV.

The Higgs potential and vacuum both respects the electromagnetic symmetry and after EWSB, the gauge group becomes :

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$$

The mass of fermions in the standard model is generated by Yukawa term.The guiding principle for any term in Lagrangian is that it should be real, gauge invariant and Lorentz invariant. The Yukawa term follows all these principles and the minimal requirement for this term to be gauge invariant is a $SU(2)$ Higgs doublet.

In the Standard Model, we have three families of fermions which are shown as:

$$l_R^j = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad l_L^j = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \qquad \nu_i = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$d_R^j = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad d_L^j = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \qquad u_L^j = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \qquad u_R^j = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

So, the most general Yukawa term is written as:

$$L_{e,\mu,\tau} = -\begin{pmatrix} \bar{\nu_e}^i & \bar{l_L}^i \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y^j_{ei} l^j_R + h.c. \tag{55}$$

$$L_{d,s,b} = -\begin{pmatrix} \bar{u_L}^i & \bar{d_L}^i \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y^j_{di} d^j_R + h.c. \tag{56}$$

$$L_{u,c,t} = -\begin{pmatrix} \bar{u_L}^i & \bar{d_L}^i \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix} y^j_{ui} u^j_R + h.c. \tag{57}$$

where the i, j indices run over the three families.

In general, the Yukawa couplings y^j_{ei}, y^j_{di}, y^j_{ui} are complex 3×3 matrices in gauge basis. After EWSB in unitary gauge the Yukawa term is written as:

$$\mathcal{L} = -\left(1 + \frac{h}{v}\right)\left(\bar{l'}^i_L m^j_{ei} l'_{Rj} + \bar{d'}^i_L m^j_{di} d'_{Rj} + \bar{u'}^i_L m^j_{ui} u'_{Rj}\right) + h.c. \tag{58}$$

where,

$$m^j_{fi} = \frac{v}{\sqrt{2}} y^j_{fi}$$

Now redefining the fields, we go in the mass basis where these matrices are diagonalised as:

$$\begin{array}{lll} l'_{Li} = L^j_{Li} l_{Lj}; & d'^j_{Li} = D^j_{Li} d_{Lj}; & u'_{Li} = U^j_{Li} u_{Lj}; \\ l'_{Ri} = L^j_{Ri} l_{Rj}; & d'^j_{Ri} = D^j_{Ri} d_{Rj}; & u'_{Ri} = U^j_{Ri} u_{Rj}; \end{array}$$

After diagonalisation the matrices become :

$$L_L \dagger m_e l_R = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$$D_L \dagger m_d D_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$$U_L \dagger m_u U_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

and the Yukawa term becomes,

$$L_{quarks} = -\left(1+\frac{h}{v}\right)\left(\bar{u_L}m_u u_R + \bar{c_L}m_c c_R + \bar{t_L}m_t t_R + \bar{d_L}m_d d_R + \bar{s_L}m_s s_R + \bar{b_L}m_b b_R\right) \tag{59}$$

$$L_{leptons} = -\left(1+\frac{h}{v}\right)\left(\bar{e_L}m_e e_R + \bar{\mu_L}m_\mu \mu_R + \bar{\tau_L}m_\tau \tau_R\right) \tag{60}$$

The first term in both the above expressions is the mass term for quarks and leptons while the second term represents the interaction of Higgs with fermions.

5 Drell-Yan cross-section

The production of a massive lepton pair via an intermediate $Z-$ boson or virtual photon γ^*, by the annihilation of a quark-anti quark pair (or, lepton-anti-lepton pair), $q\bar{q} \longrightarrow Z/\gamma^* \longrightarrow l\bar{l}$, is called Drell-Yan process [5, 6, 7, 8].

- Differential scattering cross-section of $e^+e^- \longrightarrow \mu^+\mu^-$ (mediated by γ^*):

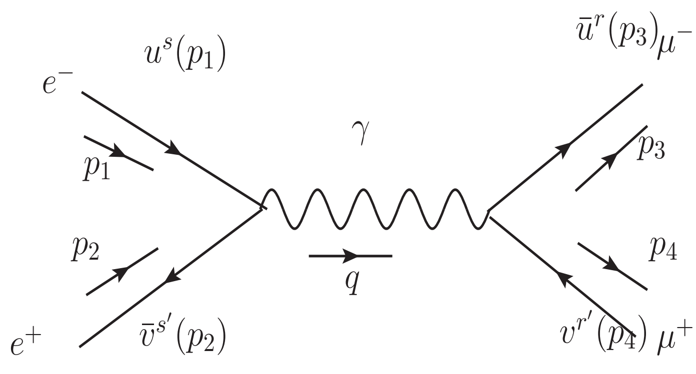


Figure 2: $e^+e^- \to \mu^+\mu^-$ scattering via photon mediation.

The amplitude is given by:

$$i\mathcal{M} = \Big(\bar{v}^{s'}_{\alpha}(p_2)\big(-ie\gamma^{\mu}_{\alpha\beta}\big)u^{s}_{\beta}(p_1)\Big)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\Big(\bar{u}^{r}_{\delta}(p_3)\big(-ie\gamma^{\nu}_{\delta\sigma}\big)v^{r'}_{\sigma}(p_4)\Big)$$

$$\implies \mathcal{M} = \frac{e^2}{q^2}\big(\bar{v}(p_2)\gamma^{\mu}u(p_1)\big)\big(\bar{u}(p_3)\gamma_{\mu}v(p_4)\big) \tag{61}$$

Considering, $(\bar{v}\gamma^{\mu}u)^{\dagger} = \bar{u}\gamma^{\mu}v$ we have,

$$\mathcal{M}^{\dagger} = \frac{e^2}{q^2}\big(\bar{v}(p_4)\gamma_{\mu}u(p_3)\big)\big(\bar{u}(p_1)\gamma^{\mu}v(p_2)\big) \tag{62}$$

$$\therefore |\mathcal{M}|^2 = \frac{e^4}{q^4}\Big(\big(\bar{v}(p_2)\gamma^{\mu}u(p_1)\big)\big(\bar{u}(p_1)\gamma^{\nu}v(p_2)\big)\Big)\Big(\big(\bar{u}(p_3)\gamma_{\mu}v(p4)\big)\big(\bar{v}(p_4)\gamma_{\nu}u(p_3)\big)\Big) \tag{63}$$

[$\because$ each terms are numbers, we can rearrange them.]

Now, $|\bar{\mathcal{M}}^2| = \frac{1}{2}\sum_s \frac{1}{2}\sum_{s'}\sum_r \sum_{r'} |\mathcal{M}^2|$

Considering,

$$\sum_s u^s_{\beta}(p_1)\bar{u}^s_{\delta}(p_1) = \sum_s \bar{u}^s_{\beta}(p_1)u^s_{\delta}(p_1) = (\not{p}_1 + m_{\mu})_{\beta\delta} \tag{64}$$

$$\sum_{s'} v^{s'}_{\alpha}(p_3)\bar{v}^{s'}_{\sigma}(p_3) = \sum_{s'} \bar{v}^{s'}_{\alpha}(p_3)v^{s'}_{\sigma}(p_3) = (\not{p}_3 - m_{\mu})_{\alpha\sigma} \tag{65}$$

we get,

$$\begin{aligned}
&\sum_s \sum_{s'} \Big(\bar{v}^{s'}_{\alpha}(p_2)\gamma^{\mu}_{\alpha\beta}u^s_{\beta}(p_1)\Big)\Big(\bar{u}^s_{\delta}(p_1)\gamma^{\nu}_{\delta\sigma}v^{s'}_{\sigma}(p_2)\Big)\\
&= (\not{p}_2 - m_e)_{\sigma\alpha}\gamma^{\mu}_{\alpha\beta}(\not{p}_1 + m_e)_{\beta\delta}\gamma^{\nu}_{\delta\sigma}\\
&= Tr\Big[(\not{p}_2 - m_e)\gamma^{\mu}(\not{p}_1 + m_e)\gamma^{\nu}\Big]\\
&= p_2^{\rho}p_1^{\sigma}Tr\Big[\gamma^{\rho}\gamma^{\sigma}\gamma^{\mu}\gamma^{\nu}\Big] - m_{\mu}^2 Tr\Big[\gamma^{\mu}\gamma^{\nu}\Big], \qquad \text{(using trace identities)}\\
&= 4\Big(p_2^{\mu}p_1^{\nu} + p_1^{\mu}p_2^{\nu} - p_1^{\sigma}p_2^{\sigma}g^{\mu\nu}\Big) - 4m_e^2 g^{\mu\nu}
\end{aligned} \tag{66}$$

Similarly,

$$
\begin{aligned}
&\sum_r \sum_{r'} \left(\bar{u}^r_\alpha(p_3)\gamma_{\alpha\beta\mu} v^{r'}_\beta(p_4)\right)\left(\bar{v}^{r'}_\delta(p_4)\gamma_{\delta\sigma\nu} u^r_\sigma(p_3)\right)\\
&= Tr\Big[(\not{p}_3 + m_\mu)\gamma_\mu(\not{p}_4 - m_\mu)\gamma_\nu\Big]\\
&= 4\Big(p_3^\mu p_4^\nu + p_4^\mu p_3^\nu - p_3^\rho p_4^\rho g^{\mu\nu}\Big) - 4m_\mu^2 g^{\mu\nu}, \qquad \text{(using trace identities)}
\end{aligned} \tag{67}
$$

So,

$$
\begin{aligned}
|\bar{\mathcal{M}}|^2 &= \frac{1}{4} \times \frac{8e^4}{q^4}\left(p_{23}p_{14} + p_{13}p_{24} + m_\mu^2 p_{12} + m_e^2 p_{34} + 2m_e^2 m_\mu^2\right)\\
&= \frac{1}{4} \times \frac{8e^4}{q^4}\left(p_{23}p_{14} + p_{13}p_{24} + m_\mu^2 p_{12}\right), \qquad (\because m_\mu >> m_e\ )
\end{aligned} \tag{68}
$$

From the Feynman diagram we can write,

$$
\begin{aligned}
q^2 = (p_1 + p_2)^2 = 4E^2; &\qquad p_{12} = 2E^2;\\
p_{13} = p_{24} = E^2 - E|\vec{p}|\cos\theta; &\qquad p_{14} = p_{23} = E^2 + E|\vec{p}|\cos\theta;
\end{aligned}
$$

Which gives,

$$
|\bar{\mathcal{M}}|^2 = e^4\left[\left(1 + \frac{m_\mu^2}{E^2}\right) + \left(1 - \frac{m_\mu^2}{E^2}\right)\cos^2\theta\right] \tag{69}
$$

The differential cross-section is:

$$
d\sigma = \frac{|\bar{\mathcal{M}}|^2\ dQ}{F} \tag{70}
$$

$$
\begin{aligned}
\text{Where, } F &= \text{Incoming flux}\\
&= 2E_1\ 2E_2\ |\vec{v_1} - \vec{v_2}|\\
&= 4\left((p_1.p_2)^2 - m_e^2 m_\mu^2\right)^{\frac{1}{2}}\\
&= 4|\vec{p_i}|\sqrt{s}, \quad \left(\text{where, Mandelstam variable, } s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = E_{cm}^2\right)
\end{aligned} \tag{71}
$$

$$
\begin{aligned}
dQ &= \text{ Differential phase space, for two particle in the final state}\\
&= (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4)\frac{d^3p_3}{(2\pi)^3}\frac{1}{2E_3}\frac{d^3p_4}{(2\pi)^3}\frac{1}{2E_4}
\end{aligned} \tag{72}
$$

Using $\delta-$function, integrating over p_4 we have:

$$\begin{aligned}
dQ &= \frac{1}{16\pi^2} d\Omega \int dp_f \frac{p_f^2}{E_3 E_4} \delta(E_3 + E_4 - E_{cm}), \quad (\text{where, } p_f = |\vec{p_3}| = |\vec{p_4}|) \\
&= \frac{1}{16\pi^2} d\Omega \frac{p_f}{E_{cm}} \Theta(E_{cm} - m_e - m_\mu) \\
&= \frac{1}{16\pi^2} \frac{p_f}{Ecm} d\Omega, \quad (\because E_{cm} >> m_e + m_\mu)
\end{aligned} \tag{73}$$

So,

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{e^4}{64\pi^2 E_{CM}^2} \frac{|\vec{p_f}|}{|\vec{p_i}|} \left[\left(1 + \frac{m_\mu^2}{E^2}\right) + \left(1 - \frac{m_\mu^2}{E^2}\right) \cos^2\theta \right] \tag{74}$$

For ultra-high energy limit $E >> m_\mu$, so $m_\mu \approx 0$.
We know $|\vec{p_i}| = \sqrt{E^2 + m_e^2}$ and, $|\vec{p_f}| = \sqrt{E^2 + m_\mu^2}$. So, $\frac{|\vec{p_f}|}{|\vec{p_i}|} \approx 1$
So,

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{CM} = \frac{e^4}{64\pi^2 E_{CM}^2} (1 + \cos^2\theta)} \tag{75}$$

6 Two body decays

$$(i) \longrightarrow (1) + (2)$$

The initial and final particle $4-$momenta are $p = (E, \vec{p})$ and $p_{f=1,2} = (E_f, \vec{p_f})$ respectively.
The decay width is given by:

$$\Gamma_{1\to 2} = (2\pi)^4 \frac{1}{2M} \int \delta^{(4)}(p_1 + p_2 - p) |\mathcal{M}|^2 \frac{d^3\vec{p_1}}{(2\pi)^2 2E_1} \frac{d^3\vec{p_2}}{(2\pi)^2 2E_2}$$

If the initial particle is in rest frame then $\Longrightarrow$
$\vec{p_1} + \vec{p_2} = 0; \quad p = (M, 0); \quad p_1 = (\sqrt{|p'^2| + m_1^2}, \vec{p'}); \quad p_2 = (\sqrt{|p'^2| + m_2^2}, -\vec{p'})$

Integrating over $\vec{p_2}$ and considering the spherical polar co-ordinate, $d^3\vec{p'} = p'^2 dp' d\Omega$ we have $\rightarrow$

$$\Gamma_{1\to 2} = \frac{1}{32\pi^2 M}\int d\Omega \int_0^\infty \frac{dp'\ \delta\left(\sqrt{|p'^2| + m_1^2} + \sqrt{|p'^2| + m_2^2} - M\right)}{\sqrt{|p'^2| + m_1^2}\sqrt{|p'^2| + m_2^2}} p'^2 |\mathcal{M}| \tag{76}$$

Changing the integration variable $p \longrightarrow E$ we have:

$$\begin{aligned}
\Gamma_{1\to 2} &= \frac{1}{32\pi^2 M}\int d\Omega \int_{m_1+m_2}^\infty \frac{dE}{E}\delta\left(E - M\right) p' |\mathcal{M}^2| \\
&= \frac{|\vec{p'}|}{32\pi^2 M^2}\int d\Omega |\mathcal{M}^2|, \quad \text{if, } M > m_1 + m_2 \\
&= 0, \quad \text{Otherwise}
\end{aligned}$$

If we assume that the initial particle is spin less, $|\mathcal{M}^2|$ doesn't depend on solid angle. Then,$\int d\Omega = 4\pi$, and,

$$\boxed{\Gamma_{1\to 2} = \frac{|\vec{p'}|}{8\pi M^2}|\mathcal{M}^2|} \tag{77}$$

If some scalar particle decays to fermion-anti-fermion pair then, let $m_1 = m_2 = m$ and, $|\mathcal{M}^2| = 4(p_1.p_2 - m^2) = 2(M^2 - 4m^2)$, so,

$$\Gamma_{1\to 2} = \frac{1}{8\pi M^2}\left(M^2 - 4m^2\right)^{\frac{3}{2}} \tag{78}$$

The lifetime is the inverse of Γ [5, 6, 7, 8].

7 Right-handed neutrinos as Majorana fermion:

Standard Model neutrinos are basically left-handed and massless also, but the evidence of neutrino flavor changing implies neutrino mass. If we assume that beside the usual left-handed neutrinos ν_l, there are right-handed neutrinos ν_R (which is not compatible with SM), then one can write the Dirac mass term for neutrinos,

$$\mathcal{L}^D_{mass} = m_D \bar{\nu_R} \nu_L + h.c., \tag{79}$$

where, *h.c.* denotes the hermitian conjugate [3, 9, 10].

We know that neutrinos have no electric charge. Hence, we can tell that the neutrino mass terms are of two different kinds: *Dirac* and *Majorana.* Dirac mass term turns a neutrino to a neutrino or an anti-neutrino into an anti-neutrino, while a Majorana mass term converts a neutrino to an anti-neutrino or vice-versa.

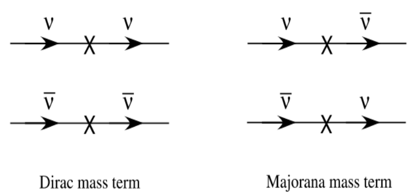


Figure 3: The effects of Dirac and Majorana mass terms. The action of the mass terms is represented by the symbol X

Dirac mass term conserve lepton number L that distinguishes leptons from anti-leptons, while the Majorana mass terms do not. The quantum number L is also conserved by the Standard Model couplings of neutrinos to other particles. Thus if we observe any L non conserve process, that would arise from Majorana mass terms.

A Majorana mass term may be constructed out of ν_L alone, in which case we have the *Left-handed Majorana mass*,

$$\mathcal{L}^L_{mass} = \frac{1}{2} m_L \nu^c_L \nu_L + h.c., \tag{80}$$

or out of ν_R alone, in which case we have the *Right-handed Majorana mass*,

$$\mathcal{L}^R_{mass} = \frac{1}{2} m_R \nu^c_R \nu_R + h.c. \tag{81}$$

Here, m_D, m_L and m_R are the mass parameters.
For any field ψ, ψ^c is the corresponding charge-conjugate field. In terms of

ψ, $\psi^c = C\bar{\psi}^T$, where C is the charge conjugation matrix (One of the representation of charge conjugation matrix is $i\gamma_2\gamma_0$) and T denotes transposition.

The electrically charged fermions can not have Majorana mass term, because such a term would convert it into an anti-quark, which will violate the electric charge conservation. In this case the mass terms are of Dirac type and arise from *Yukawa coupling* of the form,

$$-Y_q\bar{q_L}\phi q_R + h.c..$$

Here, q is some quark, ϕ is the neutral Higgs field, Y_q is Yukawa coupling constant.

When ϕ develops a vacuum expectation value $< v >$, the coupling yields a term,

$$-Y_q\frac{v}{\sqrt{2}}\bar{q_L}q_R + h.c..$$

So, the Dirac mass term for the quark q is $Y_q\frac{v}{\sqrt{2}}$.

In Standard Model if we want to include a mass term for neutrino then we have to extend the Standard Model by adding a Right-handed neutrino ν_R and a Yukawa coupling $-Y_\nu\bar{\nu_L}\phi\nu_R + h.c.$
Similarly after developing the vacuum expectation value the Lagrangian of Dirac mass term of neutrino would be

$$\mathcal{L}_D = -Y_\nu\frac{v}{\sqrt{2}}\bar{\nu_L}\nu_R + h.c. \tag{82}$$

This term indicates the neutrino mass $m\nu = Y_\nu\frac{v}{\sqrt{2}}$.

If we would like m_ν to be order of $0.05eV$, (Suggested from neutrino oscillation experiment); $\frac{v}{\sqrt{2}} = 174GeV$, then the coupling must be of the order 10^{-13}. Such an infinitesimally small coupling constant is quiet unnatural.

In Standard Model the right-handed fermion fields are weak-isospin singlet. Since, SM neutrinos are electrically charge neutral, we can add Majorana mass term here, which does not violet the weak-isospin and electric charge. If in Standard Model Dirac mass term doesn't exit then the source of neutrino mass would be the Majorana mass term only.

If the neutrino has Dirac mass and as well as the Majorana mass, then the Lagrangian of the total mass term will be,

$$\mathcal{L}_{m_\nu} = -m_D\bar{\nu_R}\nu_L - \frac{1}{2}m_R\bar{\nu_R^c}\nu_R + h.c. \tag{83}$$

(Left handed Majorana term is forbidden but the SM gauge symmetry because ν_L possesses non-zero isospin and hypercharge).

If we use the identity $\nu_L^c = \nu_R$, then $\bar{\nu_L^c} m_D \nu_R^c = \bar{\nu_R} m_D \nu_L$. This implies,

$$\mathcal{L}_{m_\nu} = \frac{1}{2} \begin{pmatrix} \bar{\nu_L^c} & \bar{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

Here the neutrino mass matrix is given by:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}$$

The Dirac mass m_D of neutrino can not be high, because, in SM all the Dirac masses come from couplings to the same Higgs field (though the Yukawa coupling is not same, but it will not affect much). But it is not required that m_R should be small. We can take $m_R >> m_D$.
The mass matrix can be diagonalised by the transformation

$$S^T M_\nu S = D_\nu,$$

where $D_\nu = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$ is a diagonal matrix whose diagonal elements are the positive-definite eigen value of M_ν. Those are $\longrightarrow$

$$\begin{aligned} m_{1,2} &= \left| \frac{1}{2} \left(m_R \mp \sqrt{m_R^2 + 4m_D^2} \right) \right| \\ &= \left| \frac{1}{2} \left(m_R \mp m_R \sqrt{1 + \frac{4m_D^2}{m_R^2}} \right) \right| \\ &= \left| \frac{1}{2} m_R \left(1 \mp \left(1 + \frac{2m_D^2}{m_R^2} \right) \right) \right| \end{aligned} \tag{84}$$

So,

$$m_1 = \frac{1}{2} m_R \frac{2m_D^2}{m_R^2} = \frac{m_D^2}{m_R} \tag{85}$$

$$m_2 = \frac{1}{2} m_R \left(2 + \frac{2m_D^2}{m_R^2} \right) = m_R \left(1 + \frac{m_D^2}{m_R^2} \right) \approx m_R \tag{86}$$

So,

$$D_\nu = \begin{pmatrix} \frac{m_D^2}{m_R} & 0 \\ 0 & m_R \end{pmatrix}$$

So, we have a neutrino mass at a scale m_R of new physics and a very light neutrino, the mass of which is $\frac{m_D^2}{m_R}$

8 Extension of Standard Model by a Right-handed neutrino Type-I seesaw

We know that two doublets can be decomposed into a triplet and a singlet ($2 \otimes 2 = 3 \oplus 1$). Hence, the left-handed leptons and the Higgs (they both are $SU(2)$ doublet) of SM can couple to a triplet and a singlet [3, 9, 10].

The 1^{st} type of the seesaw mechanism couples the lepton and the Higgs fields via the exchange of a heavy virtual fermion N_R, which is a singlet under all SM gauge group.

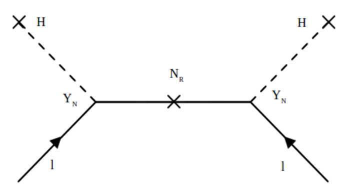


Figure 4: Type-I Seesaw Model

After electro-weak symmetry breaking of Higgs field, the neutrinos will get mass, The Lagrangian of the total system is given by:

$$\mathcal{L}^N = \underbrace{i\bar{N}_R \partial\!\!\!/ N_R}_{\text{kinetic term}} - \underbrace{\frac{1}{2}\bar{N}_R M_N N_R^c}_{\text{Majorana mass term}} - \underbrace{Y_N \bar{L}\tilde{\phi} N_R}_{\text{Yukawa interactions}} + h.c. \tag{87}$$

The covariant derivative D_μ can be replaced by ∂_μ, because the added heavy right-handed neutrino is SM singlet and do not interact with the gauge field.

Due to the additional interactions, the Yukawa couplings of the leptons will be,

$$\mathcal{L}_Y = Y_e \bar{L} \phi e_R + Y_N \bar{L} \tilde{\phi} N_R + h.c. \tag{88}$$

The total weak hypercharge $'Y'$ and the electric charge $'Q'$ has to be zero for the Lagrangian. Using the *Gell-Mann-Nishijima* formula $Q = T_3 + \frac{Y}{2}$ and the definition of $\tilde{H}$ as:

$$\tilde{H} = \begin{pmatrix} H_0^* \\ -H_- \end{pmatrix} = i\sigma_2 H^* = i\sigma_2 \begin{pmatrix} H_+ \\ H_0 \end{pmatrix}^* ;$$

we can get the following table:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
L	1	2	-1
e_R	1	1	-2
H	1	2	+1
$\tilde{H}$	1	2	-1
N_R	1	1	0

Table 1:

8.1 Decays of right-handed neutrinos:

There exits three type of predominant decay mode of SM singlet right-handed neutrino, whose strength is dictated by the neutral Dirac Yukawa couplings, as follow:

- $N_R \longrightarrow h\nu$

  Here we consider, M_N is the mass of right-handed neutrino, m_h is the mass of Higgs, m_n is the mass of SM neutrino ≈ 0
  Since, the right-handed neutrino is in rest frame, $\vec{p_1} = \vec{p_2} + \vec{p_3} = \vec{0}$, but $|\vec{p_2}| = |\vec{p_3}| = |\vec{p}|$

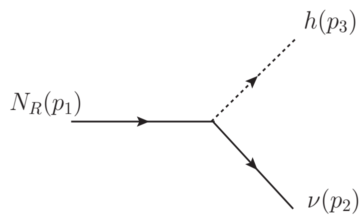


Figure 5: Decay of right handed neutrinos to Higgs boson and neutrino.

The interaction vertex between ν, N_R and $\tilde{H}$ is iY_N. The matrix element can be calculated as,

$$i\mathcal{M} = \bar{u}(p_2)u(p_1)(iY_N)\frac{h}{\sqrt{2}} \tag{89}$$

$$-i\mathcal{M}^\dagger = \bar{u}(p_1)u(p_2)(-iY_N)\frac{h^*}{\sqrt{2}} \tag{90}$$

$$\begin{aligned}
So, \quad |\mathcal{M}^2| &= \frac{1}{2}Y_N^2\ \bar{u}(p_2)u(p_1)\bar{u}(p_1)u(p_2) \\
&= \frac{1}{2}Y_N^2\ \bar{u}(p_2)_\alpha\ (\not{p}_1 + M)_{\alpha\beta}\ u(p_2)_\beta \\
&= \frac{1}{2}Y_N^2\ (\not{p}_2 + m_n)_{\beta\alpha}\ (\not{p}_1 + M)_{\alpha\beta} \\
&= \frac{1}{2}Y_N^2 Tr[(\not{p}_2)(\not{p}_1 + M)] \\
&= \frac{1}{2}Y_N^2 4(p_1.p_2) \qquad (91)
\end{aligned}$$

Averaging over incoming particle gives us, $|\bar{\mathcal{M}}^2| = Y_N^2(p_1.p_2)$
The decay width is given by:

$$\Gamma = \frac{|\vec{p}|}{8\pi M^2}|\mathcal{M}^2|$$

From energy-momentum four vector conservation relation we have,

$$M = |\vec{p}| + \sqrt{|\vec{p}|^2 + m_n^2}$$
$$\implies |\vec{p}| = \frac{M^2 - m_n^2}{2M}$$

So,

$$p_1.p_2 = M|\vec{p}|$$
$$= \frac{M^2 - m_n^2}{2}$$

So, substituting all in the decay width equation we get,

$$\Gamma = \frac{M^2 - m_h^2}{32\pi M^3} Y_N^2 (M^2 - m_h^2)$$

$$\boxed{\Gamma_{N_R \longrightarrow h\nu} = \frac{Y_N^2 M}{32\pi}\left(1 - \frac{m_h^2}{M^2}\right)^2} \tag{92}$$

- $N_R \longrightarrow e^- W^+ \ \ (or, \ e^+ W^-)$

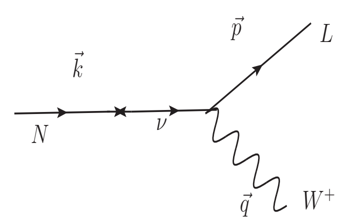


Figure 6: Decay of right handed neutrinos to charged lepton and W-boson.

From (51) we get the mass of $W-$boson as $M_W^2 = \frac{g_2^2 v^2}{4}$.
From neutrino mass mixing the mixing angle will be $\theta = \frac{Y_N v}{\sqrt{2} M_N}$.
Hence, in this case the coupling constant will be, $\lambda = \frac{g_2 Y_N v}{\sqrt{2} M_N}$

The matrix element can be written as:

$$i\mathcal{M} = -i\lambda\bar{u}(p)\gamma^{\mu}u(k)W_{\mu}^{+}(q) \tag{93}$$

$$-i\mathcal{M}^{\dagger} = i\lambda\bar{u}(k)\gamma^{\nu}u(p)W_{\nu}^{+}(q)^{*} \tag{94}$$

Hence,

$$|\mathcal{M}|^2 = \left(\lambda^2 W_{\mu}^{+}(q)W_{\nu}^{+}(q)^{*}\right)\left[u(p)\bar{u}(p)\gamma^{\mu}u(k)\bar{u}(k)\gamma^{\nu}\right] \tag{95}$$

Taking average over initial state and trace over the spinor indices we get:

$$\frac{1}{2}\sum_{spin}|\mathcal{M}|^2 = \frac{\lambda^2}{2}\sum W_{\mu}^{+}(q)W_{\nu}^{+}(q)^{*}\ \ Tr\left[\left(\not{p}+m_e\right)\gamma^{\mu}\left(\not{k}+M_N\right)\gamma^{\nu}\right] \tag{96}$$

Where,

$$\sum W_{\mu}^{+}(q)W_{\nu}^{+}(q)^{*} = -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_w^2} \tag{97}$$

and,

$$\begin{aligned} Tr\left[\left(\not{p}+m_e\right)\gamma^{\mu}\left(\not{k}+M_N\right)\gamma^{\nu}\right] &= Tr\left[\not{p}\gamma^{\mu}\not{k}\gamma^{\nu} + \underbrace{M_N\not{p}\gamma^{\mu}\gamma^{\nu}}_{=0} + \underbrace{m_e\gamma^{\mu}\not{k}\gamma^{\nu}}_{=0} + m_eM_N\ \gamma^{\mu}\gamma^{\nu}\right] \\ &= Tr\left[\not{p}\gamma^{\mu}\not{k}\gamma^{\nu}\right] + m_eM_N Tr\left[\gamma^{\mu}\gamma^{\nu}\right] \\ &= 4\left(p^{\mu}k^{\nu} + p^{\nu}k^{\mu} - g^{\mu\nu}(p.k) + m_eM_N\ g^{\mu\nu}\right) \end{aligned} \tag{98}$$

So,

$$\begin{aligned} |\bar{\mathcal{M}}|^2 &= \frac{1}{2}\sum_{spin}|\mathcal{M}|^2 \\ &= 2\lambda^2\left(g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_w^2}\right)\left(p^{\mu}k^{\nu} + p^{\nu}k^{\mu} - g^{\mu\nu}(p.k) + m_eM_N\ g^{\mu\nu}\right) \\ &= 2\lambda^2\left(2(p.k) + 2\frac{(q.p)(q.k)}{m_w^2} - \frac{q^2(p.k)}{m_w^2}\right) \\ &= 2\lambda^2\left(2(p.k) + 2\frac{(q.p)(q.k)}{m_w^2} - \frac{q^2(p.k)}{m_w^2} + \underbrace{\frac{m_eM_N(q^2-4m_w^2)}{m_w^2}}_{=0,\text{because of very small value of } m_e}\right) \end{aligned} \tag{99}$$

Since, $q^2 = m_w^2$ we have:

$$|\bar{\mathcal{M}}|^2 = 2\lambda^2 \left((p.k) + 2\frac{(q.p)(q.k)}{m_w^2} \right) \tag{100}$$

Now, we have to determine the values of $(p.k), (q.p)$ and $(q.k)$. For that we have,

$$\begin{aligned} k_\mu k^\mu &= M_N^2 = (p+q)_\mu (p+q)^\mu \\ &= (p+q)_0 (p+q)^0 \qquad \text{[Since, } |\vec{p}| = -|\vec{q}|\text{, we only have the } 0^{th} \text{ component.]} \\ &= (E_w + E_e)^2 \end{aligned}$$

So,

$$\begin{aligned} M_N &= E_w + E_e \\ &= \sqrt{|\vec{p}|^2 + m_w^2} + \sqrt{|\vec{p}|^2 + m_e^2} \\ \implies (M_N - |\vec{p}|)^2 &= |\vec{p}|^2 + m_w^2 \qquad \text{[Neglecting } m_e^2\text{]} \\ \implies |\vec{p}| &= \frac{M_N^2 - m_w^2}{2M_N} \end{aligned} \tag{101}$$

Now,

$$\begin{aligned} p.k &= p_o k_o - \vec{p}.\vec{k} \\ &= E_e E_N \qquad \text{[because in the rest frame of RHN, } \vec{k} = 0\text{]} \\ &= M_N \sqrt{|\vec{p}|^2 + m_e^2} \\ &= \frac{1}{2}\left(M_N^2 - m_w^2\right) \qquad \text{[Putting the value of } |\vec{p}|\text{]} \end{aligned} \tag{102}$$

Now,

$$p.q = \sqrt{|\vec{p}|^2 + m_w^2}.\sqrt{|\vec{p}|^2 + m_e^2} + |\vec{p}|^2$$

Calculating, $\sqrt{|\vec{p}|^2 + m_e^2} = \frac{M_N^2 - m_w^2}{2M_N}$ and, $\sqrt{|\vec{p}|^2 + m_w^2} = \frac{M_N^2 + m_w^2}{2M_N}$,

we get the final value of $p.q$ as

$$p.q = \frac{1}{2}(M_N^2 - m_w^2) \tag{103}$$

Now,

$$k.q = M_N\sqrt{|\vec{p}|^2 + m_w^2}$$
$$= \frac{M_N^2 + m_w^2}{2}$$

So, putting the values in the (100) we get,

$$|\vec{\mathcal{M}}|^2 = 2\lambda^2\left[\frac{M_N^2 - m_w^2}{2} + \frac{M_N^2 - m_w^2}{2}\frac{M_N^2 + m_w^2}{2}\frac{2}{m_w^2}\right]$$
$$= \frac{\lambda^2 M_N^2}{m_w^2}\left(1 - \frac{m_w^2}{M_N}\right)\left(1 + \frac{2m_w^2}{M_N^2}\right) \tag{104}$$

Now, we know the decay width of 2-body decay is:

$$\Gamma = \frac{|\vec{p}|}{8\pi M_N}|\bar{\mathcal{M}}|^2$$

Putting the values of $|\bar{\mathcal{M}}|$ and $|\vec{p}|$ we have,

$$\boxed{\Gamma_{N_R\longrightarrow e^- W^+} = \frac{Y_N^2 M_N}{8\pi}\left(1 - \frac{m_W^2}{M_N^2}\right)^2\left(1 + \frac{2m_W^2}{M_N^2}\right)} \tag{105}$$

- $N_R \longrightarrow Z\nu \ \ (or,\ Z\bar{\nu})$

Similarly, another decay mode is there, where the right-handed neutrino decays to one Z-boson and one light neutrino (ν).

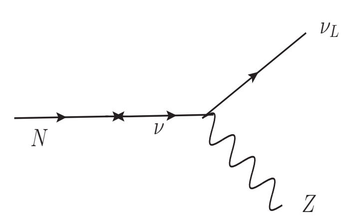
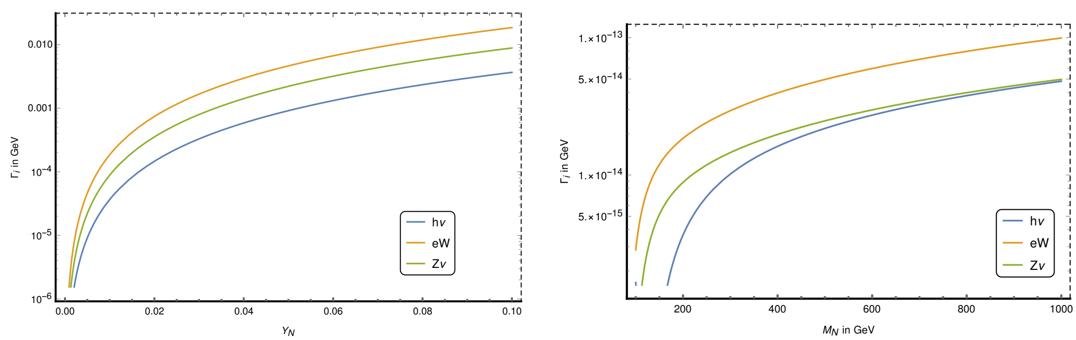


Figure 7: Decay of right handed neutrinos to Z-boson and light neutrino.

If we do the similar calculation of partial decay width (Γ_i), we get the decay width as:

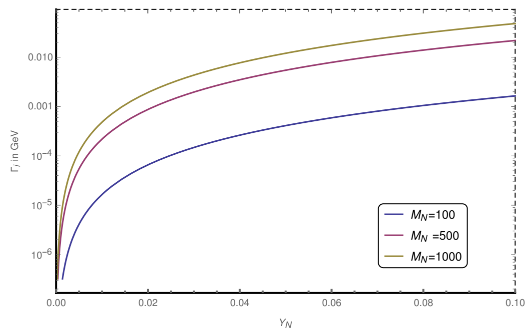
$$\boxed{\Gamma_{N_R \longrightarrow \nu Z} = \frac{Y_N^2 M_N}{16\pi}\left(1 - \frac{m_Z^2}{M_N^2}\right)^2 \left(1 + \frac{2m_Z^2}{M_N^2}\right)} \qquad (106)$$



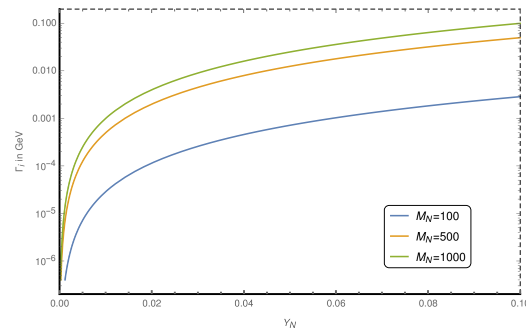
(a) (b)

Figure 8: Here we present the partial decay width of right-handed neutrino N in GeV verses Yukawa coupling Y_N for $M_N = 200GeV$ (a) and verses right-handed neutrino mass M_N for $Y_N = 10^-7$ (b)

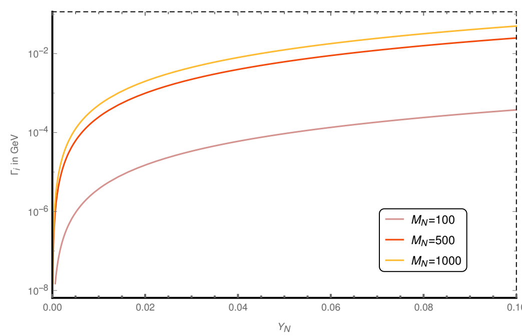
In the Figure 8 (a) we show the relation between the calculated partial decay width and the Yukawa coupling (Y_N) and in (b) there we plot the same with neutrino mass. In both the plots we can see that the decay width of the decay to $h\nu$ is smaller than the others though the mass of Higgs boson is greater than the mass of vector bosons, Z and W.



(a)



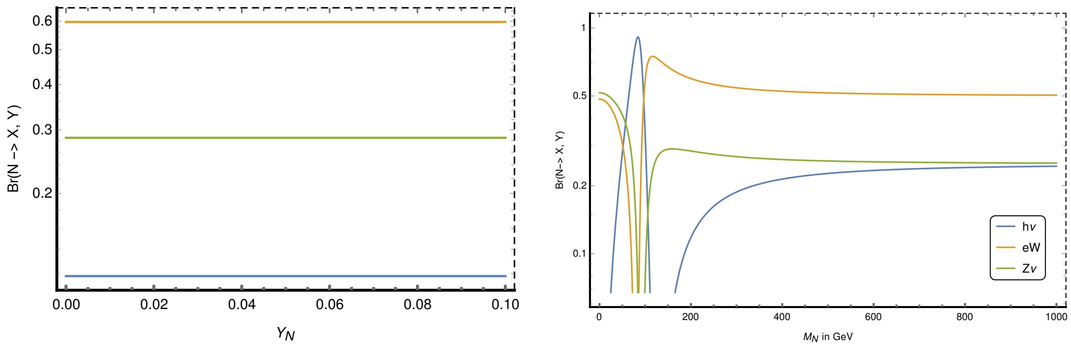
(b)



(c)

Figure 9: Here we present the partial decay widths in GeV for three choices of right-handed neutrino mass, i.e. $M_N = 100, 500, 1000$ GeV for(a) $h\nu$, (b) eW and (c) $Z\nu$ modes respectively.

In Figure 9 we plot the graph between partial decay width and Yukawa coupling for different masses (100 GeV, 500 GeV and 1000 GeV) and for three different decay modes. In all the cases we get the same pattern. If masses increases the variation between the decay widths becomes small. Hence, we can conclude that if the right-handed neutrino mass is some order of TeV (like 5 TeV or 20 TeV) there will not be any significant difference in partial decay width or decay life-time between them.



(a) (b)

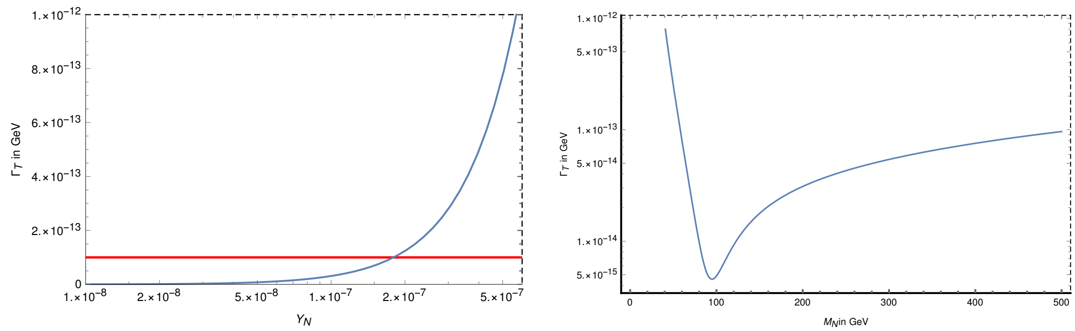
Figure 10: This represents the decay branching fraction the right-handed neutrino N verses Yukawa coupling Y_N for $m_N = 200$ GeV(a) and verses right-handed neutrino mass m_N for $Y_N = 10^{-7}$ (b).

The branching fraction or branching ratio for a decay is the ratio of a fraction of particles decays in a particular mode to the total decay. For example the branching fraction of right-handed neutrino decays to Higgs and light neutrino can be calculated as:

$$BR(N \longrightarrow h\nu) = \frac{\Gamma(N \longrightarrow h\nu)}{\Gamma(N \longrightarrow h\nu) + \Gamma(N \longrightarrow eW) + \Gamma(N \longrightarrow Z\nu)} \tag{107}$$

It is basically an unit less quantity which indicates the preferable decay modes. In Figure 10 we have shown different decay modes verses Yukawa couplings and neutrino mass. The branching ratio of the decay to $h\nu$ is greater than others if the neutrino mass is around 100 GeV but it goes down gradually beyond that and it is literally zero if the RHN mass is equal to the Higgs mass.

On the other hand as we have seen in case of partial decay width the branching ratio of the decay to eW is greater than the decay to $Z\nu$ (almost double from Figure 10 (a)) even if $m_z > m_w$.



(a) (b)

Figure 11: (a) represents the total decay width verses Yukawa coupling Y_N for $M_N = 200\ GeV$ and (b) represent Mass of the right-handed neutrino for $Y_N = 10^{-7}$.

In case of Figure 11 we have calculated the total decay width and have plotted the graph with Yukawa coupling (Y_N) and neutrino mass (M_N).

8.2 Displaced Decays of right-handed neutrinos:

The decays of a particle is known by its decay length and we classify such decays into two categories; prompt decay and displaced decays. In the prompt decays the secondary vertex matches with the primary decay vertex thus the decay length is approximately zero. However, there are cases where the secondary vertex reconstructed from the decay products are different from that of the primary one. The separation between the two is known as decay length. Such decay length can be used as standard candle in determining the nature of the particle under consideration.

In Figure 12 we describe the decay length of the right-handed neutrinos with masses of 200, 500 and 1000 GeV in the plane of Yukawa coupling Y_N and the decay length d in m. We describe the regions with displaced decay length 1 cm - 10 m, as displaced decay region within the red dashed lines. It is evident that coupling that Yukawa coupling should be $\lesssim 10^{-7}$ to have displaced decay length for $\mathcal{O}(100)$ GeV of right-handed neutrino mass [3, 4, 11].

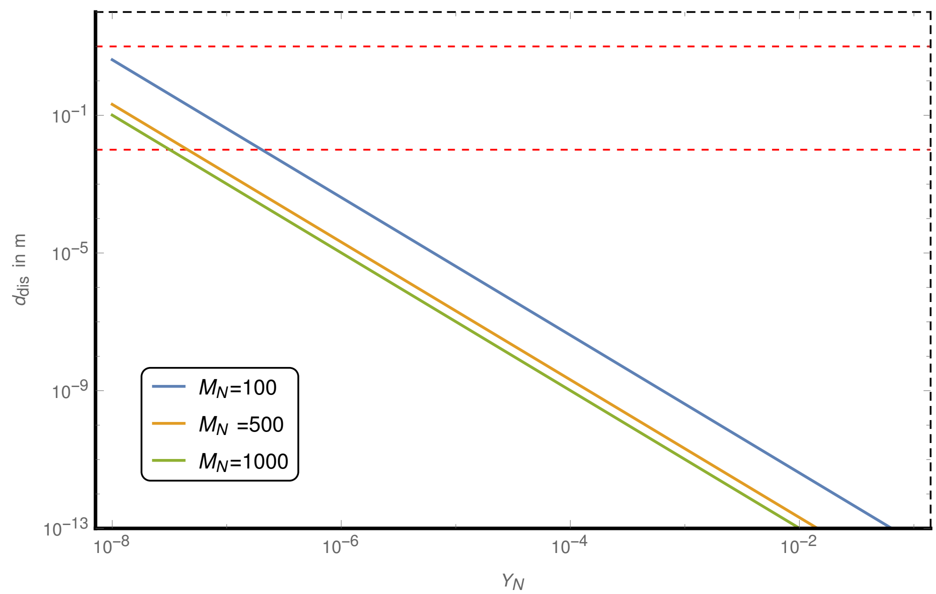


Figure 12: This represents the decay length of the right-handed neutrino N verses Yukawa coupling Y_N for $m_N = 100, 500, 1000$ GeV.

8.3 Production of right-handed neutrinos:

We implemented the model Type-I seesaw in a Mathematica [12] based program SARAH-4.14.1 [13] which generates the model files for CalcHEP-3.7.1 [14]. The RHN in type-I seesaw is SM gauge singlet thus does not have any direct coupling with SM particles. However, due to electro-weak symmetry breaking it gets mixed with left-handed neutrino which has gauge coupling. Thus production of such RHN is always proportional to the mixing angle square, i.e. the $m_D^2 = (\frac{Y_N v}{\sqrt{2}})^2$. For pair production it is even suppressed by m_D^4. Due to electro-weak nature and suppressed missing the production cross-sections of such right-handed neutrino is pretty low. We will see such cross-sections get an enhancement for the case of Type-III, as in that case the RHNs are triplet under $SU(2)_L$. We are still hopeful that LHC at high luminosity we can expect some of their signatures.

9 Extension of Standard Model by a Right-handed neutrino Type-III seesaw

The type-III seesaw model has in addition to the SM fields, $SU(2)$ triplets of fermions with zero hypercharge Σ. Basically we can get type-III seesaw mechanism by replacing the fermion singlet of type-I with a triplet.

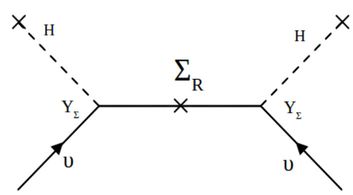


Figure 13: Type-III Seesaw Model

In terms of 2×2 notation of triplet the Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_{SM} + \underbrace{Tr[\bar{\Sigma} i \not{D} \Sigma]}_{\text{kinetic term}} - \underbrace{\frac{1}{2} Tr[\bar{\Sigma} M_{\Sigma} \Sigma^c + \bar{\Sigma}^c M_{\Sigma}^* \Sigma]}_{\text{Majorana mass term}} - \underbrace{\sqrt{2} Y_{\Sigma} \bar{\Sigma} \tilde{\phi}^{\dagger} L}_{\text{Yukawa interaction}} + h.c. \tag{108}$$

Where M_{Σ} is the mass matrix of the triplet and Y_{Σ} is the Yukawa coupling matrix.
The electromagnetic charged states are,

$$\Sigma^{\pm} = \frac{\Sigma_1 \mp i\Sigma_2}{\sqrt{2}}, \qquad \Sigma_0 = \Sigma_3$$

The fermion triplet Σ and it's conjugate $\Sigma^c \equiv C\bar{\Sigma}^T$ are given by:

$$\Sigma = \begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}, \qquad \Sigma^c = \begin{pmatrix} \Sigma^{0^c} & \sqrt{2}\Sigma^{-^c} \\ \sqrt{2}\Sigma^{+^c} & -\Sigma^{0^c} \end{pmatrix}$$

If we consider the Higgs doublet as $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ and after giving vev $H^+ = 0$ and $H^0 = v + \frac{h}{\sqrt{2}}$ (with v=174 GeV), then we get the Yukawa term as:

$$\begin{aligned}\mathcal{L}_Y &= \sqrt{2}Y_\Sigma i\tilde{H}\bar{\Sigma}L \\ &= \sqrt{2}Y_\Sigma i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0\\ v+\frac{h}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\bar{\Sigma}^0 & \sqrt{2}\bar{\Sigma}^+\\ \sqrt{2}\bar{\Sigma}^- & -\bar{\Sigma}^0\end{pmatrix}\begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + h.c. \\ &= \sqrt{2}Y_\Sigma\left(\left(v+\frac{h}{\sqrt{2}}\right)\bar{\Sigma}^0 \quad \left(v+\frac{h}{\sqrt{2}}\right)\sqrt{2}\bar{\Sigma}^-\right)\begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + h.c. \end{aligned} \tag{109}$$

So, without the mass term the Yukawa becomes,

$$\mathcal{L}'_Y = \sqrt{2}Y_\Sigma\left(\bar{\Sigma}^0\nu_l + \sqrt{2}\bar{\Sigma}^- e_L + h.c.\right)\frac{h}{\sqrt{2}} \tag{110}$$

Here also the neutrinos get a get a seesaw mass $m^\nu_{ij} = \frac{Y_{\Sigma i}Y_{\Sigma j}v^2}{m_\Sigma}$. This becomes of the order of 0.1 eV for $Y_\Sigma \approx 10^{-6}$ and $m_\Sigma \approx 1TeV$.

The neutrino Dirac mass indicates mixing between l and Σ.

The mixing angles for the neutral and charged part are $\longrightarrow$

$$\theta_\nu = \frac{Y_\Sigma v}{m_\Sigma} \text{ and } \theta_e = \sqrt{2}\frac{Y_\Sigma v}{m_\Sigma} \qquad \text{respectively.}$$

Due to the $l-\Sigma$ mixing, we get the mixed gauge interaction. The gauge interaction terms are like:
$\bar{\Sigma^0 e_l}\gamma^\mu W^+_\mu$, $\bar{\nu_l\Sigma^-}\gamma^\mu W^+_\mu$, $\bar{e_l\Sigma^0}\gamma^\mu W^-_\mu$, $\bar{\nu_l\Sigma^-}\gamma^\mu W^-_\mu$, $e_l\bar{\Sigma}^-\gamma^\mu Z_\mu$, $\bar{e}_l\Sigma^-\gamma^\mu Z_\mu$, $\nu_l\bar{\Sigma}^0\gamma Z_\mu$

Note that, there is no gauge interaction term in type-I seesaw, which is the difference between them.

9.1 Decays

Similar as the type-I seesaw model there are possible decay modes for type-III model also, where the right-handed neutrino is fermionic triplet.

The decay of Σ^0 is same as the decay of singlet right-handed neutrino of type-I, which are:

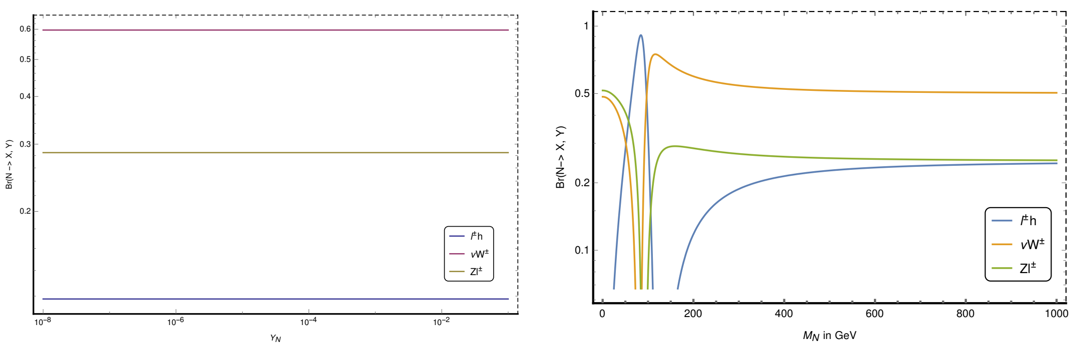
$$\Sigma^0 \longrightarrow h\nu\ (or,\ h\bar{\nu}); \quad \Sigma^0 \longrightarrow e^-W^+\ (or,\ e^+W^-); \quad \Sigma^0 \longrightarrow Z\nu\ (or,\ Z\bar{\nu})$$

Similar as type-I the decay width calculation of $\Sigma^\pm$ fermion gives us three decay modes, and decay widths are:

$$\Gamma(N^\pm \longrightarrow l^\pm h) = \frac{1}{4}\frac{Y_\Sigma^2 M_N}{8\pi}\left(1 - \frac{m_h^2}{M_N^2}\right)^2 \tag{111}$$

$$\Gamma(N^\pm \longrightarrow l^\pm Z) = \frac{1}{4}\frac{Y_\Sigma^2 M_N}{8\pi}\left(1 - \frac{M_Z^2}{M_N^2}\right)^2\left(1 + 2\frac{M_Z^2}{M_N^2}\right) \tag{112}$$

$$\Gamma(N^\pm \longrightarrow \nu_l W^\pm) = \frac{1}{2}\frac{Y_\Sigma^2 M_N}{8\pi}\left(1 - \frac{M_W^2}{M_N^2}\right)^2\left(1 + 2\frac{M_W^2}{M_N^2}\right) \tag{113}$$



(a) (b)

Figure 14: This describes decay branching fractions (a) verses Y_N for $M_N =$ 200 GeV and (b) with respect to mass M_N for $Y_N = 10^{-7}$ for type-III seesaw model.

9.2 Production:

Unlike Type-I seesaw, in this case the RHNs are triplet under $SU(2)_L$, so can be produced via $W^\pm$ exchange. However, production cross-section of two neutral pair is zero, as they are both $T_3 = Y = 0$. We have to reply on the production of the neutral one in association with the charged ones, i.e. $\Sigma^\pm\Sigma^0$ [4].

10 Discussion and conclusions

In this work we started with the abelian gauge theory and went on to study non-abelian gauge theory. In the process we studies the weak sector of Standard Model as $SU(2)_L \times U(U)_Y$ gauge theory. Standard Model is a very successful theory with the Higgs mechanism giving mass to the $W^{\pm}, Z$ bosons, quarks and the charged leptons. However, Standard Model fails to generate mass to neutrinos, which are recently observed to be massive.

A simple extension of SM with a SM gauge singlet can be achieved by the addition of a right-handed neutrino. It not only generates mass for the neutrinos but all explains why one of the neutrino will be very light keeping the other one heavy. This mechanism is known as Type-I seesaw mechanism. in the process the left- and right-handed get mixed and the right handed neutrino couplings to gauge bosons are proportional to this mixing angle. We calculated the decay widths and decay branching fractions for such right-handed neutrinos. Though their decay phenomenology is very interesting, producing such inert particles are rather challenging due to no direct gauge coupling. The production mechanism always depends on the mixing angle with the left-handed neutrinos and thus such cross-sections are very low. This results into large required luminosity to probe such right-handed neutrinos in colliders, viz. LHC.

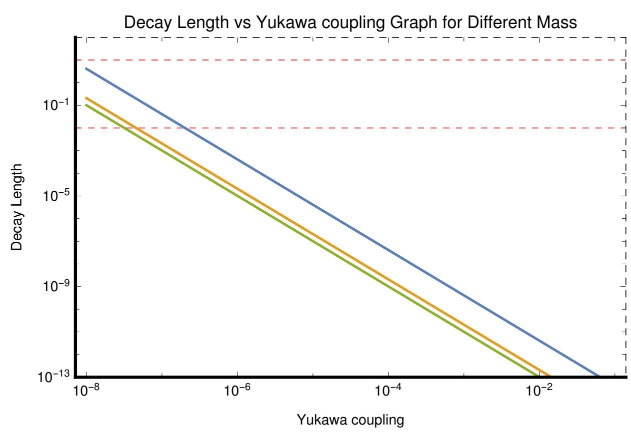
Next, we studied the extension of SM with a $SU(2)$ triplet and $Y = 0$ fermion, which results into pair of charged fermion and a neutral right-handed neutrino. Similar to type-I, type-III also decays to the gauge bosons and other leptons via the mixing angles. The phenomenology in this case enriched due to the mixings of the charged leptons. The production cross-sections in this case are larger than type-I right-handed neutrinos due to $SU(2)$ gauge coupling. Both the models are implemented in SARAH [13] and corresponding CalcHEP model files are generated [14].

Appendix A

Figure 15: **Displaced vertex calculation:**

```
In[1]:= Mw = 80.38;
        Mh = 125.5;
        Mz = 91.2;

        f[x_] = (((x^2 * Mn) / (64 * π)) * ((1 - (Mh / Mn) ^2) ^2)) ;
        f1[x_] =  (((x^2 * Mn) / (32 * π)) * ((1 - (Mw / Mn) ^2) ^2) * (1 + 2 * (Mw / Mn) ^2));
        f2[x_] =  (((x^2 * Mn) / (64 * π)) * ((1 - (Mz / Mn) ^2) ^2) * (1 + 2 * (Mz / Mn) ^2)) ;

        TD[x_] = f[x] + f1[x] + f2[x] ;   (* Total decay width *)
        Brhν[x_] = f[x] / TD[x] ;         (* Branching of hν *)
        BreW[x_] = f1[x] / TD[x]  ;       (* Branching of eW *)
        BrZν[x_] = f2[x] / TD[x]   ;      (* Branching of Zν *)

In[8]:=
        (* Decay Length vs Yukawa Coupling *)

        f[y_] = (1 / TD[x]) * 0.2 * 10^-15;
        LogLogPlot[{f[y] /. {Mw → 80.38, Mh → 125.5, Mz → 91.2, Mn → 100},
          f[y] /. {Mw → 80.38, Mh → 125.5, Mz → 91.2, Mn → 500},
          f[y] /. {Mw → 80.38, Mh → 125.5, Mz → 91.2, Mn → 1000}},
         {x, 10^-8, 0.1}, PlotRange → {10^-13, 100}, GridLines → {{}, {10, 0.01}},
         GridLinesStyle → {Red, Dashed}, Frame → True,
         FrameStyle → {{Thick, Dashed}, {Thick, Dashed}},
         FrameLabel → {"Yukawa coupling", "Decay Length"}, LabelStyle → Black,
         PlotLabel → "Decay Length vs Yukawa coupling Graph for Different Mass"]
```

Out[9]=



In[10]:=

Figure 16: **Partial decay width verses mass calculation Type I see-saw:**

In[∘]:=

```
In[∘]:= Mw = 80.38;
Mh = 125.5;
Mz = 91.2;


f[x_] = (((x^2 * Mn) / (64 * π)) * ((1 - (Mh / Mn) ^2) ^2));
LogPlot[{f[x] /. {Mw → 80.38, Mh → 125.5, Mz → 91.2, Mn → 100},
  f[x] /. {Mw → 80.38, Mh → 125.5, Mz → 91.2, Mn → 500},
  f[x] /. {Mw → 80.38, Mh → 125.5, Mz → 91.2, Mn → 1000}},
 {x, 10^-8, 0.1}, PlotRange → {{10^-8, 0.1}, Automatic},
 PlotStyle → ColorData[1, "ColorList"], PlotLegends →
  Placed[LineLegend[{"MN=100", "MN =500", "MN=1000"}, LegendFunction → Frame],
   {0.80, 0.20}], Frame → True, FrameStyle →
  {{Thick, Dashed}, {Thick, Dashed}}, FrameLabel → {"YN", "Γi in GeV"}]
```

Out[∘]=

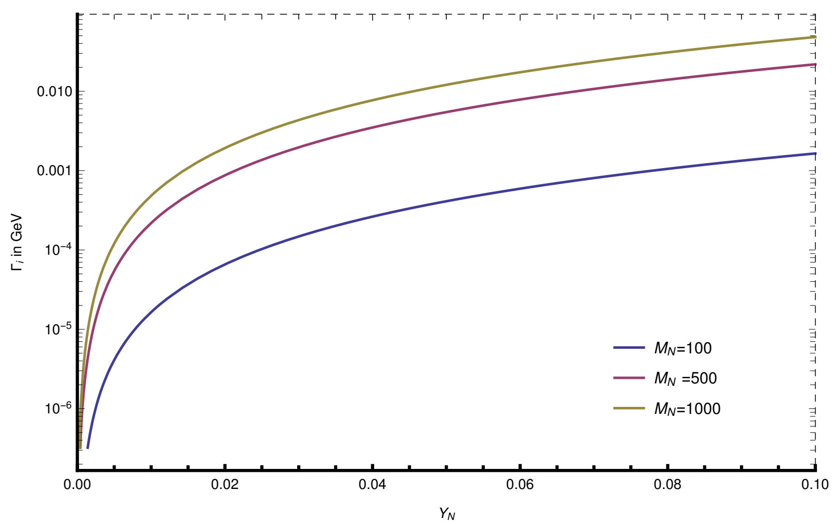


Figure 17: **Partial decay width verses Yukawa in Type I seesaw:**

```
In[•]:= (* Partial Decay Width; Branching Ratio; Total Decay Width vs Yukawa Coupling *)

Mw = 80.38;
Mh = 125.5;
Mz = 91.2;
Mn = 200;

(* Here, x=Yn; Yukawa Coupling *)

f[x_] = (((x^2 * Mn) / (64 * π)) * ((1 - (Mh / Mn) ^ 2) ^ 2))
(* Decay to hν *)
f1[x_] = (((x^2 * Mn) / (32 * π)) * ((1 - (Mw / Mn) ^ 2) ^ 2) * (1 + 2 * (Mw / Mn) ^ 2))
(* Decay to We *)
f2[x_] = (((x^2 * Mn) / (64 * π)) * ((1 - (Mz / Mn) ^ 2) ^ 2) * (1 + 2 * (Mz / Mn) ^ 2))
 (* Decay to Zν *)
Out[•]= 0.36559 x^2
Out[•]= 1.85049 x^2
Out[•]= 0.883578 x^2

In[•]:=

In[•]:= LogPlot[{f[x], f1[x], f2[x]}, {x, 10^-8, 0.1},
  Frame → True, FrameStyle → {{Thick, Dashed}, {Thick, Dashed}},
  FrameLabel → {"Y_N", "Γ_i in GeV"}, LabelStyle → Black, PlotLegends →
   Placed[LineLegend[{"hν", "eW", "Zν"}, LegendFunction → Frame], {0.80, 0.20}]]
```

Out[•]=

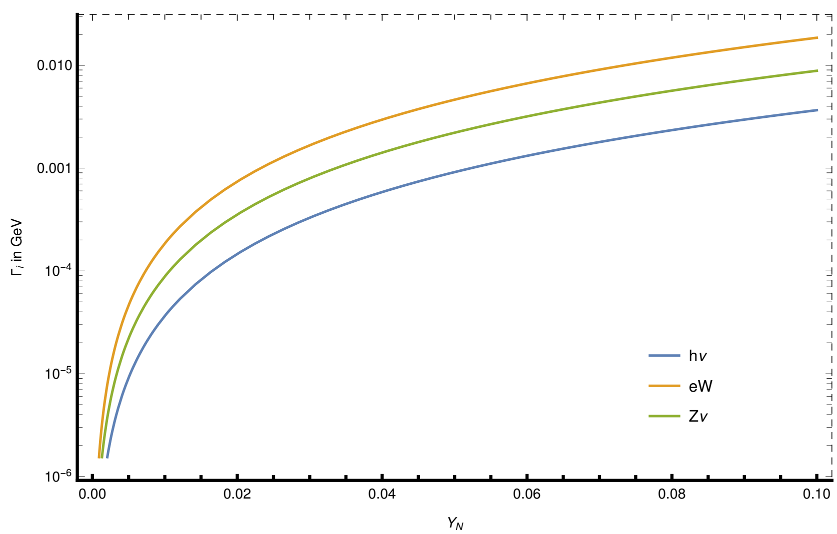


Figure 18: **Decay branching fraction verses Mass in Type I seesaw:**

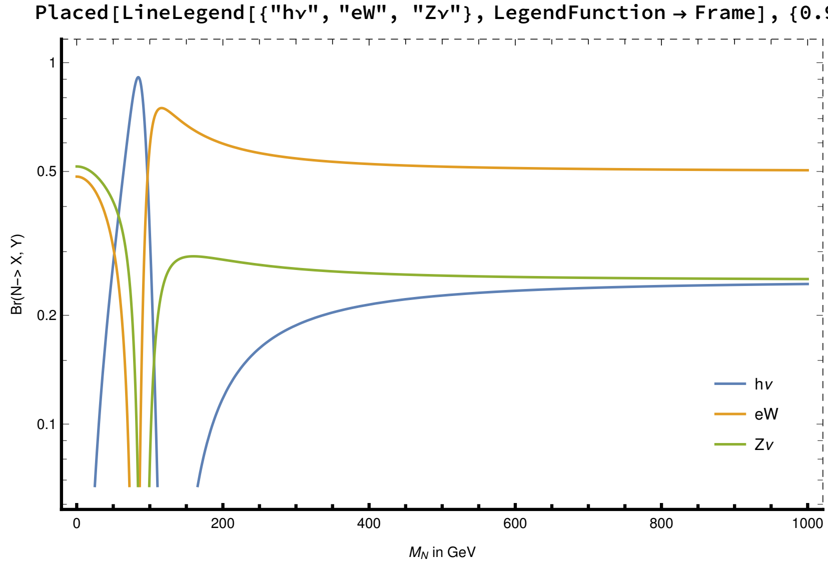
In[1]:=
```
(* Partial Decay Width; Branching Ratio vs Neutrino Mass *)
```

In[2]:=
```
Yn = 10^-7;
Mw = 80.38;
Mh = 125.5;
Mz = 91.2;
(* Here, x=Mn; Neutrino mass *)

f[x_] = (((Yn^2 * x) / (64 * π)) * ((1 - (Mh / x)^2)^2))  ;
(* Decay to hν *)
f1[x_] =  (((Yn^2 * x) / (32 * π)) * ((1 - (Mw / x)^2)^2) * (1 + 2 * (Mw / x)^2)) ;
(* Decay to We *)
f2[x_] =  (((Yn^2 * x) / (64 * π)) * ((1 - (Mz / x)^2)^2) * (1 + 2 * (Mz / x)^2))   ;
(* Decay to Zν *)
```

In[7]:=
```
TD[x_] = f[x] + f1[x] + f2[x]  ;  (* Total decay width *)
Brhν[x_] = f[x] / TD[x] ;         (* Branching of hν *)
BreW[x_] = f1[x] / TD[x]  ;       (* Branching of eW *)
BrZν[x_] = f2[x] / TD[x]   ;      (* Branching of Zν *)
```

In[9]:=
```
LogPlot[{Brhν[x], BreW[x], BrZν[x]}, {x, 0, 1000},
 Frame → True, FrameStyle → {{Thick, Dashed}, {Thick, Dashed}},
 FrameLabel → {"M_N in GeV", "Br(N-> X, Y)"},
 (*PlotLabel→"Branching Ratio vs Neutrino Mass Graph",*)
 LabelStyle → Black, PlotLegends →
  Placed[LineLegend[{"hν", "eW", "Zν"}, LegendFunction → Frame], {0.90, 0.20}]]
```

Out[9]=



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Declaration of Authorship

I, Chandrima Sen, declare that this thesis title "Phenomenology of Right-Handed Neutrino in Beyond Standard Model" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a Masters degree at Indian Institute of Technology, Hyderabad.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the sources is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have make clear exactly what was done by others and what I have contributed myself.

15/05/2019 Chandrima Sen.

Date Signature

CERTIFICATE

This is to certify that the project work titled "Phenomenology of Right-Handed Neutrino in Beyond Standard Model" submitted to the Indian Institute of Technology, Hyderabad is partial fulfillment of requirements for the award of the degree of Masters in Science in physics, is a work done by Chandrima Sen, during the period of her study, Aug 2018-April 2019, in the Indian Institute of Technology, Hyderabad, under my supervision and guidance. This thesis has not been submitted for the award of any other Degree/Diploma/Fellowship or any other similar title.

15/05/2019

Sig. of Dr. Priyotosh Bandyopadhyay.
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