DARK MATTER AND IT'S DETECTION AT TERRESTRIAL LABORATORIES

Thesis by
Akhila Kumar Pradhan
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Project Supervisor,
Dr. Narendra Sahu

Department of Physics
Indian Institute of Technology,Hyderabad

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Akhila Kumar Pradhan

Certificate

This is to certify that Akhila Kumar Pradhan has satisfactorily completed the project entitled Dark Matter And It's Detection At Terrestrial Laboratories as partial fulfillment for the award of Master Of Science(physics) degree at the Indian Institute Of Technology Hyderabad during the year 2017-2019

Narendra Sahu

(Dr Narendra Sahu,project supervisor)

Author's Declaration

I declare that the work in this thesis was carried out in accordance with the requirements of the Institute's Regulations and for the partial fulfillment of Master of Physics and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text,the work is the candidate's own work. Any views expressed in the thesis are those of the author.

Akhila Kumar Pradhan

Abstract

The work for this dissertation is two-fold, first to understand the Standard Model(SM) and then looking for physics of Dark matter(DM) which is beyond the Standard model. We discuss Standard Model as a gauge theory where abelian and non-abelian gauge theory are thoroughly discussed. Then we move to electroweak sector where we discuss the interaction of intermediate vector bososns and fermions. Also we show how using Electroweak symmetry breaking and Higgs mechanism, we can give mass to SM particles. After this first part we move to our second part which is DM physics. We derive the DM relic density in the expanding universe using Boltzmann equation. Then we concentrate our work on the direct detection of DM where we calculate the DM-nucleus scattering cross section, which is used in the terrestrial laboratories for the detection of DM.

Contents

Chapter 1

The Standard Model

1.1 Introduction

Everything in the universe is found to be made from a few basic building blocks called fundamental particles, governed by four fundamental forces. Our best understanding of how these particles and three of the forces are related to each other is encapsulated in the Standard model of particle physics. It has successfully explained almost all experimental results and precisely predicted a wide variety of phenomena. But still some of the unsolved questions of particle physics can't be explained using SM, one of them is DM. Now we know that the Standard model is a non-abelian gauge theory with the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)Y$, where $SU(3)_C$ describes the rotation in color space, $SU(2)_L$ describes the rotation in weak isospin space and $U(1)Y$ describes the rotation in hyper-charge space.

1.2 Gauge Theory

In particle physics, gauge theories are successful field theories explaining the dynamics of elementary particles. A gauge theory is a theory where the action is invariant under a continuous group symmetry. When the symmetry group depends on space time, it is called a local symmetry. The continuous symmetry is called a gauge group and this transformation is called a gauge transformation.Symmetry has a very important role in the development of physics. From the space time symmetry of special relativity up to internal and gauge symmetry, it has mapped out the route to most of the physical theory. Noether's theorem[5][7] tells us that symmetries imply conservation laws. Taking the motivation from this we can ask a question that, upon imposing to a given Lagrangian the invariance under a certain symmetry, would it be possible to determine the form of the interaction among the particles? In fact, this happens in Quantum Electrodynamics (QED), where the existence and some of the properties of the gauge field(the photon) follow from a principle of invariance under local gauge transformations of the U(1) group. Then this principle was generalized to describe other interactions.

1.2.1 Abelian gauge theory

principle

In electrodynamics, the gauge principle[6] provides a method to transform a Lagrangian that is invariant with respect to global symmetry from $U(\mathbf{1})$ group(abelian) into a Lagrangian that is invariant with respect to a local symmetry or gauge invariant. It involves replacing all conventional derivatives $\partial\mu$ by covariant derivative $D_\mu = \partial_\mu + ieA_\mu$ and adding a kinetic term -$\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

$$\mathcal{L} = (\partial_\mu\psi(x), \psi(x)) \rightarrow \mathcal{L}(D_\mu\psi(x), \psi(x)) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

A_μ is called the gauge field that transforms like $A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$ and the strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. After local gauge transformation, the final lagrangian for QED[7] is given by:

$$\mathcal{L} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi - e\overline{\psi}\gamma_\mu\psi A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

The hypothetical mass term for the gauge field (A_μ) , $\mathcal{L}^m = \frac{1}{2}m^2A^\mu A_\mu$ would not be invariant under the gauge transformation.

1.2.2 Non-Abelian gauge theory

Here we will generalize the gauge principle to the non-abelian groups, which has first been workout by YANG and MILLS in 1954,[7] by developing the SU(2)-isospin gauge theory. Then we will make generalization to other gauge groups.

As suggested by Heisenberg in 1932, under nuclear interactions proton and neutrons can be degraded as degenerated since their masses are quite similar and electromagnetic interaction is negligible.

Then any arbitrary combination of their wave function would be equivalent

$$\psi \equiv \begin{pmatrix} \psi_p \\ \psi_n. \end{pmatrix}$$

Under local gauge transformation,

$$\psi(x) \rightarrow \psi'(x) = G(x)\psi(x)$$

with $G(x) \equiv \exp(\frac{i\tau^a \alpha^a(x)}{2})$ is an element of SU(2) group where, $\tau^a, a = 1, 2, 3$, are Pauli matrices. Introducing one gauge field for each generator and defining the co-variant derivative[6]

$$D_\mu \equiv \mathbf{I}\partial_\mu + igB_\mu$$

where, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and B_μ is defined by 2×2 matrix

$$B_\mu = \frac{1}{2}\tau^a b^a_\mu = \frac{1}{2}\begin{pmatrix} b^3_\mu & b^1_\mu - ib^2_\mu \\ b^1_\mu + ib^2_\mu & -b^3_\mu \end{pmatrix}$$

where, three gauge fields are $b^a_\mu = (b^1_\mu, b^2_\mu, b^3_\mu)$. Since the covariant derivative transforms just like the matter field i.e. $D_\mu\psi \to G(D_\mu\psi)$, to hold this, gauge field transforms like

$$B'_\mu = G[B_\mu + \frac{i}{g}G^{-1}(\partial_\mu G)]G^{-1}$$

or, in infinitesimal form i.e.$G \simeq 1 + i\frac{\tau^a}{2}\alpha^a(x)$

$$b'^l_\mu = b^l_\mu - \epsilon_{jkl}\alpha^k b^l - \frac{1}{g}\partial_\mu\alpha^l.$$

Then we wish to find a field strength tensor that transforms under local gauge transformation G as $F'_{\mu\nu} = GF_{\mu\nu}G^{-1}$.
To satisfy this, for SU(2) gauge group theory a candidate field-strength tensor of the form[6]

$$F_{\mu\nu} = \frac{1}{ig}[D_\mu, D_\nu] = \partial_\mu B_\nu - \partial_\nu B_\mu + ig[B_\mu, B_\nu].$$

The gauge invariant K.E term would be

$$-\frac{1}{2}tr(F_{\mu\nu}F^{\mu\nu}).$$

Hence the Yang-Mills lagrangian

$$\mathcal{L}_{YM} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2}tr(F_{\mu\nu}F^{\mu\nu}).$$

A mass term $\frac{1}{2}m^2 B^\mu B_\mu$ is also incompatible with local gauge invariance as in electromagnetism.
To generalize other gauge groups , the levi-civita symbol ϵ_{jkl} will be replaced by the anti-symmetric structure constant f_{jkl} in the expression

$$F^l_{\mu\nu} = \partial_\mu b^l_\nu - \partial_\nu b^l_\mu + g\epsilon_{jkl}b^j_\mu b^k_\nu.$$

So far the gauge bosons are massless. To be consistent with experiment, gauge theories need massive gauge fields. So how can we introduce masses without destroying the gauge invariance of Lagrangian ? The answer is that mass terms are induced by spontaneously broken symmetry.

1.3 Spontaneous Symmetry Breaking

A necessary condition for the Higgs mechanism to take place is the non-triviality of vacuum expectation value.
Let us analyze the simple example of a scalar self-interacting real field with lagrangian ,[6]

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4, \lambda > 0.$$

$\mathcal{L}$ is invariant under the discrete transformation $\phi \to -\phi$.
The minimum of $V(\Phi)$ will be

$$\phi_0(\mu^2 + \lambda\phi_0) = 0.$$

For (a) $\mu^2 > 0$, we have just one vacuum at $\phi_0 = 0$ and it is invariant under the discrete transformation. However for (b) $\mu^2 < 0$, we have two vacua states corresponding to $\phi_0^{\pm} = \pm\sqrt{\frac{-\mu^2}{\lambda}} = \pm\nu$ in which symmetry is spontaneously broken. Defining a new field $\phi' \equiv \phi - \nu$, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi'\partial^\mu\phi' - \frac{1}{2}(\sqrt{-2\mu^2})^2\phi'^2 - \lambda\nu\phi'^3 - \frac{1}{4}\lambda\phi'^4.$$

This lagrangian describes a scalar field ϕ' with real and positive mass $M_{\phi'} = \sqrt{-2\mu^2}$ but it lost the original symmetry due to ϕ'^3 term.

When we study the case of charged self-interacting scalar field where a continuous symmetry is spontaneously broken, a new interesting phenomena happens.

$$\mathcal{L} = \partial_\mu\phi^\star\partial^\mu\phi - V(\phi^\star\phi)$$

with

$$V(\phi^\star\phi) = \mu^2(\phi^\star\phi) + \lambda(\phi^\star\phi)^2, \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}.$$

Following the previous procedure we can show that the new lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi'_1\partial^\mu\phi'_1 - \frac{1}{2}(-2\mu^2)^2\phi'^2_1 + \frac{1}{2}\partial_\mu\phi'_2\partial^\mu\phi'_2 + interaction.$$

Now we identify in the particle spectrum a scalar field ϕ'_1 with real and positive mass and a massless scalar boson ϕ'_2 which is a Goldstone boson.

1.4 Higgs Mechanism

Here the massless gauge fields correspond to each broken generator become massive by absorption of a Goldstone field. This is accomplished by requiring that the Lagrangian is also invariant under local gauge transformations.

1.4.1 Abelian case

Let us apply the local phase transformation $\phi \to \exp(iq\alpha(x))\phi$ to the charged self-interacting scalar Lagrangian.
parameterizing [7]the new field ϕ',

$$\phi = \exp(\frac{i\phi_2'}{\nu})(\frac{\phi_1' + \nu}{\sqrt{2}}) \approx \frac{1}{\sqrt{2}}(\phi_1' + \nu + i\phi_2').$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1'\partial^\mu\phi_1' - \frac{1}{2}(-2\mu^2)^2\phi_1'^2 + \frac{1}{2}\partial_\mu\phi_2'\partial^\mu\phi_2' - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{q^2\nu^2}{2}A_\mu A^\mu + q\nu A_\mu\partial^\mu\phi_2'$$

We can choose the gauge parameter $\alpha(x) = -\frac{1}{q\nu}\phi_2'(x)$. Hence

$$\phi = \exp[iq(-\frac{\phi_2'}{q\nu})]\exp(\frac{i\phi_2'}{\nu})(\frac{\phi_1' + \nu}{\sqrt{2}}) = \frac{1}{\sqrt{2}}(\phi_1' + \nu)$$

With this choice of gauge(unitary gauge) the Goldstone boson disappears and we get the Lagrangian,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1'\partial^\mu\phi_1' - \frac{1}{2}(-2\mu^2)^2\phi_1'^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{q^2\nu^2}{2}A_\mu' A^{\mu\prime} + \frac{1}{2}q^2(\phi_1' + 2\nu)\phi_1' A_\mu' A^{\mu\prime}$$
$$- \frac{\lambda}{4}\phi_1'^3(\phi_1' + 4\nu).$$

As we can see, the corresponding degree of freedom of the Goldstone boson was absorbed by the vector boson that acquires mass($M_A = q\nu$). The Goldstone turned into the longitudinal degree of freedom of the vector boson.

1.4.2 Non-Abelian case

Now can generalize the results for a non-abelian group G of dimension N_G and generators T^a[5]. Introduce N_G gauge bosons, such that $\partial\mu \to D_\mu = \partial\mu - igT^a B_\mu^a$. After spontaneous symmetry breaking , a sub group g of dimension n_g remains as a symmetry of vacuum. Hence we could expect the appearance of $(N_g - n_g)$ Goldstone bosons.

We would follw the same procedure of abelian case and parameterize the original field as

$$\phi = (\phi' + \nu)\exp(\frac{i\phi_{GB}T^a}{\nu})$$

where T^a are the $(N_g - n_g)$ broken generators that do not annihilate the vacuum. Then choose the gauge parameter $\alpha^a(x)$ in order to eliminate ϕ_{GB}. This will give rise to $(N_g - n_g)$ massive gauge bosons. The total number of degrees of freedom both before and after the spontaneous symmetry breaking will be same.

1.5 Electroweak Theory Of Leptons

The suggested the gauge group for this theory is $SU(2)_L \times U(1)_Y$, where $U(1)Y$ is associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge through the analogous of the Gell-Mann–Nishijima formula $Q = T_3 + \frac{Y}{2}$[6].
We introduce the left–handed isospin doublet ($T = \frac{1}{2}$),

$$L \equiv \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$$

where, $\nu_L = \frac{1}{2}(1 - \gamma_5)\nu$ and $l_L = \frac{1}{2}(1 - \gamma_5)l$, l is any lepton flavour($l = e, \mu, \tau$). Since there is no right-handed component for neutrino,the right-handed part of charged lepton is a singlet.

$$R \equiv l_R$$

where $l_R = \frac{1}{2}(1 + \gamma_5)l$.
From Gell-Mann Nishijima formula the weak hyper-charge of the doublet ($Y_L = -1$) and of the lepton singlet ($Y_R = -2$).
The next step is to introduce gauge fields corresponding to each generator, that is,

$$SU(2) \to W^1_\mu, W^2_\mu, W^3_\mu$$

$$U(1) \to B_\mu$$

Defining the strength tensors for the gauge fields[5]

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

We can write free lagrangian for gauge field[5]

$$\mathcal{L}_{gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}. \tag{1.1}$$

For the leptons, we write the free Lagrangian,

$$\mathcal{L}_{lepton} = \overline{R}i\gamma^{\mu}\partial_{\mu}R + \overline{L}i\gamma^{\mu}\partial_{\mu}L = \bar{l}i\gamma^{\mu}\partial_{\mu}l + \overline{\nu}i\gamma^{\mu}\partial_{\mu}\nu. \tag{1.2}$$

The next step is to introduce the fermion–gauge boson coupling via the covariant derivative, i.e.[6]

$$L : D_{\mu} = \partial_{\mu} + i\frac{g}{2}\tau^{i}W_{\mu}^{i} + i\frac{g^{'}}{2}YB_{\mu}$$

$$R : D_{\mu} = \partial_{\mu} + i\frac{g^{'}}{2}YB_{\mu}$$

where , g and g' are the coupling constant associated to the groups $SU(2)_L$ and $U(1)_Y$ respectively.
Therefore, the lepton Lagrangian (2) becomes

$$\mathcal{L}_{lepton} \to \mathcal{L}^{0}_{lepton} + \overline{L}i\gamma^{\mu}(i\frac{g^{'}}{2}YB_{\mu})L + \overline{R}i\gamma^{\mu}(i\frac{g^{'}}{2}YB_{\mu})R. \tag{1.3}$$

On simplifying we can write the charged part of equation(3) as

$$\mathcal{L}^{\pm}_{lepton} = -\frac{g}{2\sqrt{2}}[\overline{\nu}\gamma^{\mu}(1-\gamma_5)lW_{\mu}^{+} + \bar{l}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]. \tag{1.4}$$

where, the definition of charged gauged bosons as $W_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(W_{\mu}^{1} \mp iW_{\mu}^{2})$.
When we compare this equation with the Fermi lagrangian for leptons, we get $\frac{g}{2\sqrt{2}} = (\frac{M_W^2 G_F}{\sqrt{2}})^{\frac{1}{2}}$
The neutral part of equation (3) contains both right and left fermion components i.e.

$$\mathcal{L}^{0}_{lepton} = -gJ_3^{\mu}W_{\mu}^{3} - \frac{g\prime}{2}J_Y^{\mu}B_{\mu} \tag{1.5}$$

where, the currents

$$J_3^{\mu} = \frac{1}{2}(\overline{\nu_L}\gamma^{\mu}\nu_L - \overline{l_L}\gamma^{\mu}l_L).$$

$$J_Y^{\mu} = -(\overline{\nu_L}\gamma^{\mu}\nu_L + \overline{l_L}\gamma^{\mu}l_L + 2\overline{l_R}\gamma^{\mu}l_R).$$

In order to obtain the right combination of fields that couples to the electromagnetic current, let us make the rotation in the neutral fields, defining the new fields A and Z by[6],

$$\begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} cos\theta_w & sin\theta_w \\ -sin\theta_w & cos\theta_w \end{pmatrix} \begin{pmatrix} B_{\mu} \\ W_{\mu}^{3} \end{pmatrix}$$

where, θ_w is the Weinberg angle and the relation with $SU(2)$ and $U(1)$ coupling constant hold,

$$sin\theta_w = \frac{g^{'}}{\sqrt{g^2 + g'^2}}$$

$$cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

In terms of new field equation (5) becomes,

$$\mathcal{L}^0_{lepton} = -(gsin\theta_w J^\mu_3 A_\mu + \frac{g\prime}{2} cos\theta_w J^\mu_Y A_\mu) + (-gcos\theta_w J^\mu_3 + \frac{g\prime}{2} sin\theta_w J^\mu_Y) Z_\mu.$$

We can easily identify the EM current coupled to the photon field A_μ and hence electric charge $e = gsin\theta_w = g'cos\theta_w$.
Upto now we have 4 massless gauge fields $W^\pm, Z_\mu, A_\mu$ in the theory.

1.5.1 Higgs Mechanism and the W and Z mass

In order to apply the Higgs mechanism to give mass to $W^\pm$ and Z^0 let us introduce the scalar doublet[6]

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L}_{scalar} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi)$$

where $V(\Phi^\dagger\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2$.
We can choose the vacuum expectation value of the Higgs field as[5],

$$< \Phi >_0 = \begin{pmatrix} 0 \\ \frac{\nu}{\sqrt{2}} \end{pmatrix}$$

,where $\nu = \sqrt{\frac{-\mu^2}{\lambda}}$ In order to make this explicit, let us parametrize the Higgs doublet as

$$\Phi = exp(i\frac{\tau^i}{2}\frac{x_i}{\nu}) \begin{pmatrix} 0 \\ \frac{\nu+H}{\sqrt{2}} \end{pmatrix}.$$

Now, if we make a $SU(2)_L$ gauge transformation with $\alpha_i = \frac{x_i}{\nu}$(unitary gauge)[5]

$$\Phi \to \Phi' = exp(-i\frac{\tau^i}{2}\frac{x_i}{\nu})\Phi = \begin{pmatrix} 0 \\ \frac{\nu+H}{\sqrt{2}} \end{pmatrix}$$

Now,

$$\mathcal{L}_{scalar} = |(\partial_\mu + i\frac{g}{2}\tau^i W^i_\mu + i\frac{g'}{2} Y B_\mu) \begin{pmatrix} 0 \\ \frac{\nu+H}{\sqrt{2}} \end{pmatrix} |^2 - \mu^2 \frac{(\nu+H)^2}{2} - \lambda \frac{(\nu+H)^4}{4}. \quad (1.6)$$

From the first term of this lagrangian we get,
$\frac{1}{2}\partial_\mu H \partial^\mu H + \frac{g^2}{8}(\nu+H)^2 W^+_\mu W^{-\mu} + \frac{g^2}{8(cos\theta_w)^2}(\nu+H)^2 Z_\mu Z^\mu$.
Now the mass terms for charged and neutral gauge boson are $\frac{g^2}{8}(\nu)^2 W^+_\mu W^{-\mu}$

and $\frac{g^2}{8cos^2\theta_w}\nu^2 Z_\mu Z^\mu$ respectively. There is no quadratic in A_μ appears and hence photon remains massless.
The Standard Model predictions for the W and Z masses are[5]

$$M_W^2 = \frac{g^2\nu^2}{4} = \frac{e^2\nu^2}{4(sin\theta_w)^2} \sim (80GeV)^2$$

$$M_Z^2 = \frac{g^2\nu^2}{4cos^2\theta_w} = \frac{e^2\nu^2}{4(sin\theta_w cos\theta_w)^2} \sim (90GeV)^2$$

where, experimental value for $sin^2\theta_w = 0.22$.

1.5.2 Lepton masses

Spontaneous symmetry breaking will generate lepton mass, if we add a Yukawa interaction of lepton and Φ fields, which is renormalizable and invariant under $SU(2)_L \otimes (1)_Y$ gauge transformations.

$$\begin{aligned} \mathcal{L}_{Yukawa} &= Y[\bar{R}(\Phi^\dagger L + (\bar{L}\Phi)R] \\ &= Y\frac{(\nu + H)}{\sqrt{2}}\Big(l_R \begin{pmatrix}0 & \bar{1}\end{pmatrix}\begin{pmatrix}\nu_l \\ l_l\end{pmatrix} + \begin{pmatrix}\bar{\nu}_l & \bar{l}_l\end{pmatrix}\begin{pmatrix}0 \\ 1\end{pmatrix} l_R\Big) \\ &= \frac{Y\nu}{\sqrt{2}}\bar{l}l + \frac{Y}{\sqrt{2}}\bar{l}lH \end{aligned}$$

From this we have the mass term for lepton,

$$m_l = \frac{Y\nu}{\sqrt{2}}$$

Higgs lepton coupling strength is

$$C_{\bar{l}lH} = \frac{Y}{\sqrt{2}} = \frac{m_l}{\nu}$$

The neutrinos can't acquire mass or coupling to H field since there are no right handed neutrino ν_R field in the SM. The similar technique can also be extended to the quark sector to generate the masses of quarks.

Chapter 2

Search for Dark Matter

2.1 Introduction

A central task of modern cosmology is to determine what universe made of. Measurements by PLANCK and WMAP demonstrate that nearly 85% of universe's matter density is dark. Identifying the nature of dark matter (DM) remains one of the primary open questions in physics.The standard model of particle physics alone cannot explain the nature of this (DM), suggesting that the model must be extended. All evidence in favor of particle DM thus comes from observations of its gravitational effects on baryonic matter. While we have collected important clues from these results, many open questions remain: what is DM mass? what is the strength of its interactions with visible matter? How is it distributed throughout the galaxy? Fortunately, we are in the midst of a data-driven era in astroparticle physics that holds great promise towards addressing these questions. A wide variety of experiments are currently going on searching for DM interactions in the lab and sky.

2.1.1 Evidence

There are lot of evidences in support of the existence of invisible mass or DM. Some of them are

Rotation curves

The most convincing and direct evidence for dark matter on galactic scales comes from the observations of the rotation curves of galaxies, namely the graph of circular velocities of stars and gas as a function of their distance from the galactic center. From standard Newtonian gravity, we know that star's rotational velocity is

$$v(r) = \sqrt{\frac{GM}{r}}$$

where M is the enclosed mass, r is the radial distance, and G is the gravitational constant. For distances that beyond the galactic disk($r \geq R_{disk}$) ,Gauss' law tells us that M should remain constant assuming all the mass is concentrated in the disk, and $v \propto r^{-1/2}$. Instead, observations find that circular velocity curve flattens out at these distances, implying that$M(r) \propto r$. This suggests that there is an additional 'dark' component of matter beyond the visible matter in the disk.From rotation curves, we infer that DM mass density distribution is

$$\rho(r) \propto \frac{M(r)}{r^3} \sim \frac{1}{r^2}$$

The average velocity of DM in the halo can be obtained using virial theorem

$$< v >= \sqrt{\frac{GM_{halo}}{R_{halo}}} \sim 200km/s$$

Importantly notice that the DM is non-relativistic, this will end up playing an important role in predicting observational signatures.

Galaxy clusters

A cluster of galaxies gave the first hints of DM (in the modern sense). In 1933, F.Zwicky inferred from measurements of the velocity dispersion of galaxies in the coma cluster, a mass-to-light ratio of around 400 solar masses per solar luminosity, thus exceeding the ratio in the solar neighbourhood by two orders of magnitude. Today most dynamical estimates are consistent with a value $\Omega_M = 0.2 - 0.3$[3] on cluster scales. The mass of a cluster can be determined viaseveral methods, including apllication of the virial theorem to the observed distribution of radial velocities, by weak gravitational lensing, and by studying the profile of X-ray emission that traces the distribution of hot emitting gas in rich clusters.

Gravitational lensing

One of the consequences of general relativity is that massive objects(such as a cluster of galaxies) lying between a more distant source and an observer should act as a lens to bend the light from this source. By measuring the distortion geometry, the mass of the intervening cluster can be obtained.In dozens of cases where this has been done, the mass-to-light ratios obtained correspond to the dynamical DM measurements of clusters.

Cosmic Microwave Background(CMB)

The existence of background radiation originating from the propagation of photons in the early universe(once they decoupled from matter) was predicted by George Gamow and his collaborators in 1948 and discovered by Arno Penzias and Robert Wilson in 1965. After many decades of experimental effort, the CMB is known to be isotropic at the 10^{-5} level and to follow with extraordinary precision the spectrum of a blackbody corresponding to a temperature T=2.726 k. The CMB anisotropy was first discovered by COBE in 1992.From the analysis of the WMAP data alone, the following values are found for the abudance of baryons and matter in the universe $\Omega_b h^2 = 0.024 \pm 0.001$ and $\Omega_M h^2 = 0.14 \pm 0.02$ [3]. The observed CMB angular power spectrum provides powerful evidence in support of dark matter, as its precise structure is well fitted by the Lambda-CDM model.

2.1.2 Thermodynamics in the Expanding Universe

The key to understand the thermal history is the comparison between the rate of interactions Γ and the rate of expansion H. When $\Gamma \gg H$, then the time scale of particle interactions is much smaller than the characteristic time scale of expansion,

$$t_c \equiv \frac{1}{\Gamma} \ll t_H \equiv \frac{1}{H}$$

At early times, thermodynamical properties of universe were determined by local equilibrium. However, it is the departure from equilibrium that make life interesting. As the universe cools, the rate of interactions may decrease faster than the expansion rate. At time $t_c \sim t_H$, the particles decouple from the thermal bath. Different particle species may have different interaction rates and so may decouple at different times.

We consider the most popular candidate of DM that is weakly interacting massive particles(WIMP) and try to explain its thermal evolution and relic density.

Decoupling and Freeze-out

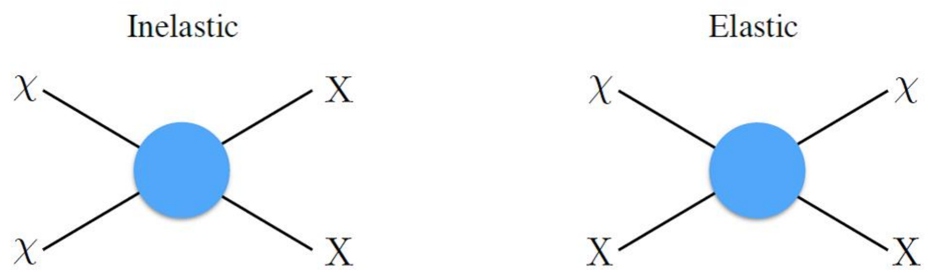
To understand the world around us, it is therefore crucial to understand the deviations from equilibrium that led to freeze-out of massive particles. Below the scale of electroweak symmetry breaking $T \leq 100GeV$, the gauge bosons of weak interactions,$W^{\pm}$ and Z, receive masses $M_W \sim M_Z$. The cross section associated with processes $2 \leftrightarrow 2$ scattering mediated by weak force becomes $\sigma \sim G_F^2 T^2$, where$G_F \sim \frac{\alpha}{M_W^2}$ is the fermi constant.

$$\Gamma = n\sigma v$$

where, n is the number density ,σ is the interaction cross section, v is average velocity of particle.
$\Gamma = n\sigma v \sim T^3 \times \frac{\alpha^2}{T^2} \sim \alpha^2 T$
Hubble rate $H \sim \frac{T^2}{M^{pl}}$, where M_{pl} is the Planck mass=10^{19}GeV.

$$\frac{\Gamma}{H} \sim \frac{\alpha^2 M_{pl} T^3}{M_W^4} \sim (\frac{T}{1MeV})^3$$

where we have used $\alpha \sim 0.01$ for numerical estimate.
Particles that interact with primordial plasma only through weak interaction therefore decouple around 1MeV.



(fig-1: An illustration of the inelastic $\chi\chi \rightarrow XX(left)$ and elastic $\chi X \rightarrow \chi X(right)$ scattering)

Figure -1 shows two possible $2 \longrightarrow 2$ interaction diagrams that are allowed with χ the DM particle and X a standard model particle. As the universe expands, it becomes increasingly harder for a DM particle to find a partner to annihilate with and the forward reaction shut off. The freeze-out time occurs when the annihilation rate $\Gamma_{inelastic}$ is in the order of Hubble rate, H:

$$\Gamma_{inelastic} = n_\chi < \sigma v > \sim H$$

Cold dark matter is non-relativistic at freeze-out with $n_\chi \sim T^{\frac{3}{2}} \exp^{\frac{m_\chi}{T}}$, hot DM is relativistic at freeze-out with $n_\chi \sim T^3$[4] where T the temperature of DM species. Warm DM falls somewhere in between these two cases.

2.1.3 DM Relic Density

The evolution of the phase space density $f(p^\mu, x^\mu)$ of a particle species is described by the Boltzmann equation which can be written as[4]

$$\hat{L}[f] = C[f] \tag{2.1}$$

where, L is the Liouville operator giving the net rate of change in time of particle phase space density f and C is the collision operator representing the number of particles per phase space volume that are lost or gained per unit time under collision with other particles.
The co-variant, relativistic generalization of the Liouville operator is[4]

$$\hat{L} = p^{\alpha}\frac{\partial}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial}{\partial p^{\alpha}}$$

Note, as expected, gravitational effects enter the equation only through the affine connection. For the Friedmann-Robertson-Walker(FRW) model, the phase space density is spatially homogeneous and isotropic:$f = f(|\overrightarrow{p}|, t)$ or equivalently f(E,t). In this case the Liouville operator becomes

$$L[f] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\overrightarrow{p}|^2\frac{\partial f}{\partial E}$$

Using the defination of number density in terms of phase space density

$$n(t) = \frac{g}{(2\pi)^3}\int f(E,t)d^3p$$

Using Boltzmann equation(1), we can write

$$\frac{g}{(2\pi)^3}\int \frac{\partial f}{\partial t}d^3p - \frac{g}{(2\pi)^3}\int H\frac{|\overrightarrow{p}|^2}{E}\frac{\partial f}{\partial E}d^3p = \frac{g}{(2\pi)^3}\int \frac{C[f]}{E}d^3p$$

Upon integration by parts,

$$\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3}\int \frac{C[f]}{E}d^3p$$

where, $H = \frac{\dot{a}}{a}$ is the expansion rate of the Universe and a is the scale factor.
Hence we can write

$$\frac{g}{(2\pi)^3}\int L[f]d^3p = \frac{1}{a^3}\frac{d}{dt}(na^3) = \frac{dn}{dt} + 3Hn \tag{2.2}$$

When there is no number changing DM interactions that is C[f]=0, then equation(2) simply shows that na^3 is constant in time.

However, the evolution of DM density is non-trivial if collision term exists. To see this explicitly, consider interactions of the form $1+2 \leftrightarrow 3+4$.
The collision term for particle 1 is then[4]

$$\frac{g_1}{(2\pi)^3}\int \frac{C[f]}{E_1}d^3p_1 = -\sum_{spin}\int [f_1 f_2(1 \pm f_3))(1 \pm f_4)|\mathcal{M}_{12\to34}|^2 - f_3 f_4(1 \pm f_1)(1 \pm f_2)$$
$$|\mathcal{M}_{34\to12}|^2] \times (2\pi)^4\delta^4(p_1 + p_2 - p_3 - p_4)d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4$$

where, g_i and f_i are the spin degrees of freedom and phase space densities respectively for particle i and $\mathcal{M}_{x\to y}$ is the matrix element for the reaction $x \to y$. Factors of form $(1 \pm f)$ represents Pauli blocking and Bose enhancement, the minus sign applies to fermions and plus sign to bosons. These terms tell the fact that it is easier(harder) for a boson(fermion) to transition to a state that already contains a boson(fermion).

The above equation contains a delta function that enforces the energy and momentum conservation, and the phase space integration factor,$d\Pi_i = \frac{d^3p_i}{(2\pi)^3 2E_i}$. The above equation is quite complicated; however, it reduces to a more manageable form after making the following assumptions:

1. The temperature of each species satisfies $Ti \ll (E_i - \mu_i)$, where μ_i is its chemical potential, so that they follow the Maxwell-Boltzmann distribution. In this case, the statistical mechanical factors in the calculation can be ignored and $(1 \pm f) \sim 1$.

2. The kinetic equilibrium is maintained and the standard model particles in the interaction are in thermal equilibrium with the photon bath.

Using the standard definition relating the cross section to the matrix element, we get

$$\sum_{spin} \int |\mathcal{M}_{ij\to kl}|^2 \times (2\pi)^4 \delta^4(p_i + p_j - p_k - p_l) d\Pi_k d\Pi_l = 4 g_i g_j \sigma_{ij} \sqrt{(p_i p_j)^2 - (m_i m_j)^2}$$

where, σ_{ij} is the cross section for the scattering process.
Substituting this into the collision term gives

$$\frac{g_1}{(2\pi)^3} \int \frac{C[f]}{E_1} d^3p_1 = -\int [(\sigma v_{m\varnothing l})_{12} dn_1 dn_2 - (\sigma v_{m\varnothing l})_{34} dn_3 dn_4]$$

where, the Moller velocity[4] is defined as

$$(\sigma v_{m\varnothing l})_{ij} = \frac{\sqrt{(p_i p_j)^2 - (m_i m_j)^2}}{E_i E_j}$$

Because $\sigma v_{m\varnothing l}$ varies slowly with changes in the number density of the initial and final state particles, it can be factored out of the integrand to give

$$\dot{n_1} + 3Hn_1 = - < \sigma v_{m\varnothing l} >_{12} n_1 n_2 + < \sigma v_{m\varnothing l} >_{34} n_3 n_4 \tag{2.3}$$

Note that the velocity that is used in the cross section average is not the relative velocity,v_{rel}, of incoming particles. This is important as $(\sigma v_{m\varnothing l})_{ij} n_i n_j$ is lorentz

invariant where $v_{rel}n_in_j$ is not.
For simplification of notation write $v_{m\varnothing l} \to v$.

Let us now return to the specific inelastic process illustrated in fig-1. In this case, particle 1 and particle 2 are identical with number density n and particle 3 and 4 are standard model particles in thermal equilibrium with the photon bath.

When the DM is also in equilibrium with standard model final states, then detailed balance says that

$$< \sigma v >_{12} n_{eq}^2 =< \sigma v >_{34} n_3^{eq} n_4^{eq}$$

which can be used to rewrite the second term of equation(3) in terms of DM density and the cross section for forward reaction.
The Boltzmann equation reduces to

$$\dot{n} + 3Hn =< \sigma v > (n_{eq}^2 - n^2)$$

The DM number density n, decreases with the expansion of the universe(in addition to any number changing effects from the collision term) and it is useful to scale out this effect by defining the quantity $N_X = \frac{n}{s}$, where s is the total entropy density of the universe and $s = \frac{2\pi^2}{4\pi} g_{\star s}(T)T^3$,$g_{\star s}$ counts the number of relativistic degrees of freedom.

Using the conservation of entropy per co-moving volume ($sa^3 = const$), it follows that $\dot{n} + 3Hn = s\dot{N}_x$ and equation (5) becomes

$$\frac{dN_x}{dt} = -s < \sigma v > (N_x^2 - (N_x^{eq})^2) \tag{2.4}$$

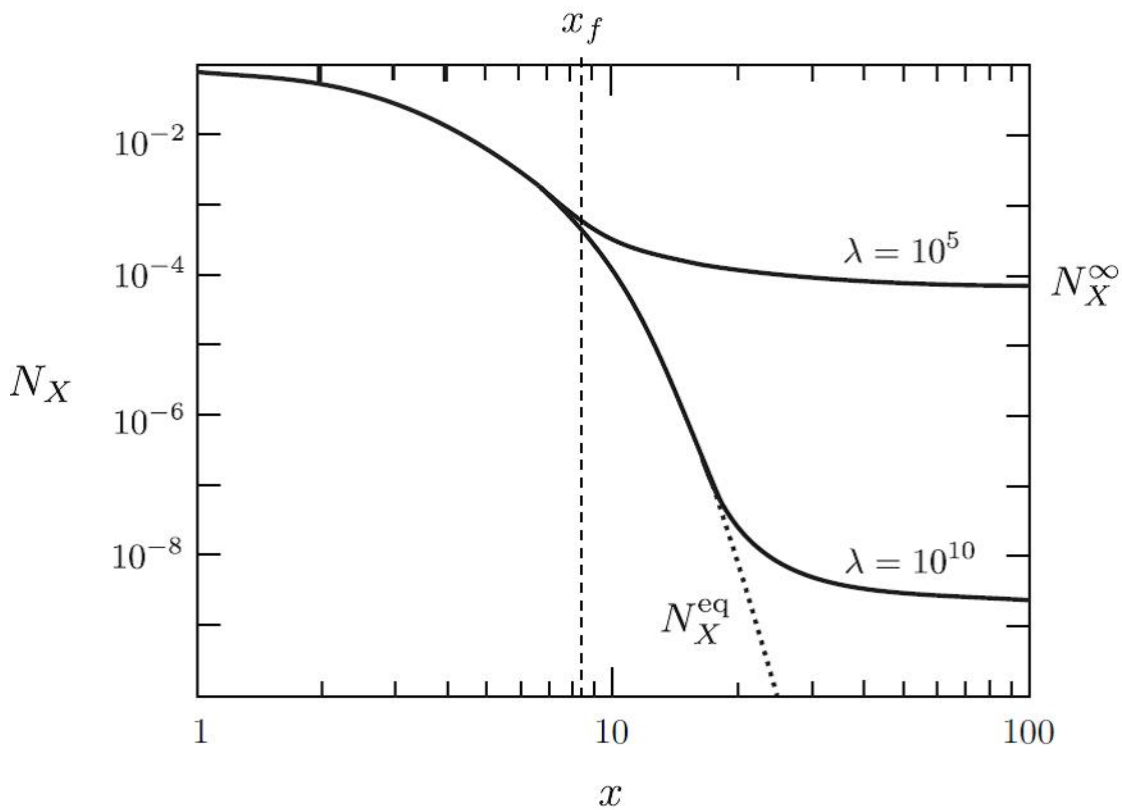
Since most of the interacting dynamics will take place when the temperature is of order of particle mass $T \sim M_x$, it is convenient to define a new measure of time, $x \equiv \frac{M_x}{T}$. To write the Boltzmann equation in terms of x rather than t, we note that

$$\frac{dx}{dt} = \frac{d}{dt}(\frac{M_x}{T}) = -\frac{1}{T}\frac{dT}{dt}x \simeq Hx,$$

where we have assumed that $T \propto a^{-1}$. We assume radiation domination so that $H = \frac{H(M_x)}{x^2}$.
The equation(2.4) becomes the so called Riccati equation

$$\frac{dN_x}{dx} = -\frac{\lambda}{x^2}[N_x^2 - (N_x^{eq})^2] \tag{2.5}$$

where we have defined $\lambda \equiv \frac{2\pi^2}{45} g_{\star s} \frac{M_x^3 <\sigma v>}{H(M_x)}$.
We will treat λ as a constant. Unfortunately even for constant λ, there are no analytic solution to equation(2.5).



(figure 2: An illustration of the DM density N_X as a function of x. Before freeze-out ($x < x_f$), the density tracks the equilibrium expectation (dashed black). After freeze-out, the density remains nearly constant as a function of time, as indicated by the solid black line. Graph taken from the article[4])

As expected at very high temperature,$x < 1$, we have $N_x \simeq N_x^{eq} \simeq 1$. However, at low temperature, $x \gg 1$, the equilibrium abundance becomes exponentially suppressed, $N_x \sim \exp(-x)$. Numerically, we find that freeze-out happens at about $x \sim 10$.This is when the solution of Boltzmann equation starts to deviate significantly from the equilibrium abundance.

The final relic abundance determines the freeze-out density of DM. Let us estimate it's magnitude as a function of λ. After freeze-out, N_x will be much larger than N_x^{eq}. Thus at later times, we can drop N_x^{eq} from the Boltzmann equation($x > x_f$),

$$\frac{dN_x}{dx} \simeq -\frac{\lambda N_x^2}{x^2}$$

Integrating from x_f to $x = \infty$, we find

$$\frac{1}{N_x^\infty} - \frac{1}{N_x^f} = \frac{\lambda}{x_f}$$

where $N_x^f \equiv N_x(x_f)$.
Typically $N_x^f \gg N_x^\infty$, so a simple analytic approximation is

$$N_x^\infty \simeq \frac{x_f}{\lambda} \tag{2.6}$$

Of course, this still depends on the unknown freeze-out time(or time)x_f

WIMP Miracle:

It just remains to relate the freeze-out abundance of DM relics to DM density today.

$$\begin{aligned}\Omega_x &\equiv \frac{\rho_{x,0}}{\rho_{crit,0}} \\ &= \frac{M_x n_{x,0}}{3M_{pl}^2 H_0^2} = \frac{M_x N_{x,0} s_0}{3M_{pl}^2 H_0^2} = \frac{M_x N_x^\infty s_0}{3M_{pl}^2 H_0^2}\end{aligned}$$

Substituting $N_x^\infty = \frac{x_f}{\lambda}$ and $s_0 \equiv s_0(T_0)$, we get

$$\begin{aligned}\Omega_x &= \frac{H(M_x)}{M_x^2}\frac{x_f}{< \sigma v >}\frac{g_{\star s}(T_0)T_0^3}{g_{\star s}(M_x M_{pl}^2 H_0^2} \\ &= \frac{\pi}{9}\frac{x_f}{< \sigma v >}\Big(\frac{g_{\star s}(M_x)}{10}\Big)^{\frac{1}{2}}\frac{g_{\star s}(T_0)}{g_{\star s}(M_x)}\frac{T_0^3}{M_{pl}^2 H_0^2}\end{aligned}$$

where we have used $H(M_x) \simeq \frac{\pi}{3}\big(\frac{g_{\star s}(M_x}{10}\big)^{\frac{1}{2}}\frac{M_x^2}{M_{pl}}$
Finally we substitute the measured values of T_0,H_0 and $g_{\star s}(M_x) = g_\star(M_x)$:

$$\Omega_x h^2 \sim 0.1(\frac{x_f}{10})\big(\frac{10}{g_\star(M_x}\big)^{\frac{1}{2}}\frac{10^{-8}GeV^{-2}}{< \sigma v >}$$

This reproduces the observed DM density if $\sqrt{< \sigma v} \sim 10^{-4}GeV^{-1} \sim 0.1\sqrt{G_F}$
The fact that a thermal relic with a cross section characteristic of weak interaction gives the right dark matter abundance is called the WIMP miracle.

2.2 Direct Detection of Dark matter

2.2.1 Introduction

Direct detection experiments appear today as one of the most promising technique to detect particle DM. The idea is very simple; if our galaxy is filled with DM,then we expect a 'DM wind' coming towards us on the earth because the sun is moving through the Milky way's DM halo. These experiments operate underground(to minimize background) and search for DM particles via their scattering with atomic nuclei in the detector(by recording the recoil energy of nuclei).

2.2.2 Ingredient

The key ingredients for the calculation of the signal in direct detection experiments are the density and the velocity distribution of DM in the solar neighbourhood and the DM-nucleon scattering cross section . With this information, it is then possible to evaluate the rate of events expected in an experiment per unit time, per unit detector material mass.

The number of expected events per unit time, the event rate

$$R \propto N_T n v \sigma$$

where, N_T=Number of target nuclei per unit mass
$n = \frac{\rho_x}{M_x}$=Number density of DM particles of mass M_x
v=average velocity DM velocity w.r.t detector
σ=DM-nucleus cross section.

But real detector never measure the total rate but it measures only a limited window of recoil energy E_R(in particular they always have a lower energy threshold). DM velocity is not unique,in fact DM particles are described by a local velocity distribution $f(\vec{v}, t)$, where $\vec{v}$ is DM velocity in the reference frame of detector. Hence we need to integrate to all possible DM velocities.

The differential rate per unit detector mass is given by

$$\frac{dR}{dE_R} = N_T \frac{\rho_x}{M_x} \int_{v_{min}}^{v_{max}} d^3v v f(\vec{v}, t) \frac{d\sigma}{dE_R} \tag{2.7}$$

2.2.3 Scattering Classifications

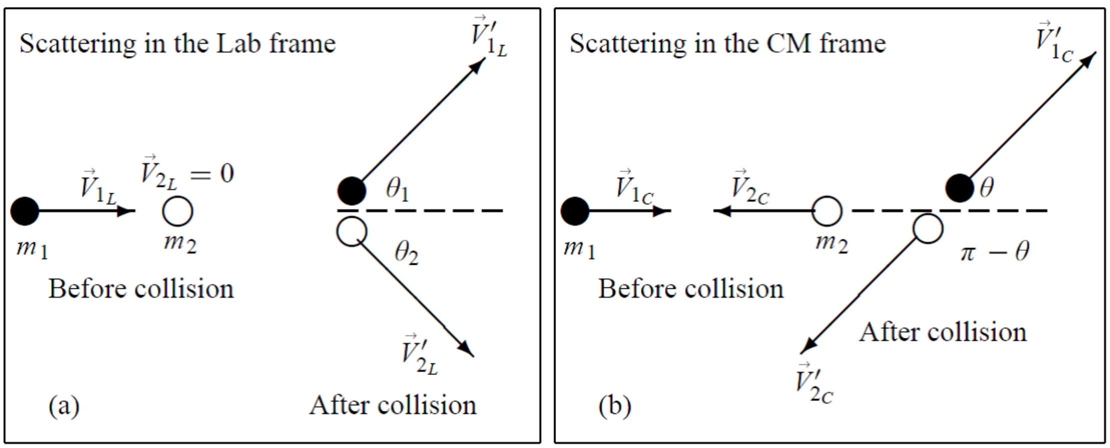
The type of scattering processes considered can be classified by two important characteristics: elastic or inelastic scattering and spin-dependent or spin-independent scattering.

Elastic and inelastic scattering: The elastic scattering of a DM particle off a nucleus in a detector is simply the interaction of the DM with a nucleus as a whole, causing it to recoil enough to measure the recoil energy spectrum in the target. Inelastic scattering, on the other hand, is not observed by the recoil of a target nuclei. Instead, the DM particle interacts with orbital electrons in the target either exciting them or ionizing the target.

spin-dependent and spin-independent scattering: These two scatterings are commonly discussed in the context of two classes of couplings. First,axial-vector(spin- dependent) interactions result from couplings to the content of a nucleon. The cross sections for spin-dependent scattering are proportional to J(J+1) rather than the number of nucleons, so little is gained by using heavier target nuclei. However, we will see for scalar(spin-independent) interaction, the cross section increases dramatically dominates over spin-dependent scattering in current experiment which use heavy atoms as targets.

2.2.4 Dark Matter Elastic Scattering off of a Nucleus

Let us consider the elastic scattering of a dark matter particle having mass m_1 moving with a non-relativistic velocity v_1 and a stationary nucleus having mass m_2.



(figure 4: Elastic scattering of DM and stationary nucleus in the Lab and CM frames)

If $\vec{r_{2L}}$ and $\vec{r_{2C}}$ denotes the position of m_2 in lab frame and CM frame respectively, and if $\vec{R}$ denotes the position of centre of mass with respect to the lab frame, then we have the relation $\vec{r_{2L}} = \vec{r_{2C}} + \vec{R}$. A time derivative of this leads to $\vec{V_{2L}} = \vec{V_{2C}} + \vec{V_{CM}}$.
After collision,

$$\vec{V'_{2L}} = \vec{V'_{2C}} + \vec{V_{CM}}.$$

From fig. we can infer the x and y components of the equation:

$$V'_{2L} cos\theta_2 = -V'_{2C} cos\theta + V_{CM}.$$

$$V'_{2L} sin\theta_2 = -V'_{2C} sin\theta.$$

In CM frame, the total momentum before and after scattering are separately zero. From this we can write $V_{CM} = V_{2C} = \frac{m_1}{m_2} V_{1C}$ and $V'_{2C} = \frac{m_1}{m_2} V'_{1C}$

From conservation of kinetic energy, it is easy to show that $V_{1C} = V'_{1C}$ and $V_{2C} = V'_{2C}$.We kn

We can derive the relation between angles in both the frames by dividing equation (7)and (8),

$$\begin{aligned} tan\theta_2 &= \frac{sin\theta}{\frac{V_{2C}}{V'_{2C}} - cos\theta} \\ &= \frac{sin\theta}{1 - cos\theta} = cot\frac{\theta}{2} \end{aligned}$$

Therefore, $\theta_2 = \frac{\pi - \theta}{2}$.

Recoil energy of Nucleus

The energy deposited in the nucleus by the DM is

$$\begin{aligned} E_R &= \frac{1}{2}m_2(V_{Lx}^2 + V_{Ly}^2) \\ &= \frac{1}{2}m_2[(V'_{2L}cos\theta_2)^2 + (V'_{2L}sin\theta_2)^2] \\ &= \frac{1}{2}m_2[(-V'_{2C}cos\theta + V_{CM})^2 + (V'_{2L}sin\theta)^2)^2] \\ &= \frac{1}{2}m_2V_{2C}^2(2 - 2cos\theta) \\ &= m_2(\frac{m_1}{m_1 + m_2}V_{1L})^2(1 - cos\theta) \\ &= \frac{\mu^2 V_{1L}^2}{m_2}(1 - cos\theta) \end{aligned}$$

where, μ=reduced mass=$\frac{m_1 m_2}{m_1+m_2}$.

Hence the expression for recoil energy is

$$E_R = \frac{\mu^2 v^2}{M_N}(1 - cos\theta) \tag{2.8}$$

where, M_N is the mass of nucleus.

The maximum recoil energy of nucleus is $\frac{2\mu^2 v^2}{M_N}$.

The velocity dispersion of DM is $v \sim 10^{-3}c$. Considering the mass of M_N is of order of 10GeV and DM is similar mass or heavier so $\mu \sim M_N$.Then the typical recoil energy should be in the range , $E_R \sim 10^{-6}M_N \sim 10keV$. Hence to detect such a very small recoil energy, our detector should be very sensitive.

2.2.5 Differential scattering cross section

The scattering takes place in the non-relativistic limit. The cross section is therefore approximately isotropic.

$$\implies \frac{d\sigma}{d(cos\theta)} = constant = \frac{\sigma}{2}$$

$$E_R = E_R^{max}(\frac{1-cos\theta}{2})$$

$$\frac{dE_R}{d(cos\theta)} = \frac{E_R^{max}}{2}$$

Therefore,

$$\frac{d\sigma}{dE_R} = \frac{d\sigma}{d(cos\theta)}\frac{dcos\theta}{dE_R} = \frac{\sigma}{2} \times \frac{2}{E_R^{max}}$$

For light nuclei, the DM particle sees the nucleus as a whole (without substructure) point like nucleus but for heavier nuclei we have to take into account a suppression factor.

$$\sigma \to \sigma_0 \times F^2(q^2)$$

where, $F(q^2)^2$ is the nuclear form factor, takes into account the finite size of nucleus and encodes dependence on momentum transfer, and σ_0 is the cross section at zero momentum transfer(point-like). Now we can write

$$\frac{d\sigma}{dE_R} = \frac{\sigma_0}{E_R^{max}}F^2(q^2) = \frac{M_N}{2\mu^2 v^2}\sigma_0 F^2(q^2) \tag{2.9}$$

We generally use the Helm form factor which is approximately the Fourier transform of the nucleus' mass distribution, given by[4]

$$F^2(q^2) = 3e^{-s^2q^2}\frac{sin(qr_n) - qr_n cos(qr_n)}{(qr_n)^3}$$

where effective nuclear radius is $r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$, with $a \simeq 0.52fm$,$s \simeq 0.9fm$, $c = 1.23^{1/3} - 0.60fm$.
q is the momentum transfer=$\sqrt{2M_N E_R}$

1. When momentum transfer is small, the DM doesn't probe the size of nucleus and coherently scatters off the entire nucleus. In this limit $FF^2(q^2) \to 1$.

2. When momentum transfer increases,the DM becomes sensitive to spatial structure of nucleus, $F^2(q^2) < 1$. This effect is strong for heavy nuclei(I,Xe etc.).

There are two relevant contributions to scattering cross section,one is spin-independent(SI) and other is spin-dependent(SD). Later we will see that spin dependent scattering cross section plays the dominant role in direct detection. Hence we can write the expression for differential cross section

$$\frac{d\sigma}{dE_R} = \frac{M_N}{2\mu^2 v^2}\Big[\sigma_0^{SI} F_{SI}^2 + \sigma_0^{SD} F_{SD}^2\Big] \tag{2.10}$$

Spin-independent(SI) scattering

Now we derive the differential scattering cross section for the DM-nucleus interaction,taking an effective operator approach. Let us assume that DM is a spin-1/2 Dirac fermion that interacts with quarks via a scalar or vector boson ϕ with mass m_ϕ.The scattering process is described by the effective four-fermion interaction:

$$\mathcal{L}_{eff} = g(q^2, m_\phi)\bar{X}\Gamma_X X\bar{Q}\Gamma_Q Q$$

where Q represents the quark field, $\Gamma_{X,Q} = \Big\{I, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, \sigma^{\mu\nu}\gamma^5\Big\}$ and $g(q^2, m_\phi)$ is an effective coupling, proportional to $\frac{1}{m_\phi}$ for contact interactions $(q^2 \ll m_\phi)$. We then proceed as follows:

1. Map the quark operator to a nucleon operator and use this to obtain the amplitude for DM-nucleus scattering.
2. Take non-relativistic limit of the scattering amplitude, $\mathcal{M}_{nr}$.
3. Relate this to the differential cross section by averaging/assuming initial and final state spins:

$$\frac{dR}{dE_R} = \frac{2M_N}{\pi v^2} < |\mathcal{M}_{nr}|^2 >$$

The spin-independent contribution can arise from scalar couplings if DM to quarks, which occurs through the operator $(\bar{X}X)(\bar{Q}Q)$,

$$\mathcal{L}_{scalar} = g_\phi \bar{X}X\bar{Q}Q$$

To write the quark fields in terms of nucleon fields (labeled n,p), we must evaluate of form $< n|\bar{Q}Q|n >$. These terms are related to nucleon mass using the trace of QCD energy momentum tensor- for further details see[2]. The fraction the proton mass accounted for by a particular quark flavour is defined as $m_p f_{Tq}^p \equiv < p|m_q\bar{Q}Q|p >$ and the coupling of the DM to the proton or neutron is given by[3]

$$f_{p,n} = \sum_{q=u,d,s} m_{p,n}\frac{g_\phi}{m_q} f_{Tq}^{p,n} + \frac{2}{27} f_{TG}^{p,n} \sum_{q=c,b,t} m_{p,n}\frac{g_\phi}{m_q}$$

where, $f_{TG}^{p,n} = 1 - \sum_{q=u,d,s} f_{Tq}^{p,n}$, ${}^{p}_{Tu} = 0.020 \pm 0.004$, $f_{Td}^{p} = 0.026 \pm 0.005$, $f_{Ts}^{p} = 0.118 \pm 0.062$, $f_{Tu}^{n} = 0.014 \pm 0.003$, $f_{Td}^{n} = 0.036 \pm 0.008$, $f_{Ts}^{n} = 0.118 \pm 0.062$.

The mass fractions $f_{Tq}^{p,n}$ are determined experimentally, so $f_{p,n}$ are constants of the theory once $g\phi$ is set. The scattering amplitude is therefore

$$\mathcal{M} = f_p \bar{X} X \bar{p} p + f_n \bar{X} X \bar{n} n$$

Because $\bar{p}p$ and $\bar{n}n$ give the proton and neutron count respectively, and taking account of the suppression factor due to size of nucleus, it is straight forward to rewrite $\mathcal{M}$ in terms of the fields for nuclei.

$$\mathcal{M} = \Big[Z f_P + (A - Z) f_n \Big] \bar{X} X \bar{N} N F(q^2)$$

Now we want to find out the non-relativistic limit of amplitude. Remember that Dirac field is given by

$$N^s(p) = \begin{pmatrix} \sqrt{p.\sigma}\xi^s \\ \sqrt{p.\bar{\sigma}}\xi^s \end{pmatrix}$$

where, s is the spin index and ξ^s is the two component spinors satisfying $\sum_{spin} \xi^{s\dagger}\xi^s = 1$.

In non-relativistic limits, $p^0 = M_N$ and $\sqrt{p.\sigma} \approx \sqrt{M_N - \vec{p}.\sigma} \approx \sqrt{M_N\Big(1 - \frac{\vec{p}.\sigma}{2M_N}\Big)}$.

The same applies for the X fields, expect with appropriate substitutions for mass and momenta. Therefore

$$\begin{aligned}
\bar{N^{s\prime}}(p\prime) N^s(p) &= \big(N^{s\prime}(p\prime)\big)^\dagger \gamma^0 N^s(p) \\
&= \big(\sqrt{p\prime.\sigma}\xi^{s\prime\dagger} \quad \sqrt{p\prime.\bar{\sigma}}\xi^{s\prime\dagger}\big) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \sqrt{p.\sigma}\xi^s \\ \sqrt{p.\bar{\sigma}}\xi^s \end{pmatrix} \\
&= \xi^{s\prime\dagger} \Big(\sqrt{p\prime.\bar{\sigma}}\sqrt{p.\sigma} + \sqrt{p\prime.\sigma}\sqrt{p.\bar{\sigma}} \Big) \\
&\approx 2M_N \xi^{s\prime\dagger}\xi^s
\end{aligned}$$

where $s(s')$ is the spin index for incoming(outgoing) nucleus.

Similarly $\bar{X}X \approx 2M_X \xi^{r\prime\dagger}\xi^r$ in the non-relativistic limit, where $r(r\prime)$ is the spin index for incoming(outgoing) DM particle. Dropping the factors of $2M_N$ and $2M_X$, which are the relativistic normalization, gives

$$\mathcal{M}_{nr} = \Big[Z f_P + (A - Z) f_n \Big] F(q^2) \xi^{s\prime\dagger}\xi^s \xi^{r\prime\dagger}\xi^r$$

The differential scattering cross section is thus

$$\Big\langle |\mathcal{M}_{nr}|^2 \Big\rangle = \frac{1}{(2J+1)(2s_X+1)} \sum_{spin} \Big[Z f_P + (A - Z) f_n \Big]^2 F^2(q^2) |\xi^{s\prime\dagger}\xi^s|^2 |\xi^{r\prime\dagger}\xi^r|^2$$

where, $J(s_x)$ is the nuclear(DM) spin.
Note that

$$\frac{1}{2s_X}\sum_{r\prime,r=1,2}|\xi^{r\prime\dagger}\xi^{r}|^2 = \frac{1}{2s_X}\sum_{r\prime,r=1,2} tr[\xi^{r\prime}\xi^{r\prime\dagger}\xi^{r}\xi^{r\dagger}] = \frac{1}{2}tr(\mathbf{1}) = 1$$

A similar result applies to the spinor product of ξ^s, leaving us with

$$\frac{d\sigma}{dE_R} = \frac{2M_N}{\pi v^2}\Big[Zf_P + (A-Z)f_n\Big]^2 F^2(q^2) \tag{2.11}$$

There are some important points to note here,

1. when $f_p = f_n$ then the differential cross section proportional to A^2. In this case, DM couples coherently to the entire nucleus and the strength of the scattering interaction increases with the mass number of the nucleus.

2. The effective interactions are referred to as spin-independent because the scattering cross section does not depend on the nuclear spin.

3. The scattering cross section is independent of the recoil energy and thus the differential rate is a falling exponential.

2.2.6 Spin dependent(SD) scattering

It arises due to the interaction of a DM particle with the spin of the nucleus. It can arises from the axial vector coupling of DM to quark field[2].

$$\mathcal{L}_{eff} \propto \bar{X}\gamma_\mu\gamma^5 X\bar{Q}\gamma^\mu\gamma^5 Q \tag{2.12}$$

with cross section

$$\frac{d\sigma}{dE_R} = \frac{16M_N}{\pi v^2}G_F^2 J(J+1)\Lambda^2 F^2(q^2)$$

where $\Lambda^2 = \frac{1}{J}[a_p < S_p > +a_n < S_n >]^2$, G_F is the fermi coupling constant, J is the spin of nucleus, $a_{p,n}$ is the effective coupling of DM to proton or neutron, and $< S_{p,n} >$ is the average spin contribution from the proton or neutron.

Notice that SD interaction is no longer coherent with the nucleus and does not scale as A^2. So it does not grow as rapidly with the size of the nucleus as the SI interaction. As a result SD interactions are more challenging to observe experimentally and the current bounds are weaker than those from SI interactions. Hence in direct detection experiment SI scattering is more significant than SD.

Let us consider only spin-independent scattering process.We have our differential cross section

$$\frac{d\sigma}{dE_R}\Big|_{SI} = \frac{M_N}{2\mu^2 v^2}\Big[\sigma_0^{SI} F_{SI}^2(q^2)\Big]$$

Comparing with equation(10) ,we have

$$\sigma_0^{SI} = \frac{4\mu^2}{\pi}\Big[Zf_P + (A-Z)f_N\Big]^2$$

To compare data from different direct detection experiment, which have different target nuclei, it is convenient to consider the DM-nucleon cross section σ_p.

$$\sigma_p = \frac{4\mu^2}{\pi}(f_p)^2$$

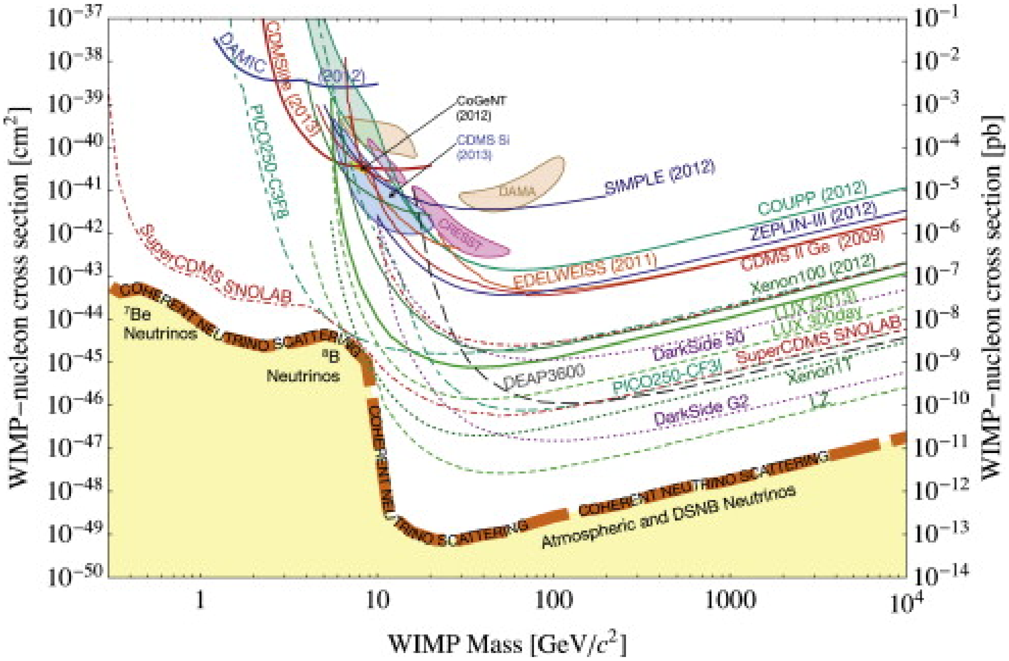
Hence

$$\sigma_0^{SI} = \sigma_p \frac{\Big[Zf_P + (A-Z)f_n\Big]^2}{f_p^2}(\frac{\mu}{\mu_p})^2$$

where, μ_p is the reduced mass of DM and nucleon. Hence the final expression for differential scattering cross section in spin-independent interaction,

$$\frac{d\sigma}{dE_R}\Big|_{SI} = \frac{M_N}{2v^2}\frac{\sigma_p}{\mu_p^2}\frac{\Big[Zf_P + (A-Z)f_n\Big]^2}{f_p^2} F_{SI}^2(q^2) \tag{2.13}$$

Figure 8 is a compilation of results from current direct detection experiments (solid lines), as well as projections for future experiments. Notice that the bounds become weaker at masses $M_x \sim 10GeV$ due to the energy thresholds of the experiments. Across all experiments, the sensitivity is optimal $\sim$ 50–100GeV, and then weakens towards higher DM mass.



(figur 4: Summary of current (solid) and projected (dotted/dashed) bounds on the spin independent WIMP-nucleon cross section. Shaded regions denote experimental anomalies, all of which are in tension with the exclusion bounds. The thick orange line denotes the cross section below which the experiments become sensitive to coherent neutrino scattering off nuclei. Figure from[1])

2.2.7 Estimating the total Rate

The differential rate per unit detector mass is given by

$$\frac{dR}{dE_R} = N_T \frac{\rho_x}{M_x} \int_{v_{min}}^{v_{max}} d^3v v f(v) \frac{d\sigma}{dE_R} \tag{2.14}$$

where,v_{min} is the threshold velocity given by $\sqrt{\frac{M_N E_R}{2\mu^2}}$ and v_{max} is set by Galactic escape velocity in frame of Earth. For calculation take $v_{max} \to \infty$. For most of the case we take $f_p \simeq f_n$. Hence the term $\frac{[Zf_P+(A-Z)f_n]^2}{f_p^2} = A^2$. Now after substituting all the values in the above equation, we get

$$\frac{dR}{dE_R} = N_T \frac{\rho_x}{M_x} A^2 F^2(q^2) \frac{\sigma_p}{2\mu^2} \int_{v_{min}}^{\infty} d^3v \frac{f(v)}{v}$$

For the purpose of illustration, consider a simple Maxwellian halo,

$$f(v)d^3v = \frac{1}{v_0^3 \pi^{3/2}} \exp{(-\frac{v^2}{v_0^2})} d^3v$$

where $v_0 \simeq 220km/s$ is the circular speed of the Sun and around the Galactic center.
If one use this (leaving out the motion of Sun and Earth), one would integrate to get

$$\int_{v_{min}}^{v_\infty} d^3v v f(v) = \frac{2}{\sqrt{\pi}}\frac{1}{v_0}\exp\left(-\frac{v_{min}^2}{v_0^2}\right)$$

Hence the differential rate becomes

$$\frac{dR}{dE_R} = \frac{1}{\sqrt{\pi}v_0} N_T M_N A^2 \frac{\sigma_p}{2M_x\mu^2}\rho \exp\left(-\frac{M_N E_R}{2\mu^2 v_0^2}\right) F^2(q^2)$$

Thus we expect to see a smooth, exponentially falling spectrum, multiplied by the form factor squared. Taking low momentum transfer limit $F^2(q^2) \to 1$ and integrating to get the total rate,

$$R = \frac{2}{\sqrt{\pi}} A^2 N_T \frac{\sigma_p}{M_x\mu^2} v_0 \rho \tag{2.15}$$

For more accurate calculation, one need to include time dependence and asymmetry velocity distribution as seen from Earth. This will change the event rate.

For a generic idea, consider a fiducial volume of 100 kg Xenon(atomic mass 132). Now question is, what WIMP-nucleon cross section do you need to see 1event/year for a 100GeV WIMP. Take $\rho \approx 0.4GeV$, $N_T \approx N_A \times 1000 = 6 \times 10^{26}$. If you do the calculation roughly it comes out to be

$$R \approx (10^{46}\sigma_p/cm^2)/yr$$

From this we can say that DM is very very weakly interacting with SM particles. The most sensitive experiments are currently starting to probe DM-nucleon cross sections $\sim 10^{-45}cm^2$, which is in the range expected for DM that interacts with the nucleus via the exchange of a Higgs boson.

2.3 Conclusion

Although we have several strong evidences for existence of DM, in fact majority of the universe is non-baryonic but still our understanding of it's nature and distribution is incomplete. Well-motivated hypothesis, such as WIMPs, have provided a starting point for experimental exploration and current experiments are reaching the necessary sensitivities to discover or exclude these candidates. However, weak-scale DM is not a guarantee, a broad range of interactions and mass scales are allowed. The current situation is complicated by the claim of a positive detection by the DAMA experiment, which has been contradicted by several other experiments. However, direct detection is not the only way to search for particle DM, there are other methods like indirect detection in space, collider searches at LHC etc. currently going on. Hopefully, these experiments will be able to detect particle DM in future and provide us a chance to know more about our universe.

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