

1 **CRACKING AND DUCTILITY ANALYSIS OF STEEL FIBER REINFORCED**
2 **PRESTRESSED CONCRETE BEAMS IN FLEXURE**
3

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ABSTRACT

9 The present study focuses on understanding the effect of steel fiber dosage on cracking and
10 ductile behavior of prestressed concrete beams (PCB) under flexure using digital image
11 correlation (DIC). Seven prestressed concrete beams were cast and tested under a shear span
12 to depth (a/d) ratio of five to simulate the flexure/flexure-shear dominant behavior. Full field
13 strain measurement with DIC technique was used to understand the effectiveness of steel
14 fibers on the crack bridging mechanisms. Three different volumetric fiber reinforcement
15 ratios viz. 0.35%, 0.70% and 1.0% were considered. Test results revealed that the strain
16 energy based ductility increased with an increase in fiber dosage from 0.35% to 1.0%. Post-
17 cracking stiffness improved by 50% due to the addition of 1.0% volume of steel fiber dosage.
18 DIC measurements of displacements and strains were found to be in good agreement with the
19 conventional linear variable displacement transducer (LVDT) measurements. DIC results
20 clearly established the effect of fibers on crack bridging and strain reduction. Concrete
21 strains and the strain in the prestressing strand reduced due to the better crack bridging of
22 steel fibers.

23

24 **Keywords:** Crack Bridging; Digital Image Correlation (DIC); Ductility; Prestressed Concrete
25 Beams; Steel Fibers;

INTRODUCTION

1
2 Usage of fiber reinforced concrete (FRC) has been continuously increasing in the
3 construction industry due to its various advantages such as improvement in post-cracking
4 stiffness, flexural toughness and ease of availability at a competitive price. Steel FRC is
5 mainly used in the seismic resistant structures, tunnel construction, blast, and impact resistant
6 structures where the post-cracking behavior is of major concern. ACI Building Code 318¹
7 permits the design engineers to use steel fiber reinforced concrete (SFRC) as a replacement to
8 conventional shear reinforcement. However, ACI codes^{1,2} mandate that SFRC beams are
9 required to have a minimum steel fiber dosage of 0.75% in volume and compressive strength
10 not greater than 42 MPa (6.091 ksi). This mandate is because current ACI provisions are
11 primarily based on experimental studies on non-prestressed concrete beams with a
12 compressive strength less than 42 MPa (6.091 ksi). However, in a prestressed concrete beam,
13 the beneficial effect of prestressing forces could further relax the minimum required fiber
14 volume fraction and can make the use of SFRC more economical. Thus, the focus of this
15 investigation is to study the effect of steel fibers on cracking and ductility behavior of
16 prestressed concrete beams under flexure/flexure-shear using full field strain measurements
17 with digital image correlation (DIC).

18
19 A number of previous works have demonstrated the use of steel fibers as secondary
20 reinforcement for concrete elements³⁻⁴. Cuenca et al.^{3,4} used steel fibers to control the crack
21 propagation in precast beams and hollow core slabs. They observed that the fibers and
22 stirrups had a synergic effect resulting in better crack control mechanisms and enhanced
23 tension stiffening. Fantilli et al.⁵ observed the increase in ductility of fiber reinforced concrete
24 both in tension and compression. They attributed the increase in ductility in compression to

1 the passive confinement provided by the steel fibers. Padmarajaih and Ramaswamy⁶
2 predicted the crack width in partially and fully prestressed concrete beams with different
3 dosage of steel fibers. They found that addition of fibers restricted the crack initiation and
4 propagation, eventually leading to an increase in the ultimate flexural strength. Harajli⁷
5 studied the bond behavior of steel fiber reinforced concrete zones under static and cyclic
6 loading. The author noted that the addition of steel fibers improved the bond strength,
7 reduced the damage and increased the energy dissipation capacity under cyclic loading.
8 Sahoo et al.⁸ evaluated the effect of steel fibers on the behavior of concrete beams with and
9 without stirrups. They noted that the flexural capacity did not significantly improve when the
10 fiber dosage was more than 0.5%. Singh⁹ presented the flexural modeling of SFRC to
11 determine the ultimate capacity, crack width of rectangular sections using strain compatibility
12 and force equilibrium equations. Additionally, a normalized design chart for strength
13 calculations considering the random distribution of fibers and other fiber parameters was also
14 presented.

15

16 Fantilli et al.¹⁰ proposed a unique function related to the post peak response of different
17 cement based materials for numerical studies. Tiberti et al.¹¹ studied the influence of concrete
18 strength on the crack development of SFRC members and found that the usage of fibers led to
19 reduced mean crack spacing in high strength concrete (HSC) when compared to normal
20 strength concrete (NSC). They also noted that the influence of HSC and the presence of
21 fibers requires further research to establish a stabilized crack pattern. Job and Ramaswamy¹²
22 developed an analytical model to predict crack width, and crack spacing of partially
23 prestressed concrete with steel fibers considering the bond slip of longitudinal reinforcement,
24 pull-out of fibers. Cuenca and Serna¹³ observed from their experimental study on hollow core

1 slabs that addition of fibers increased the ultimate loads as well as improved ductility when
2 compared with the control specimens. Marazzini et al.¹⁴ tested individual cut beams of hollow
3 core slabs with and without steel fibers. They observed improved post peak behavior due to
4 fiber addition. Campione¹⁵ presented the flexural behavior of SFRC deep beams wherein they
5 compared the RC deep beams with and without fibers, SFRC beams exhibited improved
6 strength and ductility through crack bridging. Abbas and Khan¹⁶ conducted fiber pull-out
7 tests to study Fiber-Matrix interfacial behavior of hooked end steel fiber-reinforced concrete.
8 They concluded that ultimate pull-out load was found to increase with an increase in fiber
9 size and embedment length. Gencturk et al.¹⁷ used Digital Image Correlation technique (DIC)
10 for understanding the shear behavior of prestressed concrete I-beams. They recorded the
11 values of displacement and strain using both DIC and conventional measurements. They
12 highlighted the need for further research in using DIC technique because of non-availability
13 of standard algorithms to identify cracks, crack width and loss of data in the region of
14 excessive spalling. Rimkus et al.¹⁸ proposed an algorithm to identify the crack evolution and
15 its spacing using digital images of reinforced concrete, which can eliminate the subjective
16 judgment using traditional methods. The flexural cracking behavior of reinforced concrete
17 beams made-up of plain concrete (NSC and HSC) and steel fiber reinforced high strength
18 concrete was studied using DIC technique by Hamrat et al.¹⁹. The authors were able to detect
19 the first crack with high precision and measure the crack width during the testing using DIC.
20 They have also observed an increase in the cracking load, reduction in the crack spacing and
21 crack width due to the presence of steel fibers.

22 23 **RESEARCH SIGNIFICANCE**

24 The research carried out in the past have confirmed that the addition of steel fibers

1 contributes to improved post-cracking behavior through increased ductility, multiple crack
2 formation/crack distribution, reduced crack width and enhanced toughness properties.¹⁶⁻¹⁹
3 Various factors like shear span to depth (a/d) ratio, compressive and tensile strengths of
4 concrete and fiber dosage influence the performance of fiber reinforced concrete elements.
5 However, only limited results are available on the behavior of steel fiber reinforced
6 prestressed concrete members made of concrete with strengths higher than 50 MPa. The
7 broad objective of this study is to investigate the influence of steel fibers on the response of
8 prestressed concrete beams and understand its effect on the crack propagation and post-
9 cracking behavior using full-field strain measurements with the help of DIC technique. In
10 addition, the influence of steel fiber dosage on aspects such as crack control, energy
11 absorption capacity and deflection is also presented.

12

13

EXPERIMENTAL INVESTIGATION

14 The experimental program consists of casting and testing of the full-scale prestressed
15 concrete beams designed as per IS1343-2012²⁴, having length of 3.5 m (11.48ft) and cross
16 section 200 mm (7.87 in) x 300 mm (11.81 in) containing different steel fiber dosages. Four
17 different series of beams corresponding to fiber dosages of 0, 0.35%, 0.70% and 1.0% were
18 cast. Two beams were tested at each fiber dosage to ensure the consistency of test results. All
19 these beams were tested at a shear span to depth ratio (a/d) of five to ensure flexure/ flexure
20 shear behavior. A strain energy based ductility measurement was employed to study the
21 influence of fibers. This work is part of a larger research program on investigating the effect
22 of various types of fibers and their combinations on the performance improvement of
23 prestressed concrete beams under different a/d ratios. Currently, work is in progress to
24 understand the effect of fibers on shear behavior of prestressed concrete beams at low a/d

1 ratio of 2.5. However, only the results pertaining to $a/d = 5$ are presented in this manuscript.
2 All the beams were cast on the same day and were water cured for 28 days at room
3 temperature. The overall test matrix with study parameters is detailed in **Table 1**.

4

5 **Material Properties and Behavior**

6

7 **Concrete**

8 All specimens were cast in the precast plant using ready-mix concrete designed as per IS:
9 10262-2009²⁵ to have a 28-day target compressive strength of 58 MPa (8.41ksi). Blended
10 Coarse aggregates of size 10 mm and 20 mm along with fine aggregates were used to obtain a
11 uniform mix. Mix design details are given in **Table 2**. The unit weight of concrete was 2400
12 kg/m^3 (149.82 lb/ft^3). The cube strength of concrete was 62 MPa (8.99 ksi) with a standard
13 deviation of 2.2 MPa (319.08 psi) at 28 days. The SFRC cylinders were tested using servo-
14 controlled compression testing machine for the complete stress-strain behavior (**Fig. 1**).

15

16 **Behavior of Concrete in Compression**

17 Concrete cylinders were tested (**Fig. 1a**) to obtain the stress-strain curves with and without
18 steel fibers. The addition of steel fibers was found to significantly improve the strength and
19 stiffness degradation in the post-peak region with better ductility (**Fig. 1b**). Previous work of
20 authors of this manuscript has also confirmed the same at both the room and moderate
21 temperature exposure²⁶⁻²⁸.

22

23 **Concrete tests in Tension**

24 The control and fiber reinforced concrete (FRC) dog-bone specimens were tested using
25 servo-controlled hydraulic MTS fatigue testing machine under pure tension. The loading was
26 controlled at a rate of 10 $\mu\text{m/min}$ up to a mid-span displacement of 0.8 mm (0.03 in) and then

1 at 50 $\mu\text{m}/\text{min}$ up to 1.2 mm (0.047 in) displacement followed by 1000 $\mu\text{m}/\text{min}$ for the rest of
2 the test. All the specimens with different fiber dosages were tested in uniaxial tension using
3 rigid steel plates on a servo controlled closed loop system as shown in **Fig. 2**. The failure
4 pattern in control specimen is predominantly due to a single explicit crack at the center of the
5 specimen. This observation indicates that the stress concentration in the crack region reduces
6 the post-cracking load resistance of the specimen. On the other hand, the FRC specimen
7 showed a similar large crack at failure, but the crack opening was delayed due to the presence
8 of fibers in the section. Similar observations have been made by Choun et al.³³ as shown in
9 Fig.2b.

10

11 **Internal reinforcement - prestressing steel strands**

12 Two numbers of half an inch (12.7 mm diameter) strands containing seven low relaxation
13 wires with an effective area of 200 mm^2 (0.31 in^2) were used for prestressing the beams.
14 Coupon specimens were prepared for tendons and tested under tension using a servo-
15 controlled machine. The ultimate tensile strength and modulus of elasticity were found to be
16 1860 MPa (269.77 ksi) and 196.5 GPa (28500 ksi), respectively. Jacking force is applied to
17 each of the strands to subject them to an initial strain of 0.004.

18

19 **Hooked end steel fibers**

20 Hooked end steel fibers having length and diameter of 30 mm (1.18 in) and 0.6 mm (0.023
21 in), respectively (**Fig. 3**) were used in this investigation. The tensile strength and modulus of
22 elasticity of steel fibers were found to be 1000 MPa (145 ksi) and 200 GPa (29007 ksi),
23 respectively. During casting, the fibers were added gradually to prevent the balling effect and
24 to obtain a uniform fiber distribution. Uniform fiber distribution was verified by the

1 inspection of the cracked faces of failed beams after testing.

2

3 **Instrumentation**

4 Displacements in all the specimens were recorded using Linear Variable Displacement
5 Transducers (LVDT) and DIC. LVDTs were positioned at specific locations along the length
6 of the beam to capture the entire curvature profile during testing. Two 100 mm (3.94 in)
7 LVDTs were positioned at the center of the span to capture the mid-span deflection. Six
8 LVDTs were used to form a strain rosette (**Fig. 4**) to measure surface strains near the loading
9 point. Strain gauges of 5 mm (0.197 in) gauge length were instrumented at mid-span of the
10 prestressing strands to capture the strain variation in the strands during testing.

11

12 **Digital Image Correlation (DIC)**

13 Digital image correlation (DIC) is a technique for measuring the whole-field strain and
14 displacement of specimens. DIC works by comparing two images of the specimen coated
15 with a random speckle pattern in an undeformed and deformed state³¹. Images of the object's
16 surface before and after deformation are recorded, digitized and stored in the computer.
17 These images are then analyzed to determine the displacements by invoking a pattern
18 matching principle. Since it is impossible to find matching points using single pixel, areas
19 (called as subsets) containing multiple pixels are used for the analysis³¹. The size of subset
20 varies with respect to the experimentation details. The step size controls the density of
21 analyzed data. For example, a step size of 5 will analyze every 5th point in each direction. A
22 higher step size gives faster results but coarser data. A smaller step size will return more
23 points but will take more computation time. For the present experiments, a subset size of 35
24 and a step size of five was chosen after a thorough sensitivity analysis. All the specimens had

1 speckle pattern on the surface for capturing of DIC images. The surface was initially coated
2 with non-reflective white paint, and then black speckle was sprayed on the white coat. Two
3 halogen lights were placed at an angle to the specimen as shown in **Fig. 5** to illuminate the
4 specimen. The camera was placed in front of the specimen with its axis normal to the
5 specimen. Images were taken at regular intervals and were processed using specialized
6 software (VIC- 2D) for strain analysis, crack initiation and propagation.

7

8 **Test Setup and Loading Details**

9 All beams were tested in a four-point bending configuration by which a constant moment
10 region at the mid-span was generated. **Fig. 6** illustrates the components used in the test setup.
11 A 250 kN (56.25 kips) MTS servo-hydraulic actuator was used to apply the loads. The
12 actuator load is transferred to the concrete specimens through longitudinal rigid steel spreader
13 beam and thereafter to the I-beams as shown in **Fig. 6**. Loading was applied monotonically
14 in displacement control mode at a rate of 0.05 mm/sec. Loading was paused intermittently to
15 observe and mark the cracks and study the failure progression.

16

17

TEST RESULTS AND DISCUSSION

18 **Load-deflection Behavior**

19 All the beams were tested with shear span to depth ratio of five and the load vs. mid-span
20 deflection plots were obtained as shown in **Fig. 7**. Two specimens were tested for each fiber
21 dosage to ensure the consistency of the results. The load – displacement behavior of PSC
22 beams without fibers (Control Specimen) are illustrated in **Fig. 7a**. The average cracking load
23 observed from the test was 60 kN (13.5 kips). After cracking, the beam continued to resist the
24 applied load until 142.5 kN (32.06 kips) and the corresponding deflection was 40 mm (1.57

1 in). It is worth mentioning that the beam was heavily under reinforced leading to the yielding
2 of strands just before the peak load.
3
4 The beam with 0.35% steel fibers exhibited better ductility as compared to the control
5 specimen. About 13% improvement in cracking load and 23% (average) improvement in
6 post-cracking stiffness was observed (**Fig. 7b**). However, the ultimate load did not change.
7 This beam had better crack distribution with the delayed formation of cracks as compared to
8 the control specimen before reaching its final failure load. Flexure dominant failure was
9 observed with 0.35% steel fiber dosage. The crack bridging effect was evident through the
10 formation of a number of smaller cracks and fewer major cracks. The specimen with 0.7%
11 steel fiber dosage (**Fig. 7c**) exhibited better ductility than the specimen with 0.35% fiber
12 dosage. However, there was no substantial improvement in ultimate strength due to higher
13 fiber dosage. Additionally, significant improvement in the post-cracking stiffness (50%),
14 ductility and ultimate load (11.6%) was observed with the increase in fiber dosage from 0%
15 to 1.0%. The specimen had an ultimate load of 159 kN (35.77 kips) with the first crack
16 forming at a load of 70 kN (15.75 kips). Test was terminated at 100 mm (3.94 in)
17 displacement due to a limitation in stroke capacity of the actuator. However, the specimen
18 could have resisted loads at even higher displacements which is evident from the overall load
19 displacement curves (**Fig. 7d**). It was evident from the test results that presence of steel fiber
20 increases the displacement corresponding to cracking load. The addition of fibers also
21 converted the brittle flexure-shear failure of control beams into ductile flexure dominant
22 failure in fiber reinforced beams. One specimen with 1.0% steel fiber dosage had
23 honeycombing due to improper compaction and hence it was excluded from comparisons.

24

1 **Crack distribution and failure modes**

2 The crack distribution of tested specimens is illustrated in **Fig. 8**. For control specimens, the
3 first flexural crack occurred at the tension fiber in the constant moment region. Soon after
4 cracking, the formation of multiple cracks continued at the mid-span location below the
5 loading point. All the specimens failed in flexure mode except the control specimen which
6 failed in the flexure-shear mode soon-after the yielding of prestressing strands. The failure
7 mode in control specimen was sudden due to the crushing of concrete just after the yielding
8 of strands. The presence of steel fibers resulted in more distribution of cracks and converted
9 sudden brittle failure to a gradual ductile failure mode.

10

11 **Load vs. Strain Behavior**

12 Strain gauges were installed on the strand at the mid length of the specimen to measure the
13 strain variation during pre-tensioning and testing. Strain gauges recorded the value of around
14 4000 $\mu\text{m/m}$ during pre-tensioning. Load vs. strain graphs presented in **Fig. 9** shows a trilinear
15 behavior. The first change in slope can be attributed to the cracking of beam. The second
16 change in slope can be correlated to the beginning of crushing of concrete in compression.
17 **Fig. 9** indicates that strain in the strands was nominal before cracking. After cracking, as
18 expected, the strain increased significantly indicating the higher contribution of strands in
19 load resistance. In general, the strain values at particular load reduced with an increase in
20 fiber dosage indicating the steel fiber contribution in load resistance after cracking (Fig. 9).
21 The strands yielded in all the specimens before they could reach their ultimate load.
22 However, strain values were not recorded after a certain load value level due to
23 malfunctioning of the strain gauges.

24

1 **Fiber Distribution on the fractured surface**

2 The fiber distribution on the cracked section was verified after testing of PSC beams to
3 complete failure. **Fig. 10** shows the distribution of steel fibers on the failure surface. It
4 presents the random and uniform distribution of fibers along the depth of the section.
5 Moreover, the elongation of fibers and fiber pull-out (bond failure) illustrates the
6 contribution of fibers in load resistance. Nevertheless, the fiber contribution in crack bridging
7 was evident from the fractured surfaces of the failed beams.

8

9 **Energy Absorption capacity**

10 The strain energy developed to resist the applied external load was calculated for comparison.
11 Energy absorption capacity of PSC beams can be obtained from the area under load -
12 deflection curve. All the specimens were tested at same a/d ratio but with different fiber
13 dosages. Their energy absorption capacity is compared and is presented in **Table 3**. The
14 addition of steel fibers resulted in an increase of energy absorption capacity due to better
15 crack bridging. Energy capacity doubled when 1.0% by volume of steel fibers were added to
16 the control specimen.

17

18 **ANALYSIS OF RESULTS USING DIGITAL IMAGE CORRELATION**

19 **Load vs. Deflection analysis**

20 Displacements and strains from DIC analysis can be correlated only until major crack
21 formation. After significant cracking, the correlation breaks due to excessive cracking and
22 spalling of concrete leading to the difficulty in pattern matching. The load - deflection
23 response of the specimens using DIC analysis is compared with the LVDT measurements for
24 beams with different fiber dosages (**Fig. 11**). The results from DIC analysis showed a close

1 agreement with the LVDT measurements.

2

3 **Effect of fibers on the tensile strain in concrete**

4 Full field strains on the surfaces were measured using the VIC-2D software. Strain
5 distribution was evaluated to see the effect of steel fiber contribution in resisting the crack
6 propagation. A horizontal line a-b was chosen (**Fig. 12a**) in the tensile zone of all the tested
7 beams. **Fig. 12b** shows the tensile strain variation along the line a-b at a load level of 60kN
8 (13.48 kips), which is slightly less than the cracking load. It indicates that the peak strain
9 along the considered horizontal line is same for beams of different series. This elucidates the
10 fact that fibers do not play a vital role in load carrying mechanisms before cracking. On the
11 other hand, **Fig. 12c** shows the tensile strain variation along line a-b at a load of 70 kN (15.73
12 kips), which is greater than the cracking load. It presents the decrease in peak strain at same
13 load level after cracking with the increase in fiber dosage. The reduction in strain indicates
14 that fibers resist the crack widening, localization and helps to redistribute the stresses and
15 increase the number of cracks. The extent of reduction in strains mainly depends on the
16 dosage of steel fibers. In the **Fig. 12a-c** a sudden jump in the strain contour indicates the
17 presence of a crack in that region. **Fig. 13** illustrates the reduction in average strain along the
18 level of crack tip at failure due to addition of fiber. Additionally, **Fig. 13** also indicates that
19 the presence of steel fiber reduces the maximum strain as much as by 50%. The strain values
20 corresponding to the post-cracking loads for different specimen highlights that the presence
21 of steel fibers reduces the strain at the crack locations (**Fig. 14**).

22

23

PREDICTIONS USING ANALYTICAL MODELS

24 The behavior of SFRC under compressive and tensile loads are predicted theoretically based

1 on the models available in literature^{32, 34, 35}. The following sections discuss the methodology
 2 followed to obtain the constitutive relationships of SFRC. Later, the models developed are
 3 utilised to perform the moment-curvature analysis of the section considered in this research.

4 **Constitutive relationships for SFRC under Compression**

5 Several analytical models are available in the literature to predict the behavior of SFRC under
 6 compression. Ou et al.³⁴ proposed an analytical model for the development of compressive
 7 stress-strain relationship of SFRC, which is an extension of the stress - strain model
 8 previously proposed by Carreira and Chu³² for plain concrete under compression:

$$\frac{\sigma_c}{f'_{cf}} = \frac{\beta \left(\frac{\epsilon_c}{\epsilon_{cf}} \right)}{\beta - 1 + \left(\frac{\epsilon_c}{\epsilon_{cf}} \right)^\beta} \quad (1)$$

9
 10 in which σ_c is compressive stress; f'_{cf} is the compressive strength; ϵ_{cf} is the peak strain; ϵ_c is
 11 the compressive strain, and β defines the shape of the stress-strain curve which is a function
 12 of the reinforcement index (RI_v).

$$f'_{cf} = f'_c + 2.35(RI_v) \quad (2)$$

$$\epsilon_{cf} = \epsilon_{c0} + 0.0007(RI_v) \quad (3)$$

$$\beta = 0.71(RI_v)^2 - 2.00(RI_v) + 3.05 \quad (4)$$

$$RI_v = V_f \frac{l}{\phi} \quad (5)$$

14
 15
 16 f'_c, ϵ_{c0} are compressive strength (in MPa) and the strain (mm/mm) at the peak stress of plain
 17 concrete; V_f, l, ϕ are volume fraction of fibers (in %), length and diameter of fibers,
 18 respectively.

19

1 The cylinder compressive strength (43 MPa) of control specimen is used for the development
 2 of stress-strain behavior of SFRC with different volume fraction of fibers employing the
 3 model defined by equations 1-5.

4

5 **Constitutive relationships for SFRC under Tension**

6 The tensile behavior of SFRC usually depends on the strength of the plain concrete, type of
 7 fiber and fiber fraction. Neocleous et al.³⁵ developed an analytical model to predict the
 8 behaviour of SFRC under tension, adopting recommendations of RILEM³⁶ for predicting the
 9 tensile strength of plain concrete. Typical behavior of SFRC is shown in the **Fig. 15**.

10 Various salient points in the tensile constitutive relationships for SFRC are computed as
 11 follows:

$$f'_c = 0.8 * (f_{ck}) \quad (6)$$

$$f_{ctm} = 0.3 * (f'_c)^{\frac{2}{3}} \quad (7)$$

$$f_{fcm} = f'_c + k_x S_p \quad (8)$$

$$E_c = 9500 * (f_{fcm})^{\frac{2}{3}} \quad (9)$$

$$\sigma_1 = 0.7 * f_{ctm} * \frac{(1600 - d)}{1000} \quad (10)$$

$$\sigma_{1f} = \sigma_1 * (1 + 0.32 * V_{fw}) \quad (11)$$

$$\sigma_{2f} = \sigma_{1f} * e^{\xi(\epsilon_1 - 0.002)10^3} \quad (12)$$

$$\sigma_{3f} = 0 \quad (13)$$

12

$$\varepsilon_{1f} = \frac{\sigma_{1f}}{E_c} \quad (14)$$

$$\varepsilon_{2f} = \varepsilon_{1f} + \left(\frac{2}{1000} \right) \quad (15)$$

$$\varepsilon_{3f} = 0.04 \quad (16)$$

1

2 Where f_{ck} , f_c' are the characteristic compressive strength of cube and cylinder (in MPa)
 3 respectively; f_{ctm} , f_{fcm} , E_c are the characteristic tensile strength, mean tensile strength and
 4 mean secant modulus of concrete (in MPa); V_{fw} is the weight fraction of fibers (in %); σ_{1f} , σ_{2f} ,
 5 σ_{3f} are the salient points of the tensile stress–strain model corresponding to strains of ε_{1f} , ε_{2f} ,
 6 ε_{3f} respectively.

7

8 **Moment Curvature Analysis**

9 A layer-by-layer method of sectional analysis was carried out to establish the moment-
 10 curvature behavior of steel fiber reinforced prestressed concrete sections using an iterative
 11 procedure. To maximize the accuracy of the results and to minimize the computational
 12 effort, the cross section and corresponding stress-strain curves were discretized into a number
 13 of layers of small thickness so as to get the uniform strain throughout the thickness (**Fig. 16**).
 14 The constitutive relationship for fiber reinforced concrete in compression and tension
 15 according to the model^{34, 35} (**Fig. 15**) is used. A compressive top fiber strain value is fixed,
 16 and the neutral axis depth is iterated to satisfy the force equilibrium. Moment resistance is
 17 calculated from the moment equilibrium, and the corresponding curvature can be arrived
 18 using the assumed top compressive strain and the neutral axis depth.

19

20 The analytical results are compared with experimentally measured moment-curvature
 21 response in **Fig. 17**. It can be observed that post cracking stiffness and ultimate strength

1 increases with increase in fiber dosage. The cracking and ultimate moment were found to be
2 matching reasonably well with the test results (**Fig. 17**). However after certain loading
3 curvature was not measured due to instrumentation error. Hence overall response was not
4 presented. RILEM recommendations³⁶ were also utilized to compare the experimental results
5 at ultimate capacity as shown in **Table.3**.

7 **SUMMARY AND CONCLUSIONS**

8 Pre-tensioned prestressed concrete beams with and without steel fibers were cast and tested
9 four point bending to evaluate the role of steel fibers in crack bridging and improvement in
10 serviceability performance. Based on the test results of seven prestressed concrete beams
11 presented in this study, the following major conclusions can be drawn:

- 12 i) Though the addition of steel fibers showed marginal improvement in the peak strength, it
13 resulted in very good improvement in strain energy based ductility of the prestressed
14 concrete beams. The major contribution of steel fibers was in bridging the cracked
15 surfaces which resulted in improved post-cracking stiffness and higher ultimate
16 deflection. The strain in the strand reduced due to the addition of steel fibers in the post-
17 cracking regime.
- 18 ii) Distribution of cracks increased with increase in the addition of steel fibers. More
19 importantly, the failure mode changed from flexure-shear to flexure resulting in higher
20 ductility and ultimate deflections.
- 21 iii) No increase in ultimate flexural strength was observed at low fiber dosages. The
22 maximum increase in load capacity was about 11.6% at a higher fiber dosage of 1.0%.
- 23 iv) Energy dissipation capacity increased with increase in fiber dosage. It doubled when
24 1.0% by volume of steel fibers was added.

1 v) DIC measurements correlated well with conventional measurements of displacement
2 sensors. The reduction in peak strain in tensile fibers was up to 50% due to the crack
3 bridging mechanism of steel fibers.

4 **ACKNOWLEDGEMENTS**

5 This experimental work was carried out as part of the project funded through Utchattar
6 Avishkar Yojana Scheme of Indian Government. PRECA India Pvt. Ltd. and Grenix India
7 Ltd., donated the materials required for this research. PRECA provided the labor required for
8 the casting of specimens in their factory. All of their support is duly acknowledged.

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15 concrete”. *Materials and structures*, 35(9), 579-582.

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24 TABLES AND FIGURES

- 25 **List of Tables:**
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27 **Table 2 - Mix Design Details**
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1
2 **Table 1: Details of Test Specimens**
3

Specimen Number	Label	Fiber content (Volume fraction in %)	a/d	Compressive Strength of cube (f _{ck}) in MPa (ksi)
1	SF00-1	0		
2	SF00-2	0		
3	SF35-1	0.35		
4	SF35-2	0.35	5	62 (8.99)
5	SF70-1	0.7		
6	SF70-2	0.7		
7	SF100-2	1.0		

4
5 **Table 2: Mix Design Details**
6

Material	Quantity (kg/m ³)
20 mm aggregate	754 kg
10 mm aggregate	355 kg
Crushed Stone Sand (CSS)	415 kg
Natural River Sand (NRS)	313 kg
Cement (OPC 53)	428 kg
Flyash	22 kg
Water	165 kg
Admixture	2.5 kg

7
8 (16.01 kg/m³ = 1 lb/ft³ , 1 kg= 2.2 lb)

1 **Table 3: Summary of Test results**

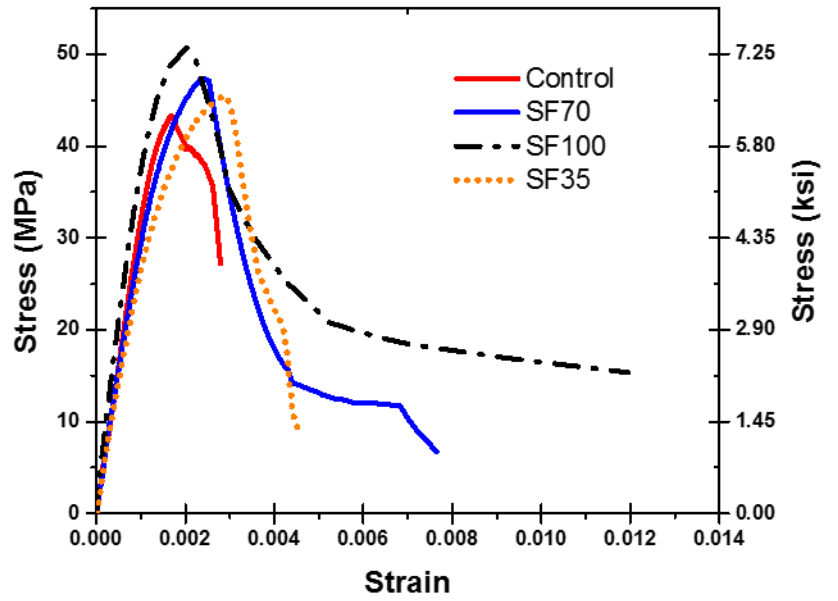
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Specimen	SF00-1	SF00-2	SF35-1	SF35-2	SF70-1	SF70-2	SF100-2
Cracking load, P _{cr} , kN (kip)	55 (12.36)	66 (14.84)	68 (15.28)	70 (15.73)	70 (15.73)	67 (15.06)	70.3 (15.80)
Deflection at Cracking load, Δ _{cr} ,mm (in)	2.5 (0.098)	2.97 (0.117)	3.02 (0.119)	3.28 (0.13)	4.03 (0.158)	4.02 (0.158)	4.66 (0.183)
Peak load, P _{pl} ,kN (kip)	141 (31.7)	144 (32.37)	143 (32.14)	143.5 (32.26)	148.6 (33.4)	146 (32.82)	159 (35.75)
Increase in P _{pl} (%)	-	-	0.35	0.7	4.28	2.45	11.57
Deflection at peak load, Δ _{pl} ,mm (in)	36.65 (1.44)	42.2 (1.66)	34.4 (1.35)	29.6 (1.17)	45.0 (1.77)	32.4 (1.27)	36.6 (1.44)
Mid span deflection at failure, mm (in)	65.0 (2.56)	53.3 (2.09)	97.5 (3.83)	78.8 (3.10)	99.0* (3.89)	97.8* (3.85)	99.7* (3.92)
Post cracking stiffness , kN/mm (kips/in)	2.57 (14.73)	2.59 (14.79)	2.903 (16.578)	3.457 (19.744)	3.076 (17.567)	3.335 (19.046)	3.871 (22.107)
Increase in Post cracking stiffness (%)	-	-	12.54	34.03	19.26	29.3	50.07
Experimental- cracking moment , kN-m (lb-ft)	34.4 (25372)	37.5 (27658)	42.5 (31346)	43.7 (32231)	43.7 (32231)	41.9 (30903)	43.9 (32378)
Experimental- peak moment , kN-m (lb-ft)	88.1 (64979)	89.4(65938)	89.4 (65938)	89.7 (66159)	92.9 (68519)	91.3 (67339)	99.4 (73313)
Cracking moment(RILEM),kN-m(lb-ft)	34.7(25593)		39(28764)		43.3(31936)		46.7(34444)
Peak moment(RILEM),kN-m(lb-ft)	81(59742)		86(63430)		91(67118)		95(70068)
Strain Energy (Joule)	6332	6358	9309	10989	11534	11774	13239
% increase in strain energy	-	-	46.7	73.2	81.8	85.6	108.7

3 * Test was terminated due to limitation in stroke capacity of the actuator



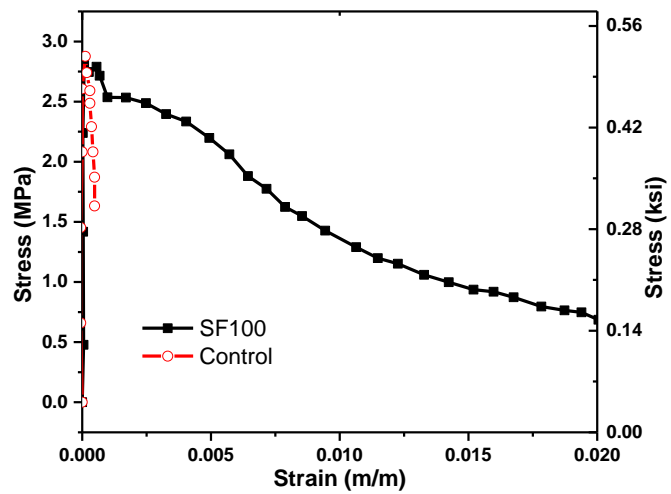
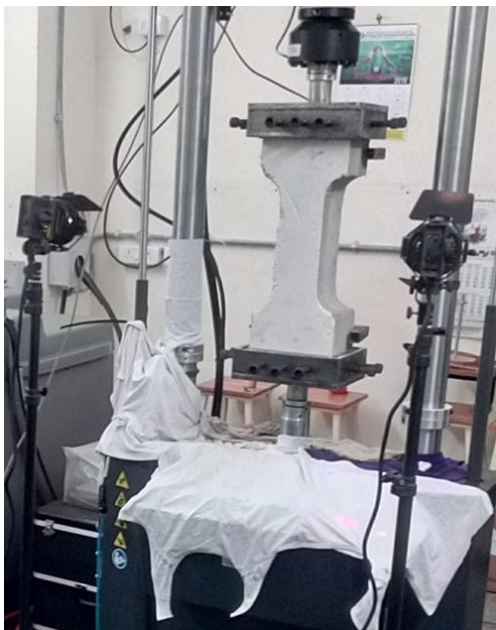
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(a) Test Setup

(b) Stress strain curves Control vs. FRC

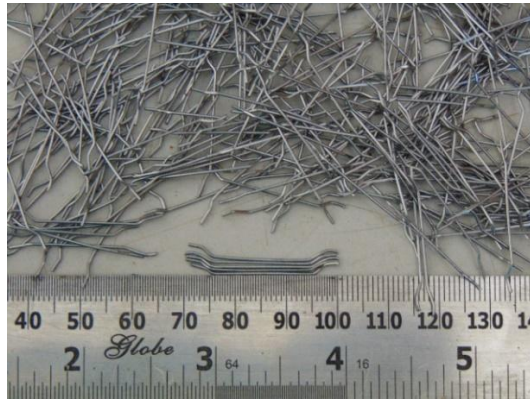
Fig. 1: Cylinder testing under compression.



(a) Test setup

(b) FRC behavior under uniaxial tension³³

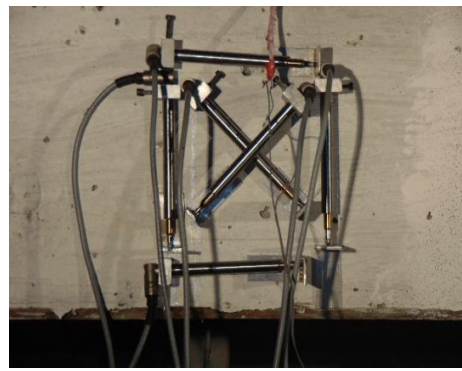
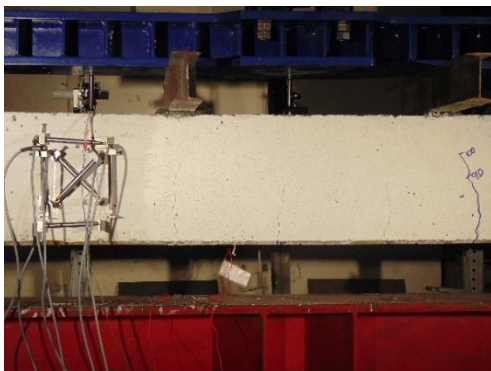
Fig. 2: Testing of FRC specimen under pure tension



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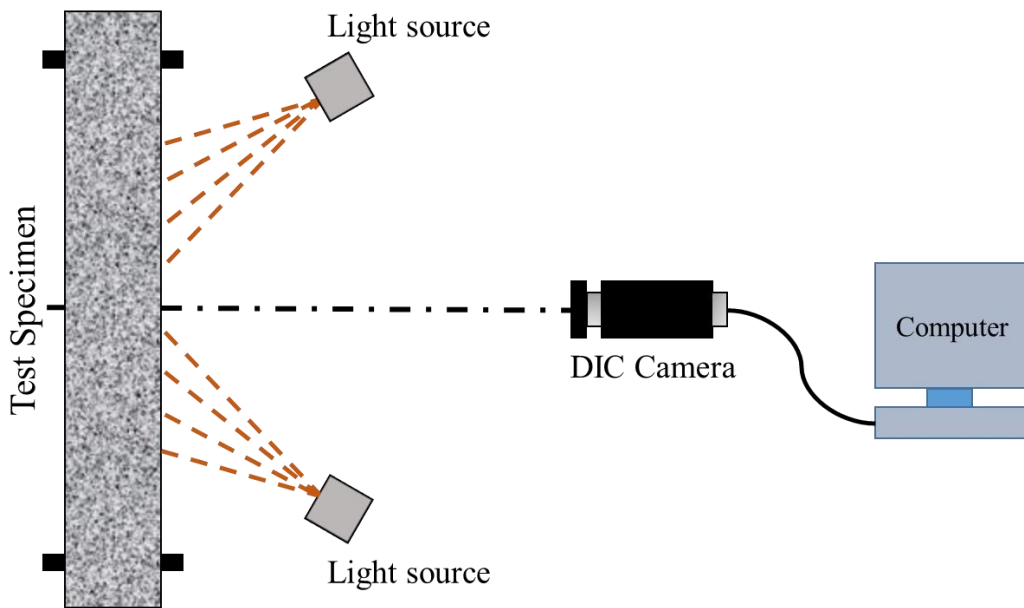
2 Fig. 3: *Hooked end steel fibers*

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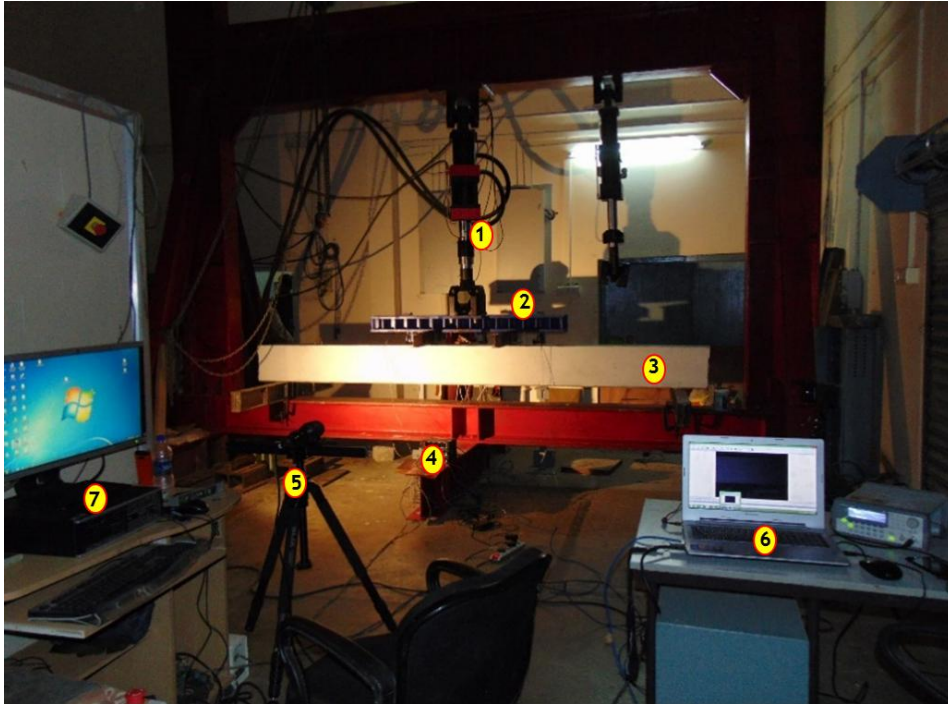
4 Fig. 4: *Instrumentation details: LVDT and strain rosette arrangement*

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7 Fig. 5: *Schematic plan view of DIC setup*



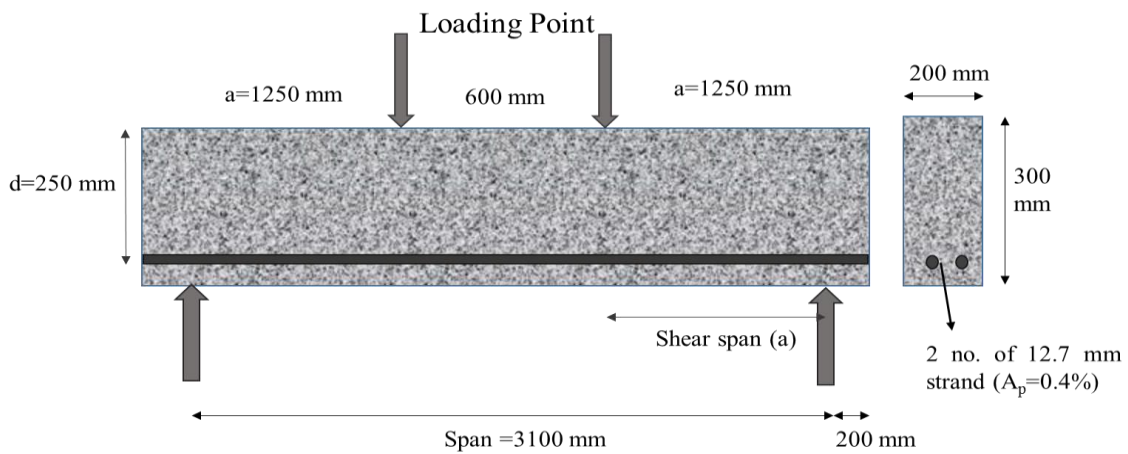
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2 1. 250 kN MTS Actuator; 2. Spreader Beam; 3. PSC Beam; 4. HBM DAQ system; 5.
 3 DIC camera; 6. Laptop; 7. MTS Controls system.

4

5 a) *Test setup and instrumentation details*

6



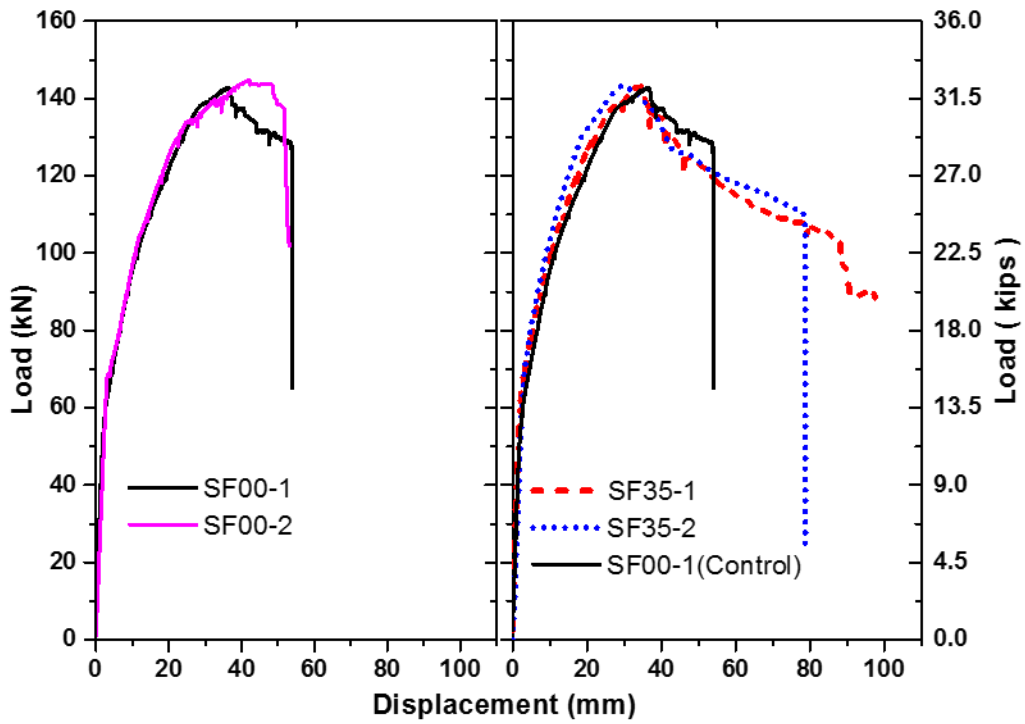
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8 b) *Schematic of loading and Sectional Details (25.4 mm = 1 in)*

9 Fig. 6: *Experimental setup and instrumentation for prestressed concrete beams*

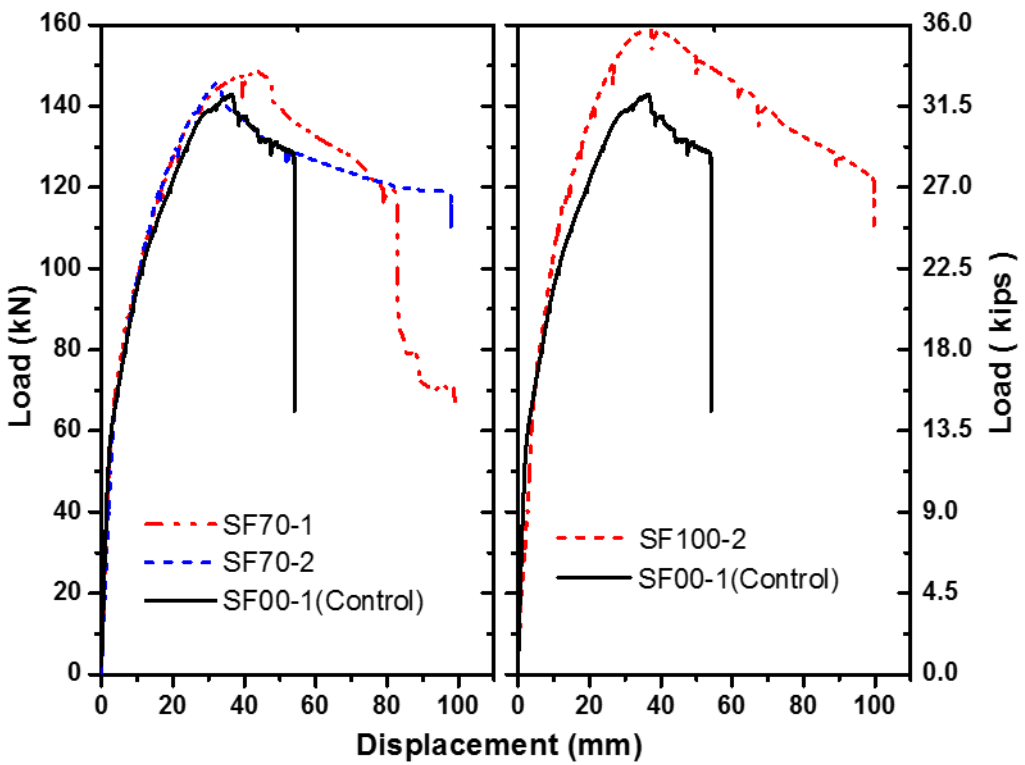
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a) Control specimen (without fibers) b) PCB with 0.35% of Steel Fibers

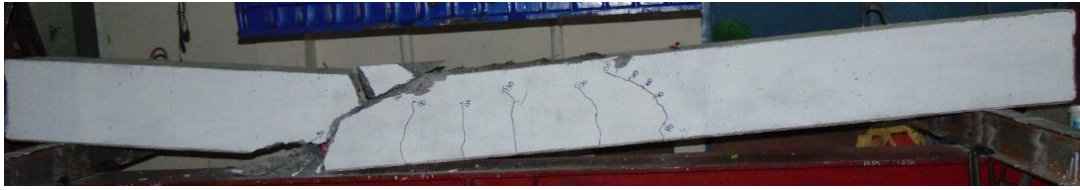


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c) PCB with 0.7% of Steel Fibers d) PCB with 1.0% of Steel Fibers

Fig. 7: Comparison of Load-Deflection behavior

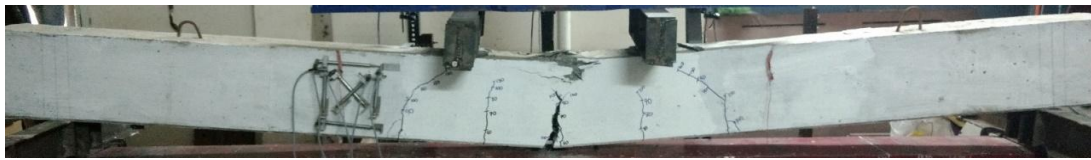
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(a) *Control specimen*



(b) *SF-0.35%*



(c) *SF-0.7%*



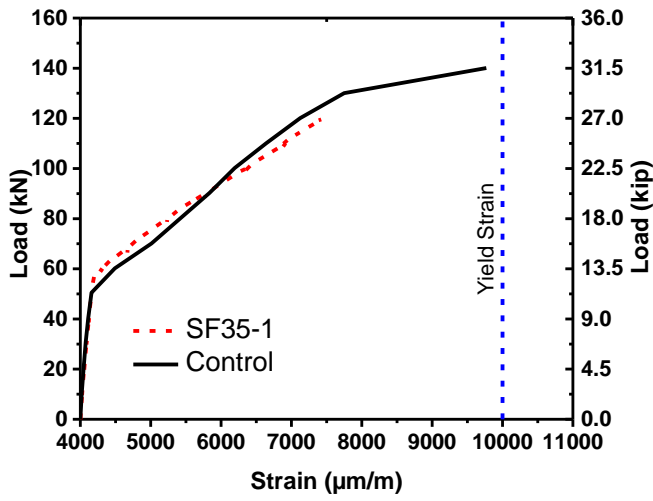
(d) *SF-1.0%*

Fig. 8: *Crack distribution and failure modes*

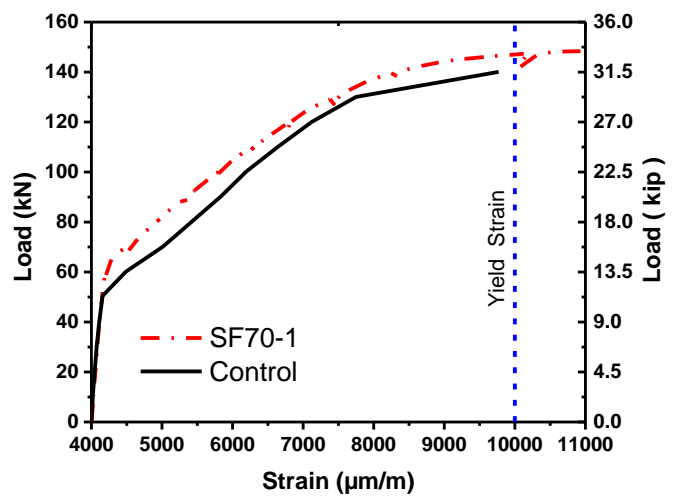
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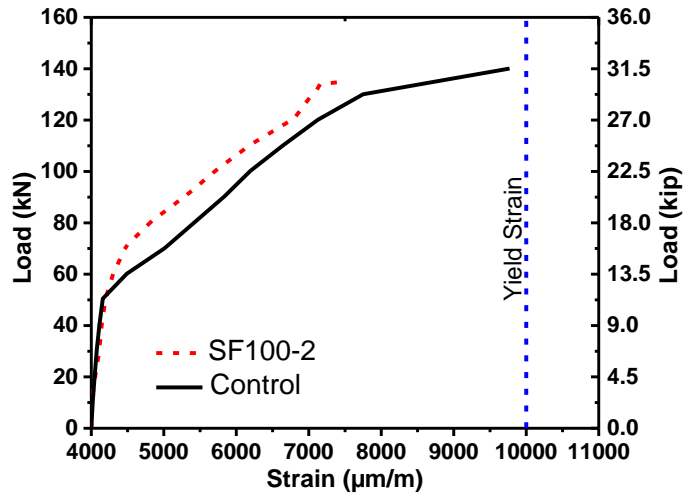
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(a) Strand strain variation in SF35 and SF00



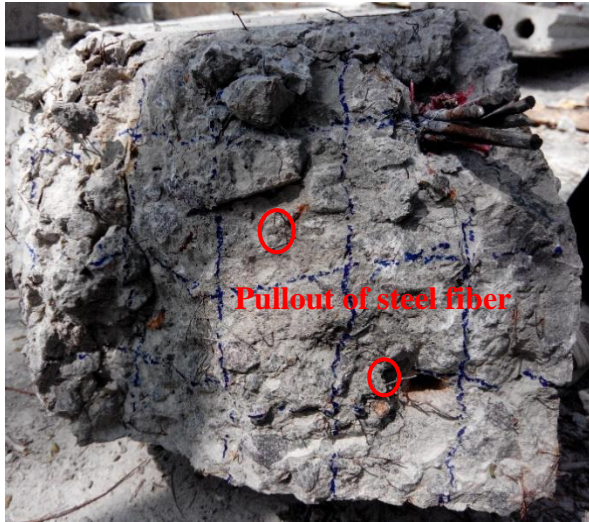
(b) Strand strain variation in SF70 and SF00



(c) Strand strain variation in SF100 and SF00

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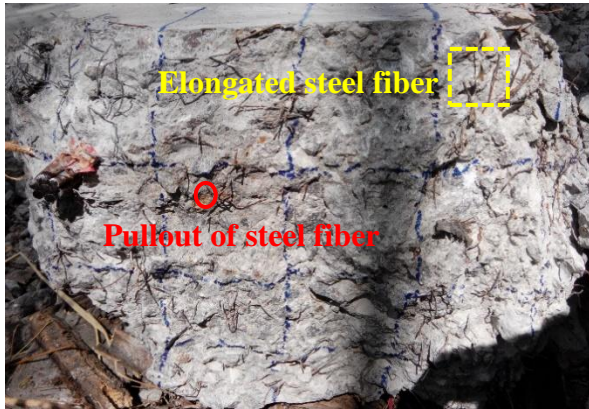
4 Fig. 9: Effect of steel fibers on strain variation of strands in PCB



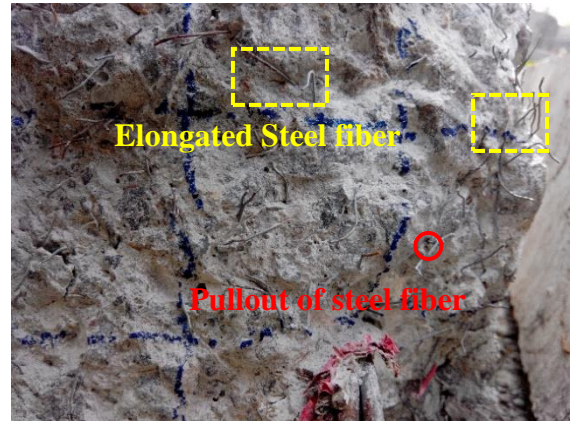
a) SF35-1



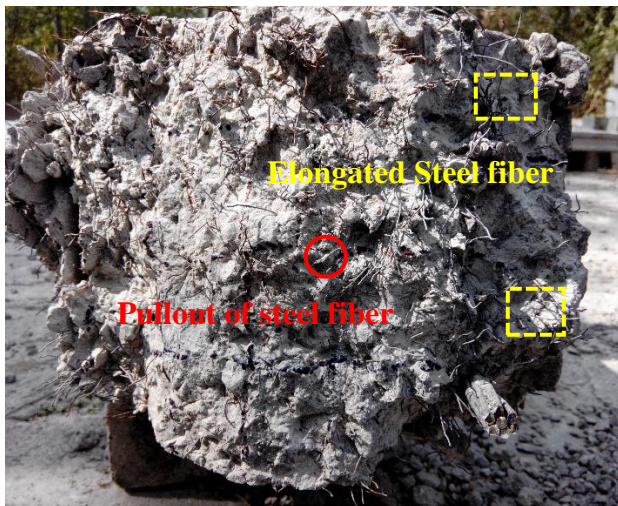
b) SF35-2



c) SF70-1



d) SF70-2

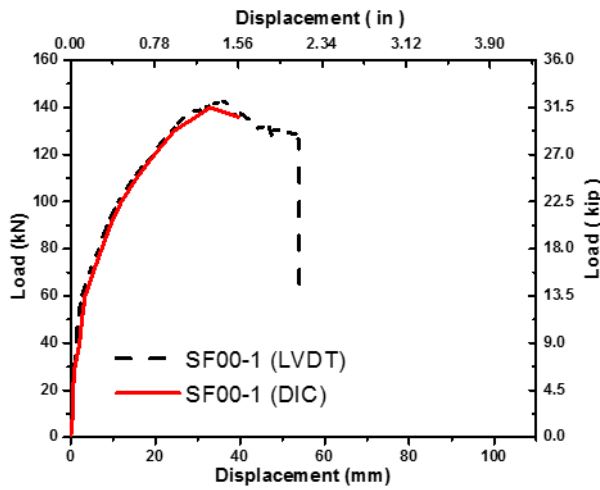


e) SF100-1

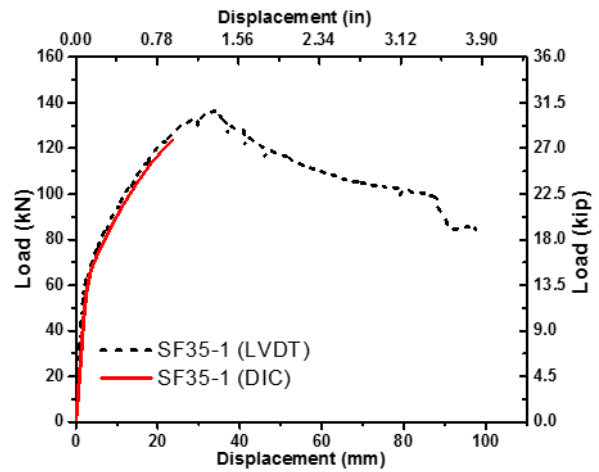


f) SF100-2

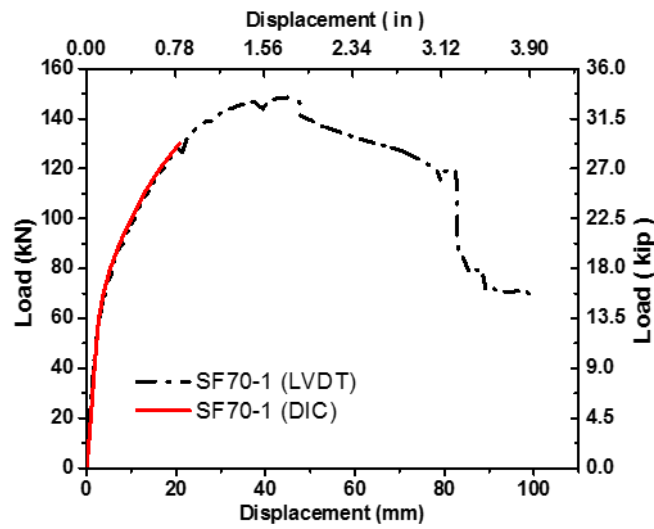
Fig. 10: Steel fibers distribution on the failure surface



a) Control specimen (DIC)

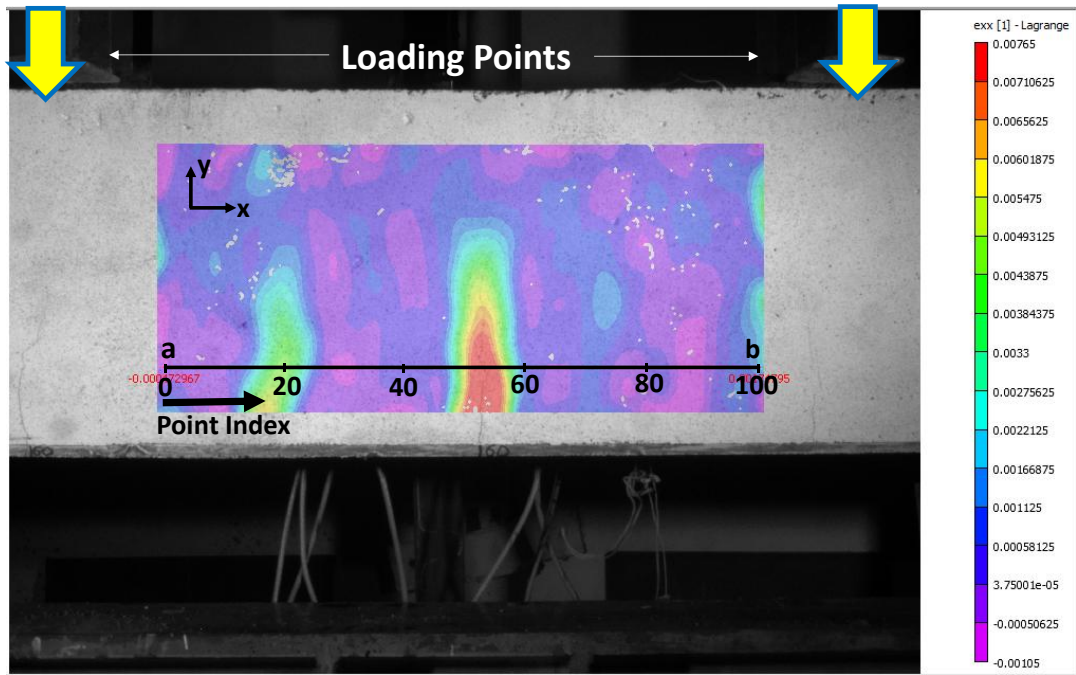


b) PCB with 0.35% steel fibers (DIC)



c) PCB with 0.7% steel fibers (DIC)

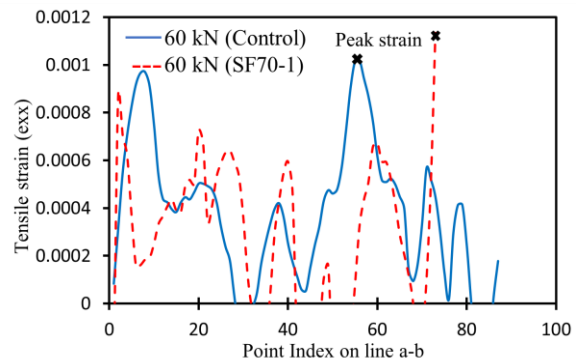
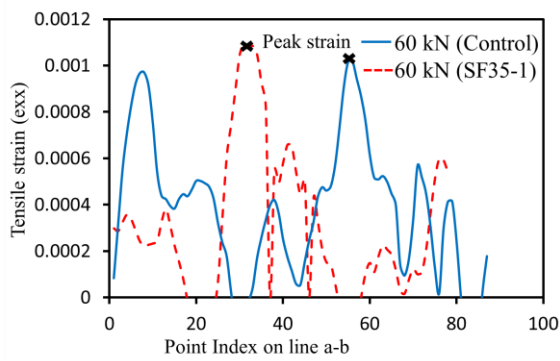
Fig. 11: Comparison of DIC results with the LVDT measurement



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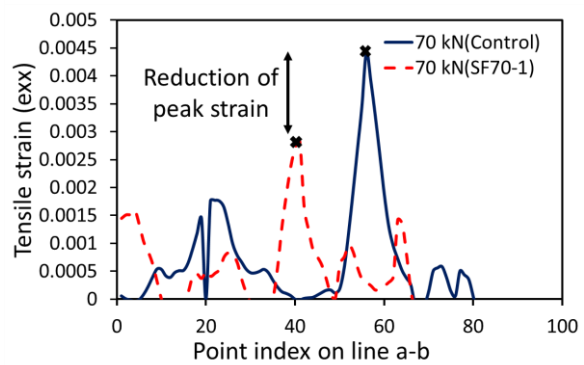
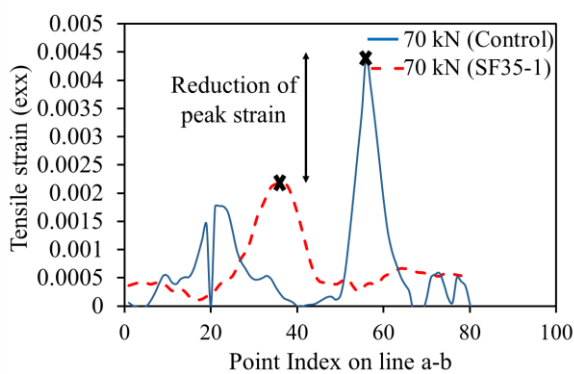
a) Horizontal line a-b in the tension zone for Analysis (Constant Moment Zone)



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b) Tensile Strain at 60 kN (13.48 kips) (before cracking)



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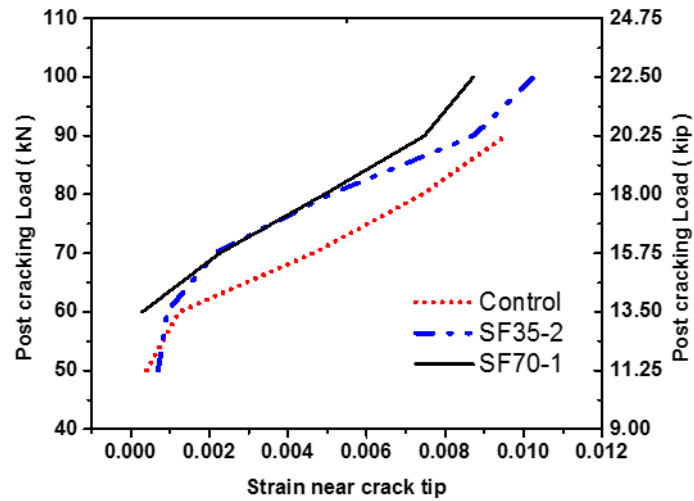
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c) Tensile Strain at 70 kN (15.73 kips) (after cracking)

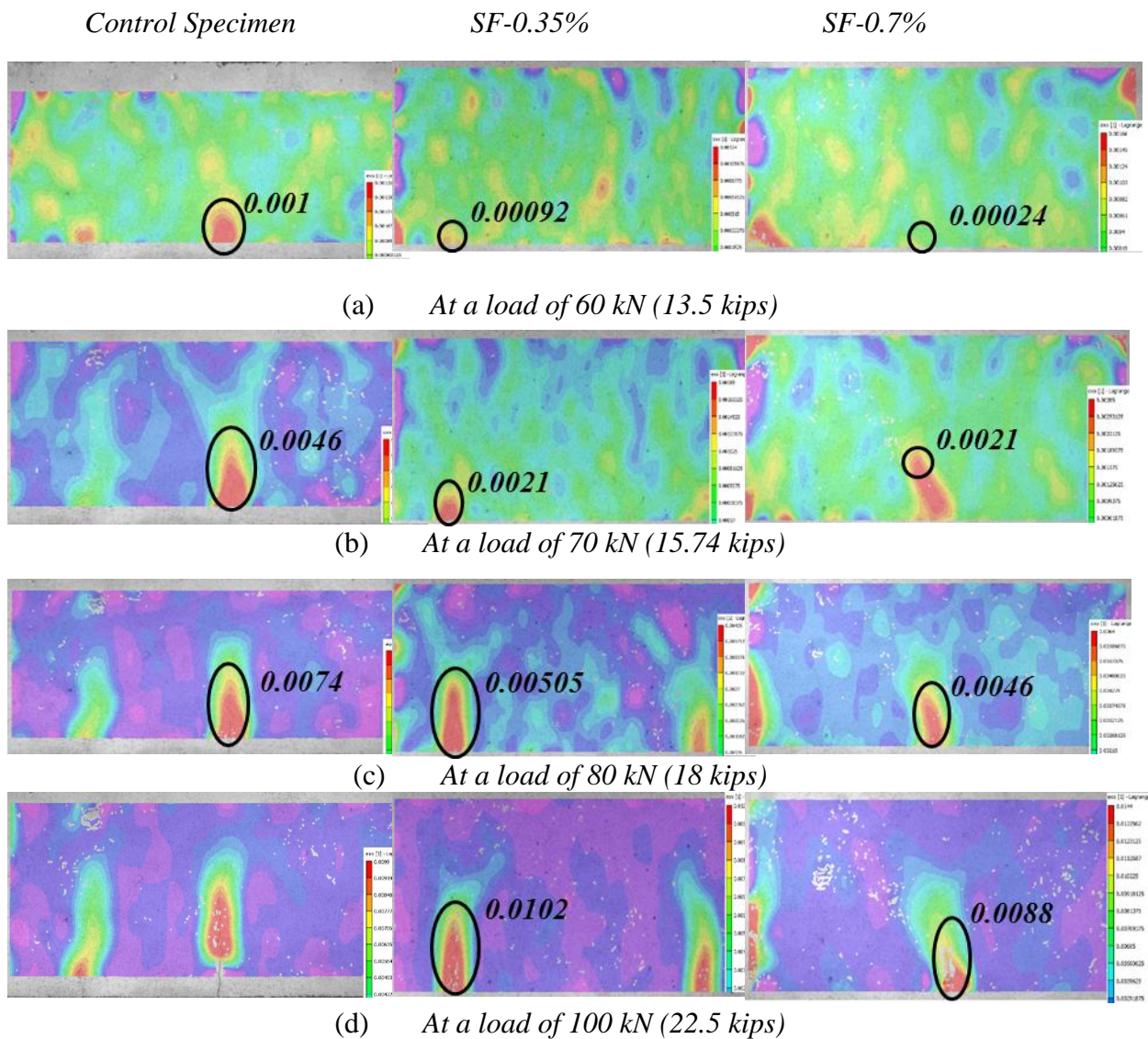
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Fig. 12: Longitudinal strain contours (DIC) on the horizontal line a-b before and after cracking

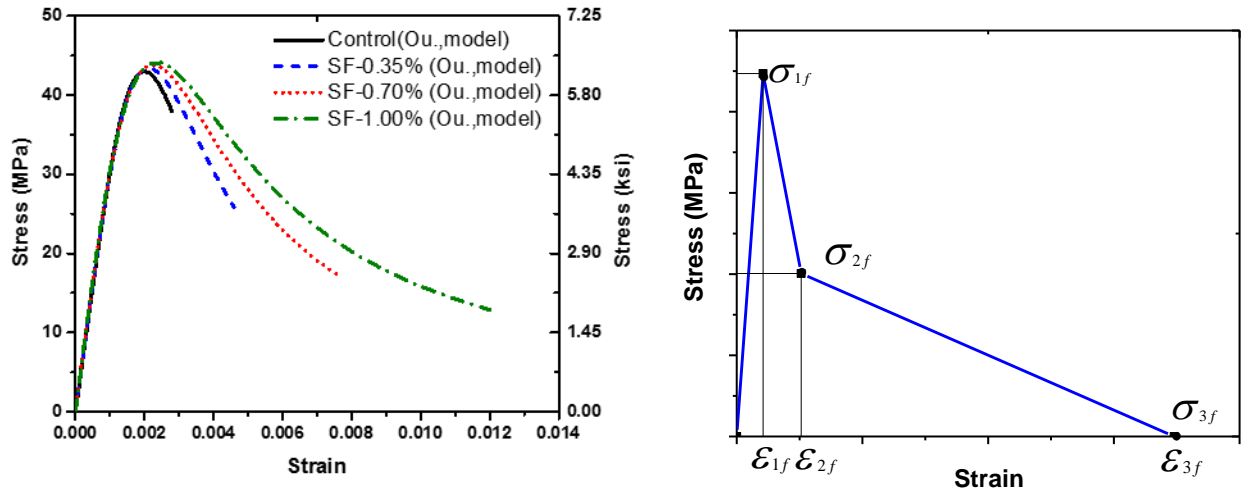
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2 Fig.13: Average Strain along the level of crack tip at failure



3 Fig. 14: DIC images showing strain contours at various load levels

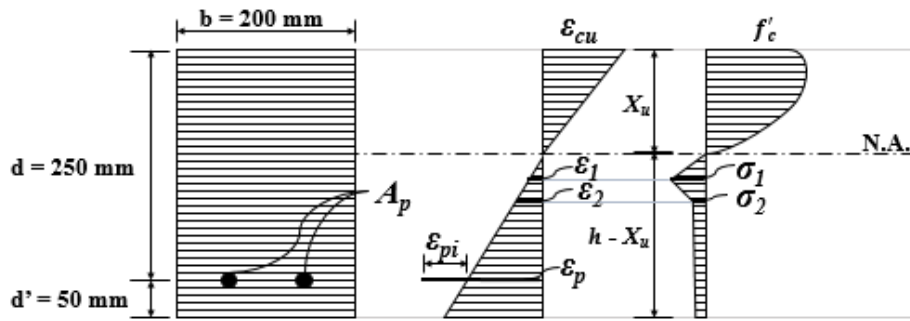


a) Compression behavior prediction

b) Tension behavior prediction

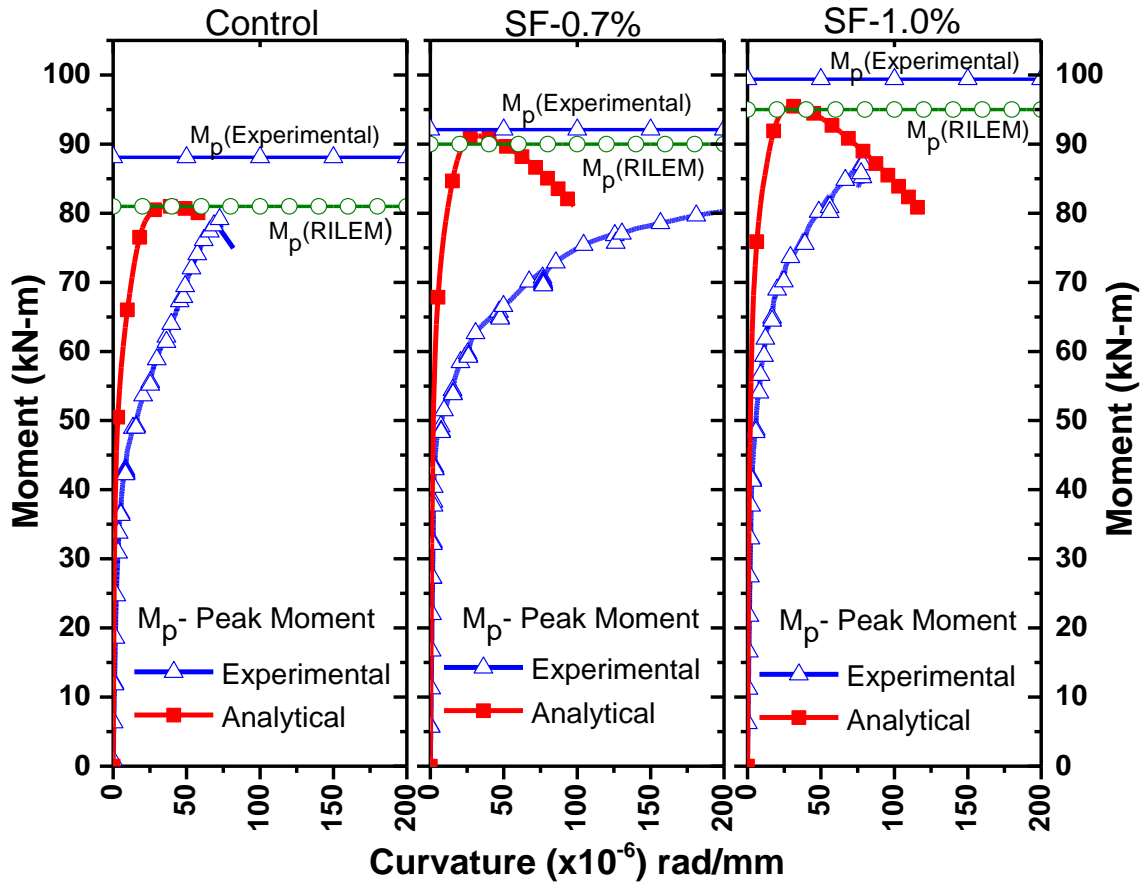
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Fig. 15: Stress-strain response of fiber-reinforced concrete in compression³⁴ and tension³⁵



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Fig.16: Layer by layer approach for sectional analysis



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 2 Fig. 17: Comparison of Analytical and Experimental results (1 kN-m = 0.73576 kip-ft)
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 4