



Dark matter assisted Dirac leptogenesis and neutrino mass

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Abstract

We propose an extension of the standard model with $U(1)_{B-L} \times Z_2$ symmetry. In this model by assuming that the neutrinos are Dirac (*i.e.* $B-L$ is an exact symmetry), we found a simultaneous solution for non zero neutrino masses and dark matter content of the universe. The observed baryon asymmetry of the universe is also explained using Dirac Leptogenesis, which is assisted by a dark sector, gauged under a $U(1)_D$ symmetry. The latter symmetry of the dark sector is broken at a TeV scale and thereby giving mass to a neutral gauge boson Z_D . The standard model Z -boson mixes with the gauge boson Z_D at one loop level and paves a way to detect the dark matter through spin independent elastic scattering at terrestrial laboratories. © 2018 Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

The standard model (SM), which is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, is a successful theory of fundamental particles of nature and their interactions. After the Higgs discovery, it seems to be complete. However, there are many unsolved issues which are not addressed within the framework of SM. In particular, the non-zero neutrino masses, baryon asym-

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metry of the Universe, existence of dark matter *etc.* These problems beg for a successful theory in physics beyond the SM.

The observed galactic rotation curve, gravitational lensing and large scale structure of the Universe collectively hint towards the existence of an invisible matter, called dark matter. In fact, the relic abundance of dark matter has been precisely determined by the satellite based experiments, such as WMAP [1] and PLANCK [2] to be $\Omega_{\text{DM}}h^2 = 0.1199 \pm 0.0027$. Hitherto the existence of dark matter is shown in a larger scale (\gtrsim a few kpc) only via its gravitational interaction. However, the particle nature of dark matter is remained elusive till today and needs to be explored in a framework of physics beyond the SM.

Within the SM, the neutrinos are exactly massless. This can be traced to a conserved $B-L$ symmetry within the SM, where B and L stands for net baryon and lepton number respectively. However, the oscillation experiments [3–5] have successfully demonstrated that the neutrinos have sub-eV masses. One attractive way to explain the small masses of active neutrinos is to introduce the lepton number violation by two units through the dimension five operator $\ell\ell HH/\Lambda$ [6], where ℓ , H are the lepton and Higgs doublet respectively and Λ is the scale at which the new physics is expected to arise. After electroweak phase transition, the neutrinos acquire a Majorana mass of the order $m_\nu = \langle H \rangle^2/\Lambda$. Naively this implies that the sub-eV masses of neutrinos indicate the scale of new physics to be $\Lambda \sim \mathcal{O}(10^{14})$ GeV. Note that the effective dimension-5 operator can be realized in many extensions of the standard model, the so called seesaw mechanisms [7–9]. In these models, the mass scale of new particles is expected to be at a scale of Λ . Therefore, it is imagined that in the early Universe, when the temperature of thermal bath is high enough, namely $T \gtrsim \Lambda$, the lepton number violation can occur rapidly, while it is suppressed today. As a result, a net lepton asymmetry [10,11] can be generated through CP violating out-of-equilibrium decay [12] of these heavy particles at $T \sim \Lambda$, which is then converted to the observed baryon asymmetry of the Universe through the electroweak sphaleron transitions. The lepton number violating interactions ($\Delta L = 2$), which also indicate Majorana nature of neutrinos, can be probed at ongoing neutrinoless double beta decay experiments [13]. But till now there is no positive result found in those experiments. So there is still a chance of hope that the neutrinos might be Dirac in nature. In other words, $B-L$ is an exact global symmetry of the SM Lagrangian.

Even the neutrinos are Dirac in nature (i.e. $B-L$ is exactly conserved), the baryon asymmetry of the Universe must be explained since it is an observed fact. It has been explored largely in the name of Dirac leptogenesis [14–20], which connect Dirac mass of neutrinos with the observed baryon asymmetry of the Universe. The key point of this mechanism is that the equilibration time between left and right-handed neutrinos mediated via SM Higgs (i.e. $Y\bar{\nu}_R H\nu_L$) is much less than the $(B+L)$ violating sphaleron transitions above electroweak phase transition. Therefore, if we demand that $B-L = B - (L_{\text{SM}} + L_{\nu_R}) = 0$ [15], then we see that a net $B - L_{\text{SM}}$ is generated in terms of L_{ν_R} . The electroweak sphalerons will not act on L_{ν_R} , as ν_R is singlet under $SU(2)_L$, while the non-zero $B - L_{\text{SM}}$ will be converted to a net B asymmetry via $B+L$ violating sphaleron transitions.

In this paper we study the consequences of Dirac nature of neutrinos to a simultaneous solution of dark matter and baryon asymmetry of the Universe. We extend the SM by introducing a dark sector constituting of two vector-like Dirac fermions: ψ , a doublet under $SU(2)_L$, and χ , a singlet under $SU(2)_L$, as shown in the Fig. 1. See also refs. [21,22]. The dark sector is gauged under a $U(1)_D$ symmetry. An over all symmetry $U(1)_{B-L} \times Z_2$ is also imposed to ensure that the neutrinos are Dirac and the lightest particle χ in the dark sector is a candidate of dark matter, being odd under the Z_2 symmetry. A heavy scalar doublet X , odd under the discrete Z_2 symme-

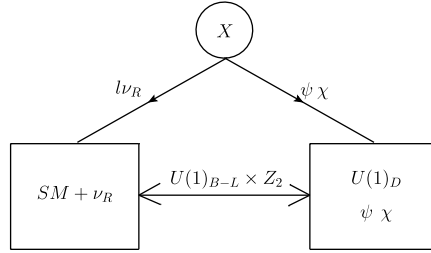


Fig. 1. Schematic diagram of a heavy scalar decay to visible and dark sectors, where the “dark sector” we mean the particles being charged under a $U(1)_D$ symmetry and constitutes two vector-like Dirac fermions: ψ (doublet under $SU(2)_L$) and χ (singlet under $SU(2)_L$). The lightest dark sector particle χ is a candidate of dark matter.

try, is introduced such that its CP-violating out-of-equilibrium decay to $\ell\nu_R$ and $\psi\chi$ can generate equal and opposite lepton asymmetry in both the channels, where ν_R is odd under Z_2 . The lepton asymmetry produced in the channel $X \rightarrow \ell\nu_R$ can be converted to a net baryon asymmetry via $B + L$ violating sphaleron transitions, whereas the asymmetry in the channel $X \rightarrow \psi\chi$ will remain intact as ψ is a vector-like Dirac fermion [23–25]. Notice that the Z_2 symmetry forbids the term $\bar{\nu}_R H \ell$, though allowed by $U(1)_{B-L}$. To generate a Dirac mass of the neutrinos we allow Z_2 to break softly by the term $\mu^2 X^\dagger H$. As a result we generate Dirac mass of the neutrinos to be $M_\nu \sim \mu^2 \langle H \rangle / M_X^2$, where M_X is the mass of X . The soft Z_2 breaking also introduces a mixing between the neutral component of the doublet ψ^0 and χ . As a result the asymmetry in ψ^0 and ψ^\pm gets converted to a net χ asymmetry. However, we will show that the χ asymmetry is significantly smaller than the symmetric component because the latter does not get annihilated efficiently to the SM particles through the mixing between the gauge bosons Z and Z_D , where Z_D is the gauge boson in the dark sector. As a result the relic of χ constitutes an admixture of dominant symmetric component with a small asymmetric component.

The paper is organized as follows. In sec. 2, we discuss the proposed model, while in sec. 2.1, we explain the Dirac masses of light neutrinos. A brief description about observed baryon asymmetry and DM abundance is given in sec. 3. Section 4 is devoted to explain baryogenesis via leptogenesis from the decay of heavy particles X , while section 5 describes DM abundance from the decay of heavy X -particles. We discuss the constraints on model parameters from direct detection of DM in sec. 6 and conclude in sec. 7.

2. The model

We extend the Standard Model with a dark sector, consisting of two vector-like leptons: ψ and χ , where ψ is a doublet and χ is a singlet under $SU(2)_L$. The dark sector is gauged under a $U(1)_D$ symmetry, which breaks at TeV scales and give mass to the neutral gauge boson Z_D . We also introduce three right handed neutrinos $\nu_{R\alpha}$, $\alpha = 1, 2, 3$ and a heavy scalar doublet X . A discrete symmetry Z_2 is also introduced under which X , ν_R and χ are odd, while ψ and all other SM particles are even. Under $U(1)_D$ symmetry ψ and χ carry non-trivial quantum numbers. See Table 1. As a result the trilinear couplings: $\bar{\psi} X \nu_R$ and $\bar{\ell} X \chi$ are forbidden. Here the singlet fermion, χ is the lightest particle in the dark sector and acts as a candidate of dark matter. An overall $B-L$ global symmetry is also introduced as in the case of SM. The $B-L$ symmetry remains unbroken and hence ensures that the neutrinos are Dirac in nature. The CP-violating out-of-equilibrium decay of heavy scalar X create asymmetries simultaneously in both lepton and dark matter sectors [23]. In the visible sector, the decay of X to ℓ and ν_R , creates an equal and

Table 1
Quantum numbers of the new particles under the extended symmetry.

Parameter	$U(1)_{B-L}$	$U(1)_D$	Z_2
$X = (X^+, X^0)^T$	0	0	–
ν_R	–1	0	–
$\psi = (\psi^0, \psi^-)^T$	–1	1	+
χ	–1	1	–

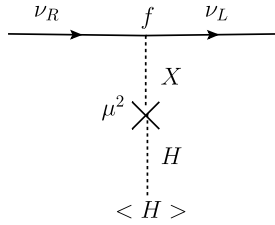


Fig. 2. Dirac mass of neutrinos generated via soft Z_2 symmetry breaking.

opposite lepton asymmetry in both left and right-handed sectors. The lepton asymmetry in the left-handed sector gets converted to a net baryon asymmetry through $B + L$ violating sphaleron transitions, while the asymmetry in the right-handed sector remains unaffected until the temperature falls much below the electroweak phase transition. Note that the coupling $\bar{\ell}\nu_R H$ and $\bar{\psi} H \chi$ are forbidden due to Z_2 symmetry. The relevant Lagrangian can be written as:

$$\mathcal{L} \supset \bar{\psi} i \gamma^\mu D_\mu \psi + \bar{\chi} i \gamma^\mu D'_\mu \chi + M_\psi \bar{\psi} \psi + M_\chi \bar{\chi} \chi + \left[f_{kl} \bar{\ell}_k \tilde{X} \nu_{Rl} + \lambda \bar{\psi} \tilde{X} \chi + \text{h.c.} \right] - V(H, X), \tag{1}$$

where

$$D_\mu = \partial_\mu - i \frac{g}{2} \tau^i W_\mu^i - i \frac{g'}{2} Y B_\mu - i g_D Y_D (Z_D)_\mu$$

$$D'_\mu = \partial_\mu - i g_D Y_D (Z_D)_\mu \tag{2}$$

and

$$V(H, X) = -M_H^2 H^\dagger H + M_X^2 X^\dagger X + \lambda_H (H^\dagger H)^2 + \lambda_X (X^\dagger X)^2 + \lambda_{HX} (H^\dagger H) (X^\dagger X) \tag{3}$$

2.1. Dirac mass of neutrinos

We allow the Z_2 symmetry to break softly [26,27] via:

$$\mathcal{L}_{soft} = -\mu^2 H^\dagger X + \text{h.c.} \tag{4}$$

As a result the Dirac mass of the neutrinos can be generated as shown in the Fig. 2. After integrating out the heavy field X we get the Dirac neutrino mass:

$$M_\nu = \frac{f \langle H \rangle \mu^2}{M_X^2}, \tag{5}$$

where $\langle H \rangle = 174$ GeV, is the Higgs vacuum expectation value. To generate Dirac masses of the neutrinos of order 0.1 eV, we need the ratio: $\frac{\mu}{M_X} \approx 10^{-4}$ assuming that $f \sim 10^{-4}$. The smallness of μ in comparison to the mass scale of heavy scalar doublets justifies the soft breaking of Z_2 symmetry.

3. Observed baryon asymmetry and DM abundance

The observed baryon asymmetry of the present Universe, usually reported in terms of the ratio of baryon to photon number density, $\eta \equiv n_B/n_\gamma$, is given as [28]

$$5.8 \times 10^{-10} \leq \eta \leq 6.6 \times 10^{-10} \text{ (BBN) (95\%C.L)} \quad (6)$$

where $\eta = 7.04 Y_B$ with $Y_B = n_B/n_s$. The ratio of DM to baryon abundance measured by WMAP and PLANCK in the cosmic microwave background is given to be $\frac{\Omega_{\text{DM}}}{\Omega_B} \approx 5$, where $\Omega_i = \rho_i/\rho_c$, and ρ_c is the critical density of the Universe. Thus the DM to baryon ratio can be rewritten as:

$$\frac{\Omega_{\text{DM}}}{\Omega_B} = \left(\frac{m_{\text{DM}}}{m_p} \right) \left(\frac{Y_{\text{DM}}}{Y_B} \right) \quad (7)$$

where m_{DM} and m_p are respectively DM and proton mass, $Y_{\text{DM}} = n_{\text{DM}}/s$ and $Y_B = n_B/s$ are respectively DM and baryon abundance in a comoving volume. In our case, the total DM abundance is sum of asymmetric and symmetric components as the DM remains out-of-equilibrium throughout the epochs. Therefore, Eq. (7) can be rewritten as:

$$\frac{\Omega_{\text{DM}}}{\Omega_B} = \left(\frac{m_{\text{DM}}}{m_p} \right) \left(\frac{Y_{\text{DM}}^{\text{sym}}}{Y_B} + \frac{Y_{\text{DM}}^{\text{asy}}}{Y_B} \right). \quad (8)$$

As we discuss below the baryon asymmetry and DM abundance are resulted from the decay of a heavy scalar X , which we assume to be present in the early Universe. Therefore, the symmetric and asymmetric component of DM abundance as well as baryon asymmetry from X -decay can be approximately computed as:

$$\begin{aligned} Y_{\text{DM}}^{\text{asy}} &= Y_X \epsilon_\chi B_\chi \\ Y_{\text{DM}}^{\text{sym}} &= Y_X B_\chi \\ Y_B &= c Y_L = c Y_X \epsilon_L B_L \end{aligned} \quad (9)$$

where $c = -0.55$ is the fraction of lepton asymmetry that is converted to a net baryon asymmetry, $Y_X = n_X/s$ is the number density of X in a comoving volume, $\epsilon_\chi, \epsilon_L$ are the CP-asymmetry parameters resulted through the decay of X to $\psi\chi$ and $\nu_R\ell$ respectively, B_χ, B_L are the branching fractions for the decay of X to $\psi\chi$ and $\nu_R\ell$ respectively. Using Eq. (9) in (8) we get the DM to baryon ratio:

$$\frac{\Omega_{\text{DM}}}{\Omega_B} = \left(\frac{m_{\text{DM}}}{m_p} \right) \left(\frac{B_\chi}{c \epsilon_L B_L} + \frac{B_\chi \epsilon_\chi}{c \epsilon_L B_L} \right). \quad (10)$$

The branching fractions B_L, B_χ and the CP-asymmetry parameters $\epsilon_L, \epsilon_\chi$ satisfy the constraints:

$$B_L + B_\chi = 1, \quad \epsilon_L = -\epsilon_\chi = \epsilon \quad \text{and} \quad \epsilon_i \leq 2B_i \quad (11)$$

where the first constraint simply demands the unitarity of the model, while second and third constraints ensures that all the amplitudes are physical and total amount of CP-violation can not exceed 100% in each channel. Using the above constraints in Eq. (10) we get

$$\frac{\Omega_{\text{DM}}}{\Omega_B} = \left(\frac{m_{\text{DM}}}{m_p} \right) \frac{B_\chi}{cB_L} \left(\frac{1}{\epsilon_L} + 1 \right), \tag{12}$$

where the 1st term on the right-hand side is due to symmetric component while the second term comes from asymmetric component. For small Yukawa couplings (required for out-of-equilibrium condition of X) the CP asymmetry parameters, $\epsilon_i \ll 1$. This implies that the symmetric component always dominates over the asymmetric component unless in the resonance limit: $\epsilon_L \sim \mathcal{O}(1)$, where symmetric and asymmetric components contribute in similar magnitudes. Thus given the constraints (11), several comments are in order:

(1) If $B_L = B_\chi = 1/2$, then $M_\chi \approx \mathcal{O}(\epsilon)$ GeV. This implies from Eq. (9) that for an optimistic case of $\epsilon = 10^{-8}$, we get $M_\chi \approx 10$ eV.

(2) If $B_L \gg B_\chi$ and $B_\chi \gtrsim \epsilon$ then we get $M_\chi \lesssim 2.5$ GeV.

(3) If $B_L \ll B_\chi$ then from Eq. (9) we get $10 \text{ eV} < M_\chi < 2.5 \text{ GeV}$.

Thus for a wide range of DM mass, we can generate correct relic abundance. In the following we solve the required Boltzmann equations to get the correct DM abundance, satisfying Eq. (12) by taking a typical DM mass to be 2.5 GeV, and observed baryon asymmetry given by Eq. (6).

4. Lepton asymmetry from X-particles decay

We assume that the X -particle is present in the early Universe. At a temperature above its mass scale, X is in thermal equilibrium due to gauge and Yukawa mediated interactions. As the temperature falls, due to Hubble expansion, below the mass scale of X -particle, the latter goes out-of-thermal equilibrium and decay. The decay rate of X -particle can be given as:

$$\Gamma_X \simeq \frac{1}{8\pi} (f^2 + \lambda^2) M_X, \tag{13}$$

where f, λ are Yukawa couplings. Demanding that $\Gamma_X \lesssim H$ at $T = M_X$, where $H = 1.67 g_*^{1/2} T^2 / M_{\text{Pl}}$ is the Hubble scale of expansion, we get $M_X \lesssim 10^{10}$ GeV for $f \sim \lambda \lesssim 10^{-4}$. The CP violating decay of X requires at least two copies. More over, we assume a mass hierarchy between the two X -particles so that the CP-violating decay of lightest X to $\ell \nu_R$ and $\psi \chi$ generates asymmetries both in lepton and dark matter sectors. Thus the decay modes to visible sector are $X^0 \rightarrow \nu_L \nu_R$ and $X^- \rightarrow \ell^- \nu_R$, while the decay modes to dark matter sector are $X^0 \rightarrow \psi^0 \chi$ and $X^- \rightarrow \psi^- \chi$. The two decay modes of X to visible and dark sectors are equivalent and hence we will focus only one of the channels, say $X^- \rightarrow \ell^- \nu_R$ in the visible sector and $X^- \rightarrow \psi^- \chi$ to dark sector. In presence of X -particles and their interactions, the diagonal mass terms in Eq. (3) can be replaced by [23,29],

$$\frac{1}{2} X_a^\dagger (M_+^2)_{ab} X_b + \frac{1}{2} (X_a^*)^\dagger (M_-^2)_{ab} X_b^*, \tag{14}$$

where

$$M_\pm^2 = \begin{pmatrix} M_1^2 - iC_{11} & -iC_{12}^\pm \\ -iC_{21}^\pm & M_2^2 - iC_{22} \end{pmatrix} \tag{15}$$

here $C_{ab}^+ = \Gamma_{ab} M_b$, $C_{ab}^- = \Gamma_{ab}^* M_b$ and $C_{aa} = \Gamma_{aa} M_a$ with

$$\Gamma_{ab} M_b = \frac{1}{8\pi} \left(M_a M_b \lambda_a \lambda_b^* + M_a M_b \sum_{k,l} f_{akl}^* f_{bkl} \right) \tag{16}$$

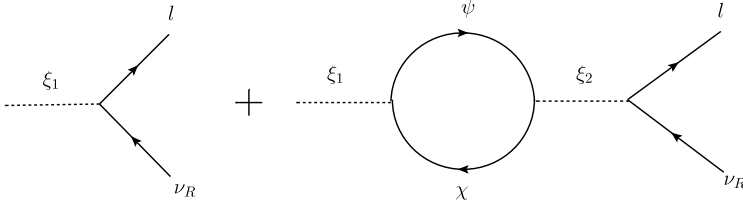


Fig. 3. Tree level and self energy correction diagrams, whose interference give rise to a net CP violation.

Diagonalizing the above mass matrix Eq. (15), we get two mass eigenvalues M_{ξ_1} and M_{ξ_2} corresponding to the two eigenstates ξ_1^\pm and ξ_2^\pm . Note that the mass eigenstates ξ_1^+ and ξ_1^- (similarly ξ_2^+ and ξ_2^-) are not CP conjugate states of each other even though they are degenerate mass eigen states. Hence their decay can give rise to CP-asymmetry. The CP-violation arises via the interference of tree level and one loop self energy correction diagrams shown in Fig. 3. The asymmetry in the visible sector is given by

$$\begin{aligned} \epsilon_L &= [B_L(\xi_1^- \rightarrow l^- \nu_R) - B_L(\xi_1^+ \rightarrow (l^-)^c \nu_R^c)] \\ &= -\frac{Im(\lambda_1^* \lambda_2 \sum_{k,l} f_{1kl}^* f_{2kl})}{8\pi^2(M_2^2 - M_1^2)} \left[\frac{M_1^2 M_2}{\Gamma_1} \right], \end{aligned} \tag{17}$$

where B_L is the branching ratio for $\xi_1^\pm \rightarrow l^\pm \nu_R$. Using the CP-asymmetry ϵ_L , we can estimate the generated lepton asymmetry $Y_L \equiv \frac{n_L}{s}$, where $s = (2\pi^2/45)g_*T^3$ is the entropy density, from the decay of ξ_1 . The relevant Boltzmann equations governing the evolution of the number density of ξ_1 , i.e. Y_{ξ_1} , and lepton asymmetry Y_L are given by:

$$\frac{dY_{\xi_1}}{dx} = -\frac{x}{H(M_{\xi_1})}s \langle \sigma |v|_{(\xi_1 \xi_1 \rightarrow All)} \rangle \left[Y_{\xi_1}^2 - Y_{\xi_1}^{eq2} \right] - \frac{x}{H(M_{\xi_1})}\Gamma_{(\xi_1 \rightarrow All)} \left[Y_{\xi_1} - Y_{\xi_1}^{eq} \right] \tag{18}$$

and

$$\frac{dY_L}{dx} = \epsilon_L \frac{x}{H(M_{\xi_1})}\Gamma_{(\xi_1 \rightarrow All)}B_L \left[Y_{\xi_1} - Y_{\xi_1}^{eq} \right], \tag{19}$$

where the $x = \frac{M_{\xi_1}}{T}$, is the dimensionless variable which ranges from $0 \rightarrow \infty$ as the temperature $T : \infty \rightarrow 0$. We have shown in Fig. 4 the lepton asymmetry Y_L and the comoving number density of ξ_1 , i.e. Y_{ξ_1} , as function of x . The decay coupling constant f is taken as 10^{-4} and the λ is taken as 0.47×10^{-7} . The typical value of the cross-section is taken as $\sigma |v|_{(\xi_1 \xi_1 \rightarrow All)} = 10^{-25} \text{ GeV}^{-2}$. The blue (dashed) line shows the abundance of ξ_1 particles. From Fig. 4, we see that as the temperature falls below the mass of ξ_1 (i.e. $x > 1$), it decouples from the thermal bath and then decays. The lepton asymmetry, which is proportional to B_L and ϵ_L , starts developing as ξ_1 decays to $l^- \nu_R$ and settles to a constant value after the decay of ξ_1 is completed. In Fig. 4 we have taken the branching ratio $B_L \sim \mathcal{O}(1)$ and the heavy scalar mass, $M_{\xi_1} = 10^{10} \text{ GeV}$. The electroweak sphalerons which violates $B + L$, but conserves $B - L$, can transfer a partial lepton asymmetry to a net baryon asymmetry $Y_B = -0.55Y_L$. For the $\epsilon_L = 2 \times 10^{-7}$ and the parameters we discussed above, we get the baryon asymmetry, $Y_B = -1 \times 10^{-10}$.

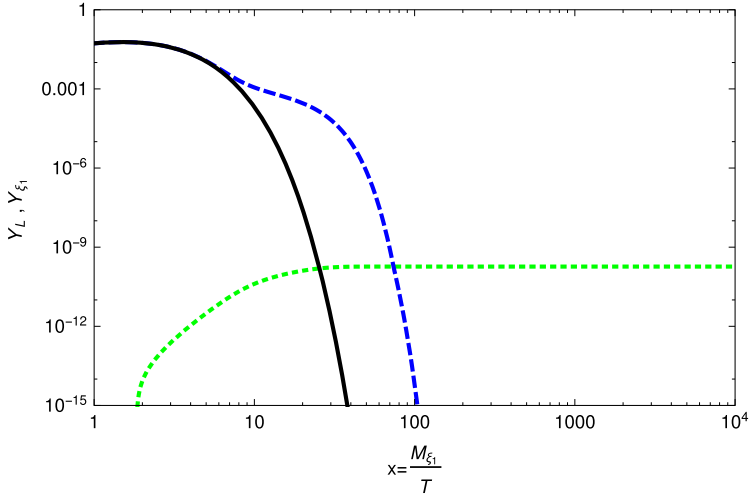


Fig. 4. The lepton asymmetry from the decay of ξ_1 . The Green (dotted) line shows the abundance of lepton asymmetry for $\epsilon_L = 2 \times 10^{-7}$. The Blue (dashed) line shows the abundance of ξ_1 particles. The Black solid line shows the equilibrium number density of ξ_1 . (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

5. Dark matter abundance from X-particles decay

The decay of X -particles to $\psi\chi$ can populate the number densities of ψ and χ . Since ψ is a doublet under $SU(2)_L$, it thermalises quickly via gauge interactions. As we will discuss in section 5.1, the symmetric component of ψ gets annihilated efficiently to the SM particles, while the asymmetric number density of ψ gets converted to a net χ density through the decay process: $\psi \rightarrow \chi \bar{f} f$, induced via the soft Z_2 symmetry breaking term $\mu^2 H^\dagger X$. Here we assume $M_\chi < M_\psi$, so that χ remains stable kinematically. On the other hand, χ is a singlet under $SU(2)_L$. The only way χ can interact with the SM particles is via the mixing of neutral gauge bosons Z_D and Z . However, as we show later that χ remains out-of-equilibrium throughout the epoch. As a result the symmetric component of χ does not get annihilated efficiently to the SM particles like ψ . Therefore, the number density of the symmetric component of χ always dominates over its asymmetric number density. In the following we discuss the symmetric and asymmetric abundance of χ produced via the decay of X -particles.

5.1. Symmetric χ -DM abundance from X-particles decay

Let us now discuss about the symmetric component of the χ -abundance. Note that χ is a singlet under electroweak interaction. Therefore, we safely assume that the thermal abundance is negligible. In this case, the number density of χ particles is produced by the CP-conserving decay of heavy scalar ξ_1 . In the early Universe, when ξ_1 goes out of thermal equilibrium and decay to ψ and χ , the ψ gets thermalised quickly through its gauge interaction, while the χ remain isolated. The abundance of χ from decay of ξ_1 and ξ_2 can be estimated from the following Boltzmann equations:

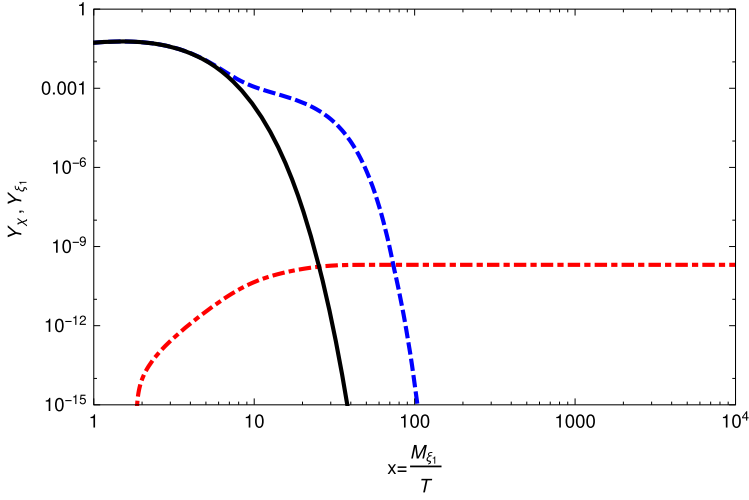


Fig. 5. The Red dotted-line shows the abundance of χ dark matter and the Blue dashed-line shows the abundance of ξ_1 particles. The Black solid-line shows the equilibrium number density of ξ_1 .

$$\begin{aligned} \frac{dY_{\xi_1}}{dx} = & -\frac{x}{H(M_{\xi_1})} s \langle \sigma |v|_{(\xi_1 \xi_1 \rightarrow All)} \rangle \left[Y_{\xi_1}^2 - Y_{\xi_1}^{eq2} \right] \\ & - \frac{x}{H(M_{\xi_1})} \Gamma_{(\xi_1 \rightarrow All)} \left[Y_{\xi_1} - Y_{\xi_1}^{eq} \right], \end{aligned} \quad (20)$$

and

$$\frac{dY_{\chi}}{dx} = \frac{x}{H(M_{\xi_1})} \Gamma_{(\xi_1 \rightarrow All)} B_{\chi} \left[Y_{\xi_1} - Y_{\xi_1}^{eq} \right]. \quad (21)$$

In the above Eq. (21), $B_{\chi} \equiv Br(\xi_1 \rightarrow \psi \chi)$ is the branching ratio for the decay of ξ_1 to $\psi \chi$. The solutions of Eqs. (20) and (21) are shown in Fig. 5. Here we use the decay coupling constant $\lambda = 0.47 \times 10^{-7}$, $M_{\xi_1} = 10^{10}$ GeV and $M_{\chi} = 2.5$ GeV. The typical value of the cross section for $\xi_1^{\dagger} \xi_1 \rightarrow All$ particles is taken as $\sigma |v|_{(\xi_1 \xi_1 \rightarrow All)} = 10^{-25}$ GeV $^{-2}$. The blue dashed-line shows the abundance of ξ_1 particles, where as the red dot-dashed line shows the abundance of dark matter particle χ . To get the observed dark matter abundance,

$$Y_{DM} \equiv \frac{n_{DM}}{s} = 4 \times 10^{-12} \left(\frac{100 \text{ GeV}}{M_{DM}} \right) \left(\frac{\Omega_{DM} h^2}{0.11} \right) \quad (22)$$

we have used the branching ratio: $B_{\chi} = 2.2 \times 10^{-7}$. This shows that ξ_1 decay significantly to leptons and rarely to invisible sector to get the correct relic abundance of dark matter and baryon asymmetry.

As the temperature falls below the mass scale of ψ , the latter gets decoupled from the thermal bath and decay back to χ and may produce an additional abundance of dark matter. However, as we show below the freeze-out cross-section of $\psi \bar{\psi} \rightarrow SM$ particles is quite large due to its coupling with SM gauge bosons and hence produce a significantly low abundance. The relevant Boltzmann equation for the evolution of ψ number density is given by:

$$\frac{dY_{\psi}}{dz} = -\frac{zs}{H(M_{\psi})} \langle \sigma |v| \rangle_{\text{eff}} \left[Y_{\psi}^2 - Y_{\psi}^{eq2} \right], \quad (23)$$

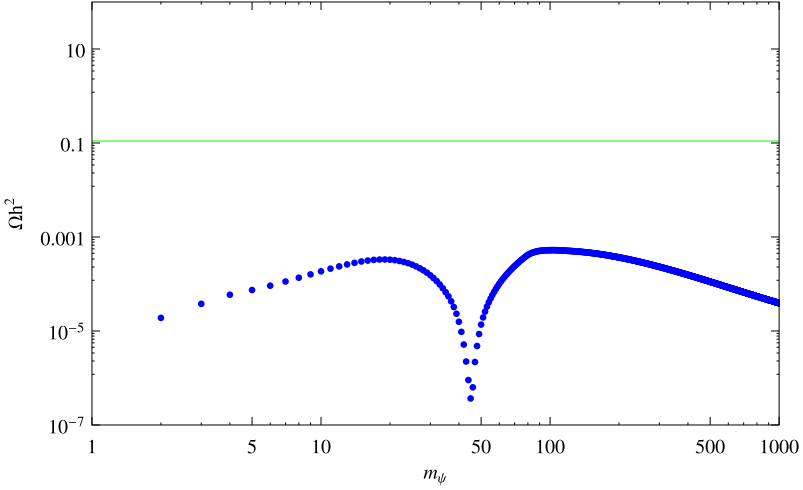


Fig. 6. Relic abundance of ψ particles (shown by dotted Blue line). Green horizontal line shows the observed relic abundance by PLANCK data.

where the $z = \frac{M_\psi}{T}$, and s is the entropy density. The relevant channels contributing to the ψ relic density are:

$$\begin{aligned} \overline{\psi^+} \psi^+ &\rightarrow \gamma Z_d, \gamma \gamma, W^+ W^- \bar{u}u, \bar{c}c, \bar{t}t, \bar{l}l, \\ \psi^+ \overline{\psi^0} &\rightarrow W^+ Z_d, \bar{b}t, \bar{d}u, \bar{s}c, \gamma W^+, \bar{l} \nu_l, Z W^+ \text{ and} \\ \overline{\psi^0} \psi^0 &\rightarrow Z Z_d, Z Z, W^+ W^-, \bar{q}q. \end{aligned}$$

We use micrOMEGAs [30] to calculate the freeze-out abundance of ψ particles. The results are shown in Fig. 6 of ref. [22].

One can clearly see in Fig. 6, that the resonance at $M_\psi = \frac{M_Z}{2}$, is due to the enhancement in the Z mediation s-channel cross section, which causes a drop in the relic density. More over we see that the relic density of ψ is much less than the observed DM abundance by WMAP [1] and PLANCK [2]. Therefore, the decay of ψ after it freezes out does not produce any significant χ abundance.

5.2. Asymmetric χ -DM abundance from X -particles decay

Similar to the lepton asymmetry, the CP-violating out-of-equilibrium decay of ξ_1 produce an asymmetry between χ and $\bar{\chi}$ as well as ψ and $\bar{\psi}$. The corresponding tree level and self energy correction diagrams are shown in Fig. 7.

The amount of CP-asymmetry can be given as:

$$\begin{aligned} \epsilon_\chi &= [Br(\xi_1^- \rightarrow \psi^- \chi) - Br(\xi_1^+ \rightarrow (\psi^-)^c \chi^c)] \\ &= \frac{Im(\lambda_1^* \lambda_2 \sum_{k,l} f_{1kl}^* f_{2kl})}{8\pi^2 (M_2^2 - M_1^2)} \left[\frac{M_1^2 M_2}{\Gamma_1} \right] = -\epsilon_L. \end{aligned} \tag{24}$$

The corresponding χ asymmetry can be computed as:

$$Y_\chi = \epsilon_\chi Y_{\xi_1} B_\chi. \tag{25}$$

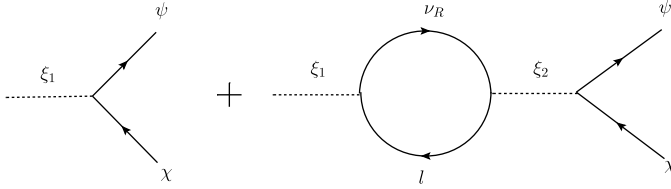


Fig. 7. Tree level and self energy correction diagrams producing the dark matter asymmetry.

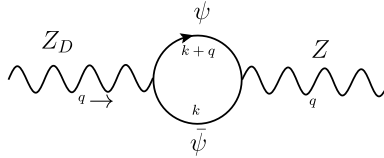


Fig. 8. The mixing between the Z and Z_D with the running of ψ particles in the loop.

Since $|\epsilon_L| = |\epsilon_\chi| = 10^{-7}$ (required for observed baryon asymmetry; see section 4) and $B_\chi = \mathcal{O}(10^{-7})$ (required for observed DM abundance; See section 5.1) we get a very small asymmetry, $\mathcal{O}(10^{-16})$. Moreover, ψ and χ are vector-like fermions. Therefore, the sphalerons don't convert this asymmetry to a net baryon asymmetry. See for instance [23–25]. Thus the observed baryon asymmetry does not get affected by the decay of ξ_1 to $\psi\chi$.

5.3. Production of χ -DM from thermal scattering

Now we check the possible scattering processes through which χ can be produced in the thermal bath apart from X -decay. The relevant processes are $\bar{\psi}\psi \rightarrow \bar{\chi}\chi$ mediated via Z_D and $f\bar{f} \rightarrow \bar{\chi}\chi$ via the exchange of Z - Z_D mixing as shown in the Fig. 8. The former one is the most relevant one as it dominates over the latter process. The scattering cross-section times velocity for the process: $\bar{\psi}\psi \rightarrow \bar{\chi}\chi$ is given by:

$$\sigma = \frac{\sqrt{s - 4M_\chi^2}}{16\pi s \sqrt{s}} \frac{g_D^4}{(s - M_{Z_D}^2)^2 + \Gamma_{Z_D}^2 M_{Z_D}^2} \times \left(s^2 + \frac{1}{3}(s - 4M_\psi^2)(s - 4M_\chi^2) + 4M_\chi^2 s + 4M_\psi^2 s \right). \tag{26}$$

In the process: $f\bar{f} \rightarrow \bar{\chi}\chi$ via the exchange of Z - Z_D mixing, the loop factor is estimated as [31]:

$$\Pi^{\mu\nu}(q) = \left(q^2 g^{\mu\nu} - q^\mu q^\nu \right) \frac{g_D}{4\pi^2} \left(\frac{g}{2} \cos\theta_W - \frac{g'}{2} \sin\theta_W \right) \times \int_0^1 dx 2x(1-x) \log \left(\frac{M_\psi^2}{M_\psi^2 - x(1-x)q^2} \right). \tag{27}$$

Where θ_W is the Weinberg angle, the M_ψ is the mass of the ψ particle running in the loop. Now the cross-section times velocity for this process is given as:

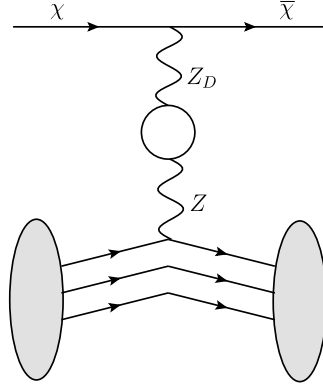


Fig. 9. Dark matter scattering with nuclei via the Z - Z_D mixing.

$$\begin{aligned} \sigma|v| = & \frac{\sqrt{s - 4M_\chi^2}}{2\pi s\sqrt{s}} \frac{\left(\frac{g}{2\cos\theta_W} g_D \hat{\Pi}_2(s)\right)^2}{\left[(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2\right] \left[(s - M_{Z_D}^2)^2 + \Gamma_{Z_D}^2 M_{Z_D}^2\right]} \\ & \times \left[\left(g_V^2 + g_A^2\right) \left\{ \frac{5}{12} s^4 - \frac{1}{3} M_\chi^2 s^3 - \frac{2}{3} M_f^2 s^3 + \frac{2}{3} M_f^2 M_\chi^2 s^2 \right\} \right. \\ & \left. + \left(g_V^2 - g_A^2\right) \left\{ M_f^2 s^3 + 2M_f^2 M_\chi^2 s^2 \right\} \right]. \end{aligned} \quad (28)$$

In the limit $s > 4M_\psi^2$,

$$\hat{\Pi}_2(s) = \frac{4g_D \left(\frac{g}{2} \cos\theta_W - \frac{g'}{2} \sin\theta_W\right)}{16\pi^2} (\pm\pi) \frac{1}{6} \sqrt{1 - \frac{4M_\psi^2}{s}} \left(1 + \frac{2M_\psi^2}{s}\right). \quad (29)$$

If the above mentioned processes are brought to thermal equilibrium then they will overpopulate the χ -number density. Therefore, we need to check the parameter space in which the above processes remain out-of-equilibrium throughout the epochs. Note that in Eqs. (26) and (28), the unknown parameters are g_D and M_{Z_D} apart from M_χ . In Fig. 10, we have shown the parameter space (given by Blue region) in the plane of g_D versus M_{Z_D} , where the above processes remain out-of-equilibrium and hence remain consistent with the dark matter relic abundance obtained from X -decay. In this case, we have chosen a typical center of mass energy: $\sqrt{s} = 1000$ GeV. The sharp deep at around $M_{Z_D} = 1000$ GeV, implies that the cross-section is large at the resonance.

6. Direct search of χ dark matter

The spin independent elastic cross-section of DM candidate with nuclei through the Z - Z_D mixing is shown in the Fig. 9.

The spin independent DM-nucleon cross-section with loop induced Z - Z_D mixing is given by [32] [33],

$$\sigma_{SI}^Z = \frac{1}{64\pi A^2} \mu_r^2 \tan^2 \theta_Z \frac{G_F}{2\sqrt{2}} \frac{g_D^2}{M_{Z_D}^2} \left[\tilde{Z} \frac{f_p}{f_n} + (A - \tilde{Z}) \right]^2 f_n^2, \quad (30)$$

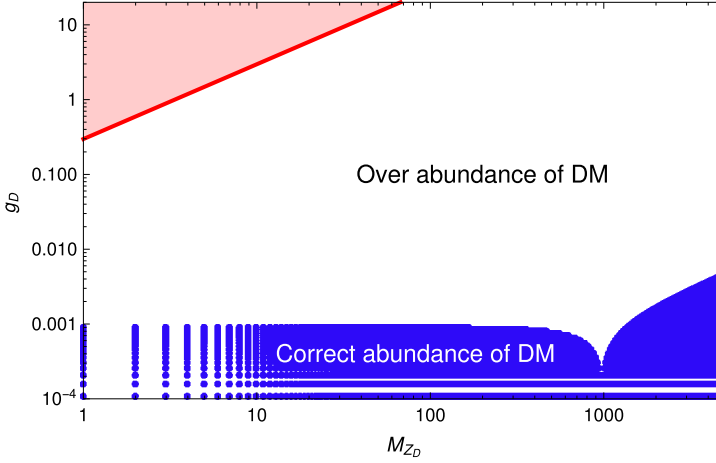


Fig. 10. LUX constraint on dark matter, arising via Z - Z_D mixing, is shown on the plane of g_D versus M_{Z_D} for a typical dark matter mass of 5 GeV (top Red line), using $\tan\theta_Z = 10^{-2}$. The blue region defines the allowed parameter space which is consistent with the dark matter relic abundance.

where A is the mass number of the target nucleus, \tilde{Z} is the atomic number of the target nucleus, θ_Z is the mixing angle between Z and Z_D , $\mu_r = M_\chi m_n / (M_\chi + m_n) \approx m_n$ is the reduced mass, m_n is the mass of nucleon (proton or neutron) and f_p and f_n are the interaction strengths of DM with proton and neutron respectively. For simplicity we assume conservation of isospin, i.e. $f_p/f_n = 1$. The value of f_n is varied within the range: $0.14 < f_n < 0.66$ [34]. If we take $f_n \simeq 1/3$, the central value, then from Eq. (30) we get the total cross-section per nucleon to be,

$$\sigma_{SI}^Z \simeq 2.171 \times 10^{-36} \text{cm}^2 \tan^2 \theta_Z \frac{g_D^2}{(M_{Z_D}/\text{GeV})^2}, \quad (31)$$

where we have used DM mass to be 5 GeV. Since the Z -boson mass puts a stringent constraint on the mixing parameter $\tan\theta_Z$ to be $\mathcal{O}(10^{-2}-10^{-4})$ [31,35,36], we choose the maximum allowed value (10^{-2}) and plot the spin independent direct DM detection cross-section, allowed by LUX [37], in the plane of g_D versus M_{Z_D} as shown in Fig. 10. The plot shows a straight line, as expected from Eq. (30), and is given by the Red lines in Fig. 10 for the DM mass of 5 GeV. Any values above that line corresponding to the DM mass of 5 GeV will not be allowed by the LUX limit.

7. Conclusions

The oscillation experiments undoubtedly shown that the neutrinos are massive. However, their nature, either Dirac or Majorana, is yet to be confirmed. In this paper, by assuming that the neutrinos are Dirac ($B-L$ is an exact symmetry), we found a way of explaining simultaneously the relic abundance of dark matter and baryon asymmetry of the Universe.

We extended the SM with a simple dark sector constituting vector-like fermions: a doublet ψ and a singlet χ , where χ is odd under the discrete Z_2 symmetry and behave as a candidate of dark matter. The same Z_2 symmetry disallowed neutrino Dirac mass by forbidding $\bar{\nu}_R \tilde{H}^\dagger \ell$ coupling, where ν_R is odd under the Z_2 symmetry. However, the discrete Z_2 symmetry was allowed to

break softly without destabilizing the dark matter component χ (i.e. we chose $M_\chi < M_\psi$). As a result, Dirac mass of the active neutrinos could be generated.

We assumed heavy Higgs doublets (X), which transform non-trivially under the discrete Z_2 symmetry, present in the early Universe. The out-of-equilibrium decay of X through $X \rightarrow \nu_R \ell$ and $X \rightarrow \chi \psi$ generated baryon asymmetry and dark matter abundance that we observe today. Since $B-L$ is considered to be an exact symmetry, the CP-violating decay of X to $\nu_R \ell$ produced equal and opposite $B-L$ asymmetries in the left and right-handed sectors. The right-handed sector coupled weakly to the SM as required by the Dirac mass of the neutrinos. Therefore, the $B-L$ asymmetry in the left-handed sector got converted to a net B -asymmetry via the $B+L$ violating sphaleron transitions, while that in the right-handed sector remained intact. The two $B-L$ asymmetries neutralized much after the electroweak phase transition when the sphaleron transitions got suppressed. Similarly the decay of $X \rightarrow \chi \psi$ also generated a net χ abundance that we observe today. Since the branching fraction $B_\chi \ll 1$, the asymmetric χ abundance is much smaller than its symmetric counterpart.

The dark matter χ is invisible at the collider. However, the signature of dark sector particle ψ^\pm can be looked at in the collider [21,22]. For example, ψ^\pm can be pair produced through Drell–Yan processes. However, their decay can give interesting signatures. In particular, three body decay of ψ^\pm can give interesting displaced vertex signatures. Here we assumed the mass of X -particles to be super heavy, namely $M_X \sim 10^{10}$ GeV. However, the mass scale of X -particles can be just above the EW scale if we assume resonant leptogenesis. In that case the decay of X -particles can give interesting signatures at collider. We will come back to these issues in a future publication [38].

References

- [1] G. Hinshaw, et al., WMAP Collaboration, *Astrophys. J. Suppl.* 208 (2013) 19, arXiv:1212.5226 [astro-ph.CO].
- [2] P.A.R. Ade, et al., Planck Collaboration, *Astron. Astrophys.* 594 (2016) A13, arXiv:1502.01589 [astro-ph.CO].
- [3] Q.R. Ahmed, et al., SNO Collaboration, *Phys. Rev. Lett.* 89 (2002) 011301;
J.N. Bahcall, C. Pena-Garay, arXiv:hep-ph/0404061.
- [4] S. Fukuda, et al., Super-Kamiokande Collaboration, *Phys. Rev. Lett.* 86 (2001) 5656.
- [5] K. Eguchi, et al., KamLAND Collaboration, *Phys. Rev. Lett.* 90 (2003) 021802.
- [6] S. Weinberg, *Phys. Rev. Lett.* 43 (1979) 1566.
- [7] P. Minkowski, *Phys. Lett. B* 67 (1977) 421;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D. Freedman (Eds.), *Supergravity*, North Holland, Amsterdam, 1979;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), *Workshop on Unified Theory and Baryon Number in the Universe*, Japan, KEK 1979, 1979;
R.N. Mohapatra, G. Senjanovic, *Phys. Rev. Lett.* 44 (1980) 912;
J. Schechter, J.W.F. Valle, *Phys. Rev. D* 22 (1980) 2227.
- [8] M. Magg, C. Wetterich, *Phys. Lett. B* 94 (1980) 61;
T.P. Cheng, L.F. Li, *Phys. Rev. D* 22 (1980) 2860;
G.B. Gelmini, M. Roncadelli, *Phys. Lett. B* 99 (1981) 411;
G. Lazarides, Q. Shafi, C. Wetterich, *Nucl. Phys. B* 181 (1981) 287;
R.N. Mohapatra, G. Senjanovic, *Phys. Rev. D* 23 (1981) 165;
J. Schechter, J.W.F. Valle, *Phys. Rev. D* 22 (1980) 2227;
E. Ma, U. Sarkar, *Phys. Rev. Lett.* 80 (1998) 5716, arXiv:hep-ph/9802445.
- [9] R. Foot, H. Lew, X.G. He, G.C. Joshi, *Z. Phys. C* 44 (1989) 441;
E. Ma, *Phys. Rev. Lett.* 81 (1998) 1171, arXiv:hep-ph/9805219.
- [10] M. Fukugita, T. Yanagida, *Phys. Lett. B* 174 (1986) 45.
- [11] M.A. Luty, *Phys. Rev. D* 45 (1992) 455;
R.N. Mohapatra, X. Zhang, *Phys. Rev. D* 46 (1992) 5331;

- A. Acker, H. Kikuchi, E. Ma, U. Sarkar, Phys. Rev. D 48 (1993) 5006;
M. Flanz, E.A. Paschos, U. Sarkar, Phys. Lett. B 345 (1995) 248;
M. Flanz, E.A. Paschos, U. Sarkar, J. Weiss, Phys. Lett. B 389 (1996) 693;
M. Plumacher, Z. Phys. C 74 (1997) 549;
W. Buchmuller, P. Di Bari, M. Plumacher, Ann. Phys. 315 (2005) 305, arXiv:hep-ph/0401240;
J. Faridani, S. Lola, P.J. O'Donnell, U. Sarkar, Eur. Phys. J. C 7 (1999) 543;
R. Barbieri, P. Creminelli, A. Strumia, N. Tetradis, Nucl. Phys. B 575 (2000) 61, arXiv:hep-ph/9911315;
G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, Nucl. Phys. B 685 (2004) 89, arXiv:hep-ph/0310123;
N. Sahu, U. Sarkar, Phys. Rev. D 74 (2006) 093002, arXiv:hep-ph/0605007.
- [12] A.D. Sakharov, JETP Lett. 5 (1967) 24.
[13] M. Agostini, et al., GERDA Collaboration, Phys. Rev. Lett. 111 (12) (2013) 122503, arXiv:1307.4720 [nucl-ex];
S.M. Bilenky, C. Giunti, Int. J. Mod. Phys. A 30 (04n05) (2015) 1530001, arXiv:1411.4791 [hep-ph].
[14] K. Dick, M. Lindner, M. Ratz, D. Wright, Phys. Rev. Lett. 84 (2000) 4039.
[15] D.G. Cerdeno, A. Dedes, T.E.J. Underwood, J. High Energy Phys. 0609 (2006) 067, arXiv:hep-ph/0607157.
[16] P.H. Gu, H.J. He, J. Cosmol. Astropart. Phys. 0612 (2006) 010, arXiv:hep-ph/0610275.
[17] P.H. Gu, H.J. He, U. Sarkar, Phys. Lett. B 659 (2008) 634, arXiv:0709.1019 [hep-ph].
[18] H. Murayama, A. Pierce, Phys. Rev. Lett. 89 (2002) 271601, arXiv:hep-ph/0206177.
[19] B. Thomas, M. Toharia, Phys. Rev. D 73 (2006) 063512, arXiv:hep-ph/0511206.
[20] D. Borah, A. Dasgupta, J. Cosmol. Astropart. Phys. 1612 (12) (2016) 034, arXiv:1608.03872 [hep-ph].
[21] S. Bhattacharya, N. Sahoo, N. Sahu, Phys. Rev. D 93 (11) (2016) 115040, arXiv:1510.02760 [hep-ph].
[22] S. Bhattacharya, N. Sahoo, N. Sahu, Phys. Rev. D 96 (3) (2017) 035010, arXiv:1704.03417 [hep-ph].
[23] C. Arina, N. Sahu, Nucl. Phys. B 854 (2012) 666, arXiv:1108.3967 [hep-ph].
[24] C. Arina, J.O. Gong, N. Sahu, Nucl. Phys. B 865 (2012) 430, arXiv:1206.0009 [hep-ph].
[25] C. Arina, R.N. Mohapatra, N. Sahu, Phys. Lett. B 720 (2013) 130, arXiv:1211.0435 [hep-ph].
[26] J. McDonald, N. Sahu, U. Sarkar, J. Cosmol. Astropart. Phys. 0804 (2008) 037, arXiv:0711.4820 [hep-ph].
[27] J. Heeck, Phys. Rev. D 88 (2013) 076004, arXiv:1307.2241 [hep-ph].
[28] C. Patrignani, et al., Particle Data Group, Chin. Phys. C 40 (10) (2016) 100001.
[29] E. Ma, U. Sarkar, Phys. Rev. Lett. 80 (1998) 5716, arXiv:hep-ph/9802445.
[30] G. Belanger, F. Boudjema, A. Pukhov, A. Semenov, Comput. Phys. Commun. 180 (2009) 747, arXiv:0803.2360 [hep-ph].
[31] S. Patra, S. Rao, N. Sahoo, N. Sahu, Nucl. Phys. B 917 (2017) 317, arXiv:1607.04046 [hep-ph].
[32] M.W. Goodman, E. Witten, Phys. Rev. D 31 (1985) 3059.
[33] R. Essig, Phys. Rev. D 78 (2008) 015004, arXiv:0710.1668 [hep-ph].
[34] R. Koch, Z. Phys. C 15 (1982) 161;
J. Gasser, H. Leutwyler, M.E. Sainio, Phys. Lett. B 253 (1991) 260;
M.M. Pavan, R.A. Arndt, I.I. Strakovski, R.L. Workman, PiN Newslett. 16 (2002) 110;
A. Bottino, F. Donato, N. Fornengo, S. Scopel, Phys. Rev. D 78 (2008) 083520, arXiv:0806.4099;
J.M. Alarcon, J. Martin Camalich, J.A. Oller, Phys. Rev. D 85 (2012) 051503, arXiv:1110.3797 [hep-ph];
J.M. Alarcon, L.S. Geng, J. Martin Camalich, J.A. Oller, Phys. Lett. B 730 (2014) 342, arXiv:1209.2870 [hep-ph].
[35] A. Hook, E. Izaguirre, J.G. Wacker, Adv. High Energy Phys. 2011 (2011) 859762, arXiv:1006.0973 [hep-ph].
[36] K.S. Babu, C.F. Kolda, J. March-Russell, Phys. Rev. D 57 (1998) 6788, arXiv:hep-ph/9710441.
[37] D.S. Akerib, et al., LUX Collaboration, Phys. Rev. Lett. 118 (2) (2017) 021303, arXiv:1608.07648 [astro-ph.CO].
[38] P.S. Bhupal Dev, N. Narendra, N. Sahu, Draft in preparation.