**On the constraint factor and Tabor coefficient pertinent to spherical indentation**

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**Abstract**

Measuring the uniaxial stress-strain response from indentation testing has been of great interest to the materials community ever since the seminal work on spherical indentation by David Tabor. In this regard, spherical indentation is the primary choice due to the ability to access a range of strains in a single test. While indentation testing is fairly simple to perform, the conversion factors required to calculate the uniaxial flow stress from hardness, which is commonly referred to as constraint factor and uniaxial strain from indentation contact radius and ball radius, which is called the Tabor coefficient, are not necessarily constant and most of the prior work involves assumptions about one of these conversion factors to calculate the other. In this work, we present a Finite Element Analysis (FEA) based approach to independently determine the constraint factor and Tabor coefficient in the fully plastic indentation regime which is a pre-requisite for this analysis. The criteria to determine whether fully plastic indentation regime is satisfied will also be presented. The proposed approach has been validated by comparing the uniaxial stress-strain response from spherical indentation tests on OFHC copper and the data obtained by conventional uniaxial testing. Excellent agreement has been found between the two approaches which can be readily applied for measuring the uniaxial stress-strain response of coatings which is otherwise difficult to determine.

*Key words*: spherical ball indentation, constraint factor, Tabor coefficient, FEM analysis of indentation, Fully plastic indentation regime

**1. Introduction**

The plastic flow behaviour of materials are normally determined on the basis of uniaxial tensile / compressive tests. However, use of this technique is not possible in the case of coatings or when volume of the material available for testing is very small. Indentation technique (especially with spherical ball or spherical tip as the indenter) is a versatile technique capable of determining the plastic flow behaviour of coatings and thin films and also small volumes in multi-phase alloys. However, it is important to ensure that the indentation tests satisfies certain criteria before the data from such tests is considered valid and the present work is concerned with developing such validity conditions.

The indentation of metallic substrate with harder spherical balls over a range of load results in generation of hardness data (*H* = Load/projected area of indentation) as a function of normalized indentation radius, *a*/*R* (*a* is indentation radius and *R* is radius of the spherical ball). Tabor [1] first demonstrated that the *H – a/R* data can be converted to uniaxial plastic flow stress – strain data using the conversion factor *C* (called constraint factor) for converting *H* to uniaxial equivalent flow stress and the second conversion factor *q* (called Tabor coefficient) for converting *a/R* to uniaxial plastic equivalent strain. Given the fact that indentation occurs under multiaxial stress state, it is rather surprising that one is able to obtain the uniaxial plastic flow stress-strain from hardness data using the two conversion factors mentioned above. However, repeated indentation experiments on a wide range of metallic materials has demonstrated the validity of the approach. Of the two conversion factors, the factor *C* has a sound theoretical basis. On the other hand, Tabor coefficient has no theoretical basis and many investigators have felt that it is an empirical fitting parameter.

The indentation behaviour of metallic materials with spherical balls has been studied exhaustively over the years. However, some aspects of the indentation have not been clarified fully and satisfactorily. It is very important that the indentation is carried out in the fully plastic indentation regime characterized by a constant values of the conversion factors (*C* and *q*) independent of depth of indentation for a given material. In addition, fully plastic indentation requires that the plastic zone beneath the indenting ball is not confined but spreads to the surface and also that the energy expended during indentation is dominated by plastic deformation. In contrast, if the indentation is carried out in the elastic-plastic indentation regime, it has been demonstrated by Johnson [2] in his seminal work that the values of *C* and plastic zone size increase continuously with increasing (*E/Y* )(*a/R*) where *E* is the elastic modulus , *Y* is the yield strength and *a/R* has been defined earlier. In addition, the plastic zone is confined within the sample.

Many investigators (see Table 1) have converted their *H* - *a/R* data obtained using spherical indentation tests to equivalent uniaxial stress- plastic strain using the two conversion factors (*C* and *q*) [1–13]. As can be noted from the Table 1, the experimentally measured / theoretically calculated *C* and *q* values covered a wide range from 2.6 to 4 and 0.1 to 0.32 respectively. Many of these investigations do not confirm whether the indentation tests were carried out in the fully plastic indentation regime wherein the constant conversion factors are applicable.

In order to systematically explore constraint factor and Tabor coefficient for a wide range of materials finite element simulations of spherical indentation can be used.

On the basis of the above discussion, the present paper has twin objectives.

1. Develop a set of validity conditions which will ensure that the indentation is being carried out in the fully plastic regime. Towards this purpose, the spherical ball indentation of a wide range of materials were carried out using FEM.
2. Of these simulations, select only those indentations which meet the criteria for fully plastic indentation regime which will be determined in the current work. Calculate the Tabor coefficient (*q*) in this regime and examine whether *q* is a constant and has the right magnitude and thereby establish a theoretical basis for the Tabor coefficient.

**2. Materials and modeling**

*2.1. Constitutive Equation*

In the present work it is proposed to carry out the indentation simulations using the Finite Element Method (FEM). A critical feature required for such a simulation of indentation is the proper choice of the constitutive equation. Many investigators have used the power-law formulation for flow stress ()-plastic strain given by,

(1)

In Eq.1, *K* is the strength coefficient and *n* is the strain hardening exponent. However, the problem with Eq. (1) is that it predicts zero flow stress at zero plastic strain in contrast to real materials characterized by a finite yield strength (*Y*) at zero strain. There have been attempts to link *K* to yield strength but the resulting *K* values are unrealistically high.

In the present study we have opted for Ludwig formulation given by,

(2)

In Eq. (2), *Y* is the yield strength, *B* and *m* are the material parameters. The advantage of this equation is that it explicitly separates out the yield strength and the strain hardening component of the flow stress and thus Eq. (2) can be fitted to the plastic flow behavior of any metal or alloy unlike Eq. (1). In addition, as will be shown later, use of Eq. (2) also allows the determination of C without inputting Tabor coefficient. In order to cover a wide material spectrum, 108 finite element simulations were carried out spanning a range of 10 to 3000 for *E/Y*, 0 to 0.775 for *m* and 0.1 to 0.7 for *a/R*. In addition to these hypothetical materials, a widely used metal (OFHC Cu) is used for simulation and subsequent validation. Table 2 illustrates the typical values of *K* and *n* (Eq. (1)) and *Y*, *B* and *m* (Eq. (2)) for typical metals and alloys.

*2.2. Finite Element Analysis (FEA)*

Axisymmetric two-dimensional finite element analysis was performed using commercial finite element package ABAQUS 6.9. Indentation problem is modelled assuming the ball to be rigid while the material being indented is considered as elastic-plastic. In addition, since a spherical ball is the indenter, the deformation can be treated as axi-symmetric leading to a simplification of the analysis.

Indentation tests were simulated by specifying a displacement boundary condition on the rigid indenter. The displacements were chosen such that indents having four different *a/R* ratios (0.1, 0.3, 0.5 and 0.7) were formed for the various combinations of material properties (*E, Y, H, B* and *m*). A fixed value of 0.1 for the friction coefficient between the indenter and the specimen contact surface was assumed. Four-node axisymmetric linear quadrilateral elements (CAX4R) with reduced integration was used to mesh the model as shown in Fig. 1. The model has 82503 elements. To analyze the deformation behaviour more accurately as well as to reduce the simulation time, finer mesh was used in the contact region. The overall dimensions of the model were chosen such that the edge effects are negligible. Output of simulations were recorded at the full load condition. Typical output of the simulations recorded at the end of loading include, indentation load, indentation depth, plastic deformation zone size and the various energies (Elastic, plastic and total energy / work done) involved in the process of indentation.

**3. Results and discussions**

*3.1. Validation of finite element model for indentation*

Prior to carrying out the FEM modelling of 108 indentations on hypothetical materials, it was validated by comparing the model predictions with the experimental results on OFHC copper. In order to model the indentation on OFHC Cu, the constitutive equation (Eq.2) with the values of *E, Y, B* and *m* from Table 2 was given as the input. The indentation experiments were carried out on OFHC samples (50mm diameter and 15mm height) with Tungsten Carbide ball (1mm diameter) as the indenter. Indentations were carried out at different loads (*F*) and at each load, the radius of the indentation (*a*) was measured. The load (*F*) was converted to Hardness (*H*) using the relation,

(3)

Fig.2 compares the FEM model prediction (solid line) with the experiment (filled circles). It is clear that the FEM model predicts the variation of *H* with *a/R* very well thus validating the model.

*3.2. Criteria for fully developed plastic flow*

The strain conversion factor (*q*) proposed by Tabor only applies to fully developed plastic flow [1]. Hence, it is important to determine the criteria for fully developed plastic flow. In this section, we present the criteria for fully developed plastic flow based on constraint factor, plastic work fraction and plastic zone size.

*3.2.1. Constraint factor*

Constraint factor (*C*) is the conversion factor used to convert the indentation hardness (*H*) to the uniaxial equivalent flow stress () as shown below

(4)

For the constitutive relationship for flow stress chosen in the present work (Eq. (2)), can be substituted in the Eq. (4), and will give the following form

(5)

Tabor has proposed an empirical relation for an average strain that induces during the indentation process as a function of *a/R* as shown below

(6)

The Tabor relationship for strain (Eq. (6)) can be substituted in the Eq. (5), to obtain a relationship for hardness in terms of *a*/*R* as shown below

(7)

From the above equation, it can be observed that the constraint factor can be calculated only if the Tabor coefficient (*q*) and the material parameters (*Y*, *B* and *m*) are known. While the material parameters can be obtained from uniaxial testing, the Tabor coefficient is usually assumed to be around 0.2 [1] to determine the constraint factor. In order to determine the constraint factor without prior knowledge of Tabor coefficient, one can assume that the constraint factor is independent of strain hardening exponent (*m*) and substitute *m* = 0 in Eq. (7) to obtain a simple relationship for constraint factor in terms of hardness in the absence of hardening () and the material parameters *Y* and *B* as shown below.

(8)

From the above equation it can be observed that constraint factor can be calculated without prior knowledge of Tabor coefficient by assuming that it is independent of strain hardening exponent. Note that this analysis also assumes that the hardness in the absence of hardening () is representative of fully plastic indentation regime which is an inherent assumption in Tabor’s analysis.

The constraint factor calculated using Eq. (8) is plotted as a function of non-dimensional parameters (*E*/*Y*)(*a*/*R*) in Fig. 3(a). The choice of the dimensionless parameter is based on the work of Johnson [2] which in turn is based on the well-known expanding cavity model. The non-dimensional parameter (*E*/*Y*)(*a*/*R*) is interpreted as the ratio of the indentation strain *a*/*R* to the elastic strain at yielding of the material *Y*/*E* and is found to determine the pressure in the hydrostatic core beneath the indenter [2] and there by the spread of the zone. From the figure it can be observed that the constraint factor increases with increasing (*E*/*Y*)(*a*/*R*) and reaches a constant value of around 2.8 which is consistent with previous observations [1,2]. However, the initial rise is slightly dependent on the value of the strain hardening exponent. In order to rationalize the data at different strain hardening exponents, constraint factor is plotted as a function of a non-dimensional parameter (*E*/*H*)(*a*/*R*) in Fig. 3(b), wherein the hardness (*H*) is intended to account for the effect of strain hardening, if any, at different levels of *a*/*R*. From the figure it can be observed that, unlike the case of Fig. 3(a) where (*E*/*Y*)(*a*/*R*) was used as the non-dimensional parameter, using (*E*/*H*)(*a*/*R*) as the non-dimensional parameter results in the entire data collapsing on to a single master curve. The overall trend is similar to Fig. 3(a) wherein the constraint factor increases in the elastic-plastic region followed by the fully plastic region where the constraint factor is constant and around 2.8. The transition from elastic-plastic deformation to fully developed plastic deformations is found to occur around a (*E*/*H*)(*a*/*R*) value of 20 (indicated by a dashed vertical line). This value is consistent with the observation of Johnson [2] where the transition to fully developed plastic flow is reported to be at a (*E*/*Y*)(*a*/*R*) value of 50 which is equivalent to (*E*/*H*)(*a*/*R*) value of 17.9 at a constraint factor of 2.8.

In order to demonstrate the difference in the plastic zone beneath the indenter in the elastic-plastic and fully plastic region, plastic zone at representative points in each of these regimes is shown in Fig. 4. The location of the points ((I) elastic-plastic regime (E/Y = 100, m = 0.1 and a/R = 0.3) and (II) fully plastic regime (E/Y = 1000, m = 0.6 and a/R = 0.3)) is marked in Fig. 3(b). The plot clearly shows the difference in the size and shape of the plastic zone for the two cases. In the case of the elastic-plastic regime the plastic zone is completely confined by the surrounding material, whereas in the case of the fully-plastic regime the zone has broken out onto the free surface and has spread radially further. The details of the minimum zone size for fully plastic indentation regime will be discussed in detail in section 3.2.3.

*3.2.2. Plastic work fraction*

In this section, the transition from elastic-plastic indentation regime to fully plastic indentation regime will be presented from a simple analysis. As discussed in the previous section, much of Tabor’s analysis is only applicable to fully plastic indentation regime and it is expected that this transition happens when the plastic work contribution to the total energy rises above a certain threshold. In order to test this hypothesis, the plastic work fraction, which is the ratio of the plastic work to the total work for the different model materials and indentation sizes presented in the previous section needs to be determined.

Fig. 5 shows a schematic of the elastic-plastic and elastic zones under a spherical indent. During the process of indentation, elastic as well as plastic deformation occurs in the material that results in a nearly hemispherical elastic-plastic zone which is commonly referred to as plastic zone and a surrounding elastic region. Note that the elastic effects in the plastic zone is usually insignificant, especially for materials that have a high value of *E*/*Y* such as metals. The total work done for creating an indent is the sum of three different energies induced below the indent and can be written in the following form,

(9)

where, is the total work done to create an indent, is the plastic work in the plastic zone, is the elastic strain energy in the plastic zone and is the elastic strain energy in the elastic zone. The plastic work fraction can then be calculated from the ratio of plastic work to total work (*Epl* /*Wind*).

The plastic work fraction calculated as detailed above is plotted as a function of the two non-dimensional parameters (*E*/*Y*)(*a*/*R*) and (*E*/*H*)(*a*/*R*) in Fig. 6(a) and 6(b) respectively. Similar to the case of constraint factor discussed in the previous section, the plastic fraction shows a steep increase initially in the elastic-plastic regime followed by a gradual increase in the fully plastic regime, with both the non-dimensional parameters. This is to be expected as the contribution of the plastic work is expected to be significant in the fully plastic indentation regime. Interestingly, the plastic fraction for all the strain hardening exponents converge on to a single master curve when plotted against (*E*/*H*)(*a*/*R*) as shown in Fig. 6(b), clearly demonstrating the scaling relationship for the plastic work fraction. The transition to fully plastic indentation regime determined from the constraint factor occurs at a (*E*/*H*)(*a*/*R*) value of 20, which is shown by a dashed vertical line in Fig. 6(b). The corresponding value of plastic work fraction for transition to fully plastic indentation regime is 0.78.

*3.2.3. Plastic zone size*

In this section, the minimum plastic zone size required to enter into fully plastic indentation regime will be presented. As discussed earlier, from the onset of yielding, the contact transitions from elastic-plastic regime to fully plastic regime, during which the plastic zone beneath the indent increases in size. As shown the Fig. 4, the plastic zone is significantly confined within the elastic zone in the elastic-plastic regime, whereas, it breaks out onto the free surface and spreads considerably in the fully plastic-regime. The boundary of the plastic zone is obtained from yield state of the elements in the FEA model, i.e., all the elements that have met the yielding criteria. As the plastic zone is roughly hemispherical, the radial extent of the plastic zone is sufficient to describe the zone and in the present work, it is taken along the free surface, i.e., the distance from the center of the indent to the zone boundary along the free surface. In order to compare the plastic zone size for different test cases, the zone size is normalized with the contact radius.

Fig. 7 shows the normalized plastic zone size as a function of the non-dimensional parameter (*E*/*H*)(*a*/*R*) for different strain hardening exponents. Except for the data points close to 0, where the elastic effects are significant, the normalized plastic zone increases from an initial value of 1 to around 7 depending on the material property which is consistent with prior studies [14]. The transition to fully plastic indentation regime determined from constraint factor is shown by the dashed vertical line. The normalized zone size is independent of the strain hardening exponent in the elastic-plastic regime and diverges in the fully plastic regime depending on the strain hardening exponent. In the absence of strain hardening (*m* = 0), the normalized zone size in the fully plastic regime remains constant around a value of 2 which is consistent with the predictions of the classical expanding cavity model that was developed for perfectly plastic materials (*m* = 0). With increasing strain hardening exponent, the zone extends further [2,14] due to the hardening capacity of the materials and results in the normalized zone size increasing with (*E*/*H*)(*a*/*R*) as shown in Fig. 7. The normalized zone size corresponding to the transition to fully plastic regime determined from constraint factor is 2.2 except for the case of *m* = 0 where the transition occurs at 2.

In summary, given that Tabor’s analysis applies to fully plastic indentation regime, the criteria for achieving the same has been explored based on different variables such as constraint factor, plastic work fraction and plastic zone size. As all these variables are found to generally scale with (*E*/*H*)(*a*/*R*), a minimum value of 20 for this parameter is sufficient to enter into fully plastic indentation regime. The corresponding values of different variables at the onset of this transition is summarized in Table. 3.

*3.3. Tabor coefficient in the fully plastic indentation regime*

As the criteria to enter into fully plastic indentation regime was established in the previous section, the Tabor coefficient (*q*) can now be calculated for the different test cases using the following equation which is obtained by rearranging the terms in Eq. (7).

(10)

As the Tabor coefficient is only valid in the fully plastic indentation regime, the above equation is applied for the test cases that meet that criterion ((*E*/*H*)(*a*/*R*) > 20. In all, out of the 118 test cases, 67 cases meet the fully plastic criteria and the Tabor coefficient calculated for these cases is plotted in Fig. 8. From the figure, it is clear that the Tabor coefficient is fairly constant with (*E*/*H*)(*a*/*R*) and independent of the strain hardening exponent. The average value of the Tabor coefficient is found to be 0.235 ± 0.035 which is close to the value suggested by Tabor [1]. In order to further explore the concept of a constant Tabor coefficient in the fully plastic regime, strain calculated from the uniaxial constitutive relationship shown in Eq. (7) is plotted against *a*/*R* for the different test cases in the fully plastic regime in Fig. 9. A trend line with a slope of 0.235, which is the average value of Tabor coefficient in the fully plastic regime, is also shown in the plot. It can be observed that all the data fall on a fairly linear trend indicating that the indentation parameter *a*/*R* scales with uniaxial strain and the strain conversion factor in the fully plastic regime is 0.235.

*3.4. Uniaxial stress strain response from spherical indentation*

Based on the analysis presented in this work, uniaxial stress-strain response can be determined from spherical indentation. The uniaxial equivalent stress is obtained by dividing the hardness by 2.87 (average constraint factor) and the uniaxial equivalent strain is obtained by multiplying *a*/*R* with 0.235 (average Tabor coefficient). Uniaxial stress strain data was obtained by performing uniaxial compression tests in Universal Testing Machine (INSTRON, 5500R, UK) at an initial displacement rate of 0.2 mm/min. Figure 10 shows a comparison of the experimentally determined uniaxial stress-strain curve as described above with that calculated from indentation. Excellent agreement can be found in the stress-strain response obtained by uniaxial testing and the indentation testing. This clearly demonstrates the validity of the Tabor coefficient determined in the present work.

**4. Summary and conclusions**

Spherical indentation is an attractive method to obtain equivalent uniaxial stress-strain response from indentation-based technique. In order to obtain the equivalent uniaxial stress and strain from spherical indentation, the hardness and indentation strain *a*/*R* are multiplied by the respective conversion factors. Traditionally constraint factor and Tabor coefficient have been used as the stress and strain conversion factors, respectively. However, they are not independent of each other. In this work, a simple FEA based model is presented to determine the constraint factor and the Tabor coefficient independently in the fully plastic regime. Fully plastic indentation regime was found to occur for (*E*/*H*)(*a*/*R*) > 20 which also corresponds to a constraint factor > 2.8, plastic work fraction of 0.78 and normalized plastic zone size of 2.2. In the fully plastic regime the average Tabor coefficient was found to be 0.235 and average constraint factor is 2.87. The model predictions have been experimentally validated on OFHC copper wherein the equivalent uniaxial response determined from indentation matches well with the traditional uniaxial testing.

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Tables and figures:

Table 1 Summary of constraint factor and Tabor coefficient obtained using spherical indentation tests.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Investigator  [Ref No] | Type of Work | Material | Constraint factor (*C*) | Tabor coefficient (*q*) |
| Tabor [1] | Experimental | Perfectly Plastic | 2.6 to 3.0 | 0.2 |
| Johnson [2] | Theoretical | Rigid perfectly plastic | 2.8 | 0.2 |
| Richmond et.al. [3] | Theoretical | Rigid perfectly plastic | 3.0 | 0.32 |
| Herbert et.al. [4] | Experimental | Al alloy | 3.7 | 0.2 |
| Tirupataiah et.al. [5] | Experimental | Iron, steel, Cu and Al alloys | 2.4 to 3.05 | 0.2 |
| Ahn et.al. [6] | Experimental | Various types steels | 3 | 0.1  (for small strains) |
| Xu et.al. [7] | Experimental | Steel and  Al alloy | 4  3.9 | 0.11  0.12  (for small strains) |
| Francis [8] | Experimental |  | 2.87 | 0.215  (for small strains) |
| Jeon et.al. [9] | Simulation | Various types steels | 3 | 0.14  (for small strains) |
| Surya et.al. [10,11] | Experimental and simulation | Tungsten and Aluminum | 2.86 and 2.65 | 0.21  (for small strains) |
| Chang et.al. [12] | Experimental | Carbon steel, Brass and Al alloy | 3.2 | 0.2 |
| Lee et.al. [13] | Experimental and FEM | Heat treated steel | 2.8 | 0.2 to 0.12 |

Table 2 Material parameters for typical metals and alloys as per Eqns. 1 and 2.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Material | *E* (MPa) | *Y* (MPa) | *B* (MPa) | *m* | *K* (MPa) | *n* |
| Steel 3V | 210000 | 1540 | 757 | 0.274 | 2213 | 0.056 |
| Steel 1V | 210000 | 525 | 721 | 0.335 | 1137 | 0.12 |
| Armco iron | 211000 | 114 | 522 | 0.43 | 568 | 0.26 |
| Nickel | 200000 | 110 | 804 | 0.63 | 737 | 0.39 |
| OFHC Cu | 117000 | 31.7 | 561.4 | 0.6995 | 503.1 | 0.538 |

Table 3 Criteria for fully developed plastic flow.

|  |  |
| --- | --- |
| Parameter | Criteria |
| (*E*/*H*)(*a*/*R*) | ≥ 20 |
| Constraint factor | ≥ 2.8 |
| Plastic work fraction | ≥ 0.78 |
| Normalized plastic zone size | ≥ 2.2 for *m* > 0  ≥ 2.0 for *m* = 0 |

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Fig. 1. Geometry and mesh of the FE model.

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Fig. 2. Comparison of experimental results and finite element model predictions for hardness as a function of a/R.

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Fig. 3. Variation in constraint factor with the dimensionless parameter (a) (*E/Y*)(*a/R*)and(b) (*E/H*)(*a/R*)for materials with different strain hardening exponents.

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Fig. 4. Plastic deformation zone size for (I) elastic plastic regime (II) fully plastic regime.

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Fig. 5. Schematic illustration of elastic and elastic-plastic zones under a spherical indent.

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Fig. 6. Variation in plastic work fraction with the non-dimensional parameters (a) (*E*/*Y*)(*a*/*R*) and (b) (*E*/*H*)(*a*/*R*) for materials with different strain hardening exponents.

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Fig. 7. Variation in normalized plastic zone with (*E*/*H*)(*a*/*R*)for materials with different strain hardening exponents.

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Fig. 8. Tabor coefficient as a function of dimensionless parameter (*E*/*H*)(*a*/*R*)in the fully plastic indentation regime.

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Fig. 9. Uniaxial strain as a function of *a/R* in the fully plastic indentation regime.

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Fig. 10. Comparison of stress strain curve obtained by uniaxial compression and indentation.