

# Study of motion of the drops on inclined plane

D Vamshidhar Reddy

A Thesis Submitted to  
Indian Institute of Technology Hyderabad  
In Partial Fulfillment of the Requirements for  
The Degree of Master of Technology



Department of Chemical Engineering

June 2018

## Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

D. Vamshidhar Reddy  
(Signature)

D VAMSHIDHAR REDDY  
( D Vamshidhar Reddy)

CH16MTECH11006  
(Roll No.)

## Approval Sheet

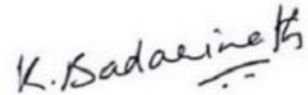
This Thesis entitled **Study of motion of drops on inclined plane** by **D Vamshidhar Reddy** is approved for the degree of **Master of Technology** from IIT Hyderabad



(Dr. Vinod Janardhanan) Examiner  
Dept. of Chem Eng  
IITH



(Dr. Kirti Chandra Sahu) Adviser  
Dept. of Chem Eng  
IITH



(Dr. Badarinath Karri) Co-Adviser  
Dept. of Mech Eng  
IITH



(Dr. B. Saravanan) Chairman  
Dept. of Mech Eng  
IITH

## Acknowledgements

First and mostly I would like to thank and express my gratitude to my advisor Prof.Kirti Chandra Sahu who supported me through out my M.Tech study. Secondly I would like to thank my co-advisor Dr.Badarinath Karri for his support and guidance through out my M.Tech study. Both their guidance helped me lot in research and writing of this thesis.

I would like to thank my lab mates Y.S.Kannan for helping me in experimental study and A.R.Premlatha and B.Mounika for helping me in analyzing results.

Finally I am thankful to IIT Hyderabad for providing the infrastructure and facilities.

## Dedication

"I would like to dedicate my work to parents for supporting me through out my life".

## Abstract

We conducted experiments for water drops and glycerol drops sliding down on inclined surface. The shapes of the water drop of same size for different inclinations are investigated. When water drop is stationary on inclined plane, it exhibit oval shape and when the drop starts to move, its shape changes to corner. This transition can be explained by front and rear curvatures. As velocity of the drop increases, its shape changes to rivulet and at higher velocities, the drops assume tail shape.

Experiments for four different sized water drops are conducted. We reported experimentally obtained critical angle of inclination, which is defined as angle of inclination at which a drop starts to slide. We calculated theoretical critical angle of inclination and compared them with experimental values. The values obtained are almost same and therefore, the results are satisfactory. The velocities for these four different sized drops are calculated and plotted against different inclination angles. We also reported height, length and width of the drops for different inclinations. We found scaling law that predict size, velocity of the drops for given physical properties like viscosity, surface tension and density of the drops.

We reported shapes of glycerol drop of same size for different inclination angles and we found that the shapes of water drops and glycerol drops are similar. We reported experimentally obtained critical angle of inclination. We calculated theoretical critical angle of inclination and compared them with experimental values. The values obtained are almost same and therefore, the results are satisfactory. We observed that critical angle of inclination glycerol drops is less than water drops of similar size. This phenomenon is observed due to glycerol drop has less surface tension, which results in less capillary forces. These low capillary forces are responsible for low critical angle for glycerol drops.

At low inclinations, the glycerol drop has high velocities than water drops due to lower capillary forces. As inclination increases, the water drop has higher velocities due to low viscous forces.

# Contents

Declaration . . . . .	ii
Approval Sheet . . . . .	iii
Acknowledgements . . . . .	iv
Abstract . . . . .	vi
<b>1 Introduction</b>	<b>1</b>
<b>2 Experimental setup</b>	<b>3</b>
<b>3 Results and discussion</b>	<b>5</b>
3.1 Experimental results for water drops . . . . .	5
3.1.1 Shape of water drops . . . . .	5
3.1.2 Critical angle of inclination . . . . .	7
3.1.3 Velocity profiles of water drops . . . . .	8
3.1.4 Aspect ratio of water drops . . . . .	9
3.2 Experimental results for glycerol drops . . . . .	10
3.2.1 Shape of glycerol drops . . . . .	10
3.2.2 Critical angle of inclination . . . . .	11
3.2.3 Velocity profiles of glycerol drops . . . . .	13
3.2.4 Aspect ratio of the drops . . . . .	14
3.3 Comparison of critical angle of inclination, velocity and shapes of water and glycerol drops . . . . .	15
<b>4 Conclusions</b>	<b>17</b>
<b>References</b>	<b>18</b>

# List of Figures

2.1	Experimental setup for study of motion of the drops on inclined plane . . . . .	3
3.1	Different shapes of water drops of volume $52.26 \text{ mm}^3$ . . . . .	6
3.2	Different shapes of water drops of volume $62.25 \text{ mm}^3$ . . . . .	6
3.3	Critical angle of inclination of water drop of different sizes . . . . .	7
3.4	Velocity profiles for different sized drops . . . . .	8
3.5	Capillary number versus inclination angle for different sized drops . . . . .	9
3.6	Variation of aspect ratios for different size drops . . . . .	9
3.7	Different shapes of glycerol drops of volume $52.26 \text{ mm}^3$ . . . . .	10
3.8	Different shapes of glycerol drops of volume $62.25 \text{ mm}^3$ . . . . .	11
3.9	Critical angle of inclination of glycerol drop of different sizes . . . . .	12
3.10	Velocity profiles for different sized glycerol drops . . . . .	13
3.11	Capillary number versus inclination angle for different sized drops . . . . .	14
3.12	Variation of aspect ratios for different size drops . . . . .	14
3.13	Variation of aspect ratios for different size drops . . . . .	15



# Chapter 1

## Introduction

The study of motion of the drops has major applications in coating techniques, self cleaning hydrophobic droplets, cooling hot spots in electric circuits and controlled micro reactions. In coating techniques like insecticide sprays and paints, the wetting of surface is required which can be achieved by motion of the drops. Self cleaning hydrophobic droplets do not wet the surface and they roll down the surface by cleaning particles on the surface. Unreachable places in electric circuits can be cooled by sliding of the drops.

On inclined surface, when drop is placed, we can observe three types of motion. When the drop is placed on inclined surface at low angles, it remains stationary. When the inclination angle increases, the drop starts to slide. When the surface is hydrophobic, the drop will roll down the inclined plane.

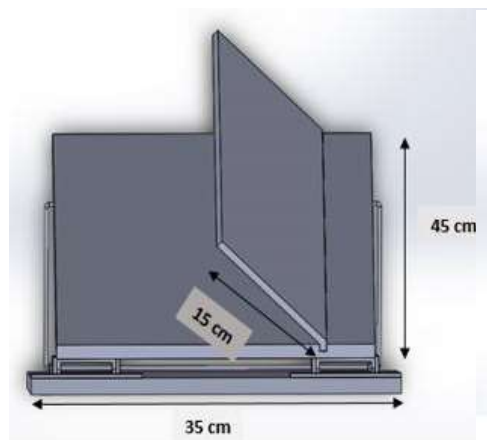
The studies that are available mostly focus on stationary drops and critical angle of inclination. The critical angle of inclination is defined as angle of inclination at which a drop starts to slide. The experimental results of various drops are reported. These critical angles depends on advancing contact angle and receding contact angle. These angles are also reported [1, 2, 3, 4].

Recently the studies of the drops sliding down an inclined plane has drawn lot of attention and experiments for various fluids are conducted. They reported sliding velocity of the drops on inclined surface. They constructed scaling law used to predict the velocity for different parameters like inclination, viscosity, surface tension and density [5]. During sliding on inclined surface the silicone drops adopt different shapes for different capillary numbers. Capillary number is defined as ratio of viscous forces to surface tension forces. At low capillary numbers, the drops assume oval shape. When the velocity is increased, the shape changes to corner. As capillary number is further increased, drop shape changes to cusp and at higher velocities, the drops assume pearl shape. They reported advancing contact angles and receding contact angles. They compared these contact angles with Gennes wetting model and Molecular kinetic model of wetting and found out that results were satisfactory [6, 7]. Experiments for water and mercury drops are conducted and derived general expression for variation of dimensionless velocity with inclination. They reported shapes of water and mercury with change in inclination and compared both shapes [8]. Experiments for water and xanthan solution drops which is non-newtonian fluid are conducted and compared experimental results of water and non-newtonian fluid. They found linear variation of capillary number of water drop with inclination and non-linear variation of capillary number of xanthan solution drop with inclination [9].

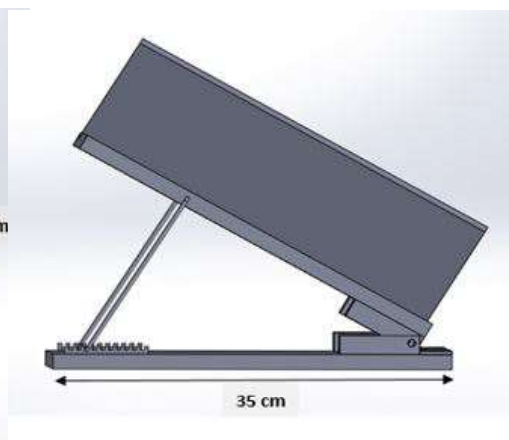
Till now experiments are performed for several fluids but there is no detailed data for water drops and different sized drops. So we conducted experiments for water drops and glycerol drops and compared both results.

## Chapter 2

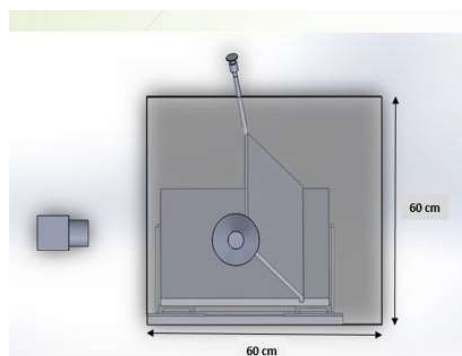
# Experimental setup



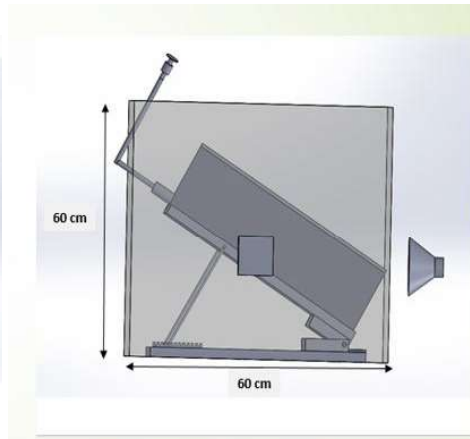
(a) Front view



(b) Side view



(c) Front view



(d) Side view

Figure 2.1: Experimental setup for study of motion of the drops on inclined plane

For studying motion of the drops, the experimental setup shown in Figure 2.1 is designed and fabricated. An acrylic plate of dimensions  $45 \times 35$  cm was fixed to another plate of same dimensions using rod and joints, so that different angles can be produced as shown in Figure 2.1.b. On the upper plate mirror of dimensions  $45 \times 15$  cm was fixed at  $45^\circ$  as shown in Figure 2.1.a. This setup was placed in closed acrylic box of dimensions  $60 \times 60$  cm as shown in Figure 2.1.c. To upper plate a small hole was induced so that capillary tubes are placed on inclined plate and other side of capillary tube syringe is fixed. When drop is injected into capillary tube it falls directly on inclined plane.

High speed camera fixed on left side to setup as shown in figure.2.1. Two light sources are used, one placed front of setup and another is placed at bottom. Side view of drop is directly recorded in the camera and top view can recorded from mirror placed at  $45^\circ$ . Videos obtained are used for calculations of velocities and aspect ratios. Shape of image are processed in Adobe photoshop.

Experiments are conducted for two different fluids that are water and glycerol and for four different sizes which are  $62.25 \text{ mm}^3$ ,  $52.26 \text{ mm}^3$ ,  $43.35 \text{ mm}^3$  and  $34.48 \text{ mm}^3$

# Chapter 3

## Results and discussion

Experiments have been conducted for water and glycerol for four different size drops and following results were produced. Shape of water drops for different inclinations and sizes. Experimental and theoretical values of critical inclination of different size water drops. Velocity profiles of water drops of different sizes. Aspect ratios of water drops for different inclinations. Shape of glycerol drops for different inclinations and sizes. Experimental and theoretical values of critical inclination of different size glycerol drops. Velocity profiles of glycerol drops of different sizes. Aspect ratios of glycerol drops for different inclinations. Comparison of velocity profiles and shapes of water drops and glycerol drops

### 3.1 Experimental results for water drops

#### 3.1.1 Shape of water drops

Water drops assume different shapes at different speeds which can be seen from the top view. Different shapes of the water drop for different velocities and sizes are shown in Figure 3.1. For different size and inclinations the side and top views of water are shown in Figure 3.1.

The four different shapes identified are shown as follows

- Oval shape
- corner shape
- rivulet shape
- tail shape

Oval shape is observed for water drops which are in stationary condition and when velocities are increased, its shape changes to corner. This transition from oval to corner are found from the advancing and receding curvatures. The advancing and receding curvatures are defined as the inverse of the radius of the contact line in top views. For a stationary or low speed drop the rear and front curvatures are equal. When velocity increases this curvature at rear increases, which is reason for transition from oval to corner shape. As velocity is further increased, the rear curvature of the drop increases until a rivulet shape is obtained. At even higher velocities water drops forms tail shape

and as velocities increases size of the tail increases and small droplets are left behind as shown in Figure 3.1.

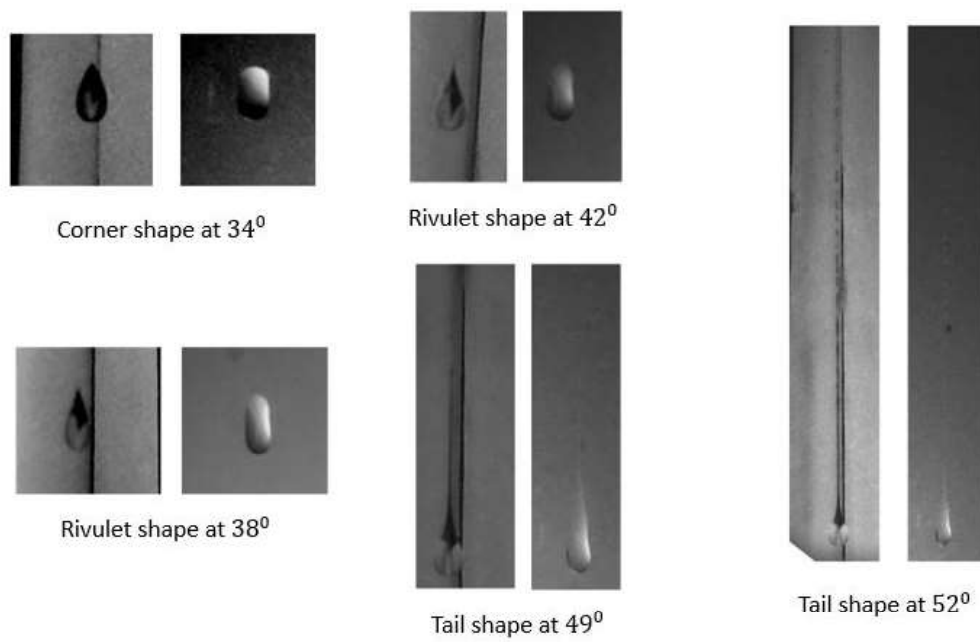


Figure 3.1: Different shapes of water drops of volume  $52.26 \text{ mm}^3$

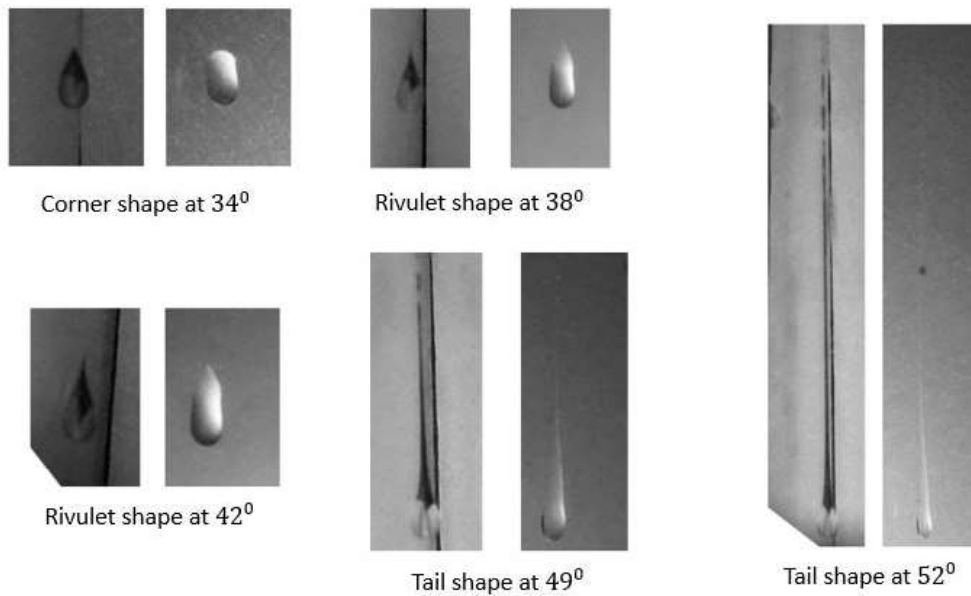


Figure 3.2: Different shapes of water drops of volume  $62.25 \text{ mm}^3$

### 3.1.2 Critical angle of inclination

At low inclinations, the drops are found stationary and they start sliding down the inclined plane above a certain angle which is known as critical angle of inclination. In this work we reported experimental critical angle and calculated theoretical critical angle. Both experimental and theoretical critical angle of inclinations for water drops of different sizes are shown in Figure 3.3.

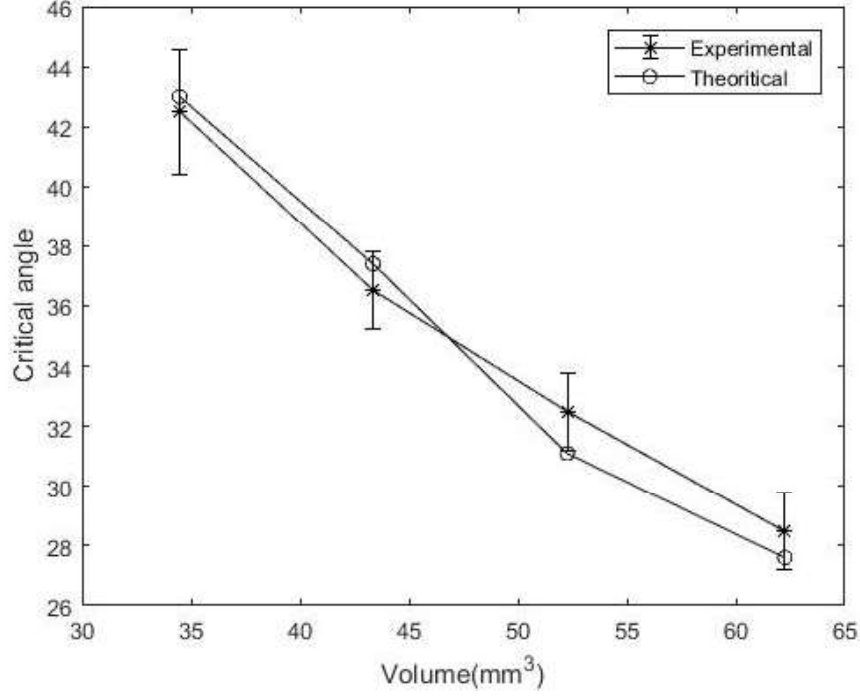


Figure 3.3: Critical angle of inclination of water drop of different sizes

Theoretical value of critical angle of inclination are calculated from the equation

$$\rho v g \sin(\alpha_c) = \sigma w c_1 (\cos(\theta_r) - \cos(\theta_a)) \quad (3.1)$$

where,  $\rho v g \sin(\alpha_c)$  - gravitational force  
 $\sigma w c_1 (\cos(\theta_r) - \cos(\theta_a))$  - capillary force  
 $\rho$  - Density of water drop  
 $v$  - volume of water drop  
 $g$  - Acceleration due to gravity  
 $(\alpha_c)$  - Critical angle of inclination  
 $\sigma$  - Surface tension between drop and air  
 $w$  - Width of water drop  
 $\theta_r$  - Receding contact angle  
 $\theta_a$  - Advancing contact angle

At critical angle of inclination, the capillary force is balanced by gravitational force. From the Figure 3.3, for the drop of size 62.25 mm<sup>3</sup>, the theoretical value of inclination is 27.6 and experimental

has a range of  $26^{\circ}$  to  $30^{\circ}$ . For the drop of size  $52.26 \text{ mm}^3$ , the theoretical value of inclination is  $31.1$  and experimental has a range of  $30^{\circ}$  to  $34^{\circ}$ . For the drop of size  $43.35 \text{ mm}^3$ , the theoretical value of inclination is  $37.4$  and experimental has a range of  $34^{\circ}$  to  $38^{\circ}$ . For the drop of size  $34.48 \text{ mm}^3$ , the theoretical value of inclination is  $43$  and experimental has a range of  $40^{\circ}$  to  $45^{\circ}$ . From the Figure 3.3, we observed that experimental and theoretical critical angle of inclinations are almost equal.

### 3.1.3 Velocity profiles of water drops

Displacement of the drop in subsequent frames is used for calculation of velocity of drops. The velocities that are calculated for water drops are shown in Figure 3.4. The variation of velocities at for different inclinations are shown by error bars shown in Figure 3.4.

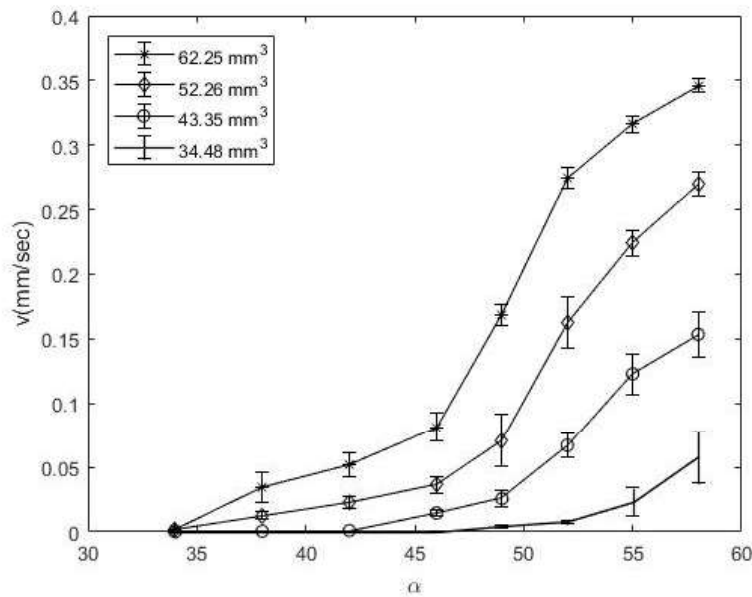


Figure 3.4: Velocity profiles for different sized drops

From the Figure 3.4, for the drops of  $62.25 \text{ mm}^3$ , for the inclination of range from  $34^{\circ}$  to  $55^{\circ}$ , the velocities of water drops are in the range of 0 to 0.34 mm/s. For the drops of  $52.26 \text{ mm}^3$ , for the inclination of range from  $34^{\circ}$  to  $55^{\circ}$ , the velocities of water drops are in the range of 0 to 0.27 mm/s. For the drops of  $43.35 \text{ mm}^3$ , for the inclination of range from  $34^{\circ}$  to  $55^{\circ}$ , the velocities of water drops are in the range of 0 to 0.15 mm/s. For the drops of  $34.48 \text{ mm}^3$ , for the inclination of range from  $34^{\circ}$  to  $55^{\circ}$ , the velocities of water drops are in the range of 0 to 0.08 mm/s. From the Figure 3.4 we can analyze that, the velocity of water drop increases with increase in both inclination and size of drops.

Capillary number versus angle of inclination is plotted as shown in Figure 3.5, to show effect of viscosity and angle of inclination on velocity. We observed that, the capillary number increases with increase in inclination angle. This scaling is done to predict the velocity of other fluids of same surface tension and different viscosities as that of water.



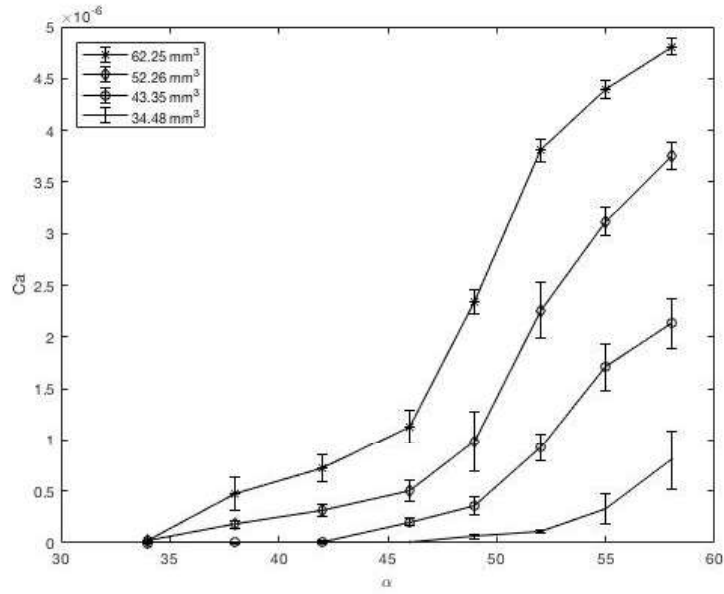


Figure 3.5: Capillary number versus inclination angle for different sized drops

### 3.1.4 Aspect ratio of water drops

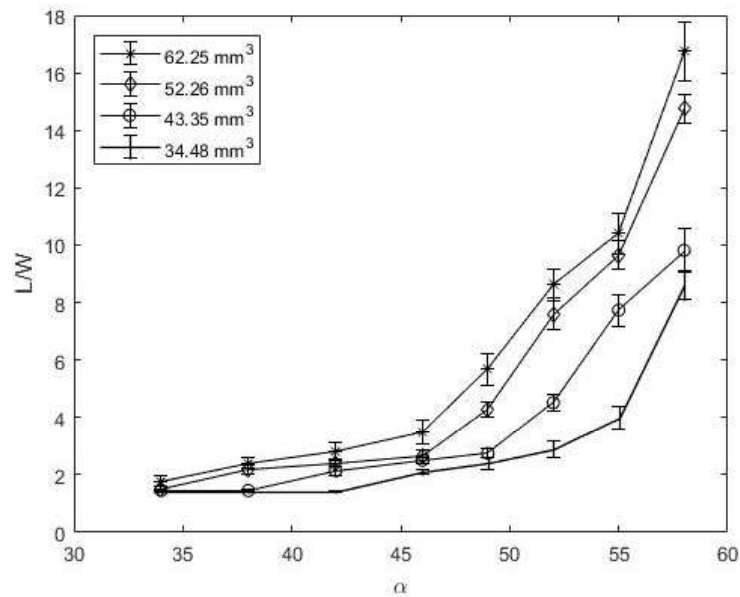


Figure 3.6: Variation of aspect ratios for different size drops

Error bars of aspect ratios of water drops for different inclinations are shown in Figure 3.5. For different inclinations and different sizes the maximum width, height and length are calculated. From

data obtained, we observe that length gradually increases and when tail shape appears, the height increases at higher rates, width of the drop decreases as inclination increases and height remains almost same.

## 3.2 Experimental results for glycerol drops

### 3.2.1 Shape of glycerol drops

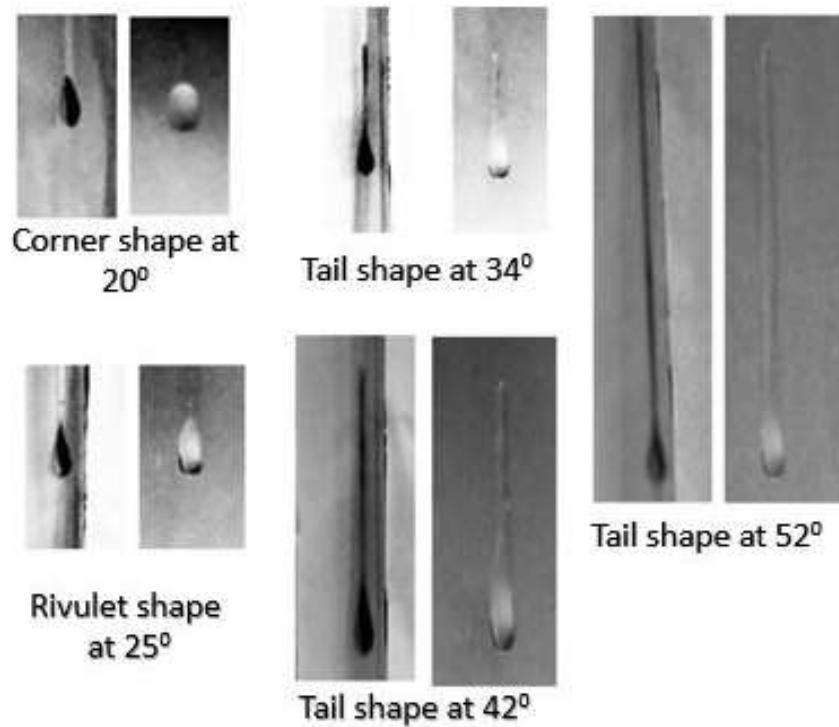


Figure 3.7: Different shapes of glycerol drops of volume  $52.26 \text{ mm}^3$

Glycerol drops assume different shapes at different speeds which can be seen from the top view. Different shapes of the glycerol drop for different velocities and sizes are shown in Figure 3.7. For different size and inclinations the side and top views of glycerol are shown in Figure 3.7.

The four different shapes identified are shown as follows

- Oval shape
- corner shape
- rivulet shape

- tail shape

Oval shape is observed for glycerol drops which are in stationary condition and when velocities are increased, its shape changes to corner. This transition from oval to corner are found from the advancing and receding curvatures. The advancing and receding curvatures are defined as the inverse of the radius of the contact line in top views. For a stationary or low speed drop the rear and front curvatures are equal. When velocity increases this curvature at rear increases, which is reason for transition from oval to corner shape. As velocity is further increased, the rear curvature of the drop increases until a rivulet shape is obtained. At even higher velocities glycerol drops forms tail shape and as velocities increases size of the tail increases and small droplets are left behind as shown in Figure 3.7.

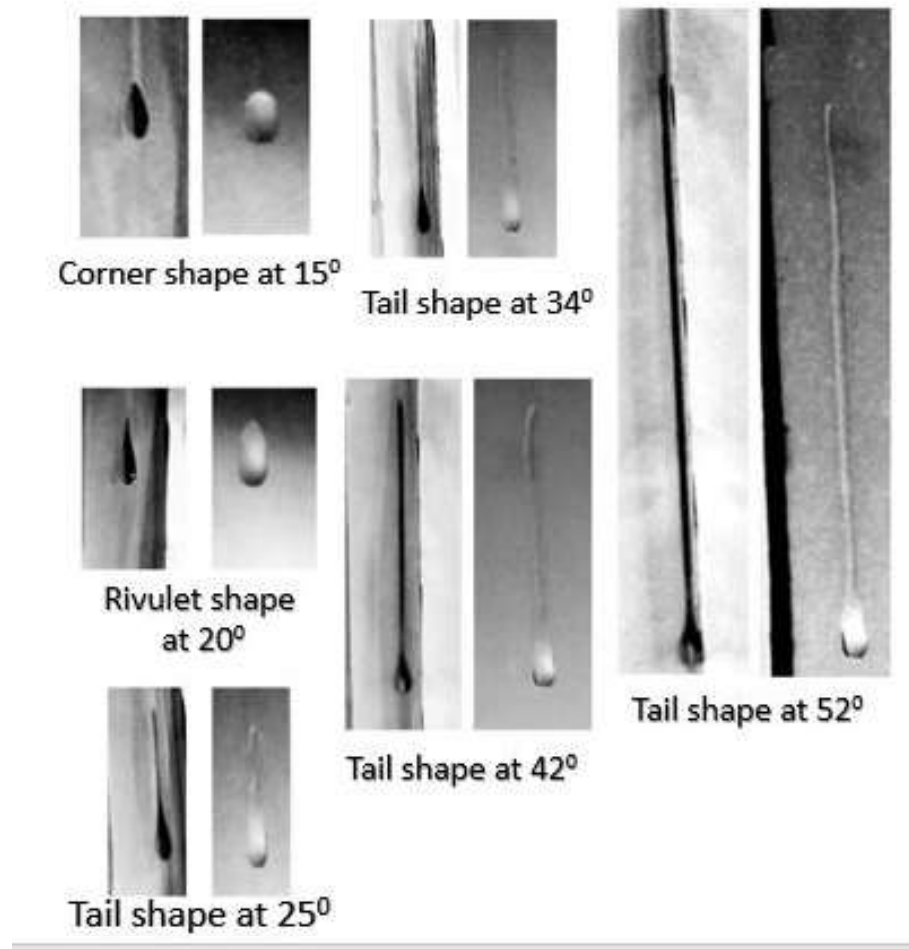


Figure 3.8: Different shapes of glycerol drops of volume  $62.25 \text{ mm}^3$

### 3.2.2 Critical angle of inclination

At low inclinations, the drops are found stationary and they start sliding down the inclined plane above a certain angle which is known as critical angle of inclination. In this work we reported ex-

perimental critical angle and calculated theoretical critical angle. Both experimental and theoretical critical angle of inclinations for glycerol drops of different sizes are shown in Figure 3.9.

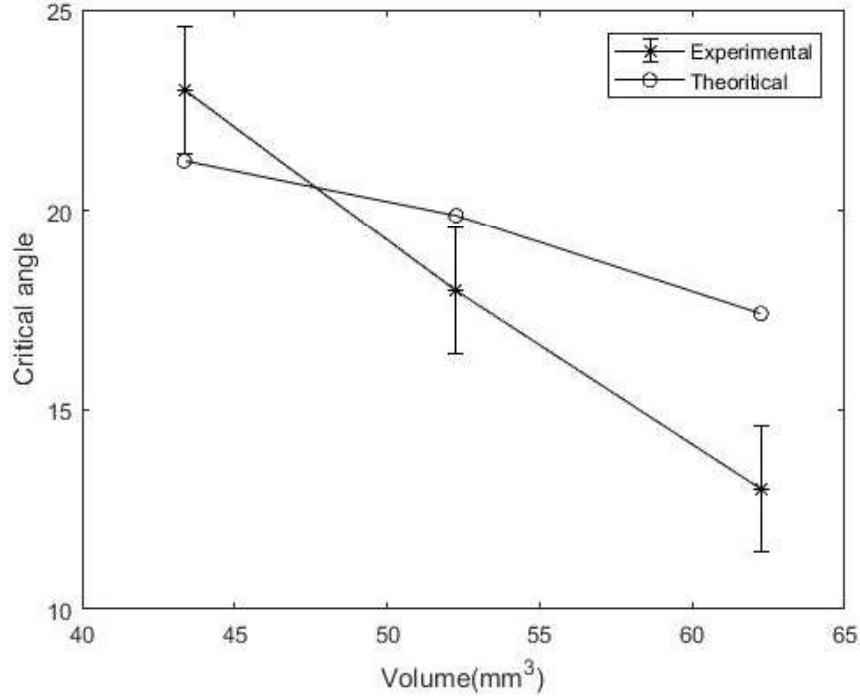


Figure 3.9: Critical angle of inclination of glycerol drop of different sizes

Theoretical value of critical angle of inclination are calculated from the equation

$$\rho v g \sin(\alpha_c) = \sigma w c_1 (\cos(\theta_r) - \cos(\theta_a)) \quad (3.2)$$

where,  $\rho v g \sin(\alpha_c)$  - gravitational force

$\sigma w c_1 (\cos(\theta_r) - \cos(\theta_a))$  - capillary force

$\rho$  - Density of glycerol drop

$v$  - volume of glycerol drop

$g$  - Acceleration due to gravity

$(\alpha_c)$  - Critical angle of inclination

$\sigma$  - Surface tension between drop and air

$w$  - Width of glycerol drop

$\theta_r$  - Receding contact angle

$\theta_a$  - Advancing contact angle

At critical angle of inclination, the capillary force is balanced by gravitational force. From the Figure 3.3, for the drop of size  $62.25 \text{ mm}^3$ , the theoretical value of inclination is  $17.4^\circ$  and experimental has a range of  $10^\circ$  to  $15^\circ$ . For the drop of size  $52.26 \text{ mm}^3$ , the theoretical value of inclination is  $19.87^\circ$  and experimental has a range of  $15^\circ$  to  $0^\circ$ . For the drop of size  $43.35 \text{ mm}^3$ , the theoretical value of

inclination is  $21.24^\circ$  and experimental has a range of  $20^\circ$  to  $25^\circ$ . From the Figure 3.3, we observed that experimental and theoretical critical angle of inclinations are almost equal.

### 3.2.3 Velocity profiles of glycerol drops

Displacement of the drop in subsequent frames is used for calculation of velocity of drops. The velocities that are calculated for glycerol drops are shown in Figure 3.10. The variation of velocities at for different inclinations are shown by error bars shown in Figure 3.10.

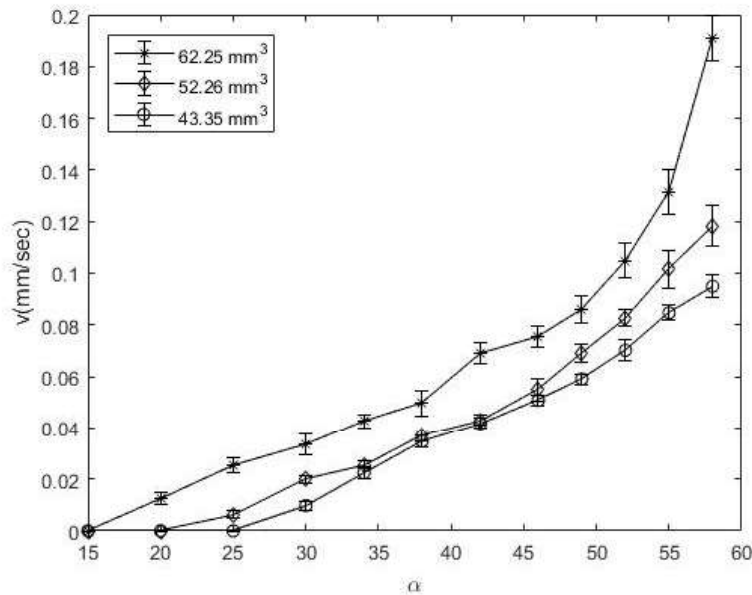


Figure 3.10: Velocity profiles for different sized glycerol drops

From the Figure 3.10, for the drops of  $62.25 \text{ mm}^3$ , for the inclination of range from  $15^\circ$  to  $55^\circ$ , the velocities of glycerol drops are in the range of 0 to 0.2 mm/s. For the drops of  $52.26 \text{ mm}^3$ , for the inclination of range from  $15^\circ$  to  $55^\circ$ , the velocities of glycerol drops are in the range of 0 to 0.12mm/s. For the drops of  $43.35 \text{ mm}^3$ , for the inclination of range from  $15^\circ$  to  $55^\circ$ , the velocities of glycerol drops are in the range of 0 to 0.1mm/s. From the Figure 3.10 we can analyze that, the velocity of glycerol drop increases with increase in both inclination and size of drops.

Capillary number versus angle of inclination is plotted as shown in Figure 3.11, to show effect of viscosity and angle of inclination on velocity. We observed that, the capillary number increases with increase in inclination angle. This scaling is done to predict the velocity of other fluids of same surface tension and different viscosities as that of glycerol.

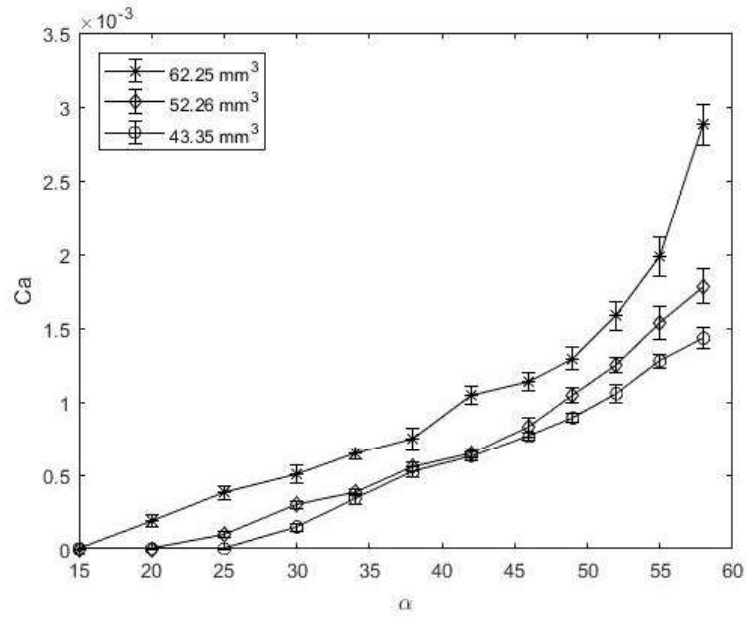


Figure 3.11: Capillary number versus inclination angle for different sized drops

### 3.2.4 Aspect ratio of the drops

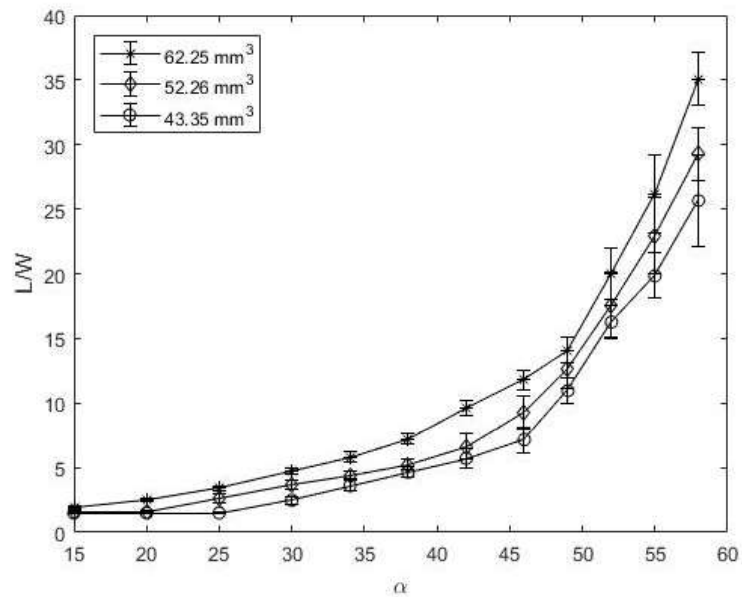


Figure 3.12: Variation of aspect ratios for different size drops

Error bars of aspect ratios of glycerol drops for different inclinations are shown in Figure 3.12. For different inclinations and different sizes the maximum width, height and length are calculated. From

data obtained, we observe that length gradually increases and when tail shape appears, the height increases at higher rates, width of the drop decreases as inclination increases and height remains almost same.

### 3.3 Comparison of critical angle of inclination, velocity and shapes of water and glycerol drops

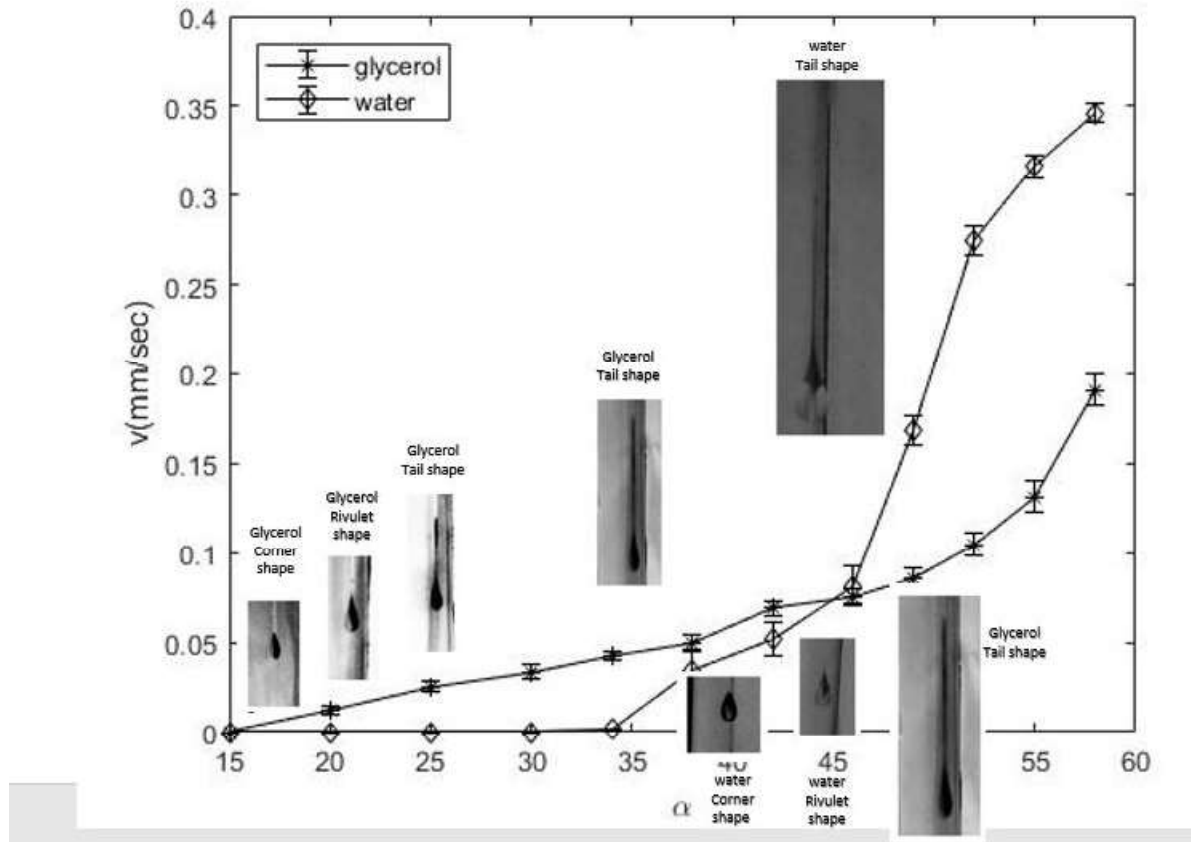


Figure 3.13: Variation of aspect ratios for different size drops

The shapes of water and glycerol drops are almost same as shown in Figure 3.1 and Figure 3.7, but there is slight variation that tail shape drops are formed for glycerol drops at longer inclinations due to wetting of surface by glycerol drops.

The measured velocities of water and glycerol drops are shown in same plot as shown in Figure 3.13. As inclination increases velocities of both drops increases. At low inclinations, the glycerol drop has high velocities than water drops due to lower capillary forces. As inclination increases, the water drop has higher velocities due to low viscous forces

Critical angle of inclination of both water and glycerol can be seen from Figures 3.3 and 3.9. We observed that critical angle for glycerol is less than water drops of same size. This phenomenon

is observed due to glycerol has less surface tension, which results in less capillary forces. These capillary forces are responsible for low critical angle for glycerol drops.



## Chapter 4

# Conclusions

We have investigated motion of water and glycerol drops on inclined plane. Our results provided clear picture of critical angle of inclination of the drops. We reported experimental and theoretical critical angles and compared them. The results obtained are found to be satisfactory.

There was concern related to drop shape and our results allowed to have brief idea regarding shape of drops. When water drop is stationary on inclined plane, it exhibit oval shape and when the drop starts to move, its shape changes to corner. This transition can be explained by front and rear curvatures. As velocity of the drop increases, its shape changes to rivulet and at higher velocities, the drops assume tail shape.

We also provided experimental data of velocities and aspect ratios of different drops. We developed scaling law so that the velocity, aspect ratio and shape change can be predicted when there is changes in viscosities of fluids and angle of inclinations.

Further experiments can be conducted for different viscosity ratios and non-newtonian fluids which lack data presently.

# References

- [1] J. Bikerman. Sliding of drops from surfaces of different roughnesses. *Journal of Colloid Science* 5, (1950) 349–359.
- [2] C. Furnidge. Studies at phase interfaces. I. The sliding of liquid drops on solid surfaces and a theory for spray retention. *Journal of colloid science* 17, (1962) 309–324.
- [3] R. T.-P. Chow et al. On the ability of drops or bubbles to stick to non-horizontal surfaces of solids. *Journal of Fluid mechanics* 137, (1983) 1–29.
- [4] E. Dussan et al. On the ability of drops or bubbles to stick to non-horizontal surfaces of solids. Part 2. Small drops or bubbles having contact angles of arbitrary size. *Journal of Fluid Mechanics* 151.
- [5] H.-Y. Kim, H. J. Lee, and B. H. Kang. Sliding of liquid drops down an inclined solid surface. *Journal of colloid and interface science* 247, (2002) 372–380.
- [6] T. Podgorski, J.-M. Flesselles, and L. Limat. Corners, cusps, and pearls in running drops. *Physical review letters* 87, (2001) 036,102.
- [7] N. Le Grand, A. Daerr, and L. Limat. Shape and motion of drops sliding down an inclined plane. *Journal of Fluid Mechanics* 541, (2005) 293–315.
- [8] B. A. Puthenveetil, V. K. Senthilkumar, and E. Hopfinger. Motion of drops on inclined surfaces in the inertial regime. *Journal of Fluid Mechanics* 726, (2013) 26–61.
- [9] S. Varagnolo, G. Mistura, M. Pierno, and M. Sbragaglia. Sliding droplets of Xanthan solutions: A joint experimental and numerical study. *The European Physical Journal E* 38, (2015) 126.