Joint Optimization of both m and K for the m-out-of-K Rule for Cooperative Spectrum Sensing

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***Abstract*—In this paper, we present closed form expressions that jointly optimizes the fusion rule (m) and the number of secondary users (K) for the m-out-of-K rule by minimizing the Bayes risk at the fusion center (FC) in the presence of erroneous reporting channels and then show that various existing and new results are special cases of the proposed solution. The results are applicable to any detector used in cooperative spectrum sensing (CSS). Numerical results are presented using energy detector (ED) which shows that CSS obtained using joint optimized values of m and K results in significant performance improvement.**

***Index Terms*—Cognitive radio, Bayes risk function, erroneous reporting channel, number of secondary users.**

I. INTRODUCTION

In cognitive radio (CR) *spectrum sensing* [1]–[3] is a fundamental component for the secondary user (SU) to detect the primary user (PU) signal. However, spectrum sensing using single SU results in poor detection performance due to multi-path and shadowing. To mitigate this problem, *cooperative spectrum sensing* (CSS) [4]–[9] has been proposed, where the observations from multiple SUs are sent over reporting channels to the fusion center (FC), where they are combined to make a final decision on activity of the PU. However, there are various combining schemes at FC such as *soft combining* [10]–[12], *quantized soft combining* [13], *weighted soft combining* [14], [15], *multi selective CSS scheme* [16] and m-out-of-K fusion rule [17]. The m-out-of-K rule detects the PU signal, if at least m out of K SUs detect the PU signal. The detection performance of CSS in the presence of erroneous reporting channels is studied in [18], under the assumption of identical SUs and identical reporting channels.

Optimizing m of the m-out-of-K rule has been presented in literature for various objective functions such as minimizing the Bayes risk [17] over error free reporting channels, *total error rate* (TER) [19] over error-free reporting channels, minimizing the TER [20] over erroneous reporting channels, minimizing the *false decision probability* (FDP) [21] over erroneous reporting channels, minimizing the TER [22] in the absence of reporting channels, maximizing the energy efficiency [23], minimizing the TER of the the multi-hop CR network [24], maximizing the secondary network throughput while satisfying protection constraint to the PU [25] and maximizing the global detection probability subject to a constraint on global false alarm probability [26]. Optimizing K has been studied to minimize the TER of OR rule [27], [28], minimizing the TER for the AND and MAJORITY rule [29], maximizing the average channel throughput of the CR network [30] and minimizing the Bayes risk [31]. In this paper, we formulate a joint optimization problem $(\mathbb{JOP})$ that jointly optimizes both m and K values by minimizing the Bayes risk of the m-out-of-K rule over erroneous reporting channels. However, to the best of our knowledge joint optimization of m and K has not been considered so far. The main contributions of this paper are listed as follows.

- We present analytical expressions for joint optimized values of m and K of the m-out-of-K rule in the presence of erroneous reporting channels. The performance of CSS obtained using joint optimized values of m and K results in significant performance improvement.
- For a given K, the $\mathbb{JOP}$ specializes to find the optimum value of m that minimizes the Bayes risk of the m-out-of-K rule over erroneous reporting channels and then we show that various existing problems [17], [19], [20] are special cases of the proposed problem.
- *Effect of erroneous reporting channels*: For a given K, it is shown that the optimality of a fusion rule is limited by *probability of error* of a reporting channel. It is shown that each rule is optimal for certain values of probability of error of a reporting channel, above which, they are never optimal. It is also observed that there is a significant difference in robustness of these fusion rules to the erroneous reporting channels due to un-equal effective weights assigned to global probability of false alarm and missed detection.

The outline of this paper is as follows. In Section II, we describe the system model for the CSS. In Section III, we presents the mathematical formulation of joint optimization problem $\mathbb{JOP}$ and its special cases. The solutions for the formulated problems are presented in Section IV. Section-V presents the numerical results using ED followed by conclusions in Section VI.

II. SYSTEM MODEL

We consider a centralized CR network as shown in [13, Fig. 2] where K SUs cooperatively detect the PU signal by reporting their local decisions to the FC over erroneous reporting channels. Each SU k, $k = 1, 2, .., K$, makes a local decision d_k based on binary hypothesis testing problem with two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, where $\mathcal{H}_0$ and $\mathcal{H}_1$ corresponding to absence and presence of the PU signal. Let $d_k = 0$ and $d_k = 1$ denote the local decisions drawn by the kth SU under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. The local probabilities of false alarm and missed detection of the kth SU denoted as $P_f^{(k)}$, $P_m^{(k)}$, respectively. The kth SU reports local decision to the FC over a erroneous reporting channel, whose probability of error is denoted as $P_e^{(k)}$. The corresponding effective probabilities of false alarm and missed detection as seen by the FC from kth SU are given, respectively, by $P_{fe}^{(k)} = P_f^{(k)}\left(1 - P_e^{(k)}\right) + \left(1 - P_f^{(k)}\right)P_e^{(k)}$, $P_{me}^{(k)} = P_m^{(k)}\left(1 - P_e^{(k)}\right) + \left(1 - P_m^{(k)}\right)P_e^{(k)}$. The FC combines the local decisions of the K SUs and makes a final decision $d_{FC} \in \{0, 1\}$ on the status of PU using m-out-of-K rule [17]. Note that $d_{FC} = \{0, 1\}$ denotes the absence and presence of the PU, respectively. Following [13], [18], [19], [28], [32], we assume that all SUs and reporting channels are identical, which implies $P_f^{(k)} = P_f, P_m^{(k)} = P_m$ and $P_e^{(k)} = P_e, \forall k$. The corresponding global probabilities of false alarm and missed detection at the FC for the m-out-of-K rule can be evaluated, respectively, by [13], [33]

$$P_F(m, K) = P(d_{FC} = 1|\mathcal{H}_0) = \mathcal{I}(m, K, P_{fe}), \quad (1)$$

$$P_M(m, K) = P(d_{FC} = 0|\mathcal{H}_1) = 1 - \mathcal{I}(m, K, 1 - P_{me}), \quad (2)$$

where

$$\mathcal{I}(m, K, P) = \sum_{k=m}^{K} \binom{K}{k} (P)^k (1 - P)^{K-k}, \ P \in [0, 1]. \quad (3)$$

The Bayes risk function or *average cost* of the m-out-of-K rule that we wish to minimize can be expressed as [17, p. 74]

$$\begin{aligned}\mathcal{R}(m, K) &= \sum_{i=0}^{1}\sum_{j=0}^{1} \alpha_{ij} P_j P(d_{FC} = i|\mathcal{H}_j) \\ &= \alpha_F P_F(m, K) + \alpha_M P_M(m, K) + \alpha_C, \end{aligned} \quad (4)$$

where $\alpha_F = P_0(\alpha_{10} - \alpha_{00})$, $\alpha_M = P_1(\alpha_{01} - \alpha_{11})$ are the effective weights of the global false alarm and missed detection probabilities, respectively, $\alpha_C = \alpha_{00}P_0 + \alpha_{11}P_1$ and where α_{ij} is the cost of deciding the final decision $d_{FC} = i$ by the FC when $\mathcal{H}_j$ is true and P_j denote the prior probability of the hypothesis $\mathcal{H}_j$, $\forall i, j \in \{0, 1\}$.

III. Problem Formulation

The mathematical formulation of $\mathbb{JOP}$ is given by

$$\mathbb{JOP}: \min_{m,K} \ \mathcal{R}(m, K), \text{ s.t } \mathcal{C}: 1 \le m \le K, \quad (5)$$

where $\mathcal{R}(m, K)$ is given in (4).

A. Special Cases of $\mathbb{JOP}$

- $\mathbb{JOP}$-I: For a fixed value of K and substitution of $P_e = 0$ in (5), the $\mathbb{JOP}$ specializes to find the optimal m that minimizes the Bayes risk over error free reporting channel [17].
- $\mathbb{JOP}$-II: For a fixed value of K and substituting appropriate values of parameters such as α_{ij}, P_j, $\forall i, j \in \{0, 1\}$ and P_e in (5), we get the optimal m of various objective functions as mentioned in [19], [20] and [21].
- $\mathbb{JOP}$-III: Choosing appropriate values of parameters in (5), we get the joint optimized values of m and K that minimizes FDP.
- $\mathbb{JOP}$-IV: Substituting appropriate values of parameters in (5), we get the joint optimized values of m and K that minimizes TER.

Note that the special cases $\mathbb{JOP}$-I and $\mathbb{JOP}$-II have been studied in the literature, while $\mathbb{JOP}$-III and $\mathbb{JOP}$-IV are new optimization problems.

IV. Solutions of the Formulated Problems

In this section, we first present the solution of $\mathbb{JOP}$ followed by solution of special cases.

Lemma 1. *For a given K, the solution of $\mathbb{JOP}$, i.e., the optimal fusion rule $m^*_{\mathcal{R}}$ that minimizes the Bayes risk is given by*

$$m^*_{\mathcal{R}} = \begin{cases} max(1, m^*), & \alpha_F < \alpha_M, \\ min(K, m^*), & \alpha_F > \alpha_M, \\ m^*, & \alpha_F = \alpha_M, \end{cases} \quad (6)$$

where

$$m^* = \left\lceil \frac{a + Kb}{b + c} \right\rceil, a = \ln\frac{\alpha_F}{\alpha_M}, b = \ln\frac{1 - P_{fe}}{P_{me}}, c = \ln\frac{1 - P_{me}}{P_{fe}} \quad (7)$$

and $\lceil . \rceil$ denotes standard ceiling function.

Proof: Please refer to Appendix ■

Theorem 1. *The solution of $\mathbb{JOP}$, i.e., the joint optimized values of m and K which are denoted as $m^*_{\mathcal{R}}$ and $K^*_{\mathcal{R}}$ respectively, and are given by*

$$\begin{cases} m^*_{\mathcal{R}} = 1, K^*_{\mathcal{R}} = \left\lceil \frac{c-a}{b} \right\rceil; \text{if } \alpha_F < \alpha_M \text{ and } m^* < 1, \\ m^*_{\mathcal{R}} = K, K^*_{\mathcal{R}} = \left\lfloor \frac{a+b}{c} \right\rfloor; \text{if } \alpha_F > \alpha_M \text{ and } m^* > K, \end{cases} \quad (8)$$

and when $1 \le m^* \le K$, irrespective of α_F, α_M, there exist a $K^*_{\mathcal{R}}$ for a given m and $m^*_{\mathcal{R}}$ for a given K, where m^*, a, b and c are given by (7).

Proof: The joint optimized values m and K can be obtained by solving optimal m and optimal K equations. Given m, the optimal K that minimizes the Bayes risk in (4), denoted as $K^*_{\mathcal{R}}$ and is given by [31, eq. 7]

$$K^*_{\mathcal{R}} = \left\lceil \frac{m(b + c) - (a + b)}{b} \right\rceil, \quad (9)$$

where a, b and c are given by (7). Therefore the joint optimized values of m and K can be obtained by solving (9) and (6) and the analaysis for three different cases is presented as follows.

Case I: From (6), if $\alpha_F < \alpha_M$ and $m^* < 1$, then $m^*_{\mathcal{R}} = 1$. The corresponding $K^*_{\mathcal{R}}$ can be obtained by substituting $m^*_{\mathcal{R}} = 1$ in (9) and is given by $K^*_{\mathcal{R}} = \left\lceil \frac{c-a}{b} \right\rceil$.

Case II: From (6), if $\alpha_F > \alpha_M$ and $m^* > K$, then $m^*_{\mathcal{R}} = K$. The corresponding $K^*_{\mathcal{R}}$ can be obtained by substituting $m^*_{\mathcal{R}} = K$ in (9), we have $K^*_{\mathcal{R}} = \left\lceil \frac{K^*_{\mathcal{R}}(b+c)-(a+b)}{b} \right\rceil$. Simplifying using the definition of ceiling function, we get $K^*_{\mathcal{R}} = \left\lfloor \frac{a+b}{c} \right\rfloor$.

Case III: From (6) when $1 \le m^* \le K$, irrespective of α_F, α_M, then $m^*_{\mathcal{R}} = m^* = \left\lceil \frac{a+Kb}{b+c} \right\rceil$. Substituting $m^*_{\mathcal{R}}$ in (9), we have

$$K^*_{\mathcal{R}} = \left\lceil \frac{b+c}{b} \left\lceil \frac{a+K^*_{\mathcal{R}}b}{b+c} \right\rceil - \frac{a+b}{b} \right\rceil .$$

Simplifying and re-arranging, we get

$$\frac{a+K^*_{\mathcal{R}}b}{b+c} < \left\lceil \frac{a+K^*_{\mathcal{R}}b}{b+c} \right\rceil \le \frac{a+K^*_{\mathcal{R}}b}{b+c} + \frac{b}{b+c}. \quad (10)$$

Substituting $K^*_{\mathcal{R}}$ into $m^*_{\mathcal{R}}$, we have

$$m^*_{\mathcal{R}} = \left\lceil \frac{a}{b+c} + \frac{b}{b+c} \left\lceil \frac{m^*_{\mathcal{R}}(b+c)-(a+b)}{b} \right\rceil \right\rceil .$$

Simplifying and re-arranging, we get

$$\frac{m^*_{\mathcal{R}}(b+c)-a-b-c}{b} < \left\lceil \frac{m^*_{\mathcal{R}}(b+c)-(a+b)}{b} \right\rceil \le \frac{m^*_{\mathcal{R}}(b+c)-a}{b}. \quad (11)$$

Note that any finite value of $K^*_{\mathcal{R}}$ satisfies (10). This means that as $K^*_{\mathcal{R}}$ increases Bayes risk deceases. Similarly (11) is satisfied for any finite value of $m^*_{\mathcal{R}}$. This imply as $m^*_{\mathcal{R}}$ increases Bayes risk decreases. This can also be observed from Fig. 5 and Fig. 6. ■

A. Solutions for Special Cases of $\mathbb{JOP}$

- The solutions of $\mathbb{JOP}$-I and $\mathbb{JOP}$-II can be obtained by substituting appropriate parameter values in (6) as mentioned in $\mathbb{JOP}$-I and $\mathbb{JOP}$-II, respectively.
- The solution for $\mathbb{JOP}$-III can be obtained by substituting $\alpha_F = P_0$ and $\alpha_M = P_1$ in (8), we get the joint optimized values of m and K that minimizes the FDP.
- The solution of $\mathbb{JOP}$-IV can be obtained substituting $\alpha_F = 1$ and $\alpha_M = 1$ in (8), we get $1 \le \left\lceil \frac{Kb}{b+c} \right\rceil \le K$. This means there exists an optimal m for a given K and an optimal K for a given m.

Note that the expressions given in (8) and (6) are general as they depends on P_f, P_m and P_e. Therefore, these results are applicable to any detector used in the CR network.

V. Numerical Results using ED

We consider the energy detector (ED) for analyzing the results obtained in this paper. The P_f and P_m of a SU using ED are given, respectively, by [34]

$$P_f = \frac{\Gamma\left(u, \frac{\beta}{2}\right)}{\Gamma(u)}, \; P_m = 1 - Q_u\left(\sqrt{2\gamma}, \sqrt{\beta}\right), \quad (12)$$

where u is the time-bandwidth product of the ED, β is the sensing threshold of the ED, γ is the *signal-to-noise ratio* (SNR) received by a SU over a sensing channel, $\Gamma(.,.)$ denotes the upper incomplete gamma function given by $\Gamma(s,t) = \int_t^\infty x^{s-1}e^{-x}dx$, $\Gamma(.)$ is the ordinary gamma function given by $\Gamma(s) = \int_0^\infty x^{s-1}e^{-x}dx$ and $Q_u(.,.)$ is the generalized Marcum Q-function $Q_u(s,t) = \frac{1}{s^{u-1}} \int_t^\infty x^u e^{-\frac{x^2+s^2}{2}} I_{u-1}(sx)\,dx$ with $I_{u-1}(.)$ is the modified Bessel function of the first kind of order $u-1$. Note that in (12), P_f is a decreasing function and P_m is a increasing function with respect to β. This means there exists a β value β_0 at which $P_f = P_m$. This in turn implies that when $\beta < \beta_0$, $\Rightarrow P_f > P_m$ and when $\beta > \beta_0$, $\Rightarrow P_f < P_m$. Also note that the plots are examined with respect to β because the β value gives the interpretation of both P_f and P_m.

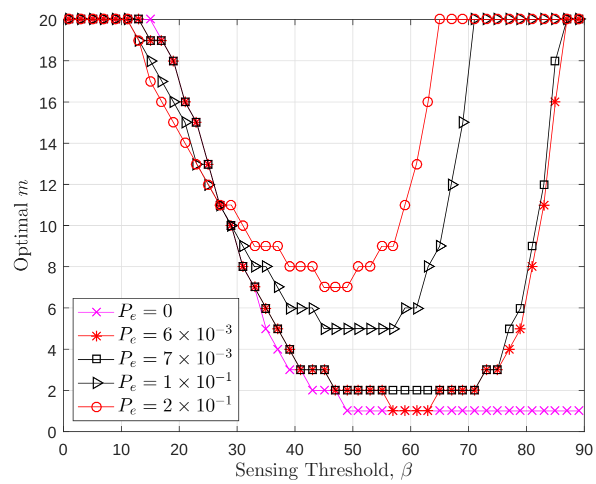

Fig. 1. When $\alpha_F > \alpha_M$: Optimal m versus β for $\alpha_{00} = 0.1$, $\alpha_{11} = 0.2$, $\alpha_{10} = 1.5$, $\alpha_{01} = 2$, $P_0 = 0.8$ $P_1 = 0.2$, $u = 10$, SNR = 10 dB and $K = 20$, using ED.

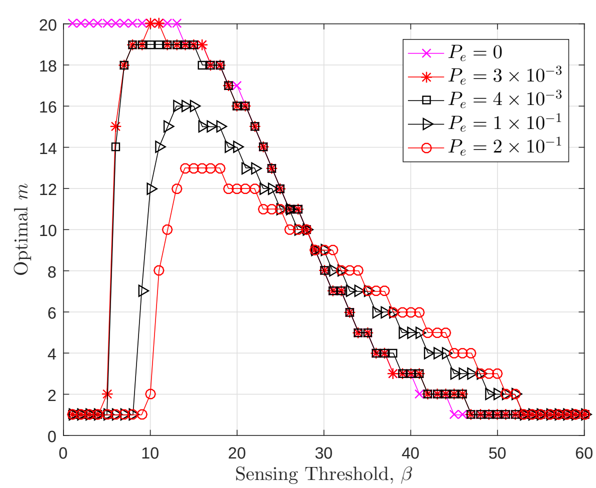

Fig. 2. When $\alpha_F < \alpha_M$: Optimal m versus β for $\alpha_{00} = 0.1$, $\alpha_{11} = 0.2$, $\alpha_{10} = 1.5$, $\alpha_{01} = 2$, $P_0 = 0.2$, $P_1 = 0.8$, $u = 10$, SNR = 10 dB and $K = 20$, using ED.

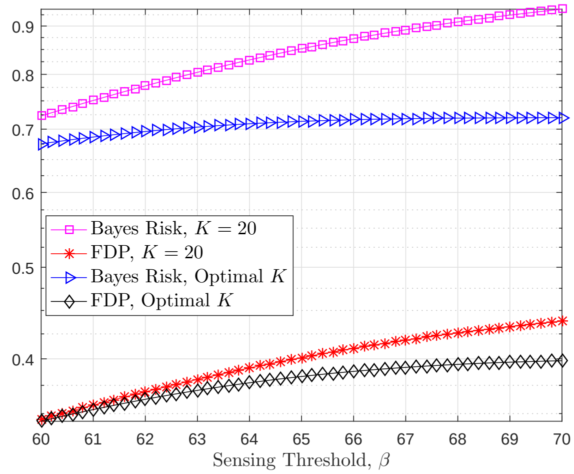

Fig. 3. When $\alpha_F < \alpha_M$ and $m^* < 1$: Various objective functions with respect to β for $\alpha_{00} = 0.1$, $\alpha_{11} = 0.2$, $\alpha_{10} = 1.5$, $\alpha_{01} = 2$, $P_0 = 0.4$, $P_1 = 0.6$, $P_e = 0.05$, $u = 10$, SNR = 10 dB and $K = 20$, using ED.

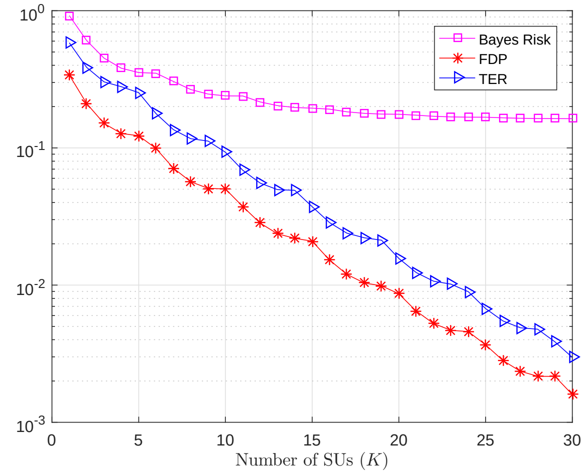

Fig. 5. When $1 \leq m^* \leq K$: Various objective functions with respect to number of SUs (K) for $\alpha_{00} = 0.1$, $\alpha_{11} = 0.2$, $\alpha_{10} = 1.5$, $\alpha_{01} = 2$, $P_1 = 0.4$, $P_0 = 0.6$, $P_e = 0.05$, $u = 10$ and SNR = 10 dB, Optimal m is applied.

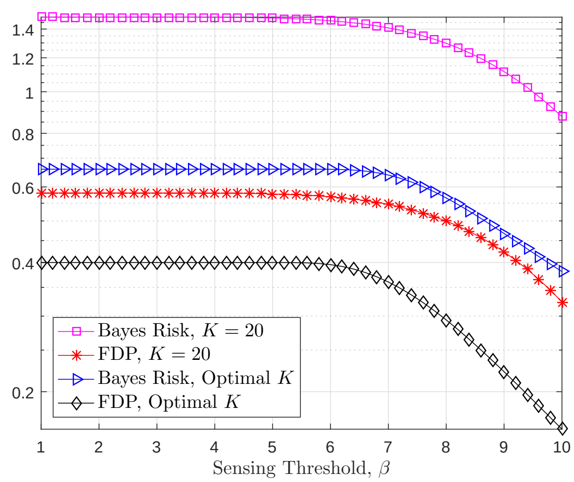

Fig. 4. When $\alpha_F > \alpha_M$ and $m^* > K$: Various objective functions with respect to β for $\alpha_{00} = 0.1$, $\alpha_{11} = 0.2$, $\alpha_{10} = 2.5$, $\alpha_{01} = 1.5$, $P_0 = 0.6$ $P_1 = 0.4$, $P_e = 0.005$, $u = 10$, SNR = 10 dB and $K = 20$, using ED.

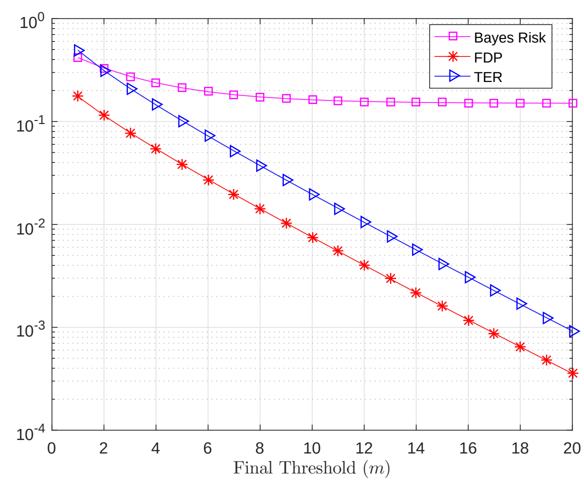

Fig. 6. When $1 \leq m^* \leq K$: Various objective functions with respect to m for $\alpha_{00} = 0.1$, $\alpha_{11} = 0.2$, $\alpha_{10} = 1.5$, $\alpha_{01} = 2$, $P_1 = 0.4$, $P_0 = 0.6$, $P_e = 0.05$, $u = 10$ and SNR = 10 dB, Optimal K is applied.

The solution of optimal m for two cases such as $\alpha_F > \alpha_M$ and $\alpha_F < \alpha_M$ shown in Fig. 1, Fig. 2, respectively. Note that the cost values of Bayes risk are chosen arbitrarily. From Fig. 1, we observe that, when $P_e = 0$, the OR rule $(m = 1)$ is optimal for large values of β at which $P_f << P_m$, while the AND rule $(m = K)$ is optimal for very low values of β at which $P_f >> P_m$. When $P_e \neq 0$, the AND rule is optimal at $P_f >> P_m$ and $P_f << P_m$. Also note that there exists a limiting value of P_e after which OR rule is never optimal. It also observed that AND rule is robust against the reporting channel errors as compared to the OR rule. This is due to lesser effective weight of the P_M as compared to effective weight of the P_F. Fig. 2 plots the optimal m versus β for five values of P_e using ED. Observe that, when $P_e = 0$ OR rule is optimal for large values of β at which $P_f << P_m$, while the AND rule is optimal for very low values of β at which $P_f >> P_m$. When $P_e \neq 0$, the AND rule is optimal at $P_f >> P_m$ and $P_f << P_m$. It also observed that OR rule is robust against the reporting channel errors as compared to AND rule. This is due to effective weight assigned to the P_F is less than the effective weight of the P_M. The observations made with respect to Fig. 1 and Fig. 2 are summarized in Table I.

Fig. 3 shows the performance of various objective functions plotted using optimized values of m and K for the case when $m^* < 1$ and $\alpha_F < \alpha_M$. Note that the β values on the x - axis are chosen such that $m^* < 1$. It is observed that the performance of various objective functions obtained using joint optimized values of m and K results in significant improvement. Fig. 4 shows the performance of various objective functions plotted using optimized values of

m and K for the case when $m^* > K$ and $\alpha_F > \alpha_M$. It is observed that the performance of various objective functions obtained using joint optimized values of m and K results in significant improvement. Fig. 5 and Fig. 6 shows the performance of various objective functions with respect to K and m, respectively when $1 \leq m^* \leq K$. From Fig. 5, observe that each of three objective function, namely Bayes risk, FDP and TER decreases with K, for the optimal value of m. Similarly, in Fig. 6 each objective function decreases with m, for the optimal value of K. Therefore, when $1 \leq m^* \leq K$, there exist an optimal value of K for a given m and an optimal value of m for a given K.

TABLE I
THE RELATION BETWEEN P_f AND P_m WHERE THE OR AND AND RULES ARE OPTIMAL AND COMPARISON WITH THE EXITING RESULTS [19], [20]. NOTE THAT x DENOTES $P_f << P_m$ AND y DENOTES $P_f >> P_m$.

Special cases of m-out-of-K rule	Bayes Risk*		TER ($\alpha_F = 1, \alpha_M = 1$)
	P_f Vs P_m	α_F Vs α_M	
OR rule ($P_e = 0$)	x	all values	x [19]
OR rule ($P_e \neq 0$)	x	$\alpha_F > \alpha_M$	x [20]
	x and y	$\alpha_F < \alpha_M$	
AND rule ($P_e = 0$)	y	all values	y [19]
AND rule ($P_e \neq 0$)	y	$\alpha_F < \alpha_M$	y [20]
	x and y	$\alpha_F > \alpha_M$	

*- The observations made with respect to Bayes risk are also applicable to FDP when $\alpha_F = P_0$, $\alpha_M = P_1$ and were not observed in [21].

VI. CONCLUSIONS

In this paper, a joint optimization problem is formulated, where the expressions for joint optimized values of m and K of the m-out-of-K rule is obtained by minimizing the Bayes risk in the presence of reporting channel errors. We have shown that many existing results are the special cases of the proposed solution. The limitations of optimal fusion rules are studied in the presence of erroneous reporting channels errors. The choice of parameters of Bayes risk results in difference in robustness of optimal fusion rules to the erroneous reporting channels.

APPENDIX: PROOF OF LEMMA 1

Given K, the solution of $\mathbb{JOP}$ in (5) without constraint $\mathcal{C}$ can be obtained by solving the following two difference equations [17].

$$\mathcal{R}(m, K) - \mathcal{R}(m-1, K) < 0, \tag{13}$$

$$\mathcal{R}(m+1, K) - \mathcal{R}(m, K) \geq 0. \tag{14}$$

Simplifying (13) using (4), (1) and (2), we have

$$\Rightarrow \alpha_F \left[\mathcal{I}(m, K, P_{fe}) - \mathcal{I}(m-1, K, P_{fe})\right] + \alpha_M \left[\mathcal{I}(m, K, 1-P_{me}) - \mathcal{I}(m-1, K, 1-P_{me})\right] < 0.$$

Simplifying above equation using (3), we have

$$-\alpha_F \left[(P_{fe})^{m-1}(1-P_{fe})^{K-m+1}\right] + \alpha_M \left[(1-P_{me})^{m-1}(P_{me})^{K-m+1}\right] < 0.$$

Simplifying and re-arranging, we get

$$m < \frac{a+Kb}{b+c} + 1. \tag{15}$$

where a, b and c are given by (7). Similarly solving (14) using (4), (1) and (2), we obtain

$$m \geq \frac{a+Kb}{b+c}. \tag{16}$$

Combining (15), (16) and evaluating at $m = m^*$, we get

$$m^* = \left\lceil \frac{a+Kb}{b+c} \right\rceil. \tag{17}$$

To satisfy the constraint $\mathcal{C}$ in (5), the value of m^* must satisfies the following in-equality.

$$0 < \frac{a+Kb}{b+c} \leq K. \tag{18}$$

Usually, a detector has $P_f + P_m \leq 1$ and the probability of error of a reporting channel $P_e < 0.5$, this implies $P_{fe} + P_{me} \leq 1$. This in turn implies $b \geq 0$ and $c \geq 0$. Under these conditions, to satisfy the left hand side in-equality of (18), we choose the optimal m as $m^*_{\mathcal{R}} = max\left(1, \lceil m^* \rceil\right)$ when $a < 0$ ($\alpha_F < \alpha_M$). Similarly, to satisfy the right hand side in-equality of (18), we choose $m^*_{\mathcal{R}} = min\left(K, \lceil m^* \rceil\right)$ when $a > 0$ ($\alpha_F > \alpha_M$). When $a = 0$ ($\alpha_F = \alpha_M$), the m^* always satisfies (18) and the $m^*_{\mathcal{R}} = m^*$. In summary, for a given K, the solution of $\mathbb{JOP}$ can expressed as

$$m^*_{\mathcal{R}} = \begin{cases} max\left(1, m^*\right), & \alpha_F < \alpha_M, \\ min\left(K, m^*\right), & \alpha_F > \alpha_M, \\ m^*, & \alpha_F = \alpha_M, \end{cases}$$

where m^* is given by (17).

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