Born-Infeld gravity with a Brans-Dicke scalar

Soumya Jana* and Sayan Kar†

Department of Physics *and* Centre for Theoretical Studies

Indian Institute of Technology, Kharagpur, 721302, India

Abstract

Recently proposed Born-Infeld (BI) theories of gravity assume a constant BI parameter (κ). However, no clear consensus exists on the sign and value of κ. Recalling the Brans-Dicke (BD) approach, where a scalar field was used to generate the gravitational constant G, we suggest an extension of Born-Infeld gravity with a similar Brans-Dicke flavour. Thus, a new action, with κ elevated to a spacetime dependent real scalar field, is proposed. We illustrate this new theory in a cosmological setting with pressureless dust and radiation as matter. Assuming a functional form of $\kappa(t)$, we numerically obtain the scale factor evolution and other details of the background cosmology. It is known that BI gravity differs from GR in the strong field regime but reduces to GR for intermediate and weak fields. Our studies in cosmology demonstrate how, with this new, scalar-tensor BI gravity, deviations from GR as well as usual BI gravity, may arise in the weak field regime too. For example, we note a late-time acceleration without any dark energy contribution. Apart from such qualitative differences, we note that fixing the sign and value of κ is no longer a necessity in this theory, though the origin of the BD scalar does remain an open question.

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* soumyajana@phy.iitkgp.ernet.in

† sayan@phy.iitkgp.ernet.in

I. INTRODUCTION

General relativity (GR) is surely successful as a classical theory of gravity, and more so, with the recent detection of gravitational waves [1]. However, generic singularities are unavoidable and acceptable explanations of dark matter or dark energy do not seem to exist within the framework of GR. In order to address some of these problems, it is not unusual to construct classical theories which deviate from GR in matter distributions, or in the strong-field regime. Thus, we have various proposals on modified gravity apart from the intense pursuit of quantum gravity.

One such modified gravity is inspired by Born-Infeld (BI) electrodynamics where the infinity in the electric field at the location of a point charge is regularised [2]. With a similar determinantal structure $\left(\left[\sqrt{-det(g_{\mu\nu} + \kappa R_{\mu\nu})}\right]\right)$ in the action, a gravity theory in the metric formulation was first suggested by Deser and Gibbons [3]. In fact, a determinantal form of the gravity action existed in Eddington's affine re-formulation of GR for de Sitter spacetime [4], though matter coupling remained a problem in the Eddington approach.

Much later, Vollick [5] introduced the Palatini formulation of Born-Infeld gravity and worked on various related aspects. He also introduced a nontrivial and somewhat artificial way of coupling matter in such a theory [6, 7]. More recently, Banados and Ferreira [8] have come up with a formulation where matter coupling is different and simpler compared to Vollick's proposal. We focus here on the theory proposed in Ref. [8] and refer to it as Eddington-inspired Born–Infeld (EiBI) gravity, for obvious reasons. The EiBI theory reduces to GR in vacuum. It also falls within the class of bimetric theories of gravity (bi-gravity) [9], [10], [11, 12].

Let us first briefly recall Eddington–inspired Born–Infeld (EiBI) gravity. The central feature here is the existence of a physical metric which couples to matter and another auxiliary metric which is not used for matter couplings. One needs to solve for both metrics through the field equations. The action for the theory developed in Ref. [8] is given as

$$S_{BI}(g, \Gamma, \Psi) = \frac{c^3}{8\pi G\kappa} \int d^4x \left[\sqrt{-|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-g}\right] + S_M(g, \Psi), \tag{1}$$

where $\lambda = \kappa\Lambda + 1$, Λ being the cosmological constant. Palatini variation with respect to $g_{\mu\nu}$ and Γ, using the auxiliary metric $q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)$ gives the field equations for this theory.

In order to obtain solutions, we need to assume a $g_{\mu\nu}$ and a $q_{\mu\nu}$ with unknown functions, as well as a matter stress-energy ($T^{\mu\nu}$). Thereafter, we write down the field equations and obtain solutions using some additional assumptions about the metric functions and the stress-energy.

A lot of work on various fronts has been carried out on diverse aspects of this theory, in the last few years. Astrophysical scenarios have been widely discussed [13–21]. Spherically symmetric solutions of various types have been obtained [8, 22–28]. A domain wall brane in a higher-dimensional generalization of EiBI theory was analyzed in Ref. [29]. Generic features of the paradigm of matter-gravity couplings were analyzed in [30]. Further, in [31], the authors showed that EiBI theory admits a nongravitating matter distribution, which is not allowed in GR. Some interesting cosmological and circularly symmetric solutions in $2+1$ dimensions are obtained in [32]. In [33], a problem in the context of stellar physics, related to surface singularities in EiBI gravity, was noticed. Gravitational backreaction was suggested as a cure in [34]. A modification of EiBI theory, through a functional extension similar to $f(R)$ theory, was proposed in [35]. Recently, in [36] a new route to matter coupling was suggested via the use of the Kaluza ansatz in a five-dimensional EiBI action (in a metric formulation) and subsequent compactification to four-dimensional gravity coupled nonlinearly to electromagnetism.

A lot of the recent work on EiBI gravity is devoted to cosmology. In [8, 10, 37], the nonsingularity of the Universe filled by any ordinary matter was demonstrated. Linear perturbations have been studied in the background of homogeneous and isotropic spacetimes in the Eddington regime [38, 39]. Bouncing cosmology in EiBI gravity was emphasized as an alternative to inflation in [40]. The authors in [41], studied a model described by a scalar field with a quadratic potential, which results in a nonsingular initial state of the Universe leading naturally to inflation. They also investigated the stability of tensor perturbations in this inflationary model [42], whereas the scalar perturbations were studied in [43]. Large-scale structure formation in the Universe and the integrated Sachs-Wolfe effect were discussed in [44]. Other relevant work has been reported in [45–53].

The theory parameter κ in EiBI gravity is a constant though we have no way to know whether it is universal. The sign of κ governs the nature of solutions. There are some upper bounds on the value of κ from astrophysical and cosmological observations [13–15, 54]. For example, the existence of self-gravitating compact objects like neutron stars strongly constrains the theory with $\kappa > 0$ and $\kappa \lesssim 5 \times 10^8$ m^2 [13]. Stellar equilibrium and evolution of the Sun puts a constraint $|\kappa| \lesssim 2 \times 10^{14}$ m^2 [14]. Primordial nucleosynthesis leads to $\kappa \lesssim 10^6$ m^2 [15] where it is assumed that $\kappa > 0$. From nuclear physics constraints (i.e. requiring the electromagnetic force as dominant over the gravitational force, at the subatomic scale) one gets $|\kappa| \lesssim 6 \times 10^5$ m^2 [54]. All the numbers (for κ) mentioned above are in the unit of m^2, whereas, in most of the literature, the unit used (for $\kappa' = 8\pi G\kappa$) is kg^{-1}m^{5}s^{-2}. In summary, no consensus exists on the sign and value of κ.

In our work here, we address this issue by suggesting the possibility of κ being a non-constant, real scalar field. The advantage with κ being a scalar field is that it can take on different functional forms in different scenarios (say, cosmology, black holes, stars etc.) and a universal sign or value is not a necessity. However, one still needs to address the issue of the origin of κ.

It is known that EiBI theory differs from GR in the high energy regime. With a scalar κ a new theory of gravity emerges, which reduces to GR only in the intermediate energy scale, but may differ in the high as well as the low energy regimes. Our aim here is to formulate this theory with a scalar κ and explore its consequences. This is carried out in the subsequent sections.

II. THE EIBI ACTION WITH κ AS A REAL SCALAR FIELD

Let us begin by proposing a new action given as

$$S_{BI\kappa}\left(g,\Gamma,\kappa,\Psi\right) = \int \left[\frac{1}{\kappa}\left(\sqrt{-|g_{\alpha\beta} + \kappa R_{\alpha\beta}(\Gamma)|} - \sqrt{-g}\right) - \sqrt{-g}\tilde{\omega}(\kappa)\partial_\mu\kappa\partial^\mu\kappa\right] d^D x + S_M\left(g,\Psi\right), \qquad (2)$$

where $\kappa(t,\vec{x})$ is a scalar field and $\tilde{\omega}(\kappa)$ is a coupling function, reminiscent of scalar-tensor modifications of GR. We assume $c = 1$, $8\pi G = 1$ and spacetime of dimension D. For a constant κ we recover the standard EiBI theory of gravity [8]. If κ is constant and small in value, the action reduces to the known Einstein-Hilbert one (with cosmological constant $\Lambda = 0$). Variation of the action [Eq. (2)] with respect to 'Γ' yields the earlier definition of the auxiliary metric field,

$$q_{\alpha\beta} = g_{\alpha\beta} + \kappa R_{\alpha\beta}(q), \qquad (3)$$

where the Γs are computed using the following relation

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} q^{\alpha\beta} \left(\partial_\nu q_{\beta\mu} + \partial_\mu q_{\nu\beta} - \partial_\beta q_{\mu\nu} \right). \tag{4}$$

However variation with respect to '$g_{\alpha\beta}$' yields

$$\sqrt{-q} q^{\alpha\beta} - \sqrt{-g} g^{\alpha\beta} = -\kappa \sqrt{-g}\, T^{\alpha\beta}_{eff}, \tag{5}$$

where

$$T^{\alpha\beta}_{eff} = T^{\alpha\beta} - \tilde{\omega} g^{\alpha\beta} g^{\mu\nu} \partial_\mu \kappa \partial_\nu \kappa + 2\tilde{\omega} g^{\mu\alpha} g^{\nu\beta} \partial_\mu \kappa \partial_\nu \kappa. \tag{6}$$

$T^{\alpha\beta}$ is the usual stress-energy tensor. Variation with respect to κ gives

$$2\kappa\tilde{\omega}(\kappa) \nabla_\mu \nabla^\mu \kappa + \kappa \tilde{\omega}'(\kappa) \nabla_\mu \kappa \nabla^\mu \kappa + \frac{1}{\kappa} + \frac{\sqrt{-q}}{\sqrt{-g}} \left(\frac{1}{2} q^{\alpha\beta} R_{\alpha\beta}(q) - \frac{1}{\kappa} \right) = 0, \tag{7}$$

where the covariant derivatives are defined with respect to the physical metric (g) and $\tilde{\omega}'(\kappa)$ is a derivative of $\tilde{\omega}$ w.r.t. κ.

Using the above mentioned field equations, one can verify that the stress-energy tensor ($T^{\mu\nu}$) is conserved, i.e.

$$\nabla_\mu T^{\mu\nu} = 0. \tag{8}$$

It is important to check whether the above equations are consistent with the solutions for constant κ –particularly the equation (7). In vacuum, from Eq. (5), we have $\sqrt{-q} q^{\alpha\beta} = \sqrt{-g} g^{\alpha\beta}$ which implies $q_{\mu\nu} = g_{\mu\nu}$. Using this in Eq. (3), $R_{\alpha\beta} = 0$. Hence, Eq. (7) is satisfied. Now, to check the consistency in presence of a matter distribution ($T_{\alpha\beta} \neq 0$), we take the example of a three dimensional ($D = 3$) cosmological solution in EiBI gravity for a dust-filled ($P = 0$) Universe [32]. The physical FRW spacetime is given by $ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\theta^2]$, where $a^2(t) = \rho_0(t^2 - \kappa)$ for $\kappa > 0$ and $\kappa < 0$ as well, and ρ_0 is the present day energy density of the Universe. The corresponding auxiliary line element is $ds_q^2 = -dt^2 + b^2(t)[dr^2 + r^2 d\theta^2]$, where $b^2(t) = \rho_0 t^2$. Then, $R(q) = 2\left(\frac{\dot{b}^2}{b^2} + 2\frac{\ddot{b}}{b}\right) = 2/t^2$. Using these relations, it is now easy to verify that Eq. (7) is consistent for constant-κ.

The non-relativistic limit of the theory is different from that in EiBI gravity [8]. For a time-independent physical metric $ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi) d\vec{x} \cdot d\vec{x}$ and an energy momentum tensor $T^{\mu\nu} = \rho u^\mu u^\nu$, the full set of linearized field equations are given by the following two equations:

$$\nabla^2 \Phi = \frac{\rho}{2} + \frac{1}{4} \nabla^2 (\kappa\rho) + \frac{1}{2} \tilde{\omega} (\vec{\nabla}\kappa)^2 + \frac{1}{4} \nabla^2 \left(\kappa \tilde{\omega} (\vec{\nabla}\kappa)^2 \right), \tag{9}$$

$$2\tilde{\omega} \nabla^2 \kappa + \tilde{\omega}' (\vec{\nabla}\kappa)^2 + \frac{1}{4} \left(\rho + \tilde{\omega} (\vec{\nabla}\kappa)^2 \right)^2 = 0, \tag{10}$$

where Φ, ρ, and κ depend only on $\vec{x}$. Equation (9) is the modified Poisson equation in the new theory. For a constant κ it reduces to the Poisson equation in the original EiBI theory. We also mention that a study of gravitational waves in vacuum as well as vacuum exact solutions in this theory will be different (unlike standard EiBI gravity [8]) from usual GR because of the presence of the scalar field κ.

III. COSMOLOGY

As an application of the new theory, we now study cosmology in the $3+1$ dimensional version of the new theory. We assume a spatially flat, FRW ansatz for the physical line element:

$$ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right], \tag{11}$$

and choose an ansatz for the auxiliary line element

$$ds_q^2 = -U dt^2 + V a^2 \left[dx^2 + dy^2 + dz^2\right]. \tag{12}$$

Let us consider a Universe driven by a perfect fluid with the stress-energy tensor,

$$T^{\mu\nu} = (p+\rho)u^\mu u^\nu + p g^{\mu\nu}, \tag{13}$$

where p and ρ are pressure and energy density respectively, and $u^\mu = diag.\{1,0,0,0\}$. Using Eqs. (11) and (12), the '00' (temporal) and 'ii' (spatial; $i=1,2,3$) components of $T^{\mu\nu}_{eff}$ [Eq. (6)] become,

$$T^{00}_{eff} = \rho + \tilde{\omega}\dot{\kappa}^2 \quad \text{and} \quad T^{ii}_{eff} = (p + \tilde{\omega}\dot{\kappa}^2)/a^2. \tag{14}$$

Further use of Eq. (5) leads to expressions for U and V given by

$$U = \frac{(2 - y - \kappa\omega\rho)^{3/2}}{\sqrt{y + \kappa\rho}}, \tag{15}$$

$$V = \sqrt{(y+\kappa\rho)(2 - y - \kappa\omega\rho)}, \tag{16}$$

where we have defined a new variable $y = 1 + \kappa\tilde{\omega}\dot{\kappa}^2$ and used the equation of state $p = \omega\rho$, ω being a constant. The '00' and '11' equations resulting from 'Γ'-variation lead to:

$$\frac{\ddot{a}}{a} + \frac{\ddot{V}}{2V} - \frac{\dot{V}^2}{4V^2} + \frac{\dot{a}}{a}\frac{\dot{V}}{V} - \frac{\dot{U}}{2U}\left(\frac{\dot{a}}{a} + \frac{\dot{V}}{2V}\right) = \frac{U-1}{3\kappa}, \tag{17}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{V}}{2V} + \frac{\dot{V}^2}{4V^2} + 3\frac{\dot{a}}{a}\frac{\dot{V}}{V} - \frac{\dot{U}}{2U}\left(\frac{\dot{a}}{a} + \frac{\dot{V}}{2V}\right) + 2\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{\kappa}\left(U - \frac{U}{V}\right). \tag{18}$$

Subtracting Eq. (17) from (18) we obtain

$$\left(\frac{\dot{a}}{a}+\frac{\dot{V}}{2V}\right)^2=\frac{1}{6\kappa}\left(1+2U-3\frac{U}{V}\right). \tag{19}$$

The κ-variation equation (7) becomes:

$$\dot{y}+6(y-1)\frac{\dot{a}}{a}=\dot{\kappa}\left[\frac{1}{2\kappa}\left(y+\kappa\rho\right)\left(1+2U-3\frac{U}{V}\right)-\rho\right]. \tag{20}$$

Finally, conservation of the stress-energy tensor leads to,

$$\dot{\rho}+3(\omega+1)\rho\frac{\dot{a}}{a}=0. \tag{21}$$

Thus, we have five independent equations [Eqs. (15), (16), (19), (20), and (21)] to solve for six unknown functions (a, U, V, κ, ρ, and y). Hence we have the freedom to choose a form of $\kappa(t)$, which we assume as:

$$\kappa(t)=\kappa_0+\epsilon\exp(\mu t), \tag{22}$$

with κ_0, ϵ, and μ as constants. For $\mu>0$, $\kappa\to\kappa_0$ at $t\to-\infty$ and, for $\mu<0$, $\kappa\to\kappa_0$ at $t\to\infty$. In limiting situations, where $|\kappa_0|\gg|\epsilon\exp(\mu t)|$, we expect to recover the known EiBI gravity (for a constant κ) and, in the other regime, there may be deviations from EiBI gravity. In the following subsections, we investigate possible deviations for the three cases: (i) vacuum, (ii) dust ($p=0$), and (ii) radiation ($p=\rho/3$).

A. Vacuum

Unlike GR or standard EiBI gravity, in this new theory, we do have nontrivial vacuum FRW solutions generated primarily by the time dependent scalar field $\kappa(t)$. For $\mu>0$ (see Eq. (22)), nonsingular solutions with accelerated expansion at late times are found for both positive and negative values of κ_0 and ϵ. As an illustration, plots of the scale factor $a(t)$ and the corresponding $\kappa(t)$ for $\kappa_0>0$ and $\epsilon<0$, are shown in Fig. 1(a) and Fig. 1(b). From Fig. 1(c) and Fig. 1(d), we note that $y\to 0$ and $\dot{y}\to 0$ at late times. During this phase, $\frac{\dot{a}}{a}\simeq\frac{\dot{\kappa}}{2\kappa}$ [from Eq. (20)]. As a result, $a\propto\sqrt{|\kappa|}$, or $a\propto\exp(\mu t/2)$, since $|\epsilon\exp(\mu t)|\gg|\kappa_0|$ at large t for $\mu>0$. For $\epsilon>0$, $y\to 2$ and $\dot{y}\to 0$, and therefore $a\propto\exp(\mu t/6)$ at large t. Thus the scale factor approaches de Sitter expansion stage at late times for $\mu>0$. As we will see later, for $\epsilon<0$, a similar reasoning applies to the Universe filled with dust or radiation, which also approaches the de Sitter expansion stage at late times when $|\kappa\rho| \sim 0$. This becomes clear from the numerical plots shown later. Although we get an expression for asymptotic behaviour of $a(t)$ at late times, we need to solve the system of equations numerically to obtain the full solution.

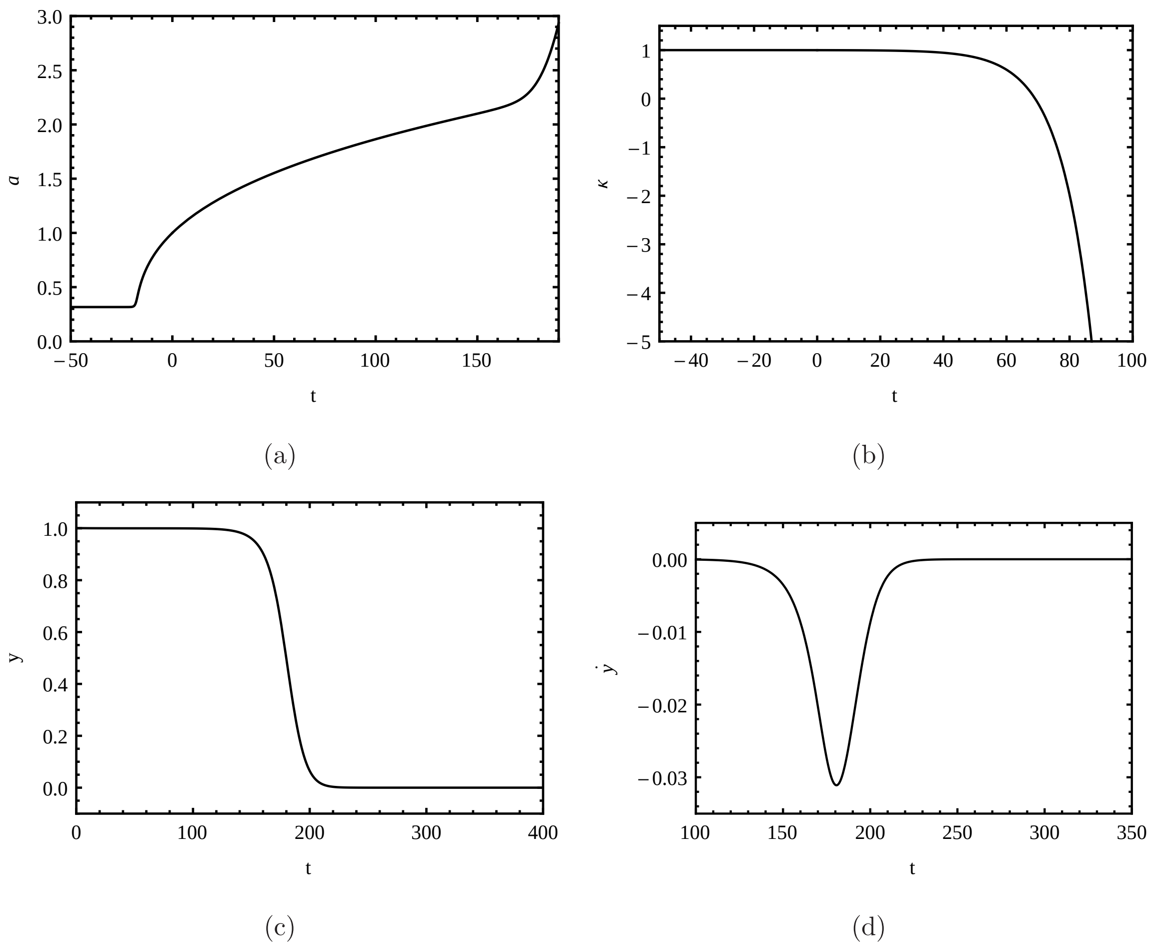


FIG. 1. Plot of (a) $a(t)$, (b) $\kappa(t)$, (c) $y(t)$, and (d) $\dot{y}(t)$ for a vacuum ($\rho = 0$ and $p = 0$) solution for $\kappa_0 > 0$, $\epsilon < 0$, and $\mu > 0$. The parameters used are $\kappa_0 = 1$, $\mu = 0.1$. We choose $a(0) = 1$, $y(0) = 1.001$, $\kappa(0) = 0.999$ for the numerical solution.

B. $p = 0$, dust

For dust ($p = 0$), $\rho = \rho_0/a^3$, where ρ_0 is a constant. Thus, U and V become

$$U = \frac{(2-y)^{3/2}}{\sqrt{y+\kappa\rho}} \quad \text{and} \quad V = \sqrt{(y+\kappa\rho)(2-y)}. \tag{23}$$

We define two new functions

$$F_1 := 1 + 2U - 3\frac{U}{V} = 1 + \frac{2(2-y)^{3/2}}{\sqrt{y+\kappa\rho}} - \frac{3(2-y)}{y+\kappa\rho}, \tag{24}$$

and

$$\beta := \frac{\dot{a}}{a} + \frac{\dot{V}}{2V} = \frac{1}{4(y+\kappa\rho)}\left[\frac{\dot{y}(2-2y-\kappa\rho)}{2-y} + \frac{\dot{a}}{a}(4y+\kappa\rho) + \mu(\kappa-\kappa_0)\rho\right], \tag{25}$$

where we have used the Eq. (22). Using Eqs. (20), (24), and (25), we get

$$\frac{\dot{a}}{a} = \frac{(y+\kappa\rho)\left[4\beta(2-y) + \mu(\kappa-\kappa_0)\left\{\frac{(2y+\kappa\rho-2)F_1}{2\kappa} - \rho\right\}\right]}{4(2y^2-4y+3)+\kappa\rho(5y-4)} \equiv H(a,\kappa,y,\beta)\,, \tag{26}$$

and

$$\dot{y} = -6(y-1)H + \mu(\kappa-\kappa_0)\left[\frac{(y+\kappa\rho)F_1}{2\kappa} - \rho\right] \equiv F_y(a,\kappa,y,\beta)\,. \tag{27}$$

Furthermore, making use of Eq. (19) we get

$$\dot{\beta} = \frac{1}{12\beta}\frac{d}{dt}\left(\frac{F_1}{\kappa}\right) \equiv F_\beta(a,\kappa,y,\beta). \tag{28}$$

Using Eq. (24), we compute

$$F_\beta = \frac{1}{12\beta\kappa}\left[\frac{\partial F_1}{\partial y}F_y - 3\rho\frac{\partial F_1}{\partial \rho}H + \mu(\kappa-\kappa_0)\left(\frac{\partial F_1}{\partial \kappa} - \frac{F_1}{\kappa}\right)\right], \tag{29}$$

where H and F_y are given by the R.H.S. of the Eqs. (26) and (27). We solve numerically the system of first order ODEs (26), (27), and (28) along with the Eq. (22). We need only three initial conditions $a(0)$, $y(0)$, and $\kappa(0)$. Then $\beta(0)$ is fixed, $\beta(0) = \pm\beta_0$, where $\beta_0^2 = (F_1/6\kappa)|_{\{a(0),y(0),\kappa(0)\}}$. However, in our solution, we choose an appropriate sign for $\beta(0)$ such that $H(0) > 0$. We also choose $\kappa(0) \sim \kappa_0$ and $y(0) \sim 1$ so that we start from an EiBI regime of the solution.

1. $\mu > 0$ case:

For $\mu > 0$, we may choose κ_0 and ϵ either positive or negative. From the analysis of our numerical solutions, we found that the solutions are nonsingular only for $\epsilon < 0$. For $\kappa_0 > 0$ and $\epsilon < 0$, κ decreases with the increase in time, changes sign from positive to negative, and becomes more and more negative with time (see Fig. 2(a)). In this case, the early Universe undergoes a loitering phase (see Fig. 2(b)). This is similar to the case of a constant positive κ in EiBI gravity [8, 37]. However, we note that the scale factor $a(t)$ never goes to zero unlike the case in EiBI gravity, where $a \to 0$ as $t \to -\infty$ for a dust filled Universe [37]. This is demonstrated in the inset of Fig. 2(b), where the dashed curve denotes the $\kappa = \kappa_0$ case and the solid curve denotes the $\kappa(t)$ case. The plot also demonstrates the accelerated expansion of the Universe at late times. This feature is absent in EiBI theory and GR, where we see deceleration of the Universe at late times for $p = 0$. Fig. 2(c) shows the plot of the deceleration parameter q. We know that, in GR, for a matter (dust) dominated Universe, $a(t) \propto t^{2/3}$ and, consequently, $q = 0.5$. In the plot of q (Fig. 2(c)), we see that there are large variations from the value in GR, both at early and late times. In the intermediate range of time scale ($t \sim 0 - 100$), we see a GR like phase. We also note that the Universe asymptotically approaches a de Sitter expansion phase ($q \to -1$) at late times (for $t > 200$ in the plot). This fact is also evident from Fig. 2(d), where the Hubble function H becomes almost a constant at late times. Fig. 2(e) shows that there is a finite maximum energy density or, conversely, a nonzero minimum scale factor. This is unlike the case in EiBI gravity, where $\rho \to \infty$ as $t \to -\infty$ for the $p = 0$ case [37]. From Fig. 2(f), we note that $U \sim 1$ and $V \sim 1$ during the GR like phase (i.e. $t \sim 0 - 100$) but varies largely at both the early ($t < 0$) and late times ($t > 100$).

For $\kappa_0 < 0$ and $\epsilon < 0$, κ is always negative and, with increase in time, $|\kappa|$ increases (see Fig. 3(a)). In this case, the Universe undergoes a bounce instead of a loitering phase at early times. This is similar to EiBI gravity. Late time accelerated expansion of the Universe occurs after a deceleration which immediately follows the bounce. This feature is understood through the plots of the scale factor $a(t)$ in Fig. 3(b) and the deceleration parameter q in Fig. 3(c). Here also, the Universe reaches, asymptotically, a de Sitter expansion at late times ($t > 280$ in the plots for q and H).

The case $\epsilon > 0$ is not shown here through plots. We have checked that our numerical

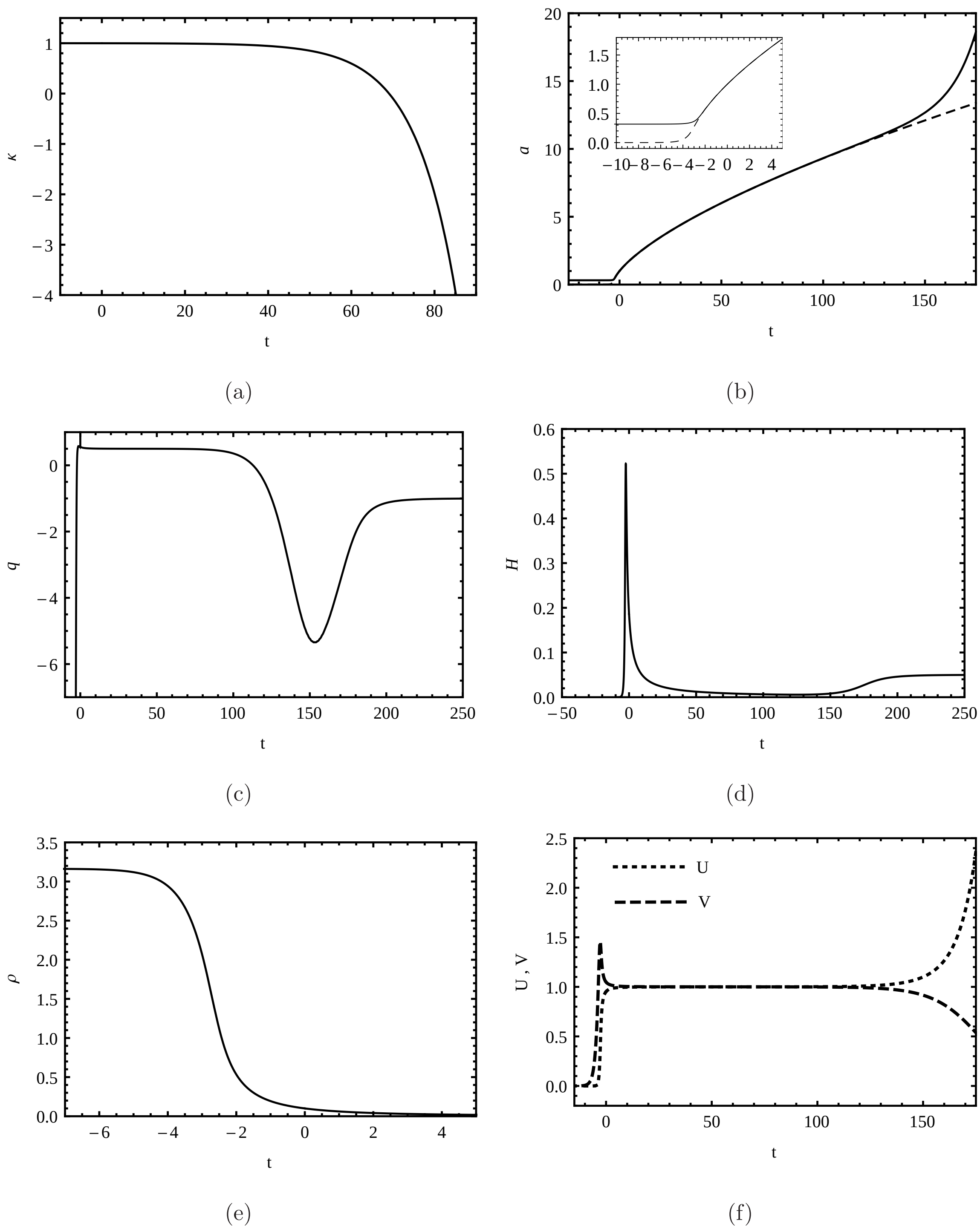


FIG. 2. Plot of (a) $\kappa(t)$, (b) $a(t)$, (c) $q(t)$, (d) $H(t)$, (e) $\rho(t)$, and (f) $U(t)$ (dotted line), $V(t)$ (dashed line) for $\kappa_0 > 0$, $\epsilon < 0$, and $\mu > 0$. The parameters used are $\kappa_0 = 1$, $\mu = 0.1$, and $\rho_0 = 0.1$. We choose $a(0) = 1$, $y(0) = 1.001$, $\kappa(0) = 0.999$ for the numerical solution. The dashed black curve in (b) corresponds to the EiBI solution with $\kappa = \kappa_0$ (constant). E.O.S. $p = 0$.

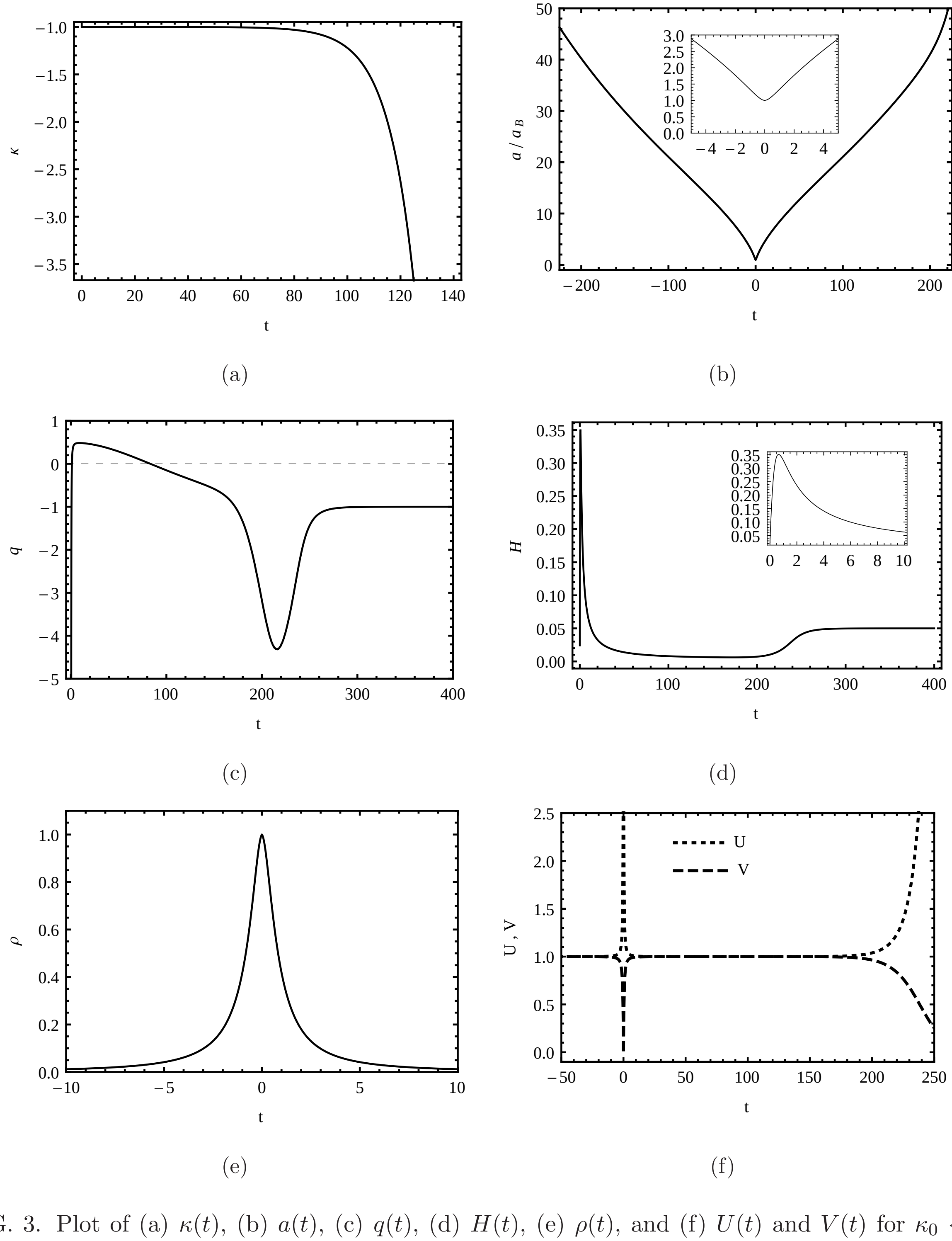


FIG. 3. Plot of (a) $\kappa(t)$, (b) $a(t)$, (c) $q(t)$, (d) $H(t)$, (e) $\rho(t)$, and (f) $U(t)$ and $V(t)$ for $\kappa_0 < 0$, $\epsilon < 0$, and $\mu > 0$. The parameters used are $\kappa_0 = -1$, $\mu = 0.1$, and $\rho_0 = 0.01$. We choose $a(0) = (-\kappa_0\rho_0)^{1/3}$, $y(0) = 0.9999$, $\kappa(0) = -1.00001$ for the numerical solution. E.O.S. $p = 0$. Evolution of $a(t)$ and $H(t)$ near the *bounce* are shown in the insets of (b) and (d).

solutions reveal an early loitering phase for $\kappa_0 > 0$ and a bounce for $\kappa_0 < 0$, as expected (κ approaches the constant value κ_0 at early times, i.e. $\kappa \to \kappa_0$ as $t \to -\infty$). Thus, the early Universe is still nonsingular. However, in both the cases, for $\epsilon > 0$, a singularity appears at a finite future time t_f where Hdiverges ($H \to -\infty$ as $t \to t_f$). The scale factor $a(t)$ and the energy density $\rho(t)$ though remain finite at t_f.

2. $\mu < 0$ case:

For $\mu < 0$, κ approaches κ_0 asymptotically as $t \to \infty$. Thus, the solutions tend to the EiBI solutions for constant κ_0, at large t. In this case also, a nonsingular Universe is found for $\epsilon < 0$. However, we do not see a loitering early stage for $\kappa_0 > 0$. A bounce occurs for both $\kappa_0 > 0$ and $\kappa_0 < 0$. We note that an accelerated contraction precedes the decelerated contraction, before the bounce occurs. These features are shown in Figs. 4 and 5. Figs. 4(c) and 5(c) show that $q \to -1$ as $t \to -\infty$. H approaches a constant negative value during this period (see the inset of Fig. 4(d) and the Fig. 5(d)). Also, we see that $q \sim 0.5$ in between the bounce and accelerated contraction phase, and throughout the future time after the bounce. Thus, evolution of the scale factor is GR like during these periods. It may also be noted that $U \sim 1$ and $V \sim 1$ in these phases.

The solutions are singular for $\epsilon > 0$. Therefore, we only mention the results, but do not show the plots. For $\kappa_0 > 0$ and $\epsilon > 0$, there may exist a big bang singularity. The Universe may also begin with a singularity at $t = -t_f$ where H diverges ($H(-t_f) \to \infty$), but a and ρ are finite. The last kind of singularity always occurs for $\kappa_0 < 0$ and $\epsilon > 0$. However, future singularities do not occur unlike the case, $\mu > 0$ and $\epsilon > 0$.

C. $p = \rho/3$ case

We now turn to a Universe filled with the radiation ($p = \rho/3$). We have $\rho = \rho_0/a^4$, and

$$U = \frac{(2 - y - \kappa\rho/3)^{3/2}}{\sqrt{y + \kappa\rho}} \quad \text{and} \quad V = \sqrt{(y + \kappa\rho)(2 - y - \kappa\rho/3)}. \tag{30}$$

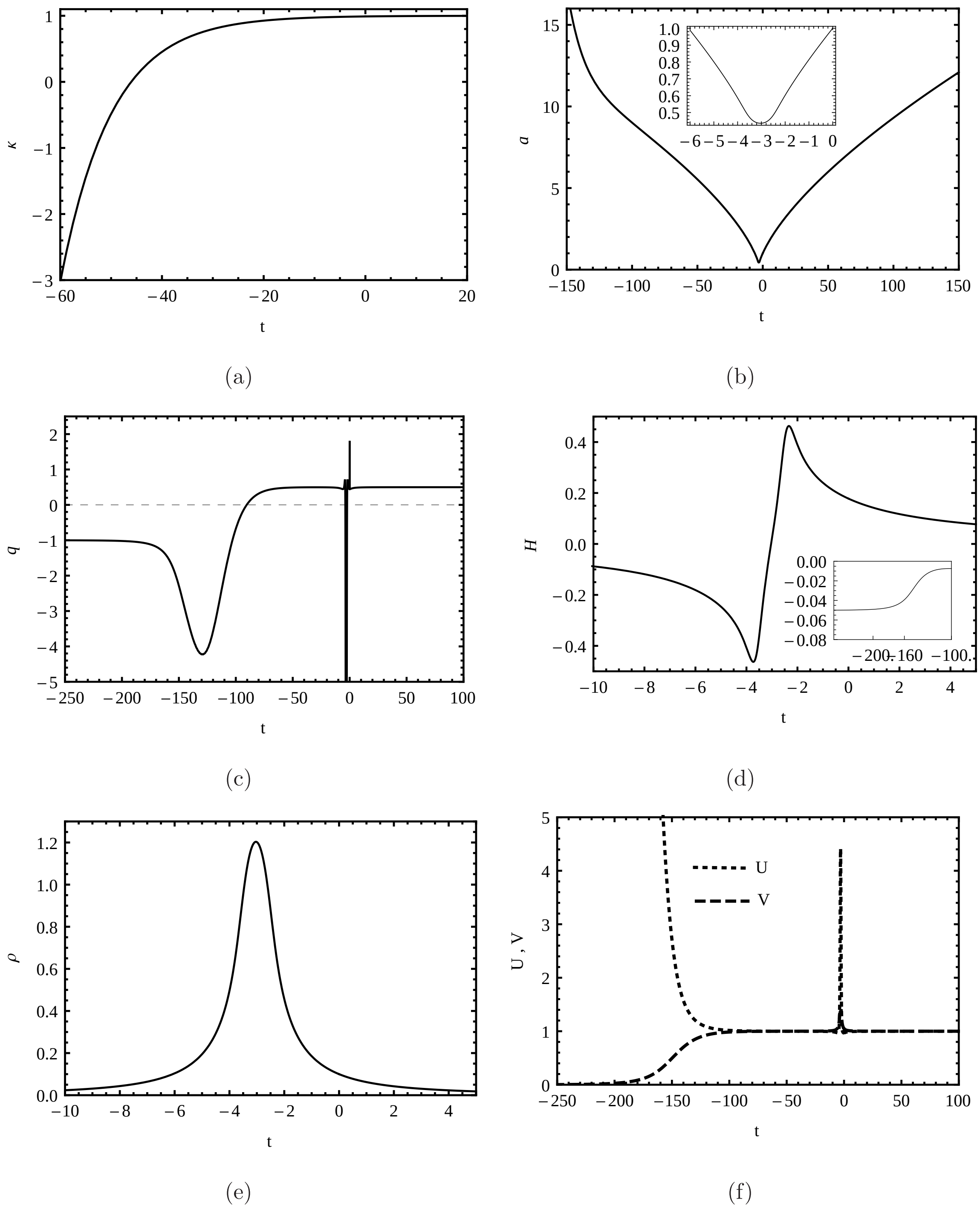


FIG. 4. Plot of (a) $\kappa(t)$, (b) $a(t)$, (c) $q(t)$, (d) $H(t)$, (e) $\rho(t)$, and (f) $U(t)$ and $V(t)$ for $\kappa_0 > 0$, $\epsilon < 0$, and $\mu < 0$. The parameters used are $\kappa_0 = 1$, $\mu = -0.1$, and $\rho_0 = 0.1$. We choose $a(0) = 1$, $y(0) = 0.99$, $\kappa(0) = 0.99$ for the numerical solution. E.O.S. $p = 0$. Evolution of $a(t)$ about the *bounce* is shown clearly in inset of (b). Inset of (d) shows that $H(t)$ approaches a constant negative value as $t \to -\infty$.

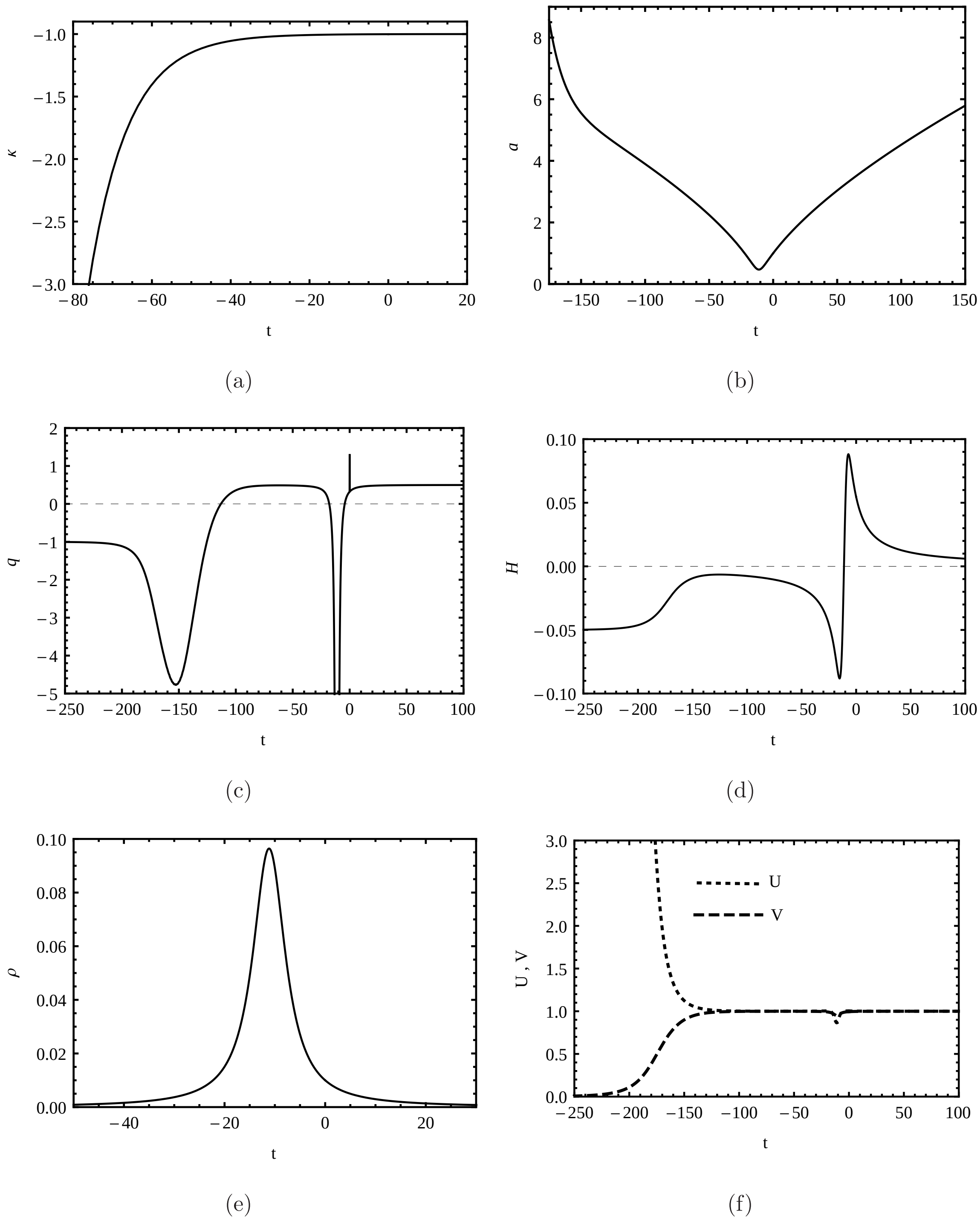


FIG. 5. Plot of (a) $\kappa(t)$, (b) $a(t)$, (c) $q(t)$, (d) $H(t)$, (e) $\rho(t)$, and (f) $U(t)$ and $V(t)$ for $\kappa_0 < 0$, $\epsilon < 0$, and $\mu < 0$. The parameters used are $\kappa_0 = -1$, $\mu = -0.1$, and $\rho_0 = 0.01$. We choose $a(0) = 1$, $y(0) = 1.001$, $\kappa(0) = -1.001$ for the numerical solution. E.O.S. $p = 0$.

Thus, F_1, H, and F_β are now given as,

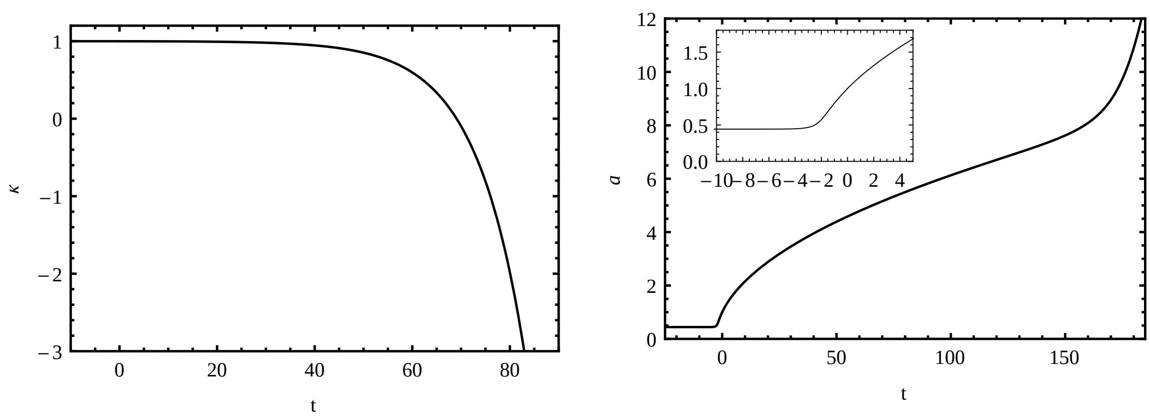
$$F_1 = \frac{4y - 6 + 2\kappa\rho + 2\sqrt{(y+\kappa\rho)(2-y-\kappa\rho/3)^3}}{y+\kappa\rho}, \tag{31}$$

$$H = \frac{(y+\kappa\rho)\left[4\beta\left(2-y-\frac{\kappa\rho}{3}\right) + \mu(\kappa-\kappa_0)\left\{\frac{F_1}{\kappa}\left(y+\frac{2\kappa\rho}{3}-1\right)-\frac{2\rho}{3}\right\}\right]}{4\left[(2y^2-4y+3)+2\kappa\rho\,(y-1)+\frac{\kappa^2\rho^2}{3}\right]}, \tag{32}$$

$$F_\beta = \frac{1}{12\beta\kappa}\left[\frac{\partial F_1}{\partial y}F_y - 4\rho\frac{\partial F_1}{\partial \rho}H + \mu(\kappa-\kappa_0)\left(\frac{\partial F_1}{\partial \kappa} - \frac{F_1}{\kappa}\right)\right]. \tag{33}$$

The expression of F_y remains unchanged (27). We solve the system of ODEs, $\dot{a} = aH$, $\dot{y} = F_y$, $\dot{\beta} = F_\beta$, and $\dot{\kappa} = \mu(\kappa - \kappa_0)$ numerically.

In the solutions, we note, qualitatively, the same features as seen in the $p = 0$ case. A notable difference is that during the GR like phases, $q \sim 1$. This is due to the fact that, in GR, for a radiation filled Universe, $a(t) \propto t^{1/2}$. Here also, the nonsingular solutions are found for $\epsilon < 0$, irrespective of the sign of μ and κ_0. We illustrate some of the nonsingular scale factors through the plots in Figs. 6 and 7.

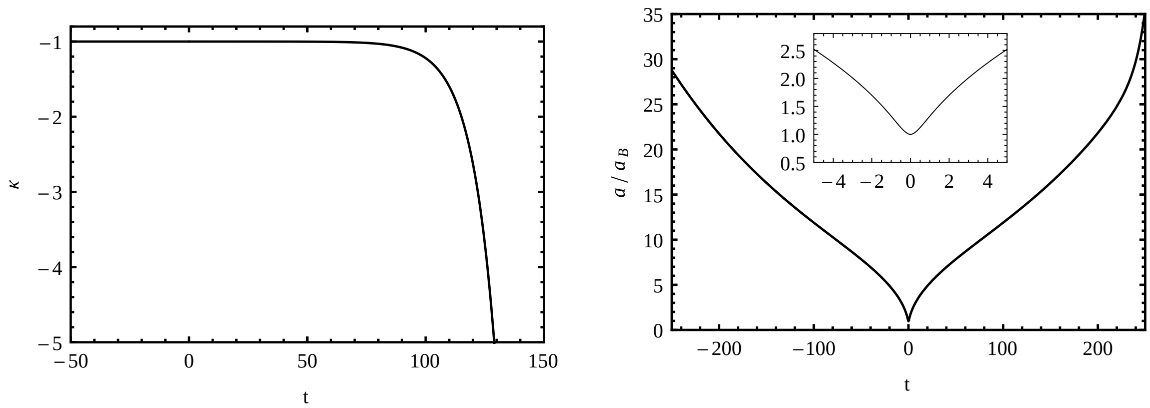


(a)

(b)

FIG. 6. Plot of (a) $\kappa(t)$ and (b) $a(t)$ for $\kappa_0 > 0$, $\epsilon < 0$, and $\mu > 0$. The parameters used are $\kappa_0 = 1$, $\mu = 0.1$, and $\rho_0 = 0.1$. We choose $a(0) = 1$, $y(0) = 1.001$, $\kappa(0) = 0.999$ for the numerical solution. E.O.S. $p = \rho/3$. Inset of (b) shows the *loitering* phase where $a(t)$ approaches a nonzero minimum value asymptotically at past.

Apart from dust and radiation, we have also looked at the vacuum case. It turns out that for $\kappa_0 > 0, \epsilon < 0, \mu > 0$, the solution for the scale factor is qualitatively the same as in the $p = 0$ or $p = \frac{\rho}{3}$ cases. However, with $\kappa_0 < 0, \epsilon < 0, \mu > 0$ we do not obtain a bounce but a big-bang singularity.



(a) (b)

FIG. 7. Plot of (a) $\kappa(t)$ and (b) $a(t)$ for $\kappa_0 < 0$, $\epsilon < 0$, and $\mu > 0$. The parameters used are $\kappa_0 = -1$, $\mu = 0.1$, and $\rho_0 = 0.01$. We choose $a(0) = (-\kappa_0\rho_0)^{1/4}$, $y(0) = 0.9999$, $\kappa(0) = -1.00001$ for the numerical solution. Evolution of $a(t)$ near the *bounce* is shown in the inset of (b). E.O.S. $p = \rho/3$.

IV. CONCLUSIONS

In this article, we have explored the possibility of κ, the Born-Infeld parameter in EiBI gravity, being a real scalar field. In this way, we have proposed a new theory of gravity by extending EiBI gravity in a manner similar to scalar-tensor theories. The action, equations of motion, energy-momentum conservation and the Newtonian limit of the theory have all been worked out.

In order to derive some of the consequences of this new theory, we studied cosmology as an example. After choosing a specific form of $\kappa(t)$, we solved the field equations numerically for spatially flat FRW spacetimes with (i) dust ($p = 0$) and (ii) radiation ($p = \rho/3$) as matter. In the case of the original EiBI theory (i.e. with a constant κ), we know that the solutions lead to a nonsingular early Universe, with a loitering phase for $\kappa > 0$ and a bounce for $\kappa < 0$. Further, the solutions reduce to those in GR at late times. In our work here, the choice of the scalar $\kappa(t) = \kappa_0 + \epsilon \exp(\mu t)$ (κ_0, ϵ, and μ are constants) broadly leads to qualitatively similar features for both $p = 0$ and $p = \rho/3$. However there are important additional features which arise. We summarize them point wise below:

- Unlike the EiBI solutions, here, the solutions are not always nonsingular. For $\epsilon < 0$, the solutions are nonsingular irrespective of the signs of κ_0 and μ. The solutions with an early loitering phase of the Universe were found for $\kappa_0 > 0$, $\epsilon < 0$, and $\mu > 0$. All other nonsingular solutions have a bounce in the early Universe.

- In EIBI gravity, with $p = 0$, the early Universe is de Sitter when the constant $\kappa > 0$. Therefore, $a \to 0$ at $t \to -\infty$. Consequently, $\rho \to \infty$ at $t \to -\infty$. In contrast, in our new theory, a never goes to zero for the solution with a loitering phase, and energy density remains finite for all t.

- Late time accelerated expansion of the Universe is an outcome for $\mu > 0$ and $\epsilon < 0$. The Universe becomes de Sitter ($q = -1$) asymptotically at large t. Note that this happens without any additional matter but only via the nature of $\kappa(t)$ and the structure of the theory.

- In the vicinity of the minimum value of the scale factor, or conversely at high energy densities, there is a deviation in the time evolution of the scale factor from that in GR. There are deviations at large values of the scale factor or conversely, low energy densities, where we noted acceleration. For intermediate values of the energy density, (or time scales), there exist GR like phases, as expected.

Our work here is a glimpse of the interesting possibilities which may arise in this new theory. Much more work is surely required to probe its feasibility. For example, we would like to investigate whether there exists any nontrivial vacuum (or non-vacuum) spherically symmetric, static spacetimes in this new theory. This would be a major difference with EiBI gravity where the vacuum solution is the Schwarzschild solution of GR. A different vacuum solution will affect the solar system tests and put bounds on the new parameters that are used in choosing $\kappa(t)$. Cosmological perturbation theory as well as the study of gravitational waves in this theory might also be useful avenues to pursue in the context of this modified theory of gravity which encodes both a Born-Infeld structure as well as a Brans-Dicke character in its formulation.

An important issue which must be dealt with is the origin of the real scalar field κ. It is not appropriate to leave it as an ad-hoc entity. However, it is possible to speculate that such a scalar may have a higher dimensional origin following work in the context of string theory and in braneworld models.

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[1] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, et al., Tests of General Relativity with GW150914, Physical Review Letters 116 (22) (2016) 221101. arXiv:1602.03841, doi:10.1103/PhysRevLett.116.221101.

[2] M. Born, L. Infeld, Foundations of the New Field Theory, Proc. R. Soc. A 144 (852) (1934) 425–451.

[3] S. Deser, G. W. Gibbons, Born - Infeld - Einstein actions?, Classical and Quantum Gravity 15 (5) (1998) L35.

[4] A. Eddington, The Mathematical Theory of Relativity, Cambridge University Press, Cambridge, England, 1924.

[5] D. N. Vollick, Palatini approach to Born-Infeld-Einstein theory and a geometric description of electrodynamics, Phys. Rev. D 69 (2004) 064030. doi:10.1103/PhysRevD.69.064030.

[6] D. N. Vollick, Born-Infeld-Einstein theory with matter, Phys. Rev. D 72 (2005) 084026. doi:10.1103/PhysRevD.72.084026.

[7] D. N. Vollick, Black hole and cosmological space-times in Born-Infeld-Einstein theory, ArXiv e-prints (gr-qc/0601136). arXiv:gr-qc/0601136.

[8] M. Bañados, P. G. Ferreira, Eddington's Theory of Gravity and Its Progeny, Phys. Rev. Lett. 105 (2010) 011101. doi:10.1103/PhysRevLett.105.011101.

[9] C. J. Isham, A. Salam, J. Strathdee, f-Dominance of Gravity, Phys. Rev. D 3 (1971) 867–873. doi:10.1103/PhysRevD.3.867.

[10] J. H. C. Scargill, M. Banados, P. G. Ferreira, Cosmology with Eddington-inspired gravity, Phys. Rev. D 86 (2012) 103533. doi:10.1103/PhysRevD.86.103533.

[11] A. Schmidt-May, M. von Strauss, A link between ghost-free bimetric and Eddington-inspired Born-Infeld theoryarXiv:1412.3812.

[12] J. B. Jimnez, L. Heisenberg, G. J. Olmo, Infrared lessons for ultraviolet gravity: the case of massive gravity and Born-Infeld, Journal of Cosmology and Astroparticle Physics 2014 (11) (2014) 004.

[13] P. Pani, V. Cardoso, T. Delsate, Compact Stars in Eddington Inspired Gravity, Phys. Rev.

Lett. 107 (2011) 031101. doi:10.1103/PhysRevLett.107.031101.

[14] J. Casanellas, P. Pani, I. Lopes, V. Cardoso, Testing Alternative Theories of Gravity Using the Sun, The Astrophysical Journal 745 (1) (2012) 15.

[15] P. P. Avelino, Eddington-inspired Born-Infeld gravity: Astrophysical and cosmological constraints, Phys. Rev. D 85 (2012) 104053. doi:10.1103/PhysRevD.85.104053.

[16] Y.-H. Sham, L.-M. Lin, P. T. Leung, Radial oscillations and stability of compact stars in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 86 (2012) 064015. doi:10.1103/PhysRevD.86.064015.

[17] Y.-H. Sham, P. T. Leung, L.-M. Lin, Compact stars in Eddington-inspired Born-Infeld gravity: Anomalies associated with phase transitions, Phys. Rev. D 87 (2013) 061503. doi:10.1103/PhysRevD.87.061503.

[18] T. Harko, F. S. N. Lobo, M. K. Mak, S. V. Sushkov, Structure of neutron, quark, and exotic stars in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 88 (2013) 044032. doi:10.1103/PhysRevD.88.044032.

[19] H. Sotani, Observational discrimination of Eddington-inspired Born-Infeld gravity from general relativity, Phys. Rev. D 89 (2014) 104005. doi:10.1103/PhysRevD.89.104005.

[20] H. Sotani, Stellar oscillations in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 89 (2014) 124037. doi:10.1103/PhysRevD.89.124037.

[21] H. Sotani, Magnetized relativistic stellar models in Eddington-inspired Born-Infeld gravity, ArXiv e-printsarXiv:1503.07942.

[22] S.-W. Wei, K. Yang, Y.-X. Liu, Black hole solution and strong gravitational lensing in Eddington-inspired Born-Infeld gravity, European Physical Journal C 75 (2015) 253. arXiv:1405.2178, doi:10.1140/epjc/s10052-015-3469-7.

[23] H. Sotani, U. Miyamoto, Properties of an electrically charged black hole in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 90 (2014) 124087. doi:10.1103/PhysRevD.90.124087.

[24] G. J. Olmo, D. Rubiera-Garcia, H. Sanchis-Alepuz, Geonic black holes and remnants in Eddington-inspired Born-Infeld gravity, The European Physical Journal C 74 (3) (2014) 2804. doi:10.1140/epjc/s10052-014-2804-8.

[25] R. Shaikh, Lorentzian wormholes in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 92 (2015) 024015. doi:10.1103/PhysRevD.92.024015.

[26] S. Jana, S. Kar, Born-Infeld gravity coupled to Born-Infeld electrodynamics, Phys. Rev. D 92

(2015) 084004. doi:10.1103/PhysRevD.92.084004.

[27] D. Bazeia, L. Losano, G. J. Olmo, D. Rubiera-Garcia, Geodesically complete BTZ-type solutions of $2+1$ Born-Infeld gravity, Classical and Quantum Gravity 34 (4) (2017) 045006.

[28] V. I. Afonso, G. J. Olmo, D. Rubiera-Garcia, Scalar geons in Born-Infeld gravity, ArXiv e-printsarXiv:1705.01065.

[29] Y.-X. Liu, K. Yang, H. Guo, Y. Zhong, Domain wall brane in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 85 (2012) 124053. doi:10.1103/PhysRevD.85.124053.

[30] T. Delsate, J. Steinhoff, New Insights on the Matter-Gravity Coupling Paradigm, Phys. Rev. Lett. 109 (2012) 021101. doi:10.1103/PhysRevLett.109.021101.

[31] I. Cho, H.-C. Kim, New synthesis of matter and gravity: A nongravitating scalar field, Phys. Rev. D 88 (2013) 064038. doi:10.1103/PhysRevD.88.064038.

[32] S. Jana, S. Kar, Three dimensional Eddington-inspired Born-Infeld gravity: Solutions, Phys. Rev. D 88 (2013) 024013. doi:10.1103/PhysRevD.88.024013.

[33] P. Pani, T. P. Sotiriou, Surface Singularities in Eddington-Inspired Born-Infeld Gravity, Phys. Rev. Lett. 109 (2012) 251102. doi:10.1103/PhysRevLett.109.251102.

[34] H.-C. Kim, Physics at the surface of a star in Eddington-inspired Born-Infeld Gravity, Phys. Rev. D 89 (2014) 064001. doi:10.1103/PhysRevD.89.064001.

[35] S. D. Odintsov, G. J. Olmo, D. Rubiera-Garcia, Born-Infeld gravity and its functional extensions, Phys. Rev. D 90 (2014) 044003. doi:10.1103/PhysRevD.90.044003.

[36] K. Fernandes, A. Lahiri, Kaluza Ansatz applied to Eddington inspired Born-Infeld gravity, Phys. Rev. D 91 (2015) 044014. doi:10.1103/PhysRevD.91.044014.

[37] I. Cho, H.-C. Kim, T. Moon, Universe driven by a perfect fluid in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 86 (2012) 084018. doi:10.1103/PhysRevD.86.084018.

[38] C. Escamilla-Rivera, M. Banados, P. G. Ferreira, Tensor instability in the Eddington-inspired Born-Infeld theory of gravity, Phys. Rev. D 85 (2012) 087302. doi:10.1103/PhysRevD.85.087302.

[39] K. Yang, X.-L. Du, Y.-X. Liu, Linear perturbations in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 88 (2013) 124037. doi:10.1103/PhysRevD.88.124037.

[40] P. P. Avelino, R. Z. Ferreira, Bouncing Eddington-inspired Born-Infeld cosmologies: An alternative to inflation?, Phys. Rev. D 86 (2012) 041501. doi:10.1103/PhysRevD.86.041501.

[41] I. Cho, H.-C. Kim, T. Moon, Precursor of Inflation, Phys. Rev. Lett. 111 (2013) 071301.

doi:10.1103/PhysRevLett.111.071301.

[42] I. Cho, H.-C. Kim, Inflationary tensor perturbation in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 90 (2014) 024063. doi:10.1103/PhysRevD.90.024063.

[43] I. Cho, N. K. Singh, Scalar perturbation produced at the pre-inflationary stage in Eddington-inspired Born-Infeld gravity, European Physical Journal C 75 (2015) 240. arXiv:1412.6344, doi:10.1140/epjc/s10052-015-3458-x.

[44] X.-L. Du, K. Yang, X.-H. Meng, Y.-X. Liu, Large scale structure formation in Eddington-inspired Born-Infeld gravity, Phys. Rev. D 90 (2014) 044054. doi:10.1103/PhysRevD.90.044054.

[45] I. Cho, J.-O. Gong, Spectral indices in Eddington-inspired Born-Infeld inflation, Phys. Rev. D 92 (2015) 064046. doi:10.1103/PhysRevD.92.064046.

[46] I. Cho, N. K. Singh, Primordial power spectra of Eddington-inspired Born-Infeld inflation in strong gravity limit, Phys. Rev. D 92 (2015) 024038. doi:10.1103/PhysRevD.92.024038.

[47] I. Cho, N. K. Singh, Tensor-to-scalar ratio in Eddington-inspired Born-Infeld inflation, European Physical Journal C 74 (2014) 3155. arXiv:1408.2652, doi:10.1140/epjc/s10052-014-3155-1.

[48] A. De Felice, B. Gumjudpai, S. Jhingan, Cosmological constraints for an Eddington-Born-Infeld field, Phys. Rev. D 86 (2012) 043525. doi:10.1103/PhysRevD.86.043525.

[49] M. Lagos, M. Bañados, P. G. Ferreira, S. Garcia-Saenz, Noether identities and gauge fixing the action for cosmological perturbations, Phys. Rev. D 89 (2014) 024034. doi:10.1103/PhysRevD.89.024034.

[50] J. B. Jimnez, L. Heisenberg, G. J. Olmo, C. Ringeval, Cascading dust inflation in Born-Infeld gravity, Journal of Cosmology and Astroparticle Physics 2015 (11) (2015) 046.

[51] T. Harko, F. S. Lobo, M. K. Mak, Bianchi Type I Cosmological Models in Eddington-inspired BornInfeld Gravity, Galaxies 2 (4) (2014) 496–519. doi:10.3390/galaxies2040496.

[52] S. Jana, S. Kar, Born-Infeld cosmology with scalar Born-Infeld matter, Phys. Rev. D 94 (2016) 064016. doi:10.1103/PhysRevD.94.064016.

[53] F. Arroja, C.-Y. Chen, P. Chen, D. han Yeom, Singular instantons in Eddington-inspired-Born-Infeld gravity, Journal of Cosmology and Astroparticle Physics 2017 (03) (2017) 044.

[54] P. Avelino, Eddington-inspired Born-Infeld gravity: nuclear physics constraints and the validity of the continuous fluid approximation, Journal of Cosmology and Astroparticle Physics

2012 (11) (2012) 022.