Power Allocation for Uniform Illumination with Stochastic LED Arrays

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Abstract: In this letter, a simpler heuristic power allocation scheme is proposed for a random LED array to obtain uniform irradiance on the projection surface. This is done by considering a binomial point process (BPP) for modeling the LED location and using the Q-factor as a performance metric. Numerical results are provided to validate the proposed model and demonstrate its simplicity over existing LED geometries.

OCIS codes: (260.0260) Physical optics; (230.3670) Light-emitting diodes.

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1. Introduction

Light has traditionally been used for making objects visible to the naked eye. Lately, there has been tremendous interest in using it for free space communication [1]. This has simultaneously been accompanied by significant interest in light emitting diodes (LEDs) that have been replacing conventional light sources in almost all applications [2–4]. Fair amount of existing literature has focused on achieving uniform irradiance over a planar surface [5–8], beginning with the problem of finding the optimal LED geometry at the light source to achieve uniform irradiance [9]. This was done by using the irradiance distributions at the closest points on the incident surface. The case of LEDs using a freeform lens with a large view angle has been considered in [10]. More literature on similar themes is available in [11, 12].

The above literature has focused on a regular geometry with equal power allocation to individual source LEDs. While uniform illuminance is desirable, optimal power consumption is an extremely important factor in the design of LED light sources. To address this, recent literature has focused on power allocation, along with flexiblity in the LED source geometry to achieve uniform irradiance.

A trial and error approach for power allocation for uniform irradiance is used in [13] for a combination of circular square geometry in order to illuminate the edges of the incident surface. An evolutionary algorithm based optimization scheme is proposed in [14] to modify the power of LED transmitters to reduce the signal power fluctuation at the receiver. In [15], a genetic algorithm is proposed to optimize the refractive indices of the concentrators on receivers to achieve a uniform distribution of the received power. An optimal LED arrangement to achieve uniform irradiance is investigated as a convex optimization problem in [16].

In all the above, computationally intensive optimization routines were used for power allocation for the source LEDs to realise uniform irradiance on the incident surface. [16] departs from the conventional model by considering arbitary locations for the source LEDs. The most practical scenario would be the case when the LEDs are placed randomly at the source with uniform illumination being achieved through power allocation, keeping the total power constant. This problem is addressed in this paper by considering a BPP based stochastic geometry [17]. Further, a simple metaheuristic power allocation scheme is proposed for uniform irradiance on the incident surface. Through numerical results, it is shown that the performance of the BPP model and the associated power allocation is comparable to the model in [13].

Rest of the paper is organized as follows. Section 2 deals with the optical signal and noise models. Section 3 contains details regarding the arrangement of different LED arrays and their performance. Heuristic power allocation for random LED sources is discussed in Section 4. The performance of the proposed model is discussed in Section 5, followed by conclusions in Section 6.

2. Preliminaries

2.1. *Irradiance*

Using the Lambertian radiation pattern to model the LED radiant irradiance, [3, 4]

$$R(\phi) = \frac{(m+1)\cos^m(\phi)}{2\pi}, \tag{1}$$

where ϕ is the angle of incident light and

$$m = \frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\cos\left(\phi_{\frac{1}{2}}\right)\right)}, \tag{2}$$

is the order of Lambertian emission, with $\phi_{\frac{1}{2}}$ being the LED semi-angle at half power, provided by the manufacturer. The channel direct current (DC) gain can then be expressed as [3, 4]

$$H = \frac{R(\phi)\cos(\theta)A}{d^2} = \frac{(m+1)\cos^m(\phi)\,A\cos(\theta)}{2\pi d^2} \tag{3}$$

where d is the distance between the LED and the photo-detector, A is the physical area of photodetector,$cos\,(\phi) = \frac{h}{d}$ and θ is the inclination of the photodector to the incident surface. Note that all the LEDs are located vertical to the plane where the photo-detector is placed. The received power at the photodetector is given by

$$P_r = HP_t \tag{4}$$

where P_t is the LED power.

2.2. *SNR*

The SNR at the output of the photo detector is

$$\Lambda = \frac{(RHP_t)^2}{\sigma^2} \tag{5}$$

where R is the responsivity of the photo-detector and σ^2 is the variance of the Gaussan noise at the photodetector [18].

3. Problem Definition

Consider the various source geometries for $N = 16$ LEDs in Fig. 1. Using (5), the respective SNR profiles for the sources are plotted in Fig. 2, when each of the LEDs has equal power. From Fig. 2d, it is obvious that the arrangement in Fig. 1d has a more uniform SNR profile, since the coverage at the edges is better. Circular geometries are limited by their inability to sufficiently illuminate the corners of the incident surface. Thus, with better power allocation, it should be possible for the source in Fig. 1d to achieve more uniformity.

3.1. *BPP*

In a BPP stochastic array, N LEDs are placed randomly within a square of length l at the points $(x_n, y_n) : x_n, y_n \sim U(-l/2, l/2), \forall n = \{1 \cdots N\}$, according to a uniform distribution U defined by

$$p_U(u) = \begin{cases} \frac{1}{l} & -\frac{l}{2} \leq u \leq \frac{l}{2} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The random BPP in Fig. 1c offers more flexiblity in the source geometry, and is useful in applications such as visible light communication [17]. However, to achieve a uniform SNR profile using a BPP, optimal power allocation is mandatory. This problem is addressed in the following.

4. Power Allocation for a BPP array

The received optical power at the photodetector j is

$$P_{r_j} = \sum_{i=1}^{N} H_{i,j}.P_{t_i} \tag{7}$$

where $H_{i,j}$, defined in (3) is dependent on the distance $d_{i,j}$ between LED i and photodetector j . The Q-factor, a metric to evaluate the fairness of the system, is defined as [13]

$$Q_{\Lambda} = \frac{\overline{\Lambda}}{2\sqrt{\mathrm{var}(\Lambda)}} \tag{8}$$

where $\overline{\Lambda}$ is average SNR and var(Λ) represents its variance. For a BPP, each LED is at a random location, so, heuristically, the power should also depend on the distance of the LED from the center of the array. The proposed power allocation is

$$P_{t_i} = \frac{r_i^{\alpha}}{\sum_{i=1}^{N} r_i^{\alpha}} P \tag{9}$$

where P is the total source power, r_i is the location of the ith LED from the centre, α is a suitable exponent and P_i is the power allocated to the ith LED. An optimal value of α can be obtained using the average mean square error metric defined below.

$$\min_{\alpha} E\left[\left(P_{r_j} - E\left[P_{r_j}\right]\right)^2\right] \tag{10}$$

A simple search routine results in $\alpha = 3.1$. This value is used in (5) and results in uniform SNR at the receiver plane.

5. Results

The noise at the photodetector is the sum of the contributions from shot noise and thermal noise, and expressed as [18]

$$\sigma^2 = \sigma^2_{shot} + \sigma^2_{thermal} \tag{11}$$

where

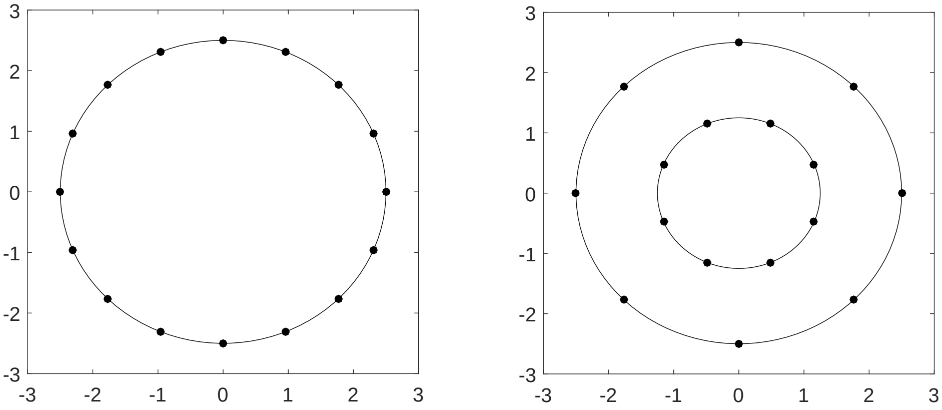
$$\sigma^2_{shot} = 2qR(P_r + P_{rISI})B + 2qI_{bg}I_2B \tag{12}$$

$$\sigma^2_{thermal} = \frac{8\pi k T_k}{G}\eta A I_2 B^2 + \frac{16\pi^2 k T_k \Gamma}{g_m}\eta^2 A^2 I_3 B^3 \tag{13}$$

with the parameters defined in Table 1.

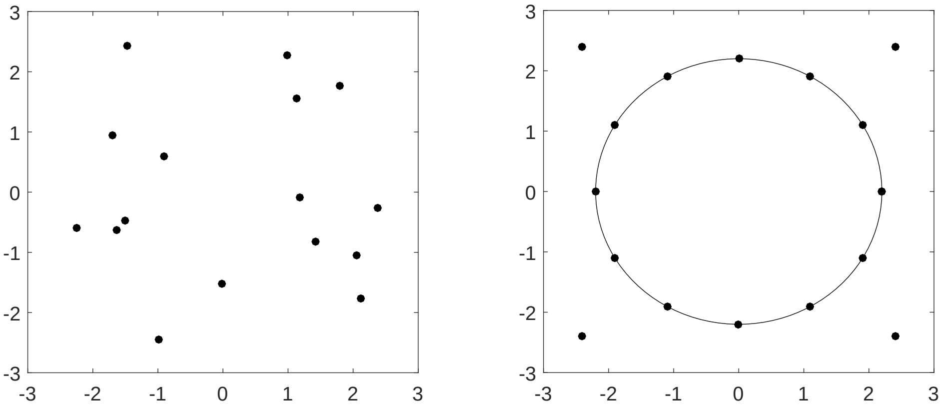
A trial and error based power allocation was done for the circle-square geometry in Fig. 1d using the average mean square error

$$\min_{P_{t_i}} E\left[\left(P_{r_j} - E\left[P_{r_j}\right]\right)^2\right] \tag{14}$$



(a) Circular geometry

(b) Concentric circular geometry



(c) BPP

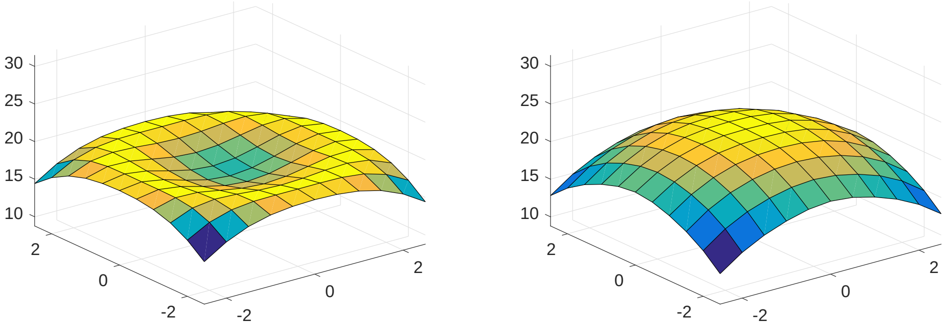
(d) Circle-square geometry in [13]

Fig. 1: Arrangement of LEDs for different geometries

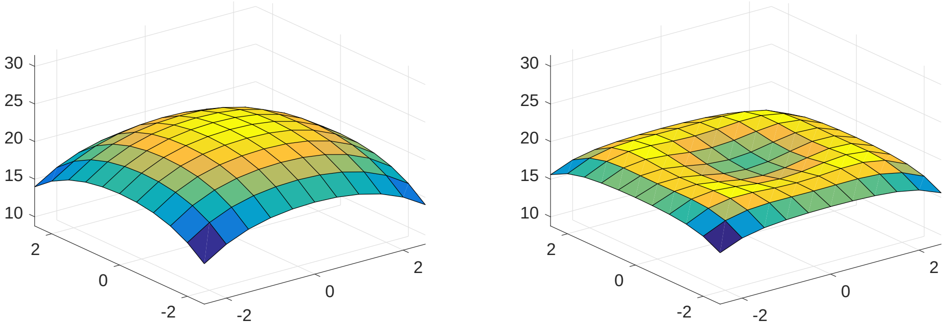
as a metric. This results in uniform illuminance, as shown in Fig. 3a. With the heuristic power allocation scheme in (9), the BPP in Fig. 1d also yields uniform illuminance as shown in Fig. 3b. The Q values defined in (8), for both geometries are listed in Table 2. It is clear that the Q value for the BPP in Fig. 1c is comparable to the one in Fig. 1d. Also, the mean SNR for random geometry is greater than that for circle-square geometry. However, the variance for the BPP model is greater, which reduces its Q value.

6. Conclusion

From the above analysis, it is clear that a BPP arrangement of LEDs can achieve uniform illumination. This makes it extremely useful in practical applications like visible light communication where the source geometry is likely to be random. Also, the proposed heuristic for power allocation is much simpler, resulting in reduced computational cost, when compared to existing optimal power allocation schemes. Finding a simple but optimal power allocation scheme for stochastic LED arrays will be the focus of future work.

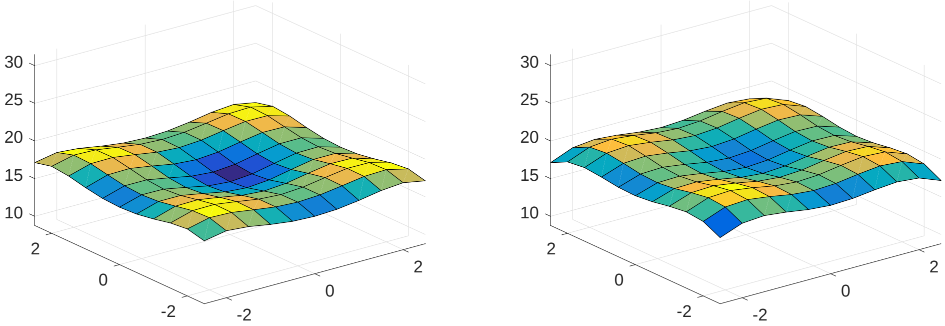


(a) Circuar geometry (b) Concentric Circular geometry



(c) BPP (d) Circle-square geometry

Fig. 2: SNR distribution with equal power allocation



(a) Circle-square geometry with optimal power allocation

(b) BPP with heuristic power allocation

Fig. 3: Uniform SNR profile with optimal/suboptimal power allocation

Parameters	**Symbol**	**Configuration**
Room size	l x b x h	$5mx5mx3m$
Height of receiver plane	H	$0.85m$
Boltzmann constant	k	$1.38064852x10^{-23}m^2kgs^{-2}K^{-1}$
electronic charge	q	1.60217662ÃŮ$10^{-19}C$
LED semiangle	$\phi_{\frac{1}{2}}$	60^o
Area of Photo detector	A	$10^{-4}m^2$
Fixed capacitance of photo-detector	ν	$112pF/cm^2$
Responsivity	R	$1A/W$
Noise bandwidth	B	$100MHz$
Background current	I_{bg}	$5100\mu A$
Noise bandwidth factor	I_2, I_3	$0.562, 0.0868$
Absolute temperature	T_k	$295K$
Open-loop voltage gain	G	10
FET cahnnel noise factor	Γ	1.5
FET transconductance	g_m	$30mS$

Table 1: Parameters of VLC system

	Circle-square		BPP	
	Equal Power	Optimal Power	Equal Power	Proposed heuristic
$\overline{\Lambda}\,(dB)$	18.2658	17.3447	20.1121	18.8510
var $(\Lambda)\,(dB)$	21.4585	17.8065	33.5970	21.1082
Q_Λ	2.8355	3.4924	1.0723	3.3780

Table 2: SNR performance