

# Reliable and Affordable Telecardiology Under Resource Constraints

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The Degree of Master of Technology



Department of Electrical Engineering

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## Declaration

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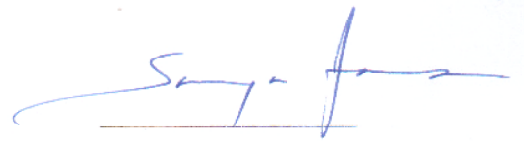
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## Approval Sheet

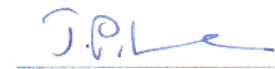
This Dissertation entitled "Reliable and Affordable Telecardiology Under Resource Constraints" by Bollepalli S Chandra is approved for the degree of Masters in Technology from IIT Hyderabad.




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## Abstract

Cardiovascular diseases (CVDs) are a leading cause of death accounting for more than 30% of global deaths. Unfortunately, traditional CVD management practices, involving hospital visits and health monitoring at professional facilities, are often too expensive or unavailable for various communities. In this context, telecardiology systems that simply records and transmits user electrocardiogram (ECG) signals to a professional diagnostic facility appears to be an attractive alternative. Conventionally, such systems transmit entire data unaltered to a professional diagnostic center, achieving diagnostic accuracy of professional bedside setup. However, such high-accuracy comes at high bandwidth cost. Further, conventional telecardiology system design ignores the infrastructural constraints in remote communities. In this backdrop, we proposed low-cost telecardiology solutions to two practical scenarios that require efficient resource utilization. Firstly, we address the problem of remote resource constrained telecardiology. Specifically, to serve the remote communities with severe infrastructural constraints (power and bandwidth), we propose a novel telecardiology framework, where resource constraints, are met by compressively sampling ECG signals, identifying anomalous signals, and transmitting only the anomalous signals. We propose compressive sampling as a low-power alternative to traditional Nyquist sampling method, which also lowers bandwidth requirement. Further, assuming ECG signals to be selfsimilar we design a practical compressive classifier to reduce bandwidth requirement. Finally, we illustrate our method by designing such a compressive classifier using ECG signals from the widely used PhysioNet database. Having demonstrated the resource-constrained telecardiology, we now address the problem of reliable low-cost telecardiology for continuous monitoring. Specifically, in subjects with heart conditions, continuous monitoring to detect various arrhythmia that interfere with normal functioning of heart assumes significance. In this context, monitoring using telecardiology systems appear attractive. However, high-cost of monitoring using conventional telecardiology systems remains a major hurdle. In this context, we propose a low-cost telecardiology framework that detects and transmits only anomalous beats to diagnostic center, where all received beats are correctly (re)classified. In this framework, high reliability is achieved by detectors with high sensitivity. We realized the desired high-sensitivity detection using a dictionary learning approach. Finally, we compare our results with reported heart-beat classifiers, and demonstrate the suitability of our approach in the context of telecardiology.



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# Chapter 1

## Introduction

Cardiovascular diseases (CVDs) are a leading cause of death across economic strata. According to World Health Organization (WHO), over 80% of world's death due to cardiovascular diseases (CVDs) takes place in developing and underdeveloped countries, where majority of the population reside in remote villages [1]. Traditional CVD management, involving consultations, testing and monitoring at medical facilities. An indispensable aid in diagnosing and managing cardiovascular diseases is electrocardiogram (ECG) that records the electrical activity of heart. In certain scenarios, including high-risk-patient care, ECG from a subject is continuously monitored to detect deviation from normal sinus rhythm. However, practical difficulties arise when only a few general physicians (or nurses), but no experts in cardiology, are available locally for on-site monitoring. In such situations, need based transportation of experts, despite being both time consuming and expensive, used to be the only recourse available in the past. With the advent of information technology, telecardiology, possibly accompanied by automated diagnostic assists, is fast becoming an attractive alternative [2,3]. Specifically, rather than physically relocating experts to the bedside of the patient, ECG signal collected from the patient is electronically transported to experts, thereby increasing overall responsiveness while bringing down cost.

### 1.1 Existing telecardiology systems

A conventional telecardiology system, depicted in Figure 1.1, acquires and transmits user ECG to the diagnostic center for professional diagnosis. A design framework for such systems has been presented, albeit in broader contexts [2,4]. In this framework, telephone based ECG transmission and associated clinical experience were investigated decades ago [5]. With growing ubiquity of mobile networks and their ability to provide pervasive health care services [6], various mobile based telecardiology systems have been reported [7, 8]. In recent years, various ZigBee based wireless systems for monitoring of elderly patients has also been presented [9]. To deliver telecardiology services in the remote and rural communities with rickety networks, a method to encode ECG signals to ASCII characters to communicate via SMS (short message service) has been reported [10].

Although, telecardiology is adopted in various generations of telecommunication technologies the underlying architecture of telecardiology system remains unchanged. Though such schemes prove to be effective in patient monitoring, they may not be directly suitable to the resource constrained

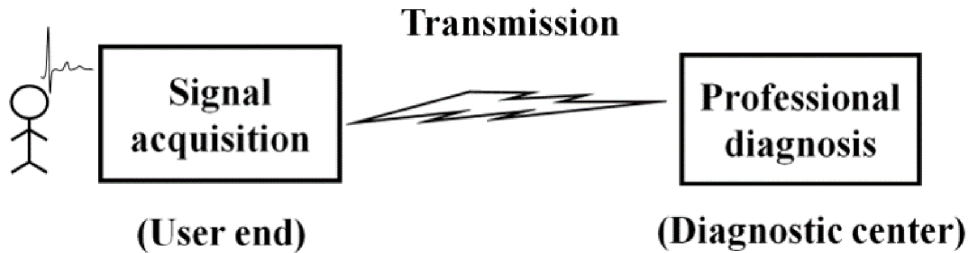


Figure 1.1: Traditional telecardiology architecture.

context. Consider a conventional telecardiology architecture for remote communities, where ECGs of remote users are transmitted over bandwidth constrained links to a diagnostic center staffed by experts to accurately detect anomalous beats. Traditionally, the entire signal would be transmitted, resulting in perfect reliability, albeit with the attendant high bandwidth requirement. In addition, manual processing of entire record is both time consuming and ineffective. In this context, we intend to tackle each of the problems to make the telecardiac management more efficient, and hence more affordable.

## 1.2 Social context

In the present work, we seek to develop a CVD management scheme that would appeal even to the economically disadvantaged communities. About 1.2 billion individuals live on less than US\$ 1.25 per day worldwide (about 276 million individuals in India alone), and have little discretionary income. To such individuals, the cost of professional monitoring could often be prohibitive. Further barriers to quality care could include travel and hospital expenses. Fortunately, high penetration of mobile phones even in remote communities has mitigated such barriers in certain scenarios [11]. In this backdrop, we ask: Can the mobile network be leveraged to provide reliable PVC monitoring at an attractive cost to the aforementioned communities living at the bottom of the economic pyramid [12]?

## 1.3 Scope of the thesis

A conventional telecardiac system, depicted in Fig. 1.1, acquires and transmits ECG signals to a diagnostic center, where a medical professional makes the diagnosis, and medical intervention is initiated, when required. Note that ECG signals are acquired by sampling faster than the Nyquist rate, which can practically be taken as about 500Hz [13]. In this framework, neither power nor bandwidth constraint is considered. Conversely, in a practical scenario, where there are power and bandwidth constraints, the aforementioned conventional scheme may not be appropriate. In this backdrop, we identified two scenarios encountered naturally that require efficient utilization of power and bandwidth resources.

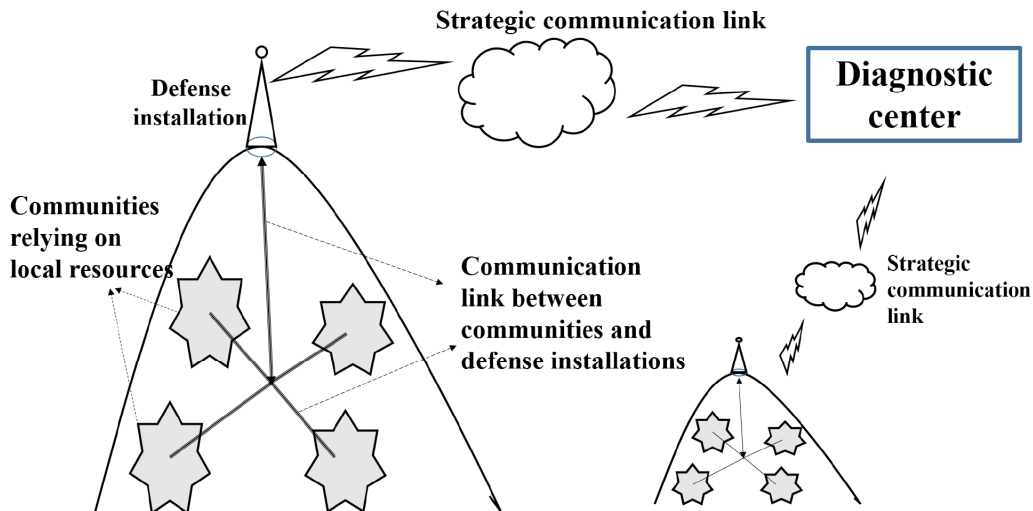


Figure 1.2: Motivating scenario for remote resource-constrained telecardiology.

### 1.3.1 Remote resource-constrained telecardiology

In the hilly northeastern part of India, cost of civic infrastructure development remains high due to general inaccessibility and remoteness. Consequently, a large population segment only has limited access to basic transportation, electricity, communication and healthcare. Generally, people are organized in small communities, each relying on local resources. Several such communities sometimes occupy one hill, but communities located across hills are often essentially isolated. In this context, one would naturally ask: Can technology be leveraged to bring basic healthcare to such subsistence communities?

At first glance, any feasible option would appear expensive. However, on closer inspection, one detects an opening. Although civic infrastructure is essentially nonexistent, interestingly (due to its geostrategic location), the region has a well developed defense infrastructure. In fact, each of a large number of hilltops has a small defense installation equipped with transmission and reception hardware, thus creating a vast communication network. Noting that such network is primarily meant for strategic communication, we reframe the aforementioned question: Can we enable affordable healthcare delivery by making sparing use of those existing communication links?

To proceed, we assume that each hill with one or more communities has a communication facility as shown in Fig. 1.2. We further assume that the communities are not connected to the electric grid, and experience electric power constraints. However, the strategic communication facility is fitted with its own adequate power source (power generator, solar power, or such like), and connected to a diagnostic center either directly or via multiple hops. In this backdrop, the objective is to make as little demand as possible on bandwidth over such links, and on the power available to the communities, while ensuring a certain level of healthcare service. From the economic angle, we also must require only marginal amount of additional infrastructure to make the scheme practical.

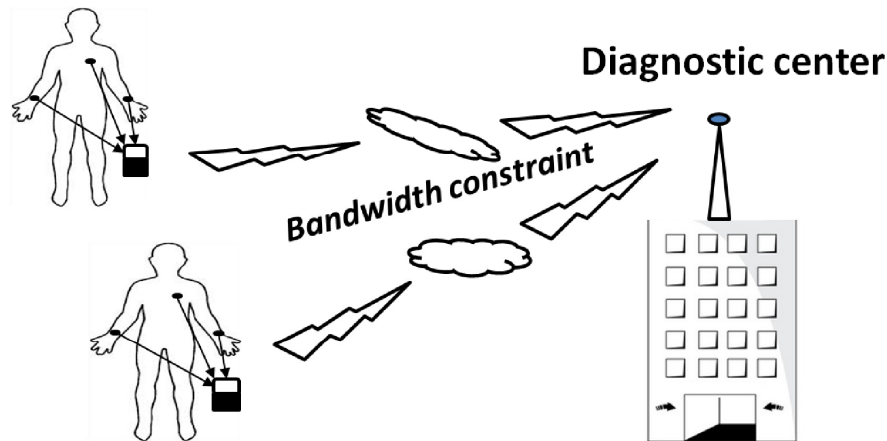


Figure 1.3: Motivating scenario for reliable low-cost telecardiology for continuous monitoring.

### 1.3.2 Reliable low-cost telecardiology for continuous monitoring

As alluded earlier, we seek to provide a low-cost telecardiology solution for individuals with average daily income of about US\$ 1.25. Consider an individual living at the economic threshold of this target population segment, who suffered heart attack in the recent past, and was successfully treated (see [14] for various treatment options). Post treatment, monitoring of the subject over long intervals has now assumed clinical significance as mentioned earlier. In this context, we shall investigate the cost associated with continuous monitoring using conventional telecardiology (depicted in Figure 1.3).

Considering a sampling rate of 360Hz and word length of 11 bits (also used in MIT/BIH arrhythmia database [15]), one would generate about 1.78MB of data per hour. Communicating the entire data to the diagnostic center would cost about US\$ 27 per hour at the rate of US\$ 1.5 per 100KB of data usage. At this rate, the cost of ten-hour monitoring would amount to US\$ 2.7. which is clearly unaffordable and subject would consider using such service for monitoring as non essential. In this backdrop, the objective is to provide reliable monitoring at cost that the user find little or no incentive to forgo it.

## 1.4 Contribution and organization

The rest of the thesis is organized as follows. In Chapter 2 we address the problem of CVD management in power and bandwidth constrained communities. Specifically, we propose a two-tier telecardiology system where automated classification is performed on the ECG signals, and only anomalous signals are transmitted for further diagnosis and intervention, thereby saving bandwidth. Additionally, we propose compressive sampling as a low-power alternative to traditional Nyquist sampling method, which also lowers bandwidth requirement. Finally, we illustrate our method by designing such a compressive classifier using ECG signals from the widely used PhysioNet database. Specifically, we demonstrate that an average down sampling factor of three leads to desirable classification performance in terms of both sensitivity and specificity while substantially saving both power and



bandwidth. The results of this chapter have been published in the following conference paper [16].

B. S. Chandra, C. S. Sastry and S. Jana, “Telecardiology: Hurst exponent based anomaly detection in compressively sampled ECG signals,” in *IEEE 15th International Conference on e-Health Networking, Applications & Services (Healthcom)*, pp. 350–354, October, 2013.

In Chapter 3, we provide a reliable telecardiology solution for continuous monitoring at low bandwidth cost. Specifically, we propose a detector at the user end so that only beats found to be anomalous are transmitted to a diagnostic center, where all received beats are correctly (re)classified. In this framework, high reliability is achieved by detectors with high sensitivity. Having laid the design framework, we then realize desired high-sensitivity detection using a dictionary learning approach. Specifically, using patient records from the MIT-BIH arrhythmia database, we detect ventricular ectopic beats (VEBs), which are known to be precursors to various serious arrhythmic conditions in the heart. We compare our results with performances a large set of reported heartbeat classifiers and demonstrate the suitability of our approach in the context of telecardiology. The results of this chapter have been published in the following conference paper [17].

B. S. Chandra, C. S. Sastry and S. Jana, “Reliable low-cost telecardiology: High-sensitivity detection of ventricular beats using dictionaries,” in *IEEE 16th International Conference on e-Health Networking, Applications & Services (Healthcom)*, pp. 305-310, October, 2014.

Finally, in Chapter 4, we summarized our contribution and discussed about the extension of the present work towards practical deployment.

## Chapter 2

# Remote resource-constrained telecardiology

A conventional telecardiac system, depicted in Figure 1.1, acquires and transmits ECG signals to a diagnostic center, where a medical professional makes the diagnosis, and medical intervention is initiated, when required. Note that ECG signals are acquired by sampling faster than the Nyquist rate, which can practically be taken as about 500Hz [13]. In this framework, neither power nor bandwidth constraint is considered. Conversely, in a scenario, where there are power and bandwidth constraints, the aforementioned conventional scheme may not be appropriate. In the face of bandwidth constraint, it would be natural to compress the recorded ECG data before transmission. At the receiver, one would then perform corresponding decompression. Unless compressed excessively, correct clinical diagnosis can be made based on the recovered data. However, if there is also a power constraint, this approach may not be suitable because executing effective compression algorithms generally requires significant power.

Interestingly, if there are both bandwidth and power constraints, one can make use of compressive sampling (CS) [18,19]. In this approach, the data can be sampled at an average rate far below the Nyquist rate, while allowing near-perfect reconstruction. This happens because CS reconstruction algorithms exploit signal sparsity rather than bandwidth limitation. Such low rate of sampling allows for low power operation while saving bandwidth. Of course, optimally designed compression algorithms would achieve higher overall compression compared to compressive sampling due to severe structural constraints put on the latter. Rather than compressing the signal with the aim of faithful reconstruction, and then classifying it based on the reconstructed signal, one can theoretically make the classification, and then transmit the classifier index. However, a classification accuracy matching human abilities would be difficult to achieve, would require intensive computation, and hence is infeasible in the face of power constraints. As an engineering middle ground, we seek to only differentiate between anomalous and normal signals. Assuming perfect classification, one need to send only the anomalous signals to the diagnostic center. In view of the report that only about 30% of the cases are anomalous [1], one could reduce the bandwidth requirement to the same percentage.

Motivated by this observation, we propose a telecardiology system, depicted in Fig. 2.1, which acquires compressively sampled ECG data, classifies those into anomalous and normals signals, and transmits the former to the diagnostic center in real time for possibly immediate response. The

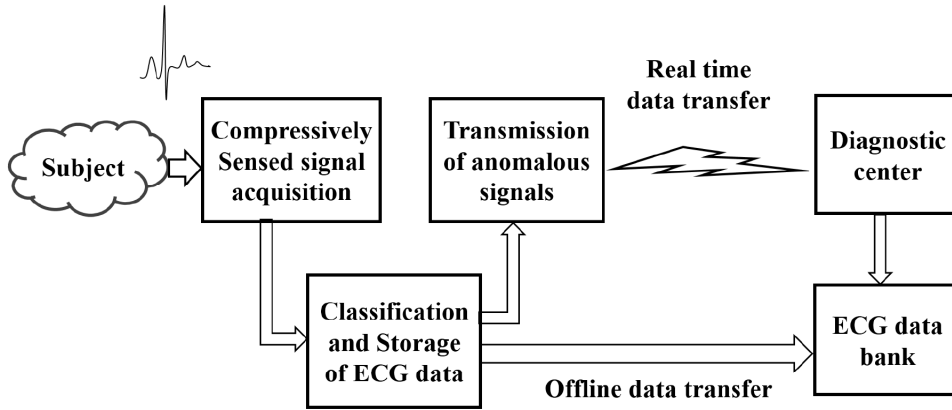


Figure 2.1: Proposed telecardiology model based on compressive sampling.

diagnostic center has the option of storing such anomalous data. If one wishes to eventually store all ECG data in a bank, one can transfer the entire data, including both anomalous and normal, to the data bank. In order to classify ECG signals, various signal processing techniques such as wavelet transform, and fractal analysis have been applied on ECG signals. A number of researchers make use of wavelets to estimate the characteristic points of ECG signals (P, Q, R, S, and T). These parameters are further exploited to identify the anomalies [20,21]. Other researchers attempt to detect anomalies based on fractal dimension and measures of self similarity such as Hurst exponent [22], [23]. We take reduction in self similarity as an indicator of anomaly, and use it to differentiate certain anomalous signals from the normal ones.

## 2.1 Telecardiology Solution

In this work, we shall confine ourselves to cardiac healthcare. Specifically, we wish to promptly intervene in a developing cardiac crisis within power and bandwidth resource constraints.

### 2.1.1 Assumptions

To achieve this end, we assume two minor investments: Each community is (i) furnished with a portable ECG device, and (ii) connected to the strategic communication facility via a dedicated local link (either wired and wireless). Also, medical professionals are available only at a remote diagnostic facility, and not locally (Figure 2.1). Further, ECG data are collected periodically, as well as in critical health situations. The goal is to respond to abnormalcy in a timely manner.<sup>1</sup> In a nutshell, we seek a tele-cardiology solution that requires (i) low power operation of portable ECG machines, and (ii) low requirement for bandwidth over the strategic communication links.

<sup>1</sup>Of course, we envisage tiered responses. At one end of the spectrum, minor aberrations can simply be addressed by medical advice, whereas at the other end, a serious condition such as heart attack may require deployment of air ambulance. Assuming that serious heart conditions arise infrequently, an air ambulance service, albeit expensive per deployment, may prove cost-effective as a healthcare measure for the geographically-dispersed remote population at hand.

### 2.1.2 Proposed architecture

Within our framework, let us first consider classical tele-cardiology. ECG signals are acquired, sampled, digitized, and transmitted to the diagnostic center, where the diagnosis is performed and response is formulated. Assuming a nominal sampling rate of 500 Hz [13], and word length of 12 bits, this requires 6000 bps. Although such bandwidth requirement is not high for an individual, total ECG data carried by the system could be substantial. Therefore, can one reduce the required bandwidth? Further, can one reduce operating power for portable ECGs?

Fortunately, the answer to both the above questions is yes, and the key to it lies in the sparsity properties of ECG signals. Specifically, uniform sampling of ECG signals warrants sampling faster than Nyquist rate, and empirically, one starts losing clinically important features below 500 Hz [13]. However, sparsity in the wavelet domain allows one to compressively (nonuniformly) sample the same signals at a lower average rate, and faithfully reconstruct from such samples. Assuming an effective downsampling ratio of  $k$ , one would reduce the effective bandwidth requirement by a factor of  $k$ . Assuming  $k = 3$ , one therefore would require only 2000 bps. Also, in the sampling circuitry, assuming that the power consumption is proportional to effective sampling rate, one would operate with only a third of original power.

At this point, note that one needs to respond to only anomalous cases, which consists of a small fraction (about 30%) of ECG signals [1]. If one could separate the anomalous signals from the normal ones, only the former would need to be transmitted leading to further savings. So, a classical telecardiology system fitted with a perfect classifier would operate at 1800 bps. Finally, if its compressively (nonuniformly) sampled cousin is fitted with a perfect classifier, then the bandwidth drops to only 600 bps (assuming affective downsampling by a factor of 3).

The next natural question is: Where do we classify? At the portable ECG machine itself, or just prior to transmission over the strategic link? Although the first option appears attractive at the first glance, note that such classification is computationally complex and hence power intensive, and may not be feasible due to power constraint. On the other hand, the option of classifying at the strategic installation suffers from no such constraints.

Summarizing, we propose an architecture, where compressively sampled ECG signals are acquired at a low effective rate, and classified based on such compressive samples prior to transmission to the diagnostic center. Although attractive from both bandwidth and power considerations, a practical issue arises: classifiers are generally not perfect, especially, those operating on compressively samples. So, one would like to analyze and optimize the performance of compressive classification. In other words, efficient design of the proposed system boils down to efficient design of compressive classifiers.

### 2.1.3 Classification using Compressive Samples: Design Criteria

An ideal classifier should classify all normal signals as normal, and all anomalous signals as anomalous. However, real classifiers are generally imperfect, with respect which all signals are divided into four subsets — (i) True positives: signals that are anomalous and classified as anomalous; (ii) True negatives: signals that are normal and classified as normal; (iii) False positives: signals that are normal and classified as anomalous; (iv) False negatives: signals that are anomalous and classified as normal. Further, a classifier performance is characterized in terms of sensitivity and specificity:

$$\text{Sensitivity} = \frac{\text{number of true positives}}{\text{number of true positives} + \text{number of false negatives}},$$

$$\text{Specificity} = \frac{\text{number of true negatives}}{\text{number of true negatives} + \text{number of false positives}}.$$

Although high sensitivity and high specificity are both desirable, these quantities enjoy an inverse relationship. Therefore, we seek a suitable tradeoff.

## 2.2 Theoretical Foundation

In this section, we provide brief accounts of various theoretical building blocks, and their interrelationship.

### 2.2.1 Self similarity

A signal  $\{x(t) : t \in (-\infty, \infty)\}$  is said to be self-similar with Hurst self-similarity exponent  $H$ , if and only if  $\{c^{-H}x(ct) : t \in (-\infty, \infty)\}$ ,  $\{x(t) : t \in (-\infty, \infty)\}$ ,  $\forall c > 0$  and  $t \in (-\infty, \infty)$  have same distributions, which is referred to as the scaling property of the process  $x$ . For a general self-similar process, the parameter  $H$  measures the degree of self-similarity. For random processes suitable for modeling self similar data,  $H$  measures of the speed of decay of the tail of the autocorrelation function.

### 2.2.2 Compressive sampling

Compressed Sensing (CS) aims at recovering high dimensional sparse vectors based on few linear measurements [24]. It refers to a problem of an economical recovery of an unknown signal  $x$  from its linear measurements  $\langle x, \phi_j \rangle$  with  $\phi_j \in R^m$ ,  $j = 1, 2, \dots, n$ . Here  $\langle x, \phi_j \rangle$  represents the inner-product of  $x$  over  $\phi_j$ . Signal recovery from measurements can be formalized as the following optimization problem:

$$\min_{\alpha} \|\alpha\|_0 \quad \text{subject to} \quad \Phi\alpha = y, \quad (2.1)$$

where  $\Phi$  is the matrix whose rows are  $\phi_j$ . The special case of compressive sampling arises when such the process of linear measurement reduces to keeping  $n$  nonuniformly spaced samples, and leaving out the rest  $m - n$  ones. In this case, the measurement matrix  $\Phi$  has rows with all entries zero except one entry of one, and the locations of those unity entries are distinct.

Signal recovery when the number  $n$  of measurements is much smaller than signal length  $m$  is of particular interest. Indeed, exact recovery is possible if (i)  $\Phi$  satisfies restricted isometry, and ii) the coherence parameter ( $\mu$ ), which is the maximum off-diagonal entry in  $\Phi^T\Phi$  (see e.g. [25]), is small.

### 2.2.3 Wavelet representation

A wavelet is a *little wave* that is both localized and oscillatory. The representation of a function in terms of wavelet basis, generated by dyadic scales and integer translates of wavelet involves a low frequency block and hierarchical high frequency blocks. A framework through which compactly

supported, orthogonal sufficiently regular and real wavelets are constructed is called *multiresolution analysis* (MRA) [26]. A signal  $x \in L^2$  has the following wavelet representation:

$$x = \sum_{k \in Z} c_{J,k} \phi_{J,k} + \sum_{j \geq J; k} d_{j,k} \psi_{j,k}. \quad (2.2)$$

In (2.2),  $\psi_{j,k}(t) = 2^{\frac{j}{2}} \psi(2^j t - k)$ ,  $c_{J,k} = \langle x, \phi_{J,k} \rangle$  and  $d_{j,k} = \langle x, \psi_{j,k} \rangle$  with  $\langle \cdot, \cdot \rangle$  denoting the standard  $L^2$  - innerproduct operation. The function  $\phi$ , called scaling function, captures the residue arising out of truncation of the level parameter  $j$  from going to  $-\infty$ .

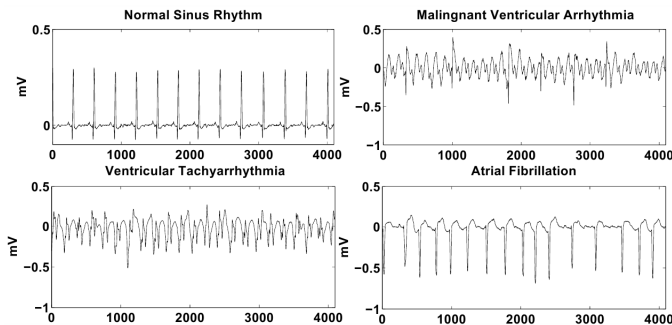


Figure 2.2: Time plots of various ECG signals. Plots indicate that the normal ECG signal exhibits self similar behavior.

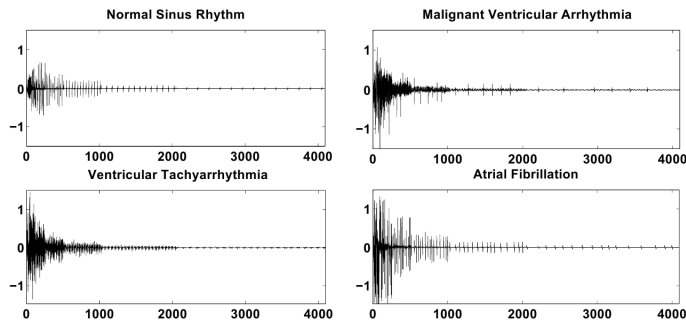


Figure 2.3: Wavelet coefficients of ECG signals in Fig. 2.2. Plots indicate that ECG signals are sparse in wavelet domain. It may be seen that most of the wavelet coefficients of Normal ECG signal are almost zero.

Since wavelet transform is linear, it can be represented as a matrix vector multiplication [27] in discrete setting. For example, if  $x \in R^n$ , the wavelet coefficients  $c$  are represented by the matrix equation  $c = Wx$  and the reconstruction formula is given by  $x = W^T c$ , where  $W$  is wavelet transform matrix. A broad family of natural signals have been found to admit sparse representation (with few nonzero coefficients) in suitable wavelet domains. Figs. 2.2 and 2.3 illustrate such sparsity for representative ECG signals.

## 2.2.4 Recovering wavelet coefficients from compressive samples

Suppose  $R$  is a row restriction matrix that picks the rows of the wavelet reconstruction matrix in tune with the compressed measurements, that is,  $Rx = RW^T c$ . Then the wavelet coefficients may

be reconstructed from the following optimization problem:

$$\hat{c} = \arg \min \| c \|_1 \quad \text{subject to} \quad \| Rx - RW^T c \|_2 \leq \epsilon, \quad (2.3)$$

provided  $c$  is sufficiently sparse, and size of  $Rc$  and  $RW^T$  satisfy aforementioned sparse recovery properties.

### 2.2.5 Estimating Hurst exponent from wavelet coefficients

Using the properties of wavelets and self-similar function, [28, 29] observe that whenever a data set satisfies scaling property, their wavelet coefficients also satisfy the same property, that is, for any integers  $j, m, n, k$  such that  $j = m + n$ , we have

$$d_{j,k} = 2^{-\frac{n(2H+1)}{2}} d_{m,k} \quad \text{provided} \quad f(2^{-n}) = 2^{-nH} f(t). \quad (2.4)$$

The equality in the above equation holds in the sense of distribution. Computing the energy  $E_j$ , at  $j^{\text{th}}$  scale, of wavelet coefficients and using (2.4), we get

$$\begin{aligned} E_j &:= \frac{1}{N_j} \sum |d_{j,k}|^2 \\ &= \frac{2^{-n(2H+1)}}{N_j} \sum_k |d_{m,k}|^2 = 2^{-n(2H+1)} E_m. \end{aligned} \quad (2.5)$$

The above equation would result in the following ‘energy scale’ formula

$$\log_2 E_j = -j(2H + 1) + \log_2 E_0. \quad (2.6)$$

The scales over which the plot is straight line are determined first to identify the scale interval over which the self-similarity possibly holds. Then from the slope of the line  $H$  is computed.

After recovering the wavelet coefficients from  $Rx$ , the restricted measurements, we adopt the method of energy-scale for computing the Hurst exponent for the classification of ECG signals for their normal and/or anomalous behaviors.

## 2.3 Experiments and results

Having laid the theoretical groundwork, now we turn to understanding ECG anomalies, and experimentally investigating how effective Hurst exponent is differentiating anomalous signals from normal ones.

### 2.3.1 ECG anomalies

Anomaly in the ECG signal arises from abnormal electrical activity in the heart. A large heterogeneous group of conditions, where the heart beat is either too fast or too slow, and may be either regular or irregular, are described as cardiac arrhythmia. Further, a subclass of conditions that start in the atria are called Atrial or Supraventricular (above the ventricles) arrhythmias. Analogously, ventricular arrhythmias begin in the ventricles. Arrhythmias originating in the atria are

Table 2.1: Hurst exponents for normal ECG signal

Patient:	16265	16272	16273	16420	16483	16539	16786	17052	17453	18177	18184	19088	19090	19830
S1	0.413	0.478	0.487	0.155	0.500	0.319	0.559	0.388	0.545	0.559	0.482	0.401	0.408	0.515
S2	0.539	0.426	0.465	0.305	0.522	0.539	0.528	0.459	0.443	0.512	0.434	0.487	0.345	0.509
S3	0.385	0.431	0.571	0.500	0.563	0.546	0.505	0.568	0.447	0.491	0.363	0.438	0.345	0.468
S4	0.413	0.511	0.351	0.437	0.564	0.502	0.589	0.363	0.478	0.311	0.443	0.546	0.375	0.455
S5	0.460	0.459	0.508	0.479	0.529	0.325	0.522	0.426	0.487	0.443	0.465	0.526	0.194	0.509
S6	0.487	0.512	0.362	0.539	0.495	0.449	0.436	0.555	0.507	0.308	0.341	0.471	0.289	0.458
S7	0.538	0.554	0.288	0.368	0.402	0.525	0.514	0.393	0.595	0.458	0.481	0.307	0.362	0.531
S8	0.431	0.583	0.462	0.405	0.476	0.429	0.500	0.263	0.540	0.144	0.364	0.461	0.343	0.453
S9	0.477	0.403	0.524	0.447	0.521	0.409	0.445	0.413	0.477	0.468	0.288	0.586	0.364	0.512
S10	0.445	0.505	0.470	0.311	0.585	0.397	0.531	0.528	0.399	0.381	0.520	0.485	0.405	0.538
S11	0.506	0.500	0.528	0.298	0.576	0.403	0.527	0.313	0.566	0.489	0.480	0.523	0.421	0.443
S12	0.464	0.553	0.459	0.300	0.481	0.465	0.475	0.577	0.528	0.523	0.502	0.496	0.374	0.546
S13	0.451	0.576	0.502	0.437	0.480	0.355	0.478	0.505	0.512	0.521	0.472	0.496	0.444	0.284
S14	0.342	0.525	0.475	0.511	0.366	0.458	0.599	0.447	0.418	0.510	0.448	0.431	0.386	0.552
S15	0.442	0.526	0.524	0.457	0.474	0.466	0.428	0.539	0.573	0.552	0.447	0.501	0.440	0.578
S16	0.493	0.549	0.433	0.523	0.590	0.242	0.525	0.579	0.501	0.524	0.491	0.642	0.450	0.567
S17	0.494	0.440	0.441	0.486	0.563	0.123	0.483	0.493	0.452	0.408	0.433	0.577	0.533	0.428
S18	0.439	0.575	0.471	0.306	0.608	0.513	0.561	0.437	0.547	0.512	0.454	0.562	0.479	0.358
S19	0.478	0.427	0.506	0.388	0.576	0.414	0.506	0.565	0.365	0.558	0.443	0.418	0.505	0.350
S20	0.507	0.447	0.485	0.361	0.417	0.331	0.530	0.474	0.385	0.469	0.550	0.545	0.450	0.483

Table 2.2: Hurst exponents for atrial fibrillation signals

Patient	4936	5121	6426
S1	0.575	0.816	0.704
S2	0.601	0.737	0.853
S3	0.781	0.831	0.858
S4	0.785	0.761	0.833
S5	0.704	0.746	0.729

further sub-categorized as Atrial fibrillations (AFIB), Atrial flutter, Supraventricular tachycardia, and those originating in ventricles as Ventricular Fibrillation (VFIB), Ventricular Tachycardia (VT) and Ventricular Flutter. Various other anomalous heart conditions exist; however, rather than taking all such conditions into account, we shall focus on three abnormal conditions, namely, AFIB, VFIB and VT, and attempt to distinguish those from the normal.

### 2.3.2 ECG databases and preprocessing

For our experiments, we use Massachusetts Institute of Technology (MIT) Beth Israel Hospital (BIH) Normal Sinus Rhythm (NSR), MIT BIH Malignant Ventricular Arrhythmia, MIT BIH Atrial Fibrillation, and Creighton University (CU) Ventricular Tachyarrhythmia Databases [15]. Since these database signals are corrupted by contact noise, power line interference, baseline drift etc., we have preprocessed data using median filters of windows of different sizes. This step normally leaves crucial signal components like P-wave, QRS complex and T-Wave in tact, but removes baseline wander.

For self-similarity analysis, we have considered signals of long enough duration by accommodating 5-6 R-peaks in them. As the database signals have different sampling frequencies, we have standardized the sampling frequencies to 500Hz using suitable multirate processing tools, and the signal length to 4096 samples (8.2 sec.).



Table 2.3: Hurst exponents for Malignant Ventricular signals

Patient	420	422	423	427	430	602
S1	0.764	0.803	0.769	0.923	0.776	0.907
S2	0.859	0.786	0.575	0.748	0.947	0.626
S3	0.836	0.668	0.726	0.743	0.888	0.477
S4	0.935	0.721	0.778	0.713	0.858	0.716
S5	0.905	0.838	0.762	0.718	0.930	0.620

Table 2.4: Hurst exponents for Ventricular Tachyarrhythmia signals

Patient:	cu01	cu07	cu08	cu10
S1	0.704	0.563	0.579	0.712
S2	0.748	0.741	0.831	0.760
S3	0.741	0.681	0.737	0.789
S4	0.770	0.733	0.930	0.754
S5	0.833	0.668	0.989	0.870

### 2.3.3 ECG signal classification results

We compute Hurst exponent of different ECG signals from preprocessed as well as compressed measurements (about 1365 samples), and furnish those in Tables 2.1, 2.2, 2.3 and 2.4, where first row of each table indicates subject number as listed in Physionet database, and first column the signal segment. Note that several signal segments of duration 8.2 seconds exist for each subject. It can be seen from these Tables that the range of Hurst exponent for Normal Sinus Rhythm is generally between 0.15 and 0.58, whereas the Hurst exponent value for other three types of anomalous ECG signals is generally greater than 0.58. Based on this observation, a threshold of 0.58 is used for classifying anomalous and normal signals based on Hurst exponent. Incorrectly classified signal segments are circled for easy reference. This choice of the threshold appears reasonable based on large proportion of correct classification as seen in Table 2.5. However, an optimized method (possibly learning based) may improve performance.

Finally, we acquire nonuniformly sampled signals by leaving the unwanted uniformly samples out. For example, for average down sampling by 2, we leave out half the original samples. For our experiments, we compare average down sampling factors of 1 (Nyquist sampling), 2, 3, and 4. Table 2.6 presents the summary of classification performance in terms of sensitivity and specificity corresponding to these four cases. A down sampling factor of 3 appears attractive as it achieves the sensitivity and the specificity of 92.31% and 96.79%, respectively, both of which appear to be acceptable. Thus based on our design principles, we would choose a compressive sampling classification system where the effective down sampling factor is 3.

Table 2.5: Confusion matrix for two class classification

	normal	abnormal
normal	271	9
abnormal	5	60

Table 2.6: Classifier performance in terms of sensitivity and specificity for various down sampling factors

Sampling rates	Sensitivity	Specificity
Nyquist Sampling	82.35	99.67
Effective down sampling factor of 2	88.23	98.66
<b>Effective down sampling factor of 3</b>	<b>92.31</b>	<b>96.79</b>
Effective down sampling factor of 4	93.84	73

## 2.4 Summary

The present work envisages a framework of telecardiology based on compressively sampled ECG signals. This system would save both bandwidth and power without significantly sacrificing diagnostic accuracy. Specifically, using sensitivity and specificity as competing dual objectives, we designed a system for ECG signals from widely used databases. Specifically, we found an effective down sampling factor of three as attractive. The system can be further improved with better classification accuracy, and in future we plan to incorporate other features to achieve such improvement.

## Chapter 3

# Reliable low-cost telecardiology for continuous monitoring

In a conventional telecardiac monitoring involving high-risk-patient care, ECG from a subject is continuously monitored to detect any deviation from normal sinus rhythm. Additional complexities arise when the subject requires remote monitoring [2]. Consider a telecardiology architecture, depicted in Figure 1.3, where ECGs of remote users are transmitted over bandwidth constrained links to a diagnostic center equipped to accurately detect anomalous beats. Traditionally, the entire signal would be transmitted, resulting in perfect reliability, albeit with the attendant high bandwidth requirement. In this context, with a view to realizing a low-cost system, one would ask: Can reliable telecardiology, in terms of accuracy of anomalous beat detection, be achieved with significantly lower bandwidth?

In response, we propose automated heartbeat classification at the user device (see Figure 3.1), and transmission of only those beats that are detected as abnormal. Indeed, assuming an (unrealizable) ideal classifier with both sensitivity and specificity unity, one would achieve perfect reliability with only  $\alpha$  fraction of the original bandwidth, where  $\alpha$  denotes the prevalence rate of anomalous beats. In practice, we shall achieve a high reliability target using suitable high-sensitivity classifiers. Not surprisingly, bandwidth requirement increases with decreasing specificity subject to sensitivity constraint. Thus, the usual sensitivity-versus-specificity tradeoff in the underlying classifier maps to the reliability-versus-bandwidth tradeoff in the telecardiology system, albeit nonlinearly. In this work, we propose a natural design framework for telecardiology system design based on the latter tradeoff, and make explicit and illustrate the aforementioned nonlinear mapping, while indicating the target high reliability (equivalently, high sensitivity) region.

Having laid down the design framework, we demonstrate high-sensitivity detection with acceptable specificity using class-specific dictionaries, and hence reliable low-cost telecardiology. In this paper, we shall consider anomaly resulting from only ventricular ectopic beats (VEB). Although such beats do occur occasionally even in healthy individuals, those could indicate onset of serious conditions, especially, in vulnerable individuals [30]. Specifically, we train individual dictionaries for normal beats and VEBs, respectively, based on well established interval and morphological features. Given a test heartbeat, such features are represented using both the dictionaries, and we assign to it that class, whose dictionary provides sparser representation. Our main idea here is that a VEB

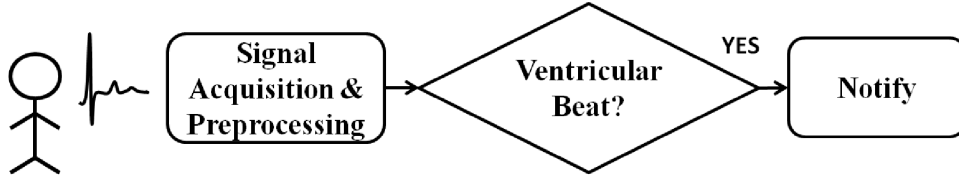


Figure 3.1: Proposed telecardiology architecture

should find a better representation in the VEB dictionary, rather than in the normal beat dictionary, and *vice versa*. Using the proposed classification rule, desired high-sensitivity detection was achieved for appropriate dictionary sizes.

### 3.1 Motivation and Contribution in Context

At this point, we provide detailed motivation by placing our contribution in medical and engineering contexts.

#### 3.1.1 Clinical Motivation

A heterogeneous set of serious conditions, symptomatized by abnormal electrical activity in the heart, are categorized as cardiac arrhythmia [30]. Arrhythmias originating in the atria include atrial fibrillation, atrial flutter, and supraventricular tachycardia, whereas those originating in the ventricles include ventricular fibrillation, ventricular tachycardia, and ventricular flutter. While a normal heartbeat is triggered by the sinoatrial node, certain abnormal ventricular conditions trigger a premature ventricular contraction (PVC) beat ahead of the usual sinoatrial trigger (Figure 3.2). Such PVC beats could be either benign, or a precursor of aforementioned serious arrhythmic conditions, especially in subjects with compromised heart. Abnormal beats also occur when the usual sinoatrial trigger does not materialize, and the contraction is instead initiated by ventricular pacemaker cells as a backup. Such a ventricular escape beat also either occurs in a healthy individual (skipped beats), or could be a harbinger of serious arrhythmic conditions in cardiac patients. Additionally, since the morphologies of both PVC and ventricular escape beats are approximately the same, the Association for the Advancement of Medical Instrumentation (AAMI EC57:1998) standard describes both as ventricular ectopic beats (VEBs) [31]. In this backdrop, we propose to detect VEBs, and use those as markers to potentially initiate medical intervention.

#### 3.1.2 Motivation for High Sensitivity Classifiers

Consider a telecardiology system depicted in Figure 1.3, where each user is equipped with a heartbeat classifier as shown in Figure 3.1, so that only beats detected as anomalous are transmitted. As mentioned earlier, we shall consider VEBs as the only anomaly. Further, denote by  $Se$  and  $Sp$ , respectively, the sensitivity and the specificity of the classifier. We also assume that the diagnostic center has the resources to validate and correct, if necessary, the class of each beat it receives. Thus one fails to detect a VEB only if that beat is originally classified as normal and never transmitted. Thus the fraction of undetected VEBs,  $1 - Se$ , measures the reliability of the system. The lower the above fraction, the more reliable is the system, and perfect reliability is achieved when such fraction

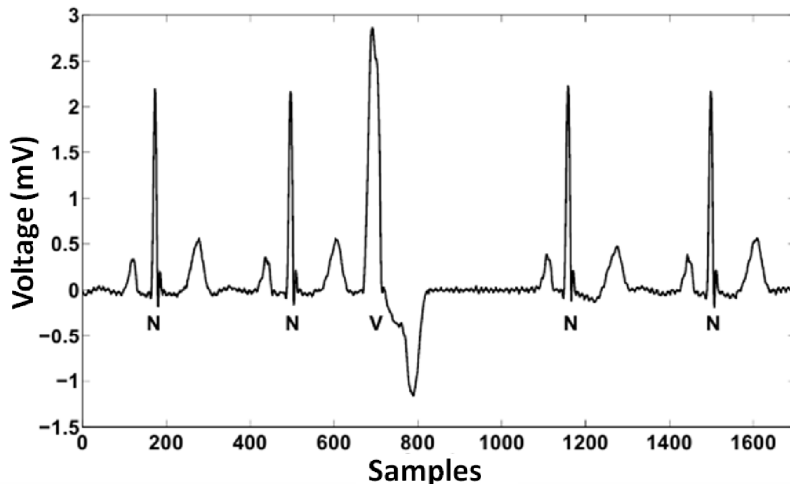


Figure 3.2: ECG record containing normal and ventricular beats. Beats annotated “N” indicate normal, and “V” indicate VEBs.

equals zero. Correspondingly, the fraction  $B$  of bandwidth usage is give by

$$B = Se \times \alpha + (1 - Sp)(1 - \alpha), \quad (3.1)$$

where  $\alpha$  is the prevalence rate of VEBs, and the bandwidth requirement without any classifier is taken as the reference. Of course, no classification is equivalent to  $Se = 1$  and  $Sp = 0$ , where, although perfect reliability is achieved ( $1 - Se = 0$ ), one does not save bandwidth ( $B = 1$ ). On the other hand, perfect reliability would be achieved by an ideal classifier ( $Se = 1$ ,  $Sp = 1$ ) with required bandwidth fraction equal to  $Se \times \alpha$ , amounting to substantial savings. Unfortunately, such an ideal classifier is not realizable. In practice, we seek to save bandwidth while still achieving high reliability (e.g., no more than two undetected VEBs in one thousand, i.e.,  $Se \geq 99.8\%$ ).

### 3.1.3 Proposed Solution vis-à-vis Engineering Choices

In classifying each heartbeat into two classes, normal and VEB, various engineering choices arise. For instance, classification algorithms have been reported based on characteristic points of ECG signals (P, Q, R, S, and T) [20,21], as well as fractal dimension and Hurst exponent [22,23]. However, we seek to design classifiers using labeled historic data, and hence limit to only machine learning techniques. In this regard, linear discriminant analysis and neural network have been employed [32,33]. Further, unsupervised methods of dimensionality reduction have been used in conjunction with compressively sampled ECG data, whence anomaly detection has been successfully demonstrated [34, 35]. In this backdrop, keeping practical implementation in view, we additionally desire a method where classification performance can seamlessly be traded off against compute requirement. Accordingly, in this paper we adopt dictionary learning so that the above tradeoff could be achieved by varying the dictionary size.

The proposed dictionary learning solution enjoys intimate theoretical connection with sparse cod-

ing, where a signal is expressed as a linear combination of relatively few basis vectors (equivalently, atoms of a dictionary) [24]. Indeed, we propose sparse coding, using dictionaries learnt using the  $K$ -SVD (singular value decomposition) algorithm [36]. Of course, other dictionary learning techniques, such as the method of optimal directions (MOD), also exist alongside  $K$ -SVD, and find applications in areas including image restoration, denoising and texture classification [37]. Specific techniques apart, effectiveness of dictionary learning has in general not been demonstrated for classification of ECG beats. The present paper fills this gap by demonstrating dictionary-based high-sensitivity classification and its effectiveness in the context of high-reliability telecardiology.

## 3.2 Proposed Dictionary based Classifier

To proceed, we need the mathematical notions and the preliminaries of compressive sensing and dictionary learning.

### 3.2.1 Problem Statement

We begin by mathematically formulating the problem of classifying an ECG beat into the normal and VEB categories. Denote by  $x$  any signal vector representing an ECG beat. A candidate classifier specifies two mutually exclusive and exhaustive subsets  $\Gamma_1$  and  $\Gamma_2$  of set  $\Gamma$  of all possible  $x$  such that if a beat  $x \in \Gamma_1$ , it is declared normal, else if  $x \in \Gamma_2$ , it is declared a VEB. We wish to find  $\Gamma_2$  (and hence  $\Gamma_1$ ) such that the sensitivity, i.e., fraction of VEB beats detected as VEB beats, is high (say, above 99.9%). Subject to this, we desire to maximize specificity, i.e., fraction of normal beats declared as normal beats. Recall that the sensitivity ( $Se$ ) determines the reliability ( $= 1 - Se$ ), whereas both sensitivity and specificity determine the bandwidth requirement according to (3.1). Generally, two approaches are taken towards designing such classifier: based on (i) stochastic model under each hypothesis (normal and VEB), and (ii) historic data making use of appropriate learning method. As mentioned earlier, we adopt the latter in view of abundant labeled data, and propose a dictionary based solution.

### 3.2.2 Mathematical preliminaries

#### Compressive Sampling

Compressive sampling (CS) aims at recovering high dimensional sparse vector  $x \in \mathcal{R}^n$  from a few of its measurements  $y = \Phi x \in \mathcal{R}^m$  with  $m < n$ , where  $\Phi$  denotes the measurement matrix [24]. Formally, we seek to solve

$$\min_x \|x\|_0 \quad \text{subject to} \quad \Phi x = y, \quad (3.2)$$

where  $\|\cdot\|_0$  indicates the  $l_0$  (counting) norm. In general, (3.2) is intractable. Fortunately, under certain technical conditions, solution to (3.2) remains unaltered if  $\|\cdot\|_0$  is replaced by the  $l_1$  norm  $\|\cdot\|_1$ , where new problem requires more tractable  $l_1$  solvers. Among the existing  $l_1$  solvers, orthogonal matching pursuit (OMP), a simple and effective (although greedy) algorithm, will be used in our paper [24]. The aforementioned technical condition relates to the sufficiency of the set of measurements as a function of signal sparsity, which is often empirically estimated through repeated experimentation.

CS theory also applies to signal recovery from noisy (inaccurate) measurements

$$y = \Phi x + e \quad \|e\|_2 < \epsilon,$$

for some  $\epsilon > 0$ . Specifically, we seek recovered signal

$$\hat{x} = \arg \min_x \|y - \Phi x\|_2 + \tau \|x\|_1, \quad (3.3)$$

for appropriate  $\tau$  under certain technical conditions. The optimization problem (3.3) is often solved by iterative soft-thresholding method [24].

### Dictionary Learning

The method of dictionary learning identifies a tunable selection of basis vectors providing sparse representation. Given a set of signals  $\{x_i\}_{i=1}^n$ ,  $K$ -SVD [36] obtains the dictionary  $D$  that provides the sparsest representation for each example in this set. It involves a two-step procedure. In the first step, for a given dictionary  $D$ , we obtain matrix  $\Psi$  with sparse columns by solving the following optimization problem:

$$\Psi = \arg \min_{\Theta} \sum_l \|\Theta_l\|_1 \quad \text{subject to } X = D\Theta, \quad (3.4)$$

where  $\Theta_l$  is the  $l^{\text{th}}$  column of  $\Theta$ , and  $X$  is the matrix whose columns are  $x_i$ 's. Using the above  $\Psi$ , the pair  $(D, \Psi)$  is then updated as

$$(\hat{D}, \hat{\Psi}) = \arg \min_{D, \Psi} \|X - D\Psi\|_F^2 \quad \text{subject to } \|\Psi_i\|_0 \leq T_0 \forall i, \quad (3.5)$$

where  $\Psi_i$  denotes the  $i^{\text{th}}$  column of  $\Psi$ ,  $T_0$  the sparsity parameter, and  $\|\cdot\|_F$  indicates the Frobenius norm. In view of CS theory, thus the  $K$ -SVD algorithm alternates between sparse coding (3.4), solved using an  $l^1$  solver such as OMP, and dictionary update (3.5), solved using iterative soft-thresholding, till there is a convergence in the dictionary so learnt.

### 3.2.3 Proposed Solution

Armed with the preceding mathematical background, we now propose a dictionary based heart-beat classification method that exploit labeled historic data. Denote such labeled dataset by  $\{\{x_{i,l}\}_{i=1}^{N_l}\}_{l=1}^K$ . Here  $l$  indicates the class label:  $l = 1$  indicates normal, and  $l = 2$  indicates VEB, with number  $K$  of classes equaling two at present. Further,  $i$  indicates the signal index and takes values up to  $N_l$ , the number of beats present in class  $l$ . Now, as detailed above, we learn the dictionary  $\hat{D}_l$  for class  $l$  from  $\{y_{i,l}\}_{i=1}^{N_l}$  for both  $l = 1$  (normal) and  $l = 2$  (VEB). All such dictionary learning is performed offline.

In real time, when a heartbeat vector  $x$  is presented, the proposed classifier assigns the class label, the dictionary corresponding which provides the sparsest representation. Specifically, we set an accuracy level  $\epsilon > 0$ , and find the sparsest representation  $\hat{\alpha}_l$  of  $x$  using each dictionary  $\hat{D}_l$

Feature	Description
Heartbeat interval features	• Number of samples between current R_peak location and Previous R_peak location
	• Number of samples between current R_peak and the next R_peak
	• QRS_offset-QRS_onset
	• R_peak-Q_peak
	• S_peak-R_peak
	• Magnitude of Q_peak
	• Magnitude of R_peak
	• Magnitude of S_peak
	• P_offset-P_onset
	• Magnitude of P_peak
	• P_peak-P_onset
	• P_offset-P_peak
	• T_offset-T_onset
	• Magnitude of T_peak
	• T_peak-T_onset
	• T_offset-T_peak
Morphological features	• 30 uniformly sampled data points within 60ms window with R_peak as center
	• 20 uniformly sampled data points within 80ms window with T_peak as center

Table 3.1: Feature vector has length 66, comprising of 16 heartbeat interval features, and 50 morphological features.

( $l = 1, \dots, K$ ) by solving

$$\hat{\alpha}_l = \arg \min \|\alpha_l\|_1 \text{ subject to } \|x - \hat{D}_l \alpha_l\|_2 < \epsilon.$$

Finally, we assign to  $x$  the class label

$$\hat{l} = \arg \min \|\hat{\alpha}_l\|_0, \quad (3.6)$$

i.e., the index of the sparsest representation. If (3.6) results in a tie between two indices  $i$  and  $j$ , we pick  $i$  such that  $\|x - \hat{D}_i \hat{\alpha}_i\|_2 < \|x - \hat{D}_j \hat{\alpha}_j\|_2$ . If dictionary size is small, it may not be possible to obtain  $\epsilon$ -accurate representation using any rival dictionary. In that case, we shall only make use of the tie-breaking mechanism. Finally, notice that one may use smaller dictionaries, potentially incurring classification accuracy loss, in order to reduce compute requirement within the proposed framework. Although our solution applies to any number  $K$  of classes, in this paper we confine to  $K = 2$ .



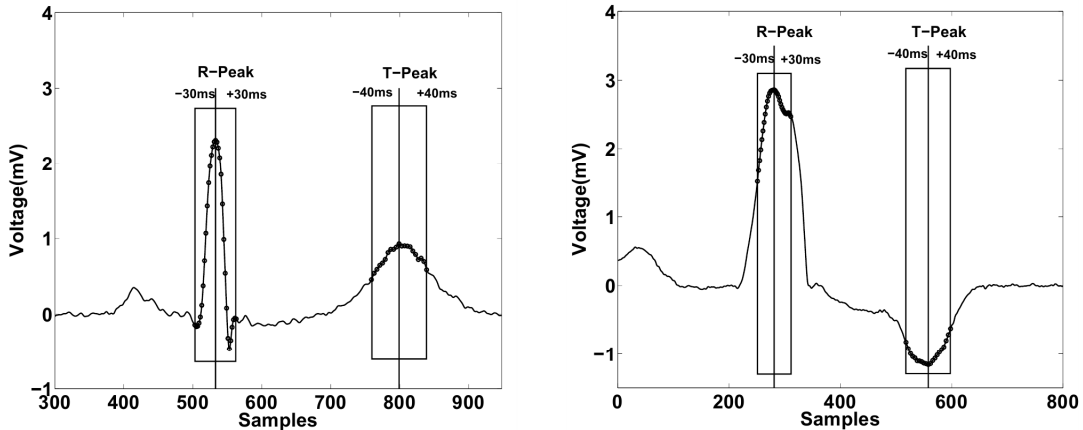


Figure 3.3: Morphological features: (left) normal beat; (right) VEB.

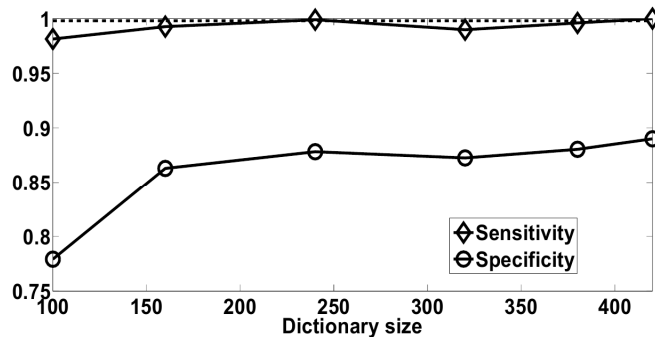


Figure 3.4: Sensitivity and Specificity of classifier for different dictionary sizes

### 3.3 Experiments and Results

We use MIT-BIH Arrhythmia Database available in the PhysioBank archives [15], consisting of 30-minute excerpts of two channel ambulatory ECG recordings digitized at 360 samples per second per channel with 11-bit resolution over a 10 mV range. Each beat in the database is annotated by two or more cardiologists independently. Prior to classification, we remove baseline wander using median filters of window size 200ms and 600ms. Such filters remove P-waves, QRS complexes and T-waves leaving behind the baseline wander [32]. We then subtract the baseline wander from the original signal.

#### 3.3.1 Proposed Features

Towards desired classification, we first generate two sets of features: (i) heartbeat interval features, and (ii) morphological features [32]. As a first step we used the following heuristic segmentation. Consider an R\_peak located at time  $t_0$ , and suppose the durations of the pre-RR and the post-RR intervals are  $T_{pre}$  and  $T_{post}$ . Then the interval  $[t_0 - 0.5T_{pre}, t_0 + 0.75T_{post}]$  provides the estimated beat segment corresponding to an R\_peak located at  $t_0$ . Here we made use of the locations of the R\_peaks given in the Physionet database annotations.

Next we obtain fiducial points of heartbeat such as, onset and offset of QRS complex, P\_wave,

	Actual	
Labled	V	N
V	1764	2
N	215	1551

(a) Dictionary size 240

	Actual	
Labled	V	N
V	1766	0
N	194	1572

(b) Dictionary size 420

Table 3.2: Confusion matrix for proposed classifiers. Here V indicate VEB and N indicates Normal classes.

and T\_wave, position and magnitude of P\_peak, Q\_peak R\_peak S\_peak and T\_peak using a fiducial point identifier algorithm [38]. In order to improve accuracy, we resampled our signals at 1024 Hz. From these points, we compute a set of heartbeat interval features given in Table 3.1. We also compute morphological feature vectors consisting of 30 uniformly spaced samples within a window of 60ms with R\_peak as center, 20 uniformly spaced samples within a window of 80ms with T\_peak as center. Such morphological features within normal and ventricular ectopic beats are depicted in Figure 3.3.

### 3.3.2 Learning Class-specific Dictionaries

The experiment is performed using ECG signals pertaining to 11 patient records in MIT-BIH Arrhythmia database. Each patient data is divided into training and test sets. For any given patient, the number of normal beats are significantly higher compared to that of VEB beats. For training, we choose the same number of normal beats as that of VEB beats for each subject. Further, we learn dictionaries for both the ECG beat classes under consideration on the basis of training data using K-SVD algorithm as described earlier. Next each test beat is projected onto both dictionaries and the beat is assigned to the class whose dictionary provides the sparser representation. The dictionaries of both normal and ventricular beats are trained using 1755 beats and the classification performance is evaluated on 1766 beats from the same set of patients.

### 3.3.3 Classification Performance

Fig. 3.4 depicts the performance of the proposed classifier in terms of sensitivity and specificity for various sizes of dictionaries. Note that our method achieves high sensitivity for a range of dictionary sizes. To highlight this, we draw a dashed line indicating a sensitivity of 99.8%, and observe multiple points above that line in the sensitivity plot. As expected [24], the specificity is acceptable when the dictionary size is about three times the feature vector length or more. For a dictionary size of 66x240, sensitivity and specificity of 99.9% and 87.8%, respectively, are achieved, and the corresponding confusion matrix is presented in Table 3.2a. For a larger dictionary size of 66x420, we achieve sensitivity and specificity of 100% and 89%, respectively and the corresponding confusion matrix is presented in Table 3.2b. Note the improvement in the classifier performance is achieved at the cost of higher compute requirement.

Table 3.3 compares the classification performance of our method with various reported algorithms in terms of sensitivity and specificity. While our technique achieves higher sensitivity than rival algorithms, the latter in general achieve higher specificity, making a fair comparison difficult. Yet, devoid of context (such as telecardiology), one sometimes wishes to keep both sensitivity and

	Sensitivity (%)	Specificity (%)
Chow <i>et al.</i> <sup>2</sup> [39]	97.4	99.2
Hu <i>et al.</i> <sup>1</sup> [40]	78.9	96.8
Christov <i>et al.</i> [41]	96.9	96.7
G Bortolan <i>et al.</i> [42]		
Neural networks (NN)	95.8	98.3
K-th nearest neighbour (kNN)	91.3	98.7
Discriminant analysis (DA)	97.0	94.4
Fuzzy logic (FL)	92.8	98.4
Chazal <i>et al.</i> <sup>1</sup> [32]	77.5	98.9
Gómez-Herrero <i>et al.</i> [43]	98.5	97.2
Inan <i>et al.</i> <sup>2</sup> [44]	85.3	99.1
Jiang <i>et al.</i> <sup>1</sup> [33]	94.3	99.4
Ince <i>et al.</i> <sup>2</sup> [45]	93.4	99.2
<b>Proposed method</b>		
Dictionary size 240	<b>99.9</b>	<b>87.8</b>
Dictionary size 420	<b>100</b>	<b>89</b>

<sup>1</sup> Classifiers proposed for multi class classification.

<sup>2</sup> Specificity calculated by assuming prevalence as 11%.

Table 3.3: Comparison of the proposed method with rival methods in terms of classification performance.

specificity roughly equal, while maximizing that equal quantity. According to such criterion, certain reported classifiers, especially, due to [39], [41], [42], [43], [33] and [45], do appear attractive. Unfortunately, an application such as telecardiology does not lead to the aforementioned criterion.

To highlight the importance of telecardiological context, in Fig. 3.5 we make comparison between the same classifiers as earlier, but now with respect to the number of VEBs undetected per one thousand beats vis-à-vis the fraction of original bandwidth used. Here we assume an 11% prevalence rate of VEBs<sup>1</sup>. As mentioned earlier, we use as reference the bandwidth requirement when no classifier is deployed. On the other hand, an ideal classifier would use only 11% of the reference bandwidth (shown by vertical dashed line). In this backdrop, notice that a number of reported classifiers do operate close to, or even less than, such ideal bandwidth. However, those do not perform close to our reliability limit of two undetected VEBs in one thousand (shown by horizontal dashed line). The nearest in this respect, the classifier proposed by Gómez-Herrero *et al.* [43], requires only 13% of the reference bandwidth, but fails to detect about 15 VEBs in 1000, which is 7.5 fold higher than the acceptable limit. In comparison, the proposed classifier with dictionary size 66x240 would use 21.8% of bandwidth, while missing only 1 anomalous beat per 1000. A larger dictionary size of 66x420 leads to only 20.8% of bandwidth with no (less than one in 1766) VEB misclassification.

<sup>1</sup>As the statistics for VEB prevalence is not directly available, we take as a representative figure the CVD prevalence rate (which is 11% in the USA [46])

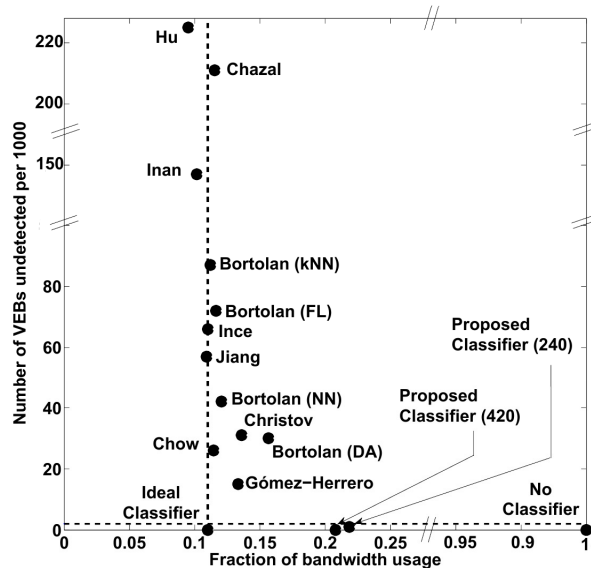


Figure 3.5: Comparison of various classifiers in the context of telecardiology.

### 3.4 Summary

In this work, we consider VEB versus normal heartbeat classification in the context of bandwidth constrained telecardiology. Specifically, we desire a high reliability of two undetected beats in one thousand or less (i.e., sensitivity greater than 99.8%). Subject to such reliability constraint, we sought to minimize the bandwidth usage. In this backdrop, we demonstrated such high-sensitivity classification (99.9% and 100%) using dictionary learning techniques, while achieving substantial bandwidth savings (78.2% and 79.2%, respectively). Additionally, proposed classifiers are scalable in terms of compute requirement (dictionary sizes of 240 and 420, respectively), and hence assume practical significance. In theory, one may achieve high classification accuracy as well as high class-specific compression and hence low transmission bandwidth, by simply enlarging the feature vector to include the entire signal vector. However, the prohibitive compute requirement for both offline training of a large dictionary, and real-time signal representation as a linear combination of large number of dictionary atoms could make such schemes impractical. In summary, the tradeoff is not merely between reliability and bandwidth, but involves the compute requirement as well.

## Chapter 4

# Conclusion and discussion

A conventional telecardiac system, that acquires and transmits user ECG signals to the diagnostic center may not be appealing to the economically disadvantaged communities living at less than US\$ 1.25 per day. Further, the design of such a system does not take into account the resource constraints in remote localities. In this backdrop, we proposed reliable and low-cost telecardiology solutions to two practical scenarios that require efficient resource utilization.

In Chapter 2, we addressed the problem of remote resource-constrained telecardiology. We proposed a two-tier telecardiology framework to tackle stringent power and bandwidth constraints. Specifically we used compressive sampling to address power constraints and compressive detection of anomaly, to communicate only anomalous signals to the diagnostic center thereby reducing the bandwidth requirement. The proposed scheme is expected to significantly reduce healthcare cost in remote areas by obviating significant personnel movement and infrastructure-related investment. Our system also reduces the burden on subjects to travel long distances to obtain expert advice. In other words, the proposed system delivers the desired benefits of classical telecardiology even with limited resources. Conveniently, the experiences of subjects and experts also remain essentially unaltered. Specifically, a subject still provides the ECG signal using the same transducers, and an expert visualizes that signal at the diagnostic center at essentially the same quality.

Further, we make experimental demonstration using annotated PhysioNet databases [15]. In particular, we designed compressive classifiers based on self-similarity property exhibited by ECG signals [22] [47]. Our method has the unique ability of operating on compressive samples, thus adapting to resource constraints. Finally, we found an effective down sampling factor of three with a Hurst threshold of 0.58 as attractive operating point. In view of the demonstrated efficacy of the proposed system, we plan its practical deployment in the future. To this end, further research on the following issues are required. (i) Compare the reconstruction accuracy from compressively sampled data using an objective measure for various downsampling factors and patterns. (ii) Design a universal downsampling pattern for a given downsampling factor that guarantees recovery from all the ECG signals. (iii) Optimize the threshold for Hurst exponent for various downsampling factors. (iv) Consider additional features of ECG (like periodicity) to improve classification performance. (v) Investigate the tradeoff between bandwidth (communication cost), classifier sensitivity (reliability) and downsampling factor (power savings). (vi) Analyze the cost-benefit of the proposed architecture compared to conventional telecardiology.

In Chapter 3, we addressed the problem of reliable and low-cost telecardiology for continuous monitoring. In this context, we proposed a novel telecardiology framework that detects anomalous beats with high accuracy (missing no more than one anomalous beat in thousand anomalous beats) and communicates only those beats to diagnostic center to achieve bandwidth savings. Note that we desired high sensitivity rather than high overall accuracy, as the diagnostic center corrects wrongly classified normal signal. To this end, we used dictionary learning technique to achieve the desired high-sensitivity classification. We demonstrated the efficacy of our method using the MIT/BIH arrhythmia database. In particular, with a reliability target of at most two undetected in one thousand, we achieve about 79.2% savings in bandwidth which translates to proportional savings in the operational cost, which is expected to be attractive to the economically marginalized.

The proposed system provides a low-cost continuous monitoring solution with reliability and user experience are on par with the conventional telecardiology. Further our system can be used in a broader setting to address preventive care and mass screening of CVDs. Current system can be further improved with the following considerations. (i) Train dictionaries with entire beat vector instead of hand-picked features would not only result in the desired classification, but also achieves compression by representing the beat vector sparsely in dictionaries. (ii) Operate at the desired classification performance by comparing the ratio of sparsity of representation to a variable threshold. (iii) Add more classes of anomalous beats to the classification algorithm. (iv) Demonstrate the performance of the proposed classification algorithm on the larger set of data. (v) Investigate the tradeoff between computational requirement, bandwidth savings and the classification accuracy. (vi) Analysis of the cost-benefit of the proposed system compared to conventional telecardiology.

Apart from the specific considerations for each of the aforementioned systems, the following guideline are applicable for both the architectures. Firstly, in this thesis, our focus has been on demonstrating the tradeoff among reliability, power and bandwidth, and we make such demonstration using only one-lead ECG signal. In contrast, professional diagnosis requires 12-lead ECG signals [48]. Thus, to realize professional grade equipment, our principles need to be applied to the 12-lead system. Secondly, to improve portability, 3-lead (generally, reduced lead) ECG systems have been suggested such that the desired 12-lead signals can be faithfully reconstructed from the observed 3-lead signals [38]. In view of this, it would be worthwhile to develop portable devices that are reliable under resource constraints. Finally, practical aspects of the desired system, including privacy and information security, and effect of network congestion and packet loss, need to be studied, and taken into account.

Towards practical deployment and large-scale adoption, one needs to also develop appropriate quality models and standards. As alluded earlier, professional evaluations are generally made based on 12-lead ECG signals, which are sampled at a rate of 500Hz or greater [13,48], and such a system can ideally be taken as the standard. In other words, while evaluating a cardiology-related system (including ours), one would seek a clinical outcome that is statistically indistinguishable from the outcome based on the standard system. This point of view has been adopted in the aforementioned work, proposing reduction in number of leads [38], where a close approximation is reported. However, such a stringent criterion could be hard to meet under resource constraints. Also, given the dire medical infrastructure generally found in rural areas, levels of expectation of various stakeholders could also be different. Various quality of service models, including the one proposed by Kastania et al. [49], take such expectations into account. Thus, towards developing a gold standard for resource-

constraint telecardiology, the expectation levels of patients, physicians and other stakeholders need to be estimated through scientifically devised surveys and trials. Such an endeavor is generally intensive and needs participation of a multitude of individuals from diverse backgrounds, and hence a standard may not evolve quickly. However, one should take heart from the fact that desired standards and guidelines have successfully evolved in related contexts [50,51].

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