

# Composition of binary compressed sensing matrices

Pradip Sasmal, R. Ramu Naidu, C. S. Sastry and Phanindra Jampana

**Abstract**—In the recent past, various methods have been proposed to construct deterministic compressed sensing (CS) matrices. Of interest has been the construction of binary sensing matrices as they are useful for multiplier-less and faster dimensionality reduction. In most of these binary constructions, the matrix size depends on primes or their powers. In the present work, we propose a composition rule which exploits sparsity and block structure of existing binary CS matrices to construct matrices of general size. We also show that these matrices satisfy optimal theoretical guarantees and have similar density compared to matrices obtained using Kronecker product. Simulation work shows that the synthesized matrices provide comparable results against Gaussian random matrices.

**Index Terms**—Compressed Sensing, RIP, Binary and ternary sensing matrices.

## I. INTRODUCTION

The objective of compressed sensing is to recover a sparse signal  $x = \{x_i\}_{i=1}^M \in \mathbb{R}^M$  from a few of its linear measurements  $y \in \mathbb{R}^m$  where  $y = \Phi x$ . Sparsity is measured by  $\|\cdot\|_0$  norm - the number of non-zero entries in  $x$ , i.e.,  $\|x\|_0 = |\{j : x_j \neq 0\}|$ . Given the pair  $(y, \Phi)$ , the problem of recovering  $x$  can be formulated as finding the sparsest solution (solution containing most number of zero entries) to the given linear system of equations  $y = \Phi x$ .

Compressed sensing has been found to have tremendous potential for several applications [12], [16], [17], [20]. Sparse representations of signals have gained importance in areas such as image/signal processing [4], [19] and numerical computation [5]. The Orthogonal Matching Pursuit (OMP) algorithm and the  $l_1$ -norm minimization (also called basis pursuit) are two widely studied CS reconstruction algorithms [21].

A sufficient condition for exact reconstruction is the restricted isometry condition on  $\Phi$  originally developed by Candes and Tao [6], [7]. An  $m \times M$  matrix  $\Phi$  is said to satisfy the Restricted Isometry Property (RIP) of order  $k$  with constant  $\delta_k$  ( $0 < \delta_k < 1$ ) if for all vectors  $x \in \mathbb{R}^M$  with  $\|x\|_0 \leq k$ , we have

$$(1 - \delta_k) \|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_k) \|x\|_2^2. \quad (1)$$

Given a matrix  $\Phi$ , a positive integer  $k$ , and  $\delta \in (0, 1)$ , verification of the restricted isometry property of  $\Phi$  with

order  $k$  and constant  $\delta$  is an NP-Hard problem [8]. An easier to check sufficient condition is provided by the concept of mutual-coherence. The mutual-coherence  $\mu(\Phi)$  of matrix  $\Phi$  is the largest absolute inner-product between its normalized columns i.e.,  $\mu(\Phi) = \max_{1 \leq i, j \leq M, i \neq j} \frac{|\phi_i^T \phi_j|}{\|\phi_i\|_2 \|\phi_j\|_2}$ , where  $\phi_k$  stands for the  $k$ -th column of  $\Phi$ . The following proposition [3] relates the RIP constant  $\delta_k$  and  $\mu$ .

**Proposition 1.** *Suppose that  $\phi_1, \dots, \phi_M$  are the unit norm columns of a matrix  $\Phi$  with coherence  $\mu$ . Then  $\Phi$  satisfies RIP of order  $k$  with constant  $\delta_k = (k - 1)\mu$ .*

To reduce the number of operations in the matrix-vector multiply ( $\Phi x$ ) it is desirable that the sensing matrix  $\Phi$  contain only a small number of non-zero entries. The density of a matrix is defined as the ratio of total number of non-zero elements to the total number of elements of the matrix.

Random matrices (e.g. Gaussian, Bernoulli) satisfy RIP with the largest possible levels on sparsity with very high probability. As the probability of failure is non-zero, there has been an interest in deterministic construction of CS matrices [10], [11], [18], [23]. Deterministic binary CS matrices are useful in multiplier-less and faster dimensionality reduction. Several existing binary constructions (i.e. containing 0 and 1 as elements) possess a block structure.

**Definition 2.** *A matrix  $\Psi_{m \times M}$  is said to be a block matrix if  $\Psi$  has  $k$  number of row partitions each of size  $n$  (i.e.  $m = kn$ ) and each column contains a single unit element in each partition with rest of the elements being zero.*

One of the first constructions of deterministic binary sensing matrices has been given by R. Devore [9]. The sizes of the constructed matrices are  $p^2 \times p^{r+1}$  with coherence  $\frac{r}{p}$  and density  $\frac{1}{p}$ , where  $p$  is a prime power and  $0 < r < p$ . This construction has  $p$  number of row partitions, each partition is of size  $p$  and each column contains a single one in each partition (i.e., in terms of the above definition,  $k = p, n = p$ ). R. Devore's construction is based on polynomials over finite fields. S. Li., F. Gao et. al. [13] have generalized this work using algebraic curves over finite fields. The size of the matrices constructed is  $|\mathcal{P}|q \times q^{\mathcal{L}(G)}$ , where  $q$  is any prime power and  $\mathcal{P}$  is the set of all rational points on algebraic curve  $\mathcal{X}$  over finite field  $\mathbb{F}_q$ . According to the previous definition, for this construction  $k = |\mathcal{P}|, p = q$ . The authors in [18] have constructed binary sensing matrices using Euler squares with sizes  $nk \times n^2$  and coherence  $\frac{1}{k}$ , where  $n, k$  are integers and  $n > k$ . This construction has  $k$  number of row partitions, each partition is of size  $n$  and each column contains single one in each partition. In [14], [15], the authors have constructed binary sensing matrices via finite geometry.

The binary CS matrices constructed in [9], [13], [18] have

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block structure as defined previously. In addition, most of these constructions, row sizes of associated matrices are given by some particular family of numbers (e.g. functions of prime or prime powers). In [1], a Binary-mixing method and Kronecker product have been used to resize existing compressed sensing matrices. While the Binary-mixing method operates on a binary matrix and a matrix containing elements with the same absolute value, the Kronecker product combines two binary matrices producing a third binary sensing matrix of a different size.

**Proposition 3.** *Given two matrices  $A_{m_1 \times n_1}$  and  $B_{m_2 \times n_2}$  with coherence  $\mu_A$  and  $\mu_B$  respectively, Kronecker product produces new matrix  $C_{m_1 m_2 \times n_1 n_2}$  with coherence  $\mu_C = \max\{\mu_A, \mu_B\}$ .*

Kronecker product does not exploit sparsity and block structure of existing binary constructions. This motivates us to propose a specialized composition rule which uses the properties of existing binary matrices efficiently in obtaining new constructions with optimal theoretical guarantees. The matrices obtained also have more general row sizes and similar densities compared to the matrices obtained by Kronecker product method.

The paper is organized in several sections. In section II, we present the new composition rule. In section III, we describe an application of the method to binary constructions presented in [9] and give a comparison with Kronecker product. Simulation results are given in section IV while section V presents the concluding remarks.

## II. PROPOSED COMPOSITION RULE

Our composition rule starts with two existing binary sensing block matrices as defined earlier. By definition, each column of the matrix contains  $k$  number of ones. If  $c$  is a column of the matrix, the support of  $c$  is defined as  $\text{supp}(c) = (i_1, \dots, i_k)$  where the indices  $i_1, \dots, i_k$  are such that  $c(i_j) \neq 0, j = 1, \dots, k$ . We treat the support as an ordered  $k$ -tuple (instead of a set) for convenience. Addition and subtraction of  $k$ -tuples is performed element-wise. Multiplying a  $k$ -tuple with a scalar is understood as multiplying each element of the  $k$ -tuple with the given scalar.

Let  $\Psi''_{m'' \times M''}$  be a binary sensing block matrix having  $k''$  row blocks, each of size  $n''$  such that each column in each block contains a single '1' and the overlap of any two columns is at most  $r''$ , that is  $|\{l|\psi''_{l,p} = 1\} \cap \{t|\psi''_{t,q} = 1\}| \leq r''$  for any two columns  $\psi''_p$  and  $\psi''_q$ . Let  $\Psi'_{m' \times M'}$  be another binary sensing block matrix having  $k'$  row blocks, each of size  $n'$  such that each column in each block contains a single '1' and the overlap of any two columns is at most  $r'$ . Assume  $r = \max\{r', r''\} < k \leq \min\{k', k''\} \leq n'' \leq n'$ . Now a new matrix can be constructed by the following steps:

**Step-1:** Let the  $i^{\text{th}}$  column of  $\Psi''$  be  $\psi''_i$ . For  $1 \leq i \leq M''$ , define  $S''_i = \text{supp}(\psi''_i) - (0, n'', \dots, n''(k'' - 1))$ . Since  $\Psi''$  has  $k''$  blocks and each block contains a single '1', it follows that  $|S''_i| = k''$  where  $S''_i$  is a  $k''$ -tuple on the set  $X = \{1, 2, \dots, n''\}$ . In other words, the  $j$ -th entry of  $S''_i$  provides the location of the single unit element in the

$j$ -th block. Since  $\Psi''$  has  $M''$  columns, we have  $M''$  such  $k''$ -tuples. For example, suppose  $m'' = 9$  and  $\Psi''$  has 3 blocks, then each block is of size 3. Now, if the  $i^{\text{th}}$  column  $\psi''_i$  is  $(1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1)^T$ ,  $\text{supp}(\psi''_i)$  is  $[1\ 5\ 9]$ , then the triplet,  $S''_i$ , corresponding to this column is  $[1\ 2\ 3]$ .

Similarly from the matrix  $\Psi'$ , define  $S'_i = \text{supp}(\psi'_i) - (0, n', \dots, n'(k' - 1))$  and we can generate  $M'$  number of  $k'$ -tuples on the set  $Y = \{1, 2, \dots, n'\}$ .

**Step-2:** From each  $k''$ -tuple of the first matrix  $\Psi''$  we remove the last  $(k'' - k)$  entries to obtain a  $k$ -tuple. We then add  $(-1)$  to each of the entries of the  $M''$  number of  $k$ -tuples that are obtained from  $\Psi''$ . Let the  $k$ -tuples be of the form  $(c''_{i1}, c''_{i2}, \dots, c''_{ik})$  for  $1 \leq i \leq M''$ .

Similarly, from each  $k'$ -tuple of second matrix  $\Psi'$ , we remove last  $(k' - k)$  entries to obtain a  $k$ -tuple. By this process, we get  $M'$  number of  $k$ -tuples from the second matrix  $\Psi'$ . Let each  $k$ -tuple have the form  $(c'_{j1}, c'_{j2}, \dots, c'_{jk})$  for  $1 \leq j \leq M'$ .

**Step-3:** We now replace each  $k$ -tuple  $(c''_{i1}, c''_{i2}, \dots, c''_{ik})$  with  $M'$  number of  $k$ -tuples (obtained from  $\Psi'$ ) by adding  $n'(c''_{i1}, c''_{i2}, \dots, c''_{ik})$  to each of the  $k$ -tuples  $(c'_{j1}, c'_{j2}, \dots, c'_{jk})$ ,  $1 \leq j \leq M'$ . This way, we get  $M''M'$  number of  $k$ -tuples on the set  $X' = \{1, 2, \dots, n''n'\}$ . Denote the set of these  $k$ -tuples by  $F$ . Let  $(a_{j1}, a_{j2}, \dots, a_{jk})$  for  $j = 1, 2, \dots, M''M'$  be the  $k$ -tuples in  $F$ .

**Step-4:** From these  $k$ -tuples we form a binary vector of length  $kn''n'$  where '1' occurs in the positions  $(l-1)n''n' + a_{jl}$  for  $l = 1, 2, \dots, k$  and rest of the positions are zeros. Using these  $M''M'$  number of  $k$ -tuples, we form a binary sensing matrix  $\Phi$  having  $k$  number of blocks where each block is of size  $n''n'$  and every block containing a single '1'. The position of the unit elements in each block is indexed by the  $k$ -tuples. So the size of the matrix  $\Phi$  becomes  $n''n'k \times M''M'$ .

**Lemma 4.** *The coherence of new binary matrix  $\Phi$  of size  $n''n'k \times M''M'$ ,  $\mu(\Phi)$ , is at most  $\frac{r}{k}$ .*

*Proof:* Let  $\phi_i, \phi_j$  be two arbitrary columns of matrix  $\Phi$ . There exist two  $k$ -tuples  $f_i, f_j \in F$  such that  $\phi_i, \phi_j$  are the corresponding vectors of  $k$ -tuples  $f_i, f_j$  as defined in the above construction. Suppose  $f_i = (c'_{k'1}, c'_{k'2}, \dots, c'_{k'k}) + n'(c''_{k''1}, c''_{k''2}, \dots, c''_{k''k})$  and  $f_j = (d'_{k'1}, d'_{k'2}, \dots, d'_{k'k}) + n'(d''_{k''1}, d''_{k''2}, \dots, d''_{k''k})$ , where  $(c'_{k'1}, c'_{k'2}, \dots, c'_{k'k}), (d'_{k'1}, d'_{k'2}, \dots, d'_{k'k})$  are two  $k$ -tuples obtained from  $\Psi'$  and  $(c''_{k''1}, c''_{k''2}, \dots, c''_{k''k}), (d''_{k''1}, d''_{k''2}, \dots, d''_{k''k})$  are  $k$ -tuples obtained from  $\Psi''$ . Set  $g_{ij} = f_i - f_j = (c'_{k'1}, c'_{k'2}, \dots, c'_{k'k}) - (d'_{k'1}, d'_{k'2}, \dots, d'_{k'k}) + n'((c''_{k''1}, c''_{k''2}, \dots, c''_{k''k}) - (d''_{k''1}, d''_{k''2}, \dots, d''_{k''k}))$ . We will show that  $g_{ij}$  has at most  $r$  number of zero elements, which implies that intersection between the supports of  $\phi_i, \phi_j$  is at most  $r$ . Now  $|c'_{k'l} - d'_{k'l}| < n'$  and  $|c''_{k''l} - d''_{k''l}| < n''$ . The  $l^{\text{th}}$  element of  $g_{ij}$  is  $(g_{ij})_l = (c'_{k'l} - d'_{k'l}) + n'(c''_{k''l} - d''_{k''l})$ . We investigate the cases wherein  $(g_{ij})_l = 0$ .

**Case 1:** Suppose  $c'_{k'l} \neq d'_{k'l}$ .

**Subcase 1.1:** If  $c''_{k''l} = d''_{k''l}$ , then  $(g_{ij})_l \neq 0$ .

**Subcase 1.2:** If  $c''_{k''l} \neq d''_{k''l}$ , then also  $(g_{ij})_l \neq 0$ .

Since  $1 \leq |c''_{k'l} - d''_{k'l}| < n''$  and  $|c'_{k'l} - d'_{k'l}| < n'$ , we have  $|(g_{ij})_l| = |(c'_{k'l} - d'_{k'l}) + n'(c''_{k'l} - d''_{k'l})| \geq |n'(c''_{k'l} - d''_{k'l})| - |c'_{k'l} - d'_{k'l}| \geq n' - (n' - 1) \geq 1$ .

**Case 2:** Suppose  $c'_{k'l} = d'_{k'l}$  then  $(g_{ij})_l = 0$  only when  $c''_{k'l} = d''_{k'l}$ .

From the above cases, we conclude that  $(g_{ij})_l = 0$  only when  $c'_{k'l} = d'_{k'l}$  and  $c''_{k'l} = d''_{k'l}$ , therefore  $(g_{ij})_l = 0$  can occur for at most  $r$ -choices of  $l$ . So  $\phi_i, \phi_j$  have at most  $r$ -intersections. Therefore,  $\mu(\Phi)$  is at most  $\frac{r}{k}$ . ■

The following theorem shows the RIP compliance of  $\Phi$ .

**Theorem 5.** *The afore-constructed matrix  $\Phi_{nn'k \times MM'}$  satisfies RIP with  $\delta_{k'} = (k' - 1)\binom{r}{k}$  for any  $k' < \frac{k}{r} + 1$ .*

*Proof:* Proof follows from the Proposition 1 and Lemma 4. ■

### III. APPLICATION TO EXISTING BINARY CONSTRUCTIONS

For distinct primes  $p_1, p_2$  and any positive integer  $r$ , suppose  $r < p_2 < p_1$ . Using the method in [9], one obtains binary matrices  $\Psi_1, \Psi_2$  of sizes  $p_1^2 \times p_1^{r+1}$  and  $p_2^2 \times p_2^{r+1}$  respectively. Thus we get  $p_1^{r+1}$  number of  $p_1$ -tuples and  $p_2^{r+1}$  number of  $p_2$ -tuples. If we apply the composition procedure on these matrices we generate a matrix  $\Phi$  of size  $p_1 p_2^2 \times (p_1 p_2)^{r+1}$  with coherence  $\frac{r}{p_2}$ . The density of  $\Phi$  is  $\frac{1}{p_1 p_2}$ , which is small compared to  $\frac{1}{p_1}$  and  $\frac{1}{p_2}$ , the densities of  $\Psi_1$  and  $\Psi_2$  respectively. The coherence  $\mu(\Phi)$  is  $\frac{r}{p_2}$ .

The row size of  $\Phi$  is  $p_1 p_2^2$  and each column of  $\Phi$  contains  $p_2$  number of ones, and the overlap of any two columns is at most  $r$ . Such systems are known in extremal set theory as  $r$ -sparse sets. If  $X$  is an  $m$ -element set and  $r < k < m$ , a family  $\mathcal{F}$  of  $k$ -element subsets of  $X$  is called  $r$ -sparse if every two members of  $\mathcal{F}$  intersect in less than  $r$  elements. A simple upper bound on the number of elements of  $\mathcal{F}$  is  $\binom{m}{r}$ . Let  $M(m, k, r)$  represent the maximum number of elements of an  $r$ -sparse family. Rodl [22] has shown that  $\lim_{m \rightarrow \infty} M(m, k, r) \binom{m}{r}^{-1} = 1$  for fixed  $r, k$ , i.e., existence of optimal  $r$ -sparse families in the asymptotic case.

For the current construction, the upper bound on the column size is  $\binom{p_1 p_2^2}{r+1}$ . Using the general bounds  $\binom{n}{k} \leq \binom{n}{k} \leq \left(\frac{ne}{k}\right)^k$

we obtain  $\left(\frac{p_1 p_2}{e}\right)^{r+1} \leq \binom{p_1 p_2^2}{r+1} \leq (p_1 p_2 e)^{r+1}$ . For fixed  $r$ ,

therefore  $\binom{p_1 p_2^2}{r+1} = O((p_1 p_2)^{r+1})$ . If  $\frac{p_2}{r} \rightarrow \infty$ , we also have

$(p_1 p_2)^{r+1} \sim \binom{p_1 p_2^2}{r+1}$  i.e.,  $\lim_{\frac{p_2}{r} \rightarrow \infty} \frac{(p_1 p_2)^{r+1}}{\binom{p_1 p_2^2}{r+1}} = 1$ . The proof

follows from a similar argument given in [2]. Consequently,  $\Phi$  attains the maximum possible column size asymptotically.

Kronecker product is a popular composition rule to generate matrices with more general row sizes. By applying Kronecker product on the matrices  $\Psi''$  and  $\Psi'$  which were used in Section II with  $r = \min\{r'', r'\}$  and  $k = \min\{k'', k'\}$ , we obtain a new matrix  $\Phi'$  of size  $n'' n' k'' k' \times M'' M'$ . The density of  $\Phi'$  is  $\frac{1}{n'' n'}$  and it can be easily shown that the coherence of  $\Phi'$  is  $\frac{r}{k}$ , where as the matrix  $\Phi$  constructed from the proposed

composition rule has density and coherence as of  $\Phi'$ , the aspect ratio of  $\Phi$  is  $\frac{M'' M'}{n'' n' k}$  which is larger than  $\frac{M'' M'}{n'' n' k'' k'}$ , the aspect ratio of  $\Phi'$ .

While applying Kronecker product on  $\Psi_1$  and  $\Psi_2$  described previously in this section, we obtain a matrix  $\Phi^{kro}$  of size  $(p_1 p_2)^2 \times (p_1 p_2)^{r+1}$ . The density of  $\Phi^{kro}$  is  $\frac{1}{p_1 p_2}$  and the coherence  $\mu(\Phi^{kro})$  is  $\frac{r}{p_2}$ . Each column of  $\Phi^{kro}$  contains  $p_1 p_2$  number of ones, and the overlap of any two columns is at most  $r p_1$ . An upper bound on the column size with the properties of  $\Phi^{kro}$  is  $\binom{p_1^2 p_2^2}{r p_1 + 1}$ , which is between  $(\frac{p_1 p_2}{e})^{r p_1 + 1}$  and  $(p_1 p_2 e)^{r p_1 + 1}$ . For a fixed  $r$ , there is a gap between  $(p_1 p_2 / e)^{r p_1 + 1}$  and  $(p_1 p_2)^{r+1}$ .

It can be noted that the proposed composition rule achieves the same performance as Kronecker product while using a factor of  $p_1$  less number of rows. More precisely, if  $p_1 = \Theta(p_2)$  and  $r, p_1$  tend to infinity,  $\log M = \Theta(m^{1/3} \mu \log m)$  for the current method whereas  $\log M = \Theta(m^{1/4} \mu \log m)$  for matrices obtained using Kronecker product.

Indeed, as we have  $m = p_1 p_2^2, M = (p_1 p_2)^{r+1}, \mu = \frac{r}{p_2}$ , we obtain  $\log M = (r+1) \log(p_1 p_2) = (\mu p_2 + 1) \log(p_1 p_2)$ . Under the condition  $r \rightarrow \infty$  we have  $\mu p_2 \rightarrow \infty$ . As  $C p_2 \leq p_1 \leq D p_2$  for some absolute constants  $C, D$ , we have  $C p_2^3 \leq m \leq D p_2^3$ , and therefore  $\mu m^{1/3} \rightarrow \infty$ . Hence,  $(D^{-1/3} \mu m^{1/3} + 1) \log(C D^{-2/3} m^{2/3}) \leq \log M \leq (C^{-1/3} \mu m^{1/3} + 1) \log(D C^{-2/3} m^{2/3})$  and therefore  $\log M = \Theta(\mu m^{1/3} \log m)$ . Similarly, it can be shown that  $\log M = \Theta(\mu m^{1/4} \log m)$  for the matrices obtained using Kronecker product. We observe an improvement of a factor of  $m^{1/12}$  in  $\log M$  with the new composition rule.

**Remark 6.** *Similar to Kronecker product, the proposed composition rule can be used to produce matrices of general row size but with better guarantees.*

### IV. SIMULATION RESULTS

In this section, we compare the numerical performance against standard Gaussian random matrices (with entries drawn from  $\mathcal{N}(0, \frac{1}{m})$ ). Binary matrices of size  $36 \times 144$  and  $64 \times 256$  are generated using the present method. The matrix  $\Phi$  of size  $36 \times 144$  has been generated by applying the composition rule on the initial matrices  $\Psi', \Psi''$  of size  $9 \times 9$  and  $16 \times 16$  respectively, which have been generated as in [9] with  $p = 3, 4$  and  $r = 1$ . Similarly, the matrix  $\Phi$  of size  $64 \times 256$  has been generated by applying the composition rule on the initial matrices  $\Psi', \Psi''$  of size  $16 \times 16$ , which have been obtained from [9] with  $p = 4$  and  $r = 1$ . Let  $\tilde{x}$  denote the recovered solution using the OMP algorithm. For purposes of comparing the two solutions, the Signal-to-noise ratio (SNR) of  $x$  is defined as

$$SNR(x) = 10 \cdot \log_{10} \left( \frac{\|x\|_2}{\|x - \tilde{x}\|_2} \right) dB.$$

For each sparsity level  $k$ , 1000  $k$ -sparse signals  $x$  (the nonzero indices chosen uniformly randomly and the entries drawn from  $\sim \mathcal{N}(0, 1)$ ) have been considered. The recovery is considered good if  $SNR(x) \geq 100dB$ . Simulation results (Figure 1) show that the matrices constructed using composition give better performance than the Gaussian random

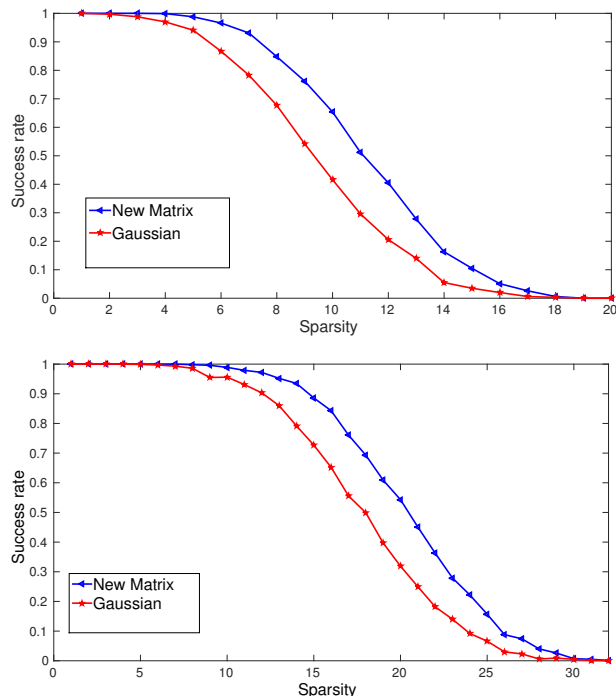


Fig. 1: Comparison of the reconstruction performances of the synthesized matrices and Gaussian random matrices when the matrices are of size (a)  $36 \times 144$  (top plot) and (b)  $64 \times 256$  (bottom plot). These plots indicate that the synthesized matrices show superior performance for some sparsity levels, while for other levels both matrices result in the same performance. The  $x$  and  $y$  axes in both plots refer respectively to the sparsity level and the success rate (in % terms).

matrices for higher sparsity levels. For lower sparsity levels both the matrices give the same recovery performance.

The efficacy of the matrices obtained using the composition rule is demonstrated using image reconstruction from lower dimensional patches. The image has been divided into smaller patches  $\{I_l | l = 1, 2, \dots, N\}$  of equal size. For each patch, the sparse vectors  $I'_l$  have been generated from the vectorized versions of  $I_l$  by decomposing them into wavelet domain. A down-sampled copy of  $I'_l$  has been generated via the binary sensing matrix  $\Phi$  as  $I''_l = \Phi I'_l$  as in [18]. If  $I'_l$  is sparse enough,  $I'_l$  (and consequently  $I_l$ ) can be recovered from the reduced vector  $I''_l$  using sparse recovery techniques.

The reconstruction shown in Figure 3 corresponds to the synthesized matrices obtained previously and Gaussian matrices of size  $64 \times 256$  and the associated reconstruction errors in terms of SNR are 16.22 and 15.01 respectively. From Figure 3 it may be concluded that the synthesized matrices provide competitive reconstruction performance when compared to Gaussian matrices.

## V. CONCLUDING REMARKS

This paper proposed a specialized composition rule which exploits sparsity and the block structure of existing binary CS matrices to construct binary CS matrices with optimal theoretical guarantees. Further, it is shown that the proposed



Fig. 2: Original image



Fig. 3: For the original image of size  $256 \times 256$  in Figure 2, the image on the left is reconstructed via the new matrix and the right image is obtained via the corresponding Gaussian matrix with a down-sampling factor of four. This figure states that the constructed matrix provides competitive reconstruction performance.

composition rule produces CS matrices with density similar to that of Kronecker product. In addition, it is also shown that the new matrix, produced by the composition rule, provides comparable results against its Gaussian counterpart.

## VI. Acknowledgments

The first author is thankful for its support (Ref No. 19-06/2011(i)EU-IV) that he receives from UGC, Govt of India. The second author gratefully acknowledges the support (Ref No. 2/40(63)/2015/R&D-II/4270) that he receives from NBHM, Govt of India. The third author is thankful to DST (SR/FTP/ETA-054/2009) for the partial support that he received. We thank Mr. Roopak R Tamboli for helping us in simulation work.

## REFERENCES

- [1] A. Amini, V. Montazerhodjat and F. Marvasti, "Matrices with small coherence using p-ary Block Codes," *IEEE Trans. Sig. Proc.*, vol. 60, no. 1, pp. 172-181, Jan. 2012.
- [2] A. Amini and F. Marvasti, "Deterministic construction of binary, bipolar and ternary compressed sensing matrices," *IEEE Trans. Inf. Theory*, vol. 57, 2360-2370, 2011.
- [3] J. Bourgain, S. Dilworth, K. Ford, S. Konyagin and D. Kutzarova, "Explicit constructions of RIP matrices and related problems," *Duke Math. J.* 159, 145-185, 2011.
- [4] J. Bowley and L. Rebollo-Neira, "Sparsity and something Else: An Approach to Encrypted Image Folding," *IEEE Signal Processing Letters*, Vol. 8 No 3, 189-192, 2011.
- [5] A. M. Bruckstein, D. L. Donoho and M. Elad, "From sparse solutions of systems of equations to sparse modeling of signals and images," *SIAM Review*, Vol. 51, No. 1, pp: 34-81, 2009.
- [6] E. Candes and T. Tao, "Decoding by linear programming," *IEEE Trans. Inform. Theory* 51, 4203-4215, 2005.

- [7] E. Candes, J. Romberg and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Comm. Pure and Appl. Math.*, 59, 1207-1223, 2006.
- [8] A.S. Bandeira, E. Dobriban, D.G. Mixon and W.F. Sawin, "Certifying the restricted isometry property is hard," *IEEE Trans. Information Theory*, 59, 3448-3450, 2012.
- [9] Ronald A. DeVore, "Deterministic constructions of compressed sensing matrices," *Journal of Complexity*, Volume 23, pp 918-925, 2007.
- [10] A. G. Dimakis, R. Smarandache, and P. O. Vontobel, "LDPC codes for compressed sensing," *IEEE Trans. Inf. Theory*, vol. 58, no. 5, pp. 3093-3114, May 2012.
- [11] P. Indyk, "Explicit constructions for compressed sensing of sparse signals," in *Proc. ACM-SIAM Symp. Discrete Algorithms*, pp. 30-33, 2008.
- [12] Rakshith Jagannath and K.V.S. Hari, "Block Sparse Estimator for Grid Matching in Single Snapshot DOA Estimation," *IEEE Signal Processing Letters*, vol. 20(11), 1038-1041, 2013.
- [13] S. Li, F. Gao, G. Ge, and S. Zhang, "Deterministic construction of compressed sensing matrices via algebraic curves," *IEEE Trans. Inf. Theory*, vol. 58, 5035-5041, 2012.
- [14] S. Li and G. Ge, "Deterministic construction of sparse sensing matrices via finite geometry," *IEEE Trans. Signal Process.*, 2014.
- [15] S. T. Xia, X. J. Liu, Y. Jiang and H. T. Zheng, "Deterministic constructions of binary measurement matrices from finite geometry," *IEEE Trans. Signal Process.*, 2015.
- [16] S. Mukherjee and C. S. Seelamantula, "Fienup algorithm with sparsity constraints: Application to frequency-domain optical-coherence tomography," *IEEE Transactions on Signal Processing*, vol. 62, 4659-4672, 2014.
- [17] R. Prasad, Chandra. R. Murthy, and B. D. Rao, "Joint Approximately Sparse Channel Estimation and Data Detection in OFDM Systems using Sparse Bayesian Learning," *IEEE Transactions on Sig. Proc.*, Vol. 62, no. 14, pp. 3591-3603, 2014.
- [18] R. Ramu Naidu, P. V. Jampana and C. S. Sastry, "Deterministic compressed sensing matrices: Construction via Euler Squares and applications," Accepted, *IEEE Transactions on Sig. Proc.*, March. 2016.
- [19] L Rebollo-Neira and Z. Xu, "Sparse signal representation by adaptive non-uniform B-spline dictionaries on a compact interval," *Signal processing*, 90 (7), pp. 2308-2313, 2010.
- [20] Y. Rivenson and A. Stern, "Compressed Imaging With a Separable Sensing Operator," *IEEE Signal Processing Letters*, 449-452, 2009.
- [21] J. A. Tropp and S. J. Wright, "Computational methods for sparse solution of linear inverse problems," *Proceedings of the IEEE*, vol. 98, no. 6, pp. 948958, 2010.
- [22] V. Rodl, "On a packing and covering problem" *European J. Combin.*, 1985, vol. 6, no. 1, pg. 69 - 78
- [23] N. Y. Yu and N. Zhao, "Deterministic construction of real valued ternary sensing matrices using optical orthogonal codes," *IEEE Signal Processing Letters*, Vol.20, 1106-1109, 2013.