Explanation of the accelerated expansion of the universe and dark energy with irreversible thermodynamics*

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In this paper, by invoking the laws of irreversible thermodynamics the accelerated expansion of the universe is explained. It is shown that the entropy of the universe, at any particular instant of time, plays a significant role in the accelerated expansion of the universe. Considering the universe to be filled with a classical mono-atomic, homogeneous and isotropic gas under classical non-equilibrium situations, two generalized forces causing the expansion of the universe are arrived at. One of the two forces, the trivial force, has affinity to volume expansion and the non-trivial force has affinity to spatial expansion. The acceleration of the expansion of the universe is due to the spatial expansion caused by the non-trivial force and which in turn might account for the presence of the dark energy. It is shown in this paper that the non-trivial generalized force and the dark energy, providing the negative pressure for spatial expansion, can be explained with irreversible thermodynamics.

1. Introduction

Since the experimental validation of the accelerated expansion of the universe and the associated dimming of the Type Ia supernovae [1], there has been a plethora of theoretical constructions to explain the dynamics of the universe, each one with increasing number of complexities. There are candidates that explain this by assuming the existence of exotic matter with negative pressure, etc. [2, 3, 4, 5, 6].

The homogeneous and isotropic cosmological models based on FLRW metric, having proved to be remarkably successful. But with the introduction of data from WMAP and distant supernova together with the the data from supernovae and galaxy distributions and cosmic microwave background anisotropies lead to introduction of cosmological constant Λ or vacuum energy Ω_Λ [7, 8]. Along these there are other approaches like that of dark energy models which attempt to provide a dynamical explanation of the cosmological constant requiring fine-tuning [6]. Together with modifications on cosmological scales of the general theory of relativity, like that of $f(R)$ gravity theory which is plagued with instabilities in the metric formalism [9]. There are also in-homogeneous cosmological models based on spherically symmetric but in-homogeneous LTB metric [10].

The relation between the pressure P, volume V and the internal energy of an ideal gas is related as $PV = \frac{2}{3}U$. The relation like this, in general, leads to the study of thermodynamics systems of the form $PV = \omega U$, where ω is a constant and can be positive or negative. The accelerating expansion of the universe and the associated dimming of the Type Ia Supernovae is explained by assuming the existence of substances that exert negative pressure, with $\omega < -\frac{1}{3}$ [3, 11]. The cold dark matter candidates of cosmological models are described phenomenologically by $PV = \omega U$, where for $\omega = -1$ corresponds to positive cosmological constant and $\omega < -1$ corresponds to phantom dark energy characterised by null chemical potential [5].

In this article I'll deal only for $\omega = \frac{2}{3}$ which implies a universe filled with monoatomic ideal gas [4]. The normal statistical thermodynamic equations for energy density, entropy density and chemical potential of the universe remains the same as that of an monoatomic ideal gas [3, 4].

2. Entropy and the accelerated expansion of the universe

In nature reversible process are fictive. Rather all the process are irreversible in nature. This comes from the second law of thermodynamics. Which states that the entropy of any isolated system always increases for any process, or mathematically we have $dS \geq 0$ where S is the entropy of the system [12, 15]. Now the universe being an isolated system the entropy of the universes always increases. The Sauckur-Tetrode equation for entropy of a gas obtained

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by following the Boltzmann statistic is given as

$$S = \frac{5Nk}{2} + Nk\ln\left[\frac{V(2\pi mkT)^{2/3}}{Nh^3}\right], \tag{1}$$

where $k = 1.381 \times 10^{-23} J/K$ is the Boltzmann constant, V is the volume, T is the temperature, N is the number of particles, m is the mass of the particles and $h = 6.626 \times 10^{-34} m^2 Kgs^{-1}$ [15]. A simple rearrangement of the above equation will yield the volume V of the gas as a function of entropy S given as

$$V = \exp\left[\frac{S}{Nk} - \frac{5}{2} - \ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}\right]. \tag{2}$$

Now, since at a given instant at some point of the universe some sort of irreversible process is always going on. As from the second law of thermodynamics it follows that for every single irreversible process the entropy of the universe increases. Due to which the entropy of the universe is a monotonic increasing function of time. The rate of change of entropy S' as $S' = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt}$. Since entropy is a non-decreasing function of time hence $S' \geq 0$. On similar grounds we have the second derivative of entropy with respect to time defined as $S'' = \frac{d^2S}{dt^2}$. From thermodynamics we have the second time derivative of entropy defined as the rate of entropy production and is negative as for any system to attain thermodynamic equilibrium has its entropy maximal. The rate of the expansion of the universe is calculated by differentiating equation (2) with respect to time:

$$V' = \frac{d}{dt}\exp\left[\frac{S}{Nk} - \frac{5}{2} - \ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}\right]. \tag{3}$$

Differentiating again gives the acceleration of expansion of the universe and is given as

$$V'' = \left[\left(\frac{S}{Nk}\right)' - \left(\ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}\right)'\right]^2 \exp\left[\frac{S}{Nk} - \frac{5}{2} - \ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}\right] + \left[\left(\frac{S}{Nk}\right)'' - \left(\ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}\right)''\right] \exp\left[\frac{S}{Nk} - \frac{5}{2} - \ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}\right]. \tag{4}$$

On analyzing equation (4) we find that the term, $\exp[\frac{S}{Nk} - \frac{5}{2} - \ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3}]$ and the squared term, $[(\frac{S}{Nk})' - (\ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3})']^2$ are always positive as the exponential of any term is always ≥ 0 and the square of any number is also positive. But the term $[(\frac{S}{Nk})'' - (\ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3})'']$ is negative. Assuming that the term $(\ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3})''$ is negligible compared to $(\frac{S}{Nk})''$ we have the the second term to be negative as the rate of decrease of entropy is negative.

Although we must have certain contributions due to the $(\ln\frac{(2\pi mkT)^{\frac{3}{2}}}{Nh^3})''$ as the universe was definitely hotter at the beginning than now and thus T' must exist and must be negative and T'' too is negative as the universe cooled rapidly at the beginning than now. Though we know the behavior of T but we don't know the behavior of N, h, k and m. As these might be different at different time of the universe.

3. Generalized force governing the dynamics of universe

In practice, we can consider the universe to be composite thermodynamic system, with identical and non-identical subsystems separated by permeable diathermal wall and non-rigid boundaries. In addition to this I'll consider non-equilibrium situations, where physical quantities like mass, temperature, pressure, etc. are a subject to change in both time and space and the flow of matter and the dynamic change of the various physical parameters of the subsystems are governed by the gradient of the parameters and their time dependence. In this non-equilibrium universe we have spatially homogeneous states, dividing the system into small cells.

To any irreversible thermodynamic process, one can in principle, associate two important types of parameters: one to describe the 'force' that drives the process and the other to describe the 'response' to the force [12, 14]. Again, from irreversible thermodynamics if the extensive parameters of two subsystems of a system are unconstrained then an equilibrium is reached when the affinity vanishes [12, 14, 15]

The universe itself works similar to a composite irreversible thermodynamic system. As in one part of the universe or the other we have some sort of irreversible process happening due to which the entropy of the universe is always increasing.

The processes in the universe being irreversible in nature we can, in principle, safely apply the laws of non-equilibrium thermodynamics. In non-equilibrium thermodynamics if an extensive parameter of two subsystems are unconstrained, an equilibrium is obtained when the affinity vanishes [12, 13, 14].

For a universe that is expanding at an accelerated rate we have the volume to be an extensive parameter that is unconstrained. Hence for an equilibrium to be reached we must have the affinity associated with volume to be zero. The affinity associated with the volume is mathematically defined as $A_v = (\frac{\partial S^0}{\partial V})_{V_0}$ which must be zero for a system to be in an equilibrium state [12]. But if A_v is non zero we have irreversible process taking place which takes the system, here the universe, to a state of equilibrium. A state of equilibrium is also defined as the state at which the entropy of the system, here the universe, is maximum. Thus we can say that the a generalized force that drives the universe can be written as A_v. Or that A_v is the force that causes the accelerated expansion of the universe.The response to the generalized force is defined as the rate of change of the extensive parameter $J_v = \frac{dV}{dt}$ [12, 13, 14].

3.1 Volume element of the expanding universe

The FLRW metric gives the volume element as [16]

$$d^3V = R^3(t)\frac{1}{\sqrt{1-ar^2}}r^2 dr \sin\theta d\theta d\phi. \tag{5}$$

The above metric can be written as

$$dV = dV(k(t,r,\theta,\phi),\tau) = k(t)\tau, \tag{6}$$

where $\tau = r^2 dr \sin\theta d\theta d\phi$ is the normal volume element and $k(t,r,\theta,\phi) = R^3(t)\frac{1}{\sqrt{1-ar^2}}$. Where k is the expansion factor and in general can be a function of both time and space. Signifying that the expansion can vary both spatially and temporally different . Thus from equation (6) can be written as $dV = (\frac{\partial V}{\partial \tau})_k d\tau + (\frac{\partial V}{\partial k})_\tau dk$.

3.2 Affinities of expansion

The rate of production of entropy is given as

$$\frac{dS}{dt} = \frac{\partial S}{\partial V}\frac{dV}{dt}. \tag{7}$$

Substituting $\frac{dV}{dt} = \frac{\partial V}{\partial \tau}_k \frac{d\tau}{dt} + \frac{\partial V}{\partial k}_\tau \frac{dk}{dt}$ in the above equation, we get

$$\frac{dS}{dt} = \frac{\partial S}{\partial V}\left[\frac{\partial V}{\partial \tau}_k \frac{d\tau}{dt} + \frac{\partial V}{\partial k}_\tau \frac{dk}{dt}\right]. \tag{8}$$

Solving further and using the mathematical definition of flux, we have equation (8) to be given as

$$\frac{dS}{dt} = \left(\frac{\partial S}{\partial \tau}\right)_k J_\tau + \left(\frac{\partial S}{\partial k}\right)_\tau J_k. \tag{9}$$

Thus from equation (7) and equation (9) we have the affinity due to volume expansion to be given as $(\frac{\partial S}{\partial \tau})_k = A_k$ and the affinity due to spatial expansion to be given as $(\frac{\partial S}{\partial k})_\tau = A_k$

Where the affinity due to volume expansion $(\frac{\partial S}{\partial \tau})_k = A_k$ is the generalized force responsible for the increase in the mean free path of the constituents due to the actual displacement of the constituents comprising the universe. The response to the affinity $J_\tau = \frac{d\tau}{dt}$ is the response to the affinity which the flux in the universe due to the actual movement of the particles.

The affinity due to spatial expansion $(\frac{\partial S}{\partial k})_\tau = A_k$ is the generalized force responsible for the increase of the space between the constituents of the universe by stretching the fabric of the space itself. The response to the affinity $J_k = \frac{dk}{dt}$ is the flux due to the expansion of the fabric of space itself.

4. Negative pressure and dark energy

From the first law of thermodynamics, we have for a quasi-static process $\delta W = \delta Q - dU$ where, δW is the work done, δQ is the heat exchanged, dU is the change in the internal energy of the system. We have $\delta W = -pdV$. In the above expression if we substitute dV with $dV = (\frac{\partial V}{\partial \tau})_k d\tau + (\frac{\partial V}{\partial k})_\tau dk$ we find,

$$\delta W = -p\left[\left(\frac{\partial V}{\partial \tau}\right)_k\right]d\tau - p\left[\left(\frac{\partial V}{\partial k}\right)_\tau dk\right]. \tag{10}$$

From equation (10) we have the first component $-p[(\frac{\partial V}{\partial \tau})_k]d\tau$ as the normal work done while the second component $-p[(\frac{\partial V}{\partial k})_\tau]dk$ is of some interest as it represents the work done to expand the fabric of the space. Or in other words it is the work done to expand the space between any two particles in an ensemble.

A comparison of the equation (10) with $\delta W = -pdV$ we have $-p[(\frac{\partial V}{\partial \tau})_k]$ is the generalized pressure due to the motion of the constituents while $-p[(\frac{\partial V}{\partial k})_\tau]$ is the generalized pressure due to the expansion of the space.

The work done to expand the space is stored in the fabric of the space. A similar analogy can be derived from the shearing of an iron rod. When one applies a force to expand an iron rod the work done by the force is stored between the molecules of the solid in the form of inter-molecular forces. a similar kind of process happens for the fabric of space. To expand the space fabric one has to do some work which is stored in the fabric of space just like in the case of the of iron rod.

5. Conclusion

The exponential factors present in equation (4) might refer to an inflationary process as the accelerated rate of expansion of the universe is proportional to an exponential factor [2]. But this doesn't refer to an inflationary process because the equation (1) is applicable only after the first elementary particles were formed i.e during the quark epoch. Referring to an era where the universe was filled with the quark gluon plasma as these are fermions and bosons and temperatures was high enough to safely assume them to be a gas of fermions and bosons i.e. the quarks and gluons couldn't have founded any bounded structures. Thus we can safely apply the Sackur-Tetrode equation of entropy for fermions and bosons respectively. Thus the total entropy will then be $S_{total} = S_{fermions} + S_{bosons}$, where S_{total} is the total entropy of the universe and $S_{fermions}$ is the entropy due to fermionic particles and S_{bosons} is the entropy due to bosonic particles. The form $S_{total} = S_{fermions} + S_{bosons}$ can be written as entropy is a extensive parameter. The Sackur-Tetrode equation for entropy thus is valid from the Quark epoch to all way through to the current state of the universe.

From the equation (4) one can predict that the active regions of the universe i.e. regions where the rate of change of entropy $\frac{dS}{dt}$ is very high those regions will have higher red shits compared to the regions where the $\frac{dS}{dt}$ is lower. Though the Hubble's law is not compromised rather along with that if the $(\frac{dS}{dt})$ is higher for a star system the faster will the system recede from us. Thus system like quasars and active galaxies have higher red shifts than the ones which have low $\frac{dS}{dt}$.

Considering the universe as a thermodynamic system under non-equilibrium conditions we see that there are two generalized forces that causes the expansion of the universe. One is a trivial force is the affinity due to volume expansion while the other, the non-trivial one is the affinity due to spatial expansion. This, affinity due to spatial expansion, is also a reason for the accelerated expansion of the universe if not the only.

Initial total energy released during the big bang was either converted into matter or was used to expand the spacetime or for other various process. And this accounts for the missing energy or the dark energy as it is also characterized by a negative generalized pressure $-p[(\frac{\partial V}{\partial k})_\tau]$.

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