Relic Abundance of Scalar Dark Matter

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A Thesis Submitted to

Indian Institute of Technology Hyderabad

In Partial Fulfillment of the Requirements for

The Degree of Master of Science

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April 2016

Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

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This thesis entitled "Relic Abundance of Scalar Dark Matter" by Shweta Dalal is approved for the degree of Master in Physics from IIT Hyderabad.

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Acknowledgements

My sincere thanks to **Dr. Narendra Sahu** for his efforts as a teacher and an advisor who has helped me grow academically. He led me to a better understanding and broader knowledge of the subject.

I would like to thank Nirakar Sahu for numerous interesting discussions, and for kindly answering all my tedious questions over and over again.

I am thankful to Sachin and Anoop for always being willing to discuss any kind of questions and always having a positive attitude in general.

Abstract

The nature of dark matter is one of the great unsolved questions in particle physics and cosmology. Infact, the hypothesis of dark matter is strongly supported by the galaxy rotation curve, gravitational lensing and large scale structure of the universe. However, there is no candidate of dark matter in the standard model of particle physics. Different extensions of the standard model provide candidates of dark matter. This thesis concerns with two models, Scalar Singlet model and Inert Higgs Doublet Model and calculating the relic abundance of dark matter in both models, to further check their suitability as a candidate of dark matter. WMAP and PLANCK measurements of anisotropies- the Cosmic Microwave Background precisely measure the relic abundance of dark matter. We compare our theoretical estimated value of the relic abundance of dark matter with the WMAP and PLANCK value of relic abundance and find the relevant constraints on the model parameters.

Contents

Chapter 1

Introduction

At present, the fundamental particles and the interaction among themselves are best understood in terms of a gauge theory called the Standard Model ($\mathcal{SM}$) of particle physics which is based on a non-abelian gauge theory with the symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$ where C stands for colour, L stands for left-handed doubled particle and Y stands for the hypercharge. We have four fundamental forces in nature: strong, gravitational, weak and electromagnetic. The fundamental particles, includes 12 fermions, 4 gauge bosons, and with the recent discovery of Higgs boson, $\mathcal{SM}$ seems to be complete. According to this model, all matter is built from spin$\frac{1}{2}$ fundamental particles, or fermions: 6 quarks and 6 leptons. Unlike bosons, all matter particles have associated antimatter particles, like anti-quarks and anti-electron (positron).

1.1 Fundamental Fermions

The fundamental matter particles are quarks and leptons. All fermions follow Fermi-Dirac statistics and Pauli exclusion principle. Figure 1.1 shows all fundamental fermions with charge, mass and flavor.

We are familiar with electron whose rest mass is $0.511 MeV/c^2$. Charged leptons such as electron (e) , muon (μ) and tauon (τ) carry unit negative charge and neutrinos (ν_e, ν_μ, ν_τ) are neutral leptons. The neutrinos are peculiar, they are neutral under both the strong and the electromagnetic interactions. As indicated by oscillation experiment, neutrinos are massive ,but they are at least six order of magnitudes lighter than all the other $\mathcal{SM}$ fermions. As neutrinos do not carry electric charge, so their motion is directly influenced only by the weak nuclear force, which makes them notoriously difficult to detect. μ and τ are heavier than electron and are both unstable. So, μ

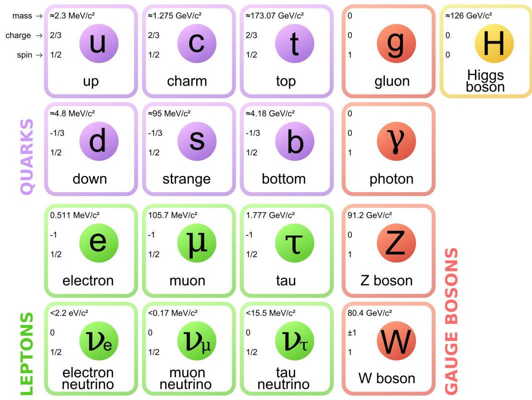

Figure 1.1: Pictorial Representation of$\mathcal{SM}$ particles [1]

and τ decays simultaneously into electron, neutrinos and other particles. All leptons are spin half particles. However, by virtue of carrying an electric charge, the electron, muon, and tau all interact electromagnetically.

There are six quarks: up, down, charm, strange, top and bottom. The up quarks carry fractional charge$\frac{2}{3}$ and down quarks carry $-\frac{1}{3}$. A quark also carry one of the primary colours (red, green, blue). They carry color charge and hence, interact with strong interactions. Strange quarks were produced prolifically in strong interactions and therefore, expected to decay in strong interaction timescale ($10^{-23}sec$) ; instead they decay via weak interaction. Quarks cannot be isolated singularly because of color confinement phenomenon and hence they form color-neutral composite particles (hadrons) containing either quark-antiquark pair (mesons) or three quarks (baryons). As, quarks also carry electric charge and weak isospin, they interact by both electromagnetic and the weak interactions.

1.2 Gauge Bosons

At the present, these four types of interactions are sufficient to explain all phenomena in physics.

Table 1.1: Gauge Bosons [5]

Force	Mediator	Spin/Parity
Strong	Gluon	1^-
Electromagnetic	Photon	1^-
Weak	$W^+, W^- and\ Z^0$	$1^-, 1^+$
Gravitational	Graviton	2^+

1.2.1 Strong Interaction

Strong interactions are responsible for strong nuclear forces. Strong forces act only at short distances, they bind quarks together to make nucleons (protons and neutrons) and bind nucleons together to make nuclei. Interaction between quarks is mediated by boson called a gluon, neutral and massless carrier of strong force. Gluon is a vector particle with spin-parity as 1^- . To understand strong force theory, we study Quantum Chromodynamics (QCD) which is a non-abelian theory based on a local (gauge) symmetry group called $SU(3)_C$.

1.2.2 Electromagnetic Interaction

Electromagnetic interaction is responsible for bound states of electrons with nuclei. It is mediated by photon and is long ranged. It acts between electrically charged particles. We study electrodynamics to understand the electromagnetic interaction. Electromagnetic interaction is responsible virtually for all the phenomena we see around like lightning and many man-made devices.

1.2.3 Weak Interaction

Weak interactions or weak nuclear force is responsible for radioactive decay of subatomic particles like β decay. Fermions like neutrinos also interact by weak interactions. The mediators of the weak interaction are $W^+, W^- and\ Z^0$ bosons. Figure 1.2 shows how elementary particles interact according to $\mathcal{SM}$.

1.2.4 Gravitational Interaction

Gravitational is the only interaction with acts on all particles having mass, energy and/or momentum. This interaction has infinite range like electromagnetic but unlike electromagnetic interaction, this is always attractive and never repulsive. Gravity is the weakest of all fundamental interactions, although it is dominant on the scale of the universe. It is supposedly mediated by the exchange of spin 2 boson, called graviton.

1.2.5 Higgs Boson

The masses of the quarks and leptons,$W^+, W^- and\ Z^0$ bosons are generated through what is known as electroweak symmetry breaking. This process leaves the photon massless thereby rendering the electromagnetic force long-ranged, macroscopic and quite different from the weak interactions. According to the Standard Model, the Higgs field exists throughout space, and breaks certain symmetry

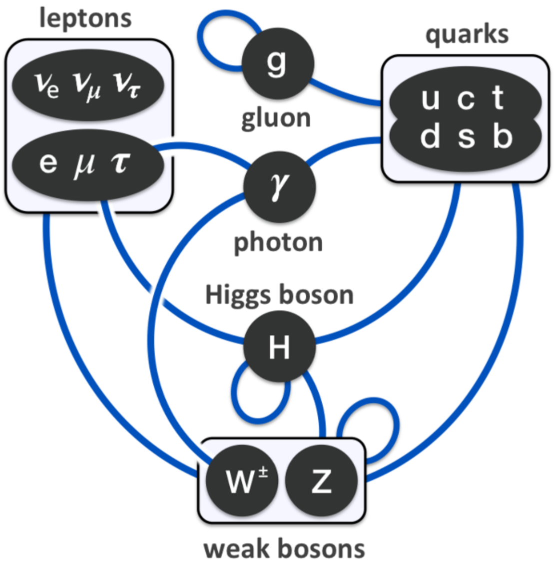

Figure 1.2: Interaction between elementary particles according to $\mathcal{SM}$ [1]

laws of the electroweak interaction. The existence of this field triggers the Higgs mechanism, causing the gauge bosons responsible for the weak force to be massive, and explaining their very short range. Experimentalists have detected the Higgs boson having mass 125 GeV which is the quantum excitation of the Higgs field.

1.3 Successes of the Standard Model

The SM has been successfully tested at an impressive level of accuracy and provides at present our best fundamental understanding of the phenomenology of particle physics.

- The biggest success of the Standard Model is the prediction of the Higgs Boson. The particle has been experimentally observed in 2012 though it was postulated 50 years ago.
- Other successes of the standard model include the prediction of the $W^{\pm} and\ Z^0$ bosons, the gluon, and the top and charm quark, before they have even been observed.
- The prediction of the masses of the W and Z boson compared with experimental data.

Table 1.2: $W^+, W^- and\ Z^0$ boson mass

Boson	$\mathcal{SM}$ Predicted mass (GeV)	Measured mass (GeV)
$W^{\pm}$	80.390±0.018	80.387±0.019
Z^0	91.1874 ± 0.0021	91.1876 ± 0.0021

1.4 Failures of the Standard Model

These include:

- The $\mathcal{SM}$ predicts neutrinos to be massless. On the other hand, neutrino oscillation experiments confirmed that neutrinos are massive however small.

- We have many evidences (as discussed below) that Universe consists of mysterious form of invisible matter which is approximately five times the visible mass, dark matter. Among all the standard model particles, none has the properties of dark matter and hence, we need to go beyond the standard model of particle physics to incorporate dark matter.

1.5 Evidence of Dark Matter

Throughout the Universe, the evidence for dark matter is compelling astrophysical environment such as in stars, gas clouds, spiral galaxies, galaxy clusters as well as at cosmological scales.

1.5.1 Evidence for dark matter in galaxy clusters

Galaxies are held together in a spinning galactic cluster by the gravitational force provided by its matter. However, gravitational force generated by the visible matter in a cluster was not enough to avoid galaxies from scattering. So, there is non-luminous matter which provides this extra gravitational force for holding galaxy clusters tightly together, which we call as Dark Matter. Observations also shows that visible mass from light of galaxies is less than mass calculated from the speed of the galaxies.

1.5.2 Rotation Curve in spiral galaxies

From the Newtonian dynamics, we expect the velocity of stars falling as we move away from the center of mass of galaxy.

$$v = \sqrt{\tfrac{GM(r)}{r}}$$

where $M(r) = 4\pi \int \rho(r)\, r^2 \mathrm{d}r$, $\rho(r)$ is the mass density profile. We would thus expect that beyond the optical disk $v(r)$ drops as $r^{-1/2}$. However, the observations of rotation curves of spiral galaxy dont decrease as expected rather it becomes flat at larger distances as shown in Figure 1.3. This could happen if large amount of invisible matter filled the entire galaxy and beyond.

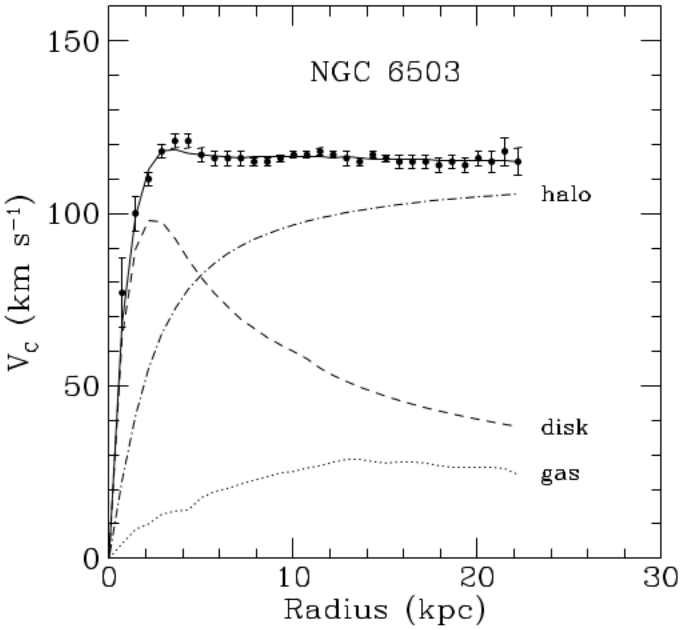

Figure 1.3: Rotation curve of NGC 6503 from [7] The measured curve shows a flat behaviour at large distances from the galatic center.

1.5.3 Bullet Cluster

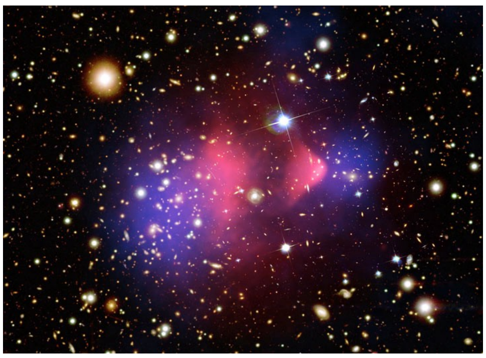

Figure 1.4: Bullet Cluster [6]

Bullet cluster (1E0657-558) is collision of two galaxy clusters in which we observe two different regions. Figure 1.4 shows bullet cluster with blue region showing the dark matter which we determine from gravitational lensing and pink showing visible mass, determined by x-ray emission. Explanation given for the presence of different regions is that in the process of collision, dark matter and interstellar gas separated, as in evidenced by gravitational lensing profile of the cluster.

1.5.4 Cosmic Microwave Background

CMB observations have a characteristic imprint on the existence of dark matter. WMAP and PLANCK observations which measures the temperature fluctuations in the CMB sky reveals that its contents include 4.6% atoms, the building blocks of stars and planets. Dark matter comprises 23% of the universe (Figure 1.5). This matter, different from atoms, does not emit or absorb light. It has only been detected indirectly via its gravity. 72% of the universe, is composed of "dark energy", that acts as a sort of an anti-gravity. This energy, distinct from dark matter, is responsible for the present-day acceleration of the universal expansion.

1.6 Dark Matter

Dark matter is not directly detected yet in the earth bound laboratory. Generally the dark matter candidate is stable and abundant in the early Universe. To find the relic abundance of dark matter, standard model of cosmology is taken into account.

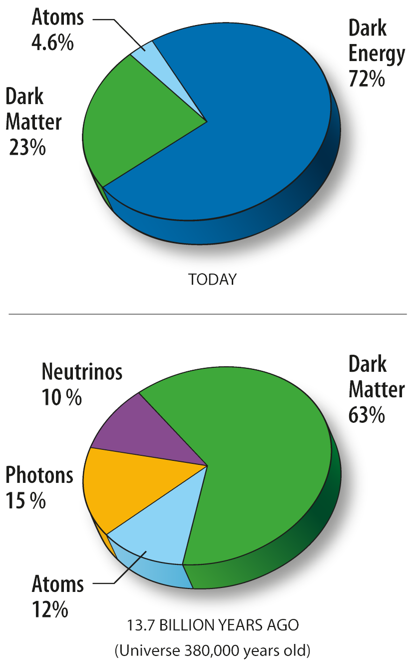

Figure 1.5: Pie Chart for energy budget of the Universe [8]

Relic density

Relic density of dark matter is also constrained by the results of PLANCK and WMAP. [4] Annihila-

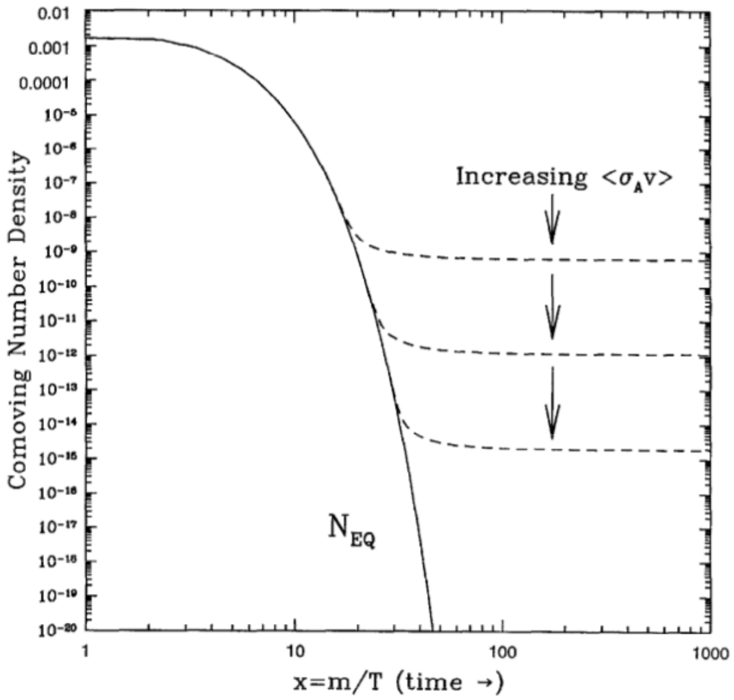

Figure 1.6: Comoving number density of a dark matter candidate in the early Universe as a function of $x = m_\chi/T$(increases with increasing time) from . If the particle remains in thermal equilibrium, its abundance is exponentially suppressed (solid line). The actual abundance after freeze-out stays constant in a comoving volume and depends on the annihilation cross section: the larger $< \sigma v >$ is, the smaller is the abundance. [2]

tion into lighter $\mathcal{SM}$ particles and the inverse processes keep the dark matter in thermal equilibrium and coupled to the plasma. If the temperature of the Universe falls below the dark matter mass, the dark matter number density falls exponentially. Therefore, remaining in equilibrium would result in a strongly suppressed todays dark matter abundance, possibly even lead to a complete disappearance. However, if the Universe expands fast enough, the annihilation rate drops below the expansion rate and dark matter decouples from the plasma. This departure from equilibrium, called freeze-out, opens a possibility of maintaining a sizeable dark matter abundance. Figure 1.6 shows that the actual abundance after freeze-out stays constant in a comoving volume and depends on the annihilation cross section. Dark matter relic abundance is evaluated by solving the evolution of Boltzmann equation given as:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - < \sigma v > [n_\chi^2 - (n_\chi^{EQ})^2] \tag{1.1}$$

where H is Hubble expansion rate and n_χ is dark matter number density. Assuming that $< \sigma v >$ is temperature independent, an estimate on the present abundance is found from the following equation:

$$\Omega h^2 = 1.07 \times 10^9 \frac{x_\chi \; GeV^{-1}}{(g_{\star s}/g_\star^{1/2}) M_{Pl} < \sigma v >} \tag{1.2}$$

Here, M_{Pl} is planck mass and $g_{\star s}$ and $g_\star$ are total number of effective massless degree of freedom. Finally we will get

$$\Omega h^2 = \frac{2.12 \times 10^{-10} \, /GeV^2}{< \sigma v >} \tag{1.3}$$

Later in calculations, this equation will be used to find relic abundance theortically. For a dark matter candidate, the correct abundance can be obtained with an annihilation cross section $< \sigma v > \approx pb$. This is the typical scale of weak interactions:$< \sigma v > \approx \alpha_2 (100 \; GeV)^{-2} \approx pb$.

As, WIMPs (weakly interacting massive particle) are particularly well suited as dark matter candidates and there is no candidate for dark matter in $\mathcal{SM}$ of particle physics. We go beyond the $\mathcal{SM}$ and consider two different models, Scalar Singlet and Inert Doublet Model in the next chapters. We intend to find cross section and relic abundance of dark matter in both models and check their suitability as a candidate of dark matter.

Chapter 2

Scalar Singlet Dark Matter Model

As mentioned in Dark Matter section of chapter 1, there is no candidate of dark matter in the $\mathcal{SM}$. A plethora of models in the beyond $\mathcal{SM}$ scenarios have been proposed. But, none can be fully accepted until they pass the experimental gauntlet. These models must satisfy the properties of dark matter. The most extreme of the minimal dark matter models is the scalar singlet model, which we explore here. This idea has been considered several times in the past to provides a baseline model for WIMP dark matter. The main attraction of the scalar singlet model is its fundamental simplicity.

2.1 Model Construction

Let us begin by writing the Lagrangian density in this model to understand the singlet interactions with the SM fields.

$$\mathcal{L} = \mathcal{L}_{\mathcal{SM}} + \partial_\mu S^\dagger \partial^\mu S - V(S, H) \tag{2.1}$$

where

$$V(S, H) = m_1^2 H^\dagger H + \lambda_h (H^\dagger H)^2 + m_0^2 S^\dagger S + \lambda_s (S^\dagger S)^2 + \lambda (S^\dagger S)(H^\dagger H) \tag{2.2}$$

$\mathcal{L}_{\mathcal{SM}}$ is the $\mathcal{SM}$ Lagrangian, 'S' is the scalar singlet, λ_s is its quartic coupling and λ is the Higgs-singlet coupling. This model has a Z_2 symmetry,i.e

$$S \to -S,$$

which gurantees the stability of the S scalars by eliminating the interaction terms involving odd powers of S and $S^\dagger$ which lead to S decay. H is $\mathcal{SM}$ Higgs doublet,which after spontaneous symmetry breaking acquires a Vaccum Expectation Value (VEV),

$$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix}$$

Here,v the vacuum expectation value of the Higgs field, is taken to be 246 GeV. The singlet scalar S does not couple to gauge bosons and fermions whereas doublet H couples with them as in the $\mathcal{SM}$.

Singlet S has a vanishing VEV which ensures the dark matter stability.

$$S = s$$

$$V(S,H) = m_0^2(s)^2 + \lambda_s(s)^4 + \frac{\lambda}{2}(s)^2(v+h)^2 + \frac{m_1^2}{4}(v+h)^2 + \frac{\lambda_h}{4}(v+h)^4 \tag{2.3}$$

$$V(S,H) \subset (m_0^2 + \frac{\lambda}{2}v^2)s^2 + \lambda_s s^4 + \frac{\lambda}{2}s^2h^2 + \lambda v s^2 h \tag{2.4}$$

First term denotes the physical singlet mass i.e $m_s^2 = m_0^2 + \frac{\lambda}{2}v^2$ and second term is singlet self-interaction term, and the last two terms represent the ssh and $sshh$ interaction vertices between the singlets and $\mathcal{SM}$ Higgs bosons.

In order to calculate the relic density arising from S scalars freezing out of thermal equilibrium, we considered the Boltzmann equation and after solving analytically, relic denisty is given as

$$\Omega h^2 = \frac{2.12 \times 10^{-10}\,/GeV^2}{< \sigma v >} \tag{2.5}$$

where it has been assumed that $< \sigma v >$is independent of temperature. To calculate cross-section, we considered all possible ways through which S annihilates to the $\mathcal{SM}$ particles. Depending on its mass, the S scalar can annihilate via Higgs exchange following the channels:

$SS \to hh$ $\quad$ $SS \to ZZ$ $\quad$ $SS \to W^+W^-$ $\quad$ $SS \to f\bar{f}$

as shown in Figure 2.1, 2.2 and 2.3.

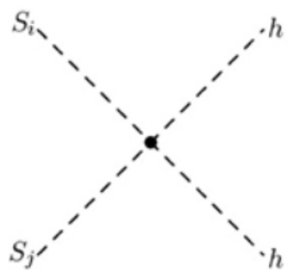

Figure 2.1: Annihilation of Dark Matter into Higgs

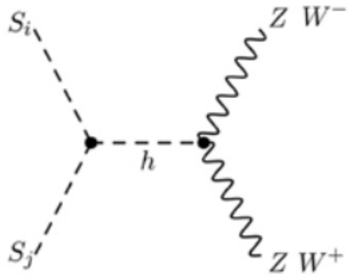

Figure 2.2: Annihilation of Dark Matter into W,Z bosons

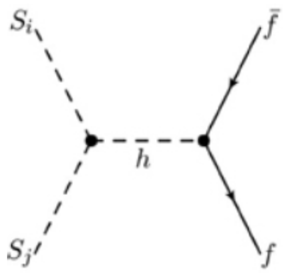

Figure 2.3: Annihilation of Dark Matter into $\mathcal{SM}$ fermions

The corresponding contribution to $< \sigma v >$ is given by:

$\underline{SS \to hh}$

$$< \sigma v >_{hh} = \frac{\lambda^2}{128\pi m_s^2}\left(1 - \frac{m_h^2}{m_s^2}\right)^{\frac{1}{2}} \tag{2.6}$$

$\underline{SS \to W^+W^-}$

$$< \sigma v >_{WW} = \left(3 + \frac{4m_s^4}{m_W^4} - \frac{m_s^2}{m_W^2}\right)\frac{\lambda^2 m_W^4}{8\pi m_s^2\big((4m_s^2 - m_h^2)^2 + m_h^2\Gamma_h^2\big)}\left(1 - \frac{m_W^2}{m_s^2}\right)^{\frac{1}{2}} \tag{2.7}$$

$\underline{SS \to ZZ}$

$$< \sigma v >_{ZZ} = \left(3 + \frac{4m_s^4}{m_Z^4} - \frac{m_s^2}{m_Z^2}\right)\frac{\lambda^2 m_Z^4}{16\pi m_s^2\big((4m_s^2 - m_h^2)^2 + m_h^2\Gamma_h^2\big)}\left(1 - \frac{m_Z^2}{m_s^2}\right)^{\frac{1}{2}} \tag{2.8}$$

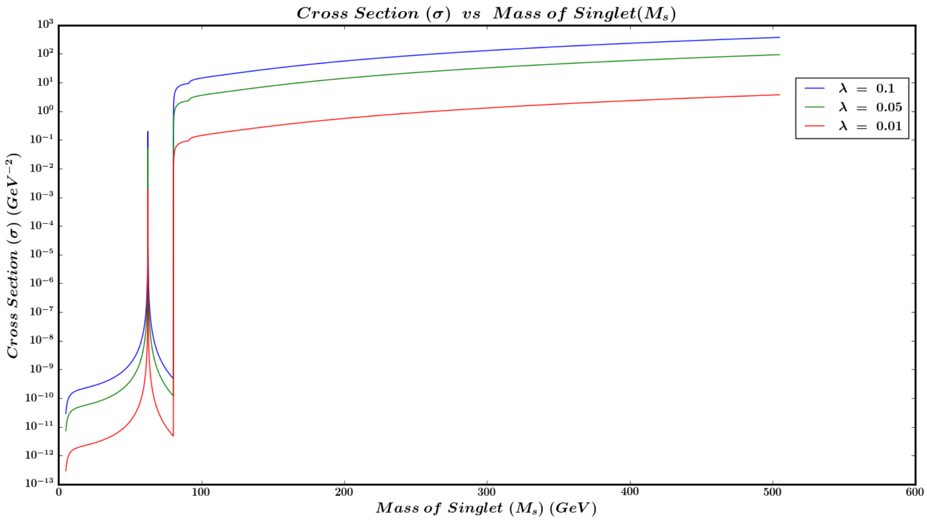

Figure 2.4: Cross section vs singlet mass

$\underline{SS \to f\bar{f}}$

$$< \sigma v >_{f\bar{f}} = N_C \frac{\lambda^2 m_f^2}{4\pi\big((4m_s^2 - m_h^2)^2 + m_h^2\Gamma_h^2\big)} \left(1 - \frac{m_f^2}{m_s^2}\right)^{\frac{3}{2}} \tag{2.9}$$

Here, m_f is the fermion mass, m_h is the Higgs boson mass, m_W is W boson mass, m_Z is Z boson mass and Γ_h^2 is the Higgs decay width, for which we use the standard model values i.e $4MeV$. Also, N_C is colour charge which is 3 for quark.

There are two free parameters, mass of dark matter m_s and coupling constant λ.

The total velocity scaled annihilation cross section as a function of singlet mass is plotted for different coupling values. We observe a resonance peak at $m_s = m_h/2$ for all coupling values as shown in Figure 2.4. At low values of singlet masses, only fermions contribute to the annihilation cross-section. When $m_s \geq 80\ GeV$, W^+, W^- and Z bosons have dominant contribution to the annihilation cross-section. At higher singlet mass, dark matter decays into Higgs boson is also possible, which further contributes to annihilation cross-section at 125 GeV. Around resonance peak, dark matter candidate have an annihilation cross section of typical scale of weak interactions: $< \sigma v > \approx \alpha_2 (100\ GeV)^{-2} \approx pb$. From equation 1.3, relic density is calculated for all the channels. Similarly, relic density is also plotted against singlet mass for different values of coupling. Figure 2.5 also include PLANCK observed data from [11].

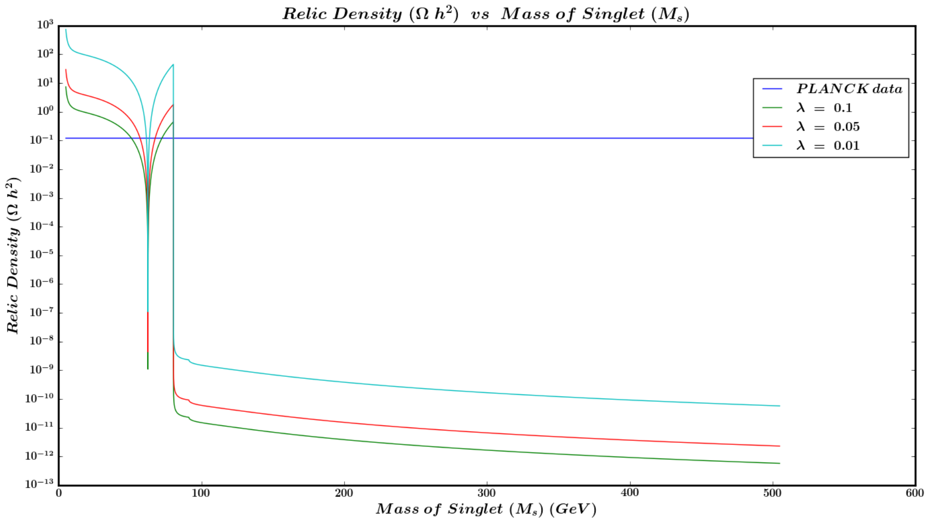

Figure 2.5: Relic Density vs singlet mass

2.2 Higgs Invisible/Unseen decay

In addition to the decays into $\mathcal{SM}$, the CP-even Higgs boson h have a number of possible invisible decays. By "invisble" we mean decays modes that contains invisible particles such as neutrinos, dark matter etc. The $h \to SS$ decay width is given by:

$$\Gamma(h \to SS) = \frac{(\lambda v)^2}{32\pi m_h}\left(1 - \frac{4m_s^2}{m_h^2}\right)^{\frac{1}{2}} \tag{2.10}$$

$$\text{Branching Ratio, } BR(h \to SS) = \frac{\Gamma(h\to SS)}{\Gamma(h\to SS)+\Gamma_h}$$

There is also an interesting feature in the branching ratio ≤ 0.3 at the Higgs mass $m_h = 125GeV$, where the Higgs production cross section peaks due to the resonance between the Higgs and singlet masses.Figure 2.6 shows all those value of λ and m_s for which $BR(h \to SS) < 0.3$.

Accoding to PLANCK data from [11], relic density of dark matter is

$$\Omega\ h^2 = 0.1199 \pm 0.0027$$

in the present universe. Figure 2.7 shows all those value of λ and m_s for which $\Omega\ h^2$ is between 0.1172 and 0.1226, i.e. satisfies the observed value of relic density.

We consider only those values of independent parameters which satisfies both the conditions, $BR(h \to SS) < 0.3$ and $\Omega\ h^2 = 0.1199 \pm 0.0027$. The range of values of λ and m_s for which these two conditions are valid is very small. Value of coupling is also of the order of 10^{-2}.

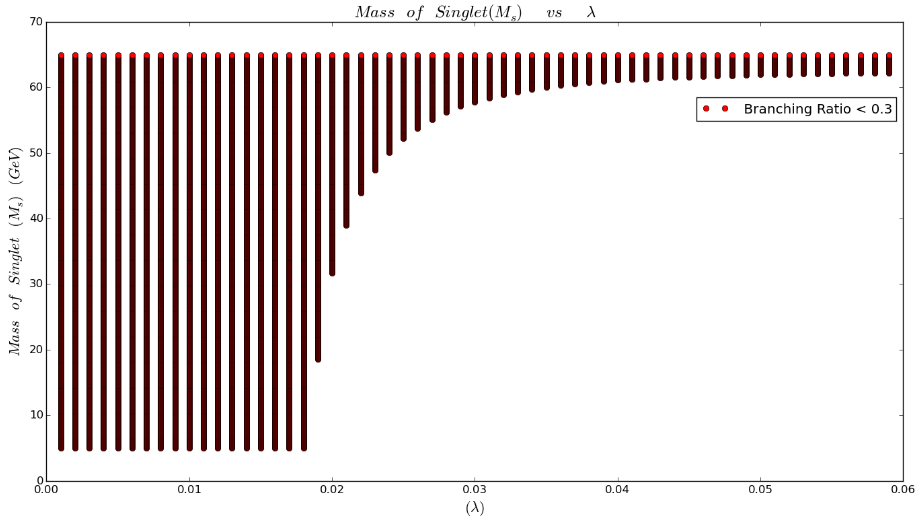

Figure 2.6: Value of coupling and singlet mass which gives branching ratio ≤ 0.3

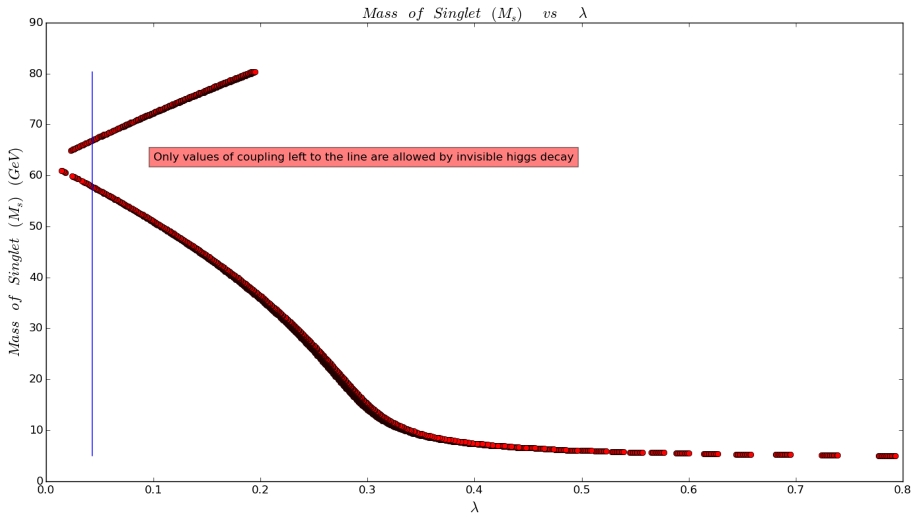

Figure 2.7: Value of coupling and singlet mass with $\Omega\ h^2$ in between 0.1172 and 0.1226,

2.3 Summary

In this chapter we discussed in detail the Scalar Singlet Model. In particular we saw that the singlet extension model has a plausible DM candidate which is stable. We calculated annihilation cross section for all possible ways in which dark matter singlet decays into standard model particles and finally plotted relic abundance against singlet mass. We analysed the plots to estimate the model parameters for correct observed relic abundance. We also took care of the higgs invisible decay channel i.e $h \to SS$. For Singlet mass,

$$57\,GeV\ <\ m_s\ <\ 65\,GeV$$

and coupling,

$$0.01\ <\ \lambda\ <\ 0.045$$

are allowed to have correct value of relic abundance.

We will consider another model, Inert Doublet model in the next chapter.

Chapter 3

Inert Scalar Doublet Model

The Inert Doublet Model is obtained by extending the standard model by an extra Z_2 symmetry and a second Higgs doublet odd under this symmetry. The major consequence is the stability of the lightest particle of this doublet which makes it a candidate for dark matter. In this chapter, we will consider all annihilation processes and find their cross-section and finally relic abundnace, after solving the potential of the model.

3.1 Potential and Parameters

We consider two Higgs doublet H_1 and H_2 version of $\mathcal{SM}$ with a Z_2 symmetry such that

$$H_1 \to H_1 \text{ and } H_2 \to -H_2$$

In this model, H_2 doesnot have any vaccum expectation value.

$$\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \qquad \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0 + \iota A^0) \end{pmatrix}$$

The most general potential of the model can be written as:

$$\begin{aligned} V(H_1, H_2) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \\ \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2) + h.c.\right] \end{aligned} \tag{3.1}$$

Solving this potential, we will get

$$V(H_1,H_2)=\frac{\mu_1^2}{2}(v+h)^2+\mu_2^2\left[H^-H^++\frac{(H^0)^2+(A^0)^2+\iota(H^0A^0-A^0H^0)}{2}\right]+\frac{\lambda_1}{4}(v+h)^4+$$
$$\lambda_2\left[H^-H^++\frac{(H^0)^2+(A^0)^2+\iota(H^0A^0-A^0H^0)}{2}\right]^2+\lambda_3\left[H^-H^++\frac{(H^0)^2+(A^0)^2+\iota(H^0A^0-A^0H^0)}{2}\right]\frac{(v+h)^2}{2}+$$
$$\frac{\lambda_4}{4}\big((H^0)^2+(A^0)^2+\iota(H^0A^0-A^0H^0)\big)(v+h)^2+\frac{\lambda_5}{2}(v+h)^2((H^0)^2-(A^0)^2)$$

where the λ_i are real quartic couplings.We know the vacuum expectation value $v = 248\ GeV$ for higgs boson. Mass terms of particles derived from the potential.

Mass of higgs boson is given by:

$$m_h^2 = 2\lambda_1 v^2$$

whereas the mass of charged, $H^\pm$, neutral H^0 and A^0 of the H_2 fiels is given by:

$$m_{H^\pm}^2 = m_2^2 + \lambda_3 \frac{v^2}{2}$$

$$m_{H^0}^2 = m_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)\frac{v^2}{2} \tag{3.2}$$

$$m_{A^0}^2 = m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)\frac{v^2}{2}$$

As a consequence of the unbroken Z_2 symmetry, the lightest odd particle is stable and therefore a candidate for dark matter. In absence of other lighter fields odd under Z_2 this will be the lightest component of the inert doublet, either H^0 or A^0. In this work, H^0 is chosen to be the lightest inert particle and a dark matter candidate studied in the following. The coupling of Higgs Boson with H^0 is given by $\lambda_l = (\lambda_3 + \lambda_4 + \lambda_5)/2$, which we can get from Equation 3.1. The mass splittings between A^0 and H^0 is defined as $\Delta m_{A^0} = m_{A^0} - m_{H^0}$ and mass splittings between $H^\pm$ and H^0 is defined as $\Delta m_{H^\pm} = m_{H^\pm} - m_{H^0}$. In the next section, we will find all possible annihilation and co-annihilation channels through which H^0 can interact with $\mathcal{SM}$ particles.

3.2 Annihilation Channels

In the following sections, different cross sections will be needed for further calculations. For the sake of a better overview, different channels are summarized in this section.

3.2.1 Annihilation into Fermions

For less massive H^0 ($m_H^0 < m_W$), the only possible annihilation channel is the one through an intermediate Higgs boson into the kinematically allowed fermions. Ffigure 3.1 shows the feynmann diagram for the annihilation process:

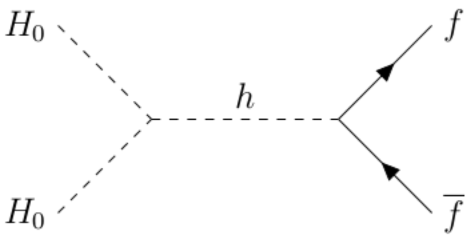

Figure 3.1: Annihilation into fermions [10]

3.2.2 Annihilation into Bosons

For $m_{H^0} \geq 80GeV$, dark matter mainly annihilates into $W^{\pm}$ and Z^0 bosons. Annihilation of massive H^0 into Higgs boson is possible when $m_{H^0} \geq 125GeV$. Figure 3.2 and 3.3 shows all possible annihilation channels into $W^{\pm}$ and Z^0 bosons and Higgs bosons.

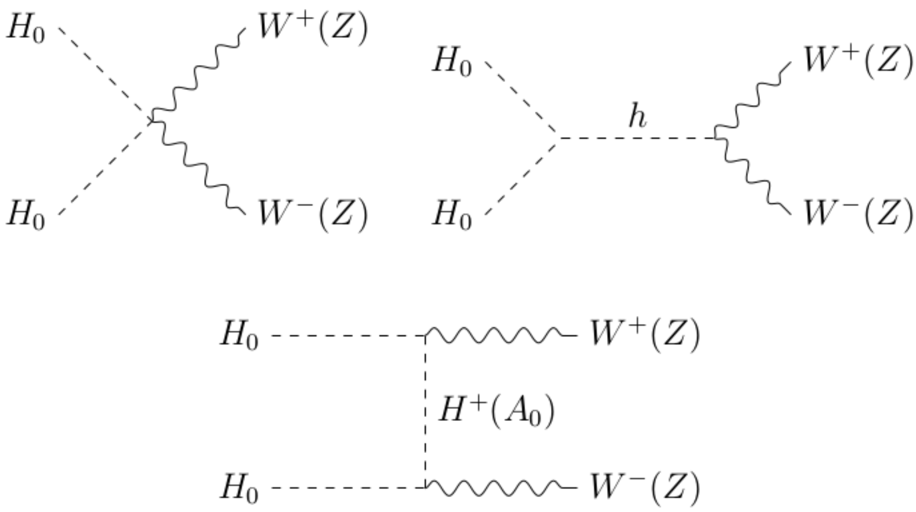

Figure 3.2: Annihilation into $W^{\pm}$ and Z^0 bosons [10]

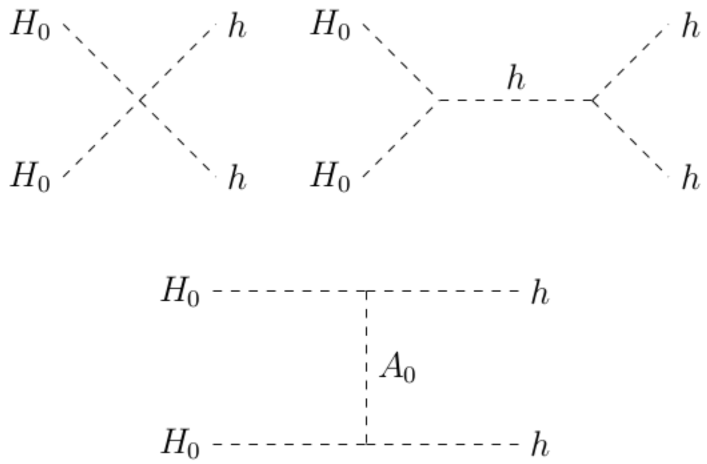

Figure 3.3: Annihilation into Higgs bosons [10]

3.3 Co-annihilation Channel

The coannihilation of H^0 and A^0 (H^+) is another interaction which is possible only in the early Universe (when A^0 is unstable). This process plays an important role for the relic abundance of dark matter especially if the mass splitting between both particles is not too large. For a mass splitting of the order $T_{fo} \sim m_{H^0}/25$, coannihilation is possible and therefore should be taken into account in the calculation of the dark matter relic abundance. A large coannihilation rate leads to a small abundance and vice versa. The coannihilation of H^0 and A^0 (H^+) takes place through an intermediate Z-boson ($W^+ boson$). This effect on the abundance will not be discussed in this chapter.

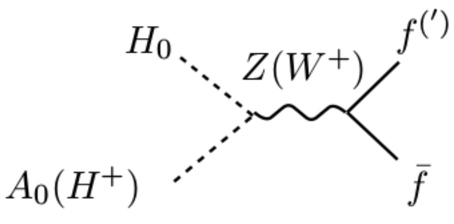

Figure 3.4: Co-Annihilation into fermion and anti-fermion [10]

3.4 Plots

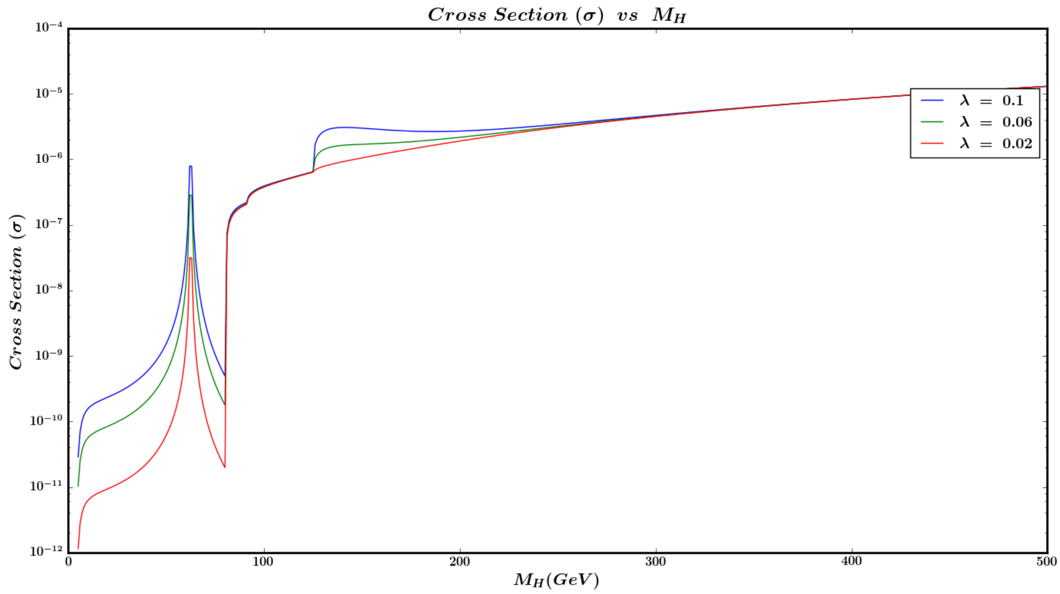

Figure 3.5: $\sigma \;\; vs \;\; M_{H^0}$

The annihilation cross section as a function of singlet mass is plotted for different coupling values. While calculating the cross section mixed terms of amplitude was not considered. In this case, we observe a resonance peak at $m_{H^0} = m_h/2$ for all coupling values as shown in Figure no.3.5. At low values of dark matter mass, only fermions contribute to the annihilation cross-section. When $m_{H^0} \geq 80\ GeV$, W^+, W^- and Z bosons have dominant contributing to the annihilation cross-section. At higher mass range, dark matter decays into Higgs boson is also possible, which further contributes to annihilation cross-section at 125 GeV. Around resonance peak, dark matter candidate have an annihilation cross section of typical scale of weak interactions:$< \sigma v > \approx \alpha_2 (100\ GeV)^{-2} \approx pb$. Similarly, for different coupling relic density is plotted against dark matter mass, m_{H^0} in Figure 3.6.

Figure 3.7 shows all those value of λ_l and m_{H^0} for which $\Omega\ h^2$ is between 0.1172 and 0.1226, i.e. satisfies the observed value of relic density. From the graph, we can estimate the range of model parameters.

3.5 Summary

This chapter concerned with how cross-section and relic abundance of dark matter varies with m_{H^0}, mass of dark matter and λ_l, coupling constant between Higgs and dark matter. We again observed

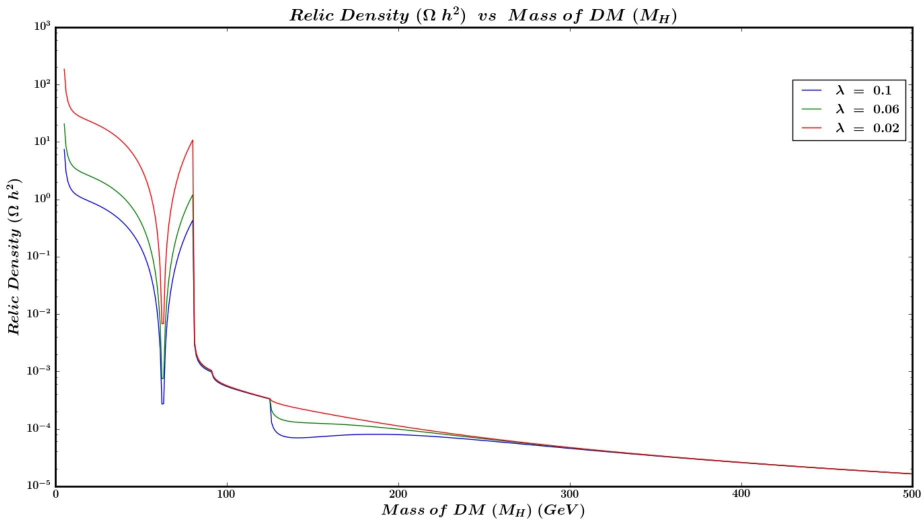

Figure 3.6: $\Omega h^2 \;\; vs \;\; M_{H^0}$

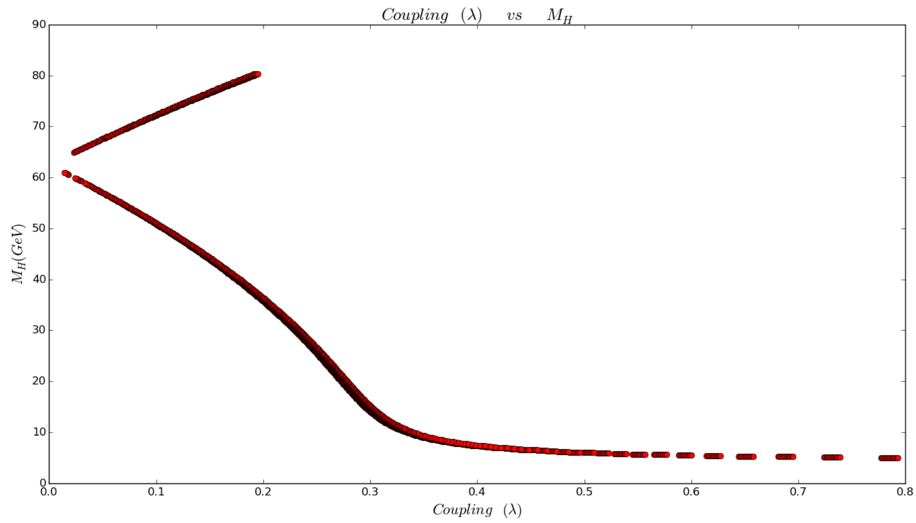

Figure 3.7: Value of coupling and dark matter mass for which $\Omega\, h^2$ is between 0.1172 and 0.1226

a resonance peak at $m_{H^0} = m_h/2$. Cross-section and relic abundance plots donot include the co-annihilation channel contribution.

$0.1172 \leq \Omega h^2 \leq 0.1226$

Chapter 4

Conclusion

We know a theory is only hypothetical until experimental measurements validate it's existence. Any given theory has to comply with experiments for it to be considered viable, needlessto say under acceptable error, where the errors may arise from statiscal analysis or detectors etc.

In the case of scalar singlet model, we found a range for singlet mass and coupling which satisfies the observed relic abundance. In this model, dominant contribution to relic abundance is from scalar particles. We considered another model, Inert doublet model and again plotted relic abundance against dark matter mass. This model has dominant contribution to relic abundance from $W^{\pm}$ and Z bosons.

In models discussed in the thesis, dark matter mass is approximately close to $m_h/2$, i.e in low mass region and coupling is very small of the order of 10^{-2}. We can also consider high mass region which will have high value of coupling, where we can still get correct relic abundance.

We can then test the model at direct, indirect and collider search methods. So far we have not succeeded yet to detect the dark matter. So we can not say about the efficacy of any of the model.

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