Relic Abundance of Singlet Doublet Fermionic Dark Matter

SAMBO SARKAR

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The Degree of Master of Science

Department of Physics

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Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

(Signature)

Sambo. Sarkar

(SAMBO SARKAR)

PH14HSCST 11011

(Roll No.)

Approval Sheet

This Thesis entitled Relic Abundance of Singlet Doublet Fermionic Dark Matter by SAMBO SARKAR is approved for the degree of Master of Science from IIT Hyderabad

() Examiner
Dept. of Physics
IITH

() Examiner
Dept. Physics
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Dept. Physics
IITH

() Examiner
Dept. Physics
IITH

() Examiner
Dept. Physics
IITH

(Dr. Narendra Sahu) Adviser
Dept. of Physics
IITH

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Dedication

I would like to dedicate this work to my beloved Parents and family,who have made it possible for me to see this day

Abstract

Recent Sattelite experiments like WMAP & PLANCK precisely measured the dark matter components to be 26.8 % of the total energy budget of the universe.Infact it is supported by the indirect observations, such as galaxy collision in the bullet cluster,gravitational lensing and the rotational curve .However the existence of dark matter cannot be explained within the Standard Model of Prticle Physics.In this thesis we studied a viable solution of dark matter by extending the standard model with vector like Leptons.In particular we studied two scenarioes.
1.In one we augmented the standard model with singlet vector like leptons which is odd under Z_2 symmetry and calculated the parameter space in which we get the correct relic abundance.
2.Secondly we extended the standard model with a vector like doublet and a singlet which is odd under Z_2 symmetry. Then we calculated this relic abundance of Dark Matter in a suitable parameter space.

Contents

Chapter 1

Introduction

1.1 Standard Model of Particle Physics

Till date the most successful theory defining the electromagnetic and weak interactions together has been presented in the standard model of particle physics .This model id based on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ where the U(1) was associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge through the analogous of the Gell-Mann-Nishijima formula ($Q = T3 + Y/2$).In 1967,Weinberg and independently Salam in 1968, employed the idea of spontaneous symmetry breaking and the Higgs mechanism to give mass to the weak bosons and, at the same time, to preserve the gauge invariance in this theory.The Glashow–Weinberg–Salam model is known, at the present time, as the Standard Model of Electroweak Interactions.

The fundamental interactions their strength and corresponding mediating particles can be summarised as

INTERACTION	STRENGTH	MEDIATOR
NUCLEAR	1	GLUON
ELECTRO-MAGNETIC	10^{-2}	PHOTON
WEAK	10^{-7}	BOSONS(W,Z)
GRAVITATIONAL	10^{-36}	GRAVITON

1.2 The Electro-Weak Interctions

In the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry

$$C = Colour; Y = Hypercharge; L = Left - handed$$

The fundamental particles are categorised into Three types
$(i).Spin\frac{1}{2} - Thefermions$
(ii). Spin 0 particles-The bosons

(iii).Spin 1 particles =The Gauge bosons

1.2.1 Spin $\frac{1}{2}$ Fermions

Leptons

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

These particles forming the doublet show a left handedness and a hupercharge of(Y=-1). The right handedness is exhibited by the singlet having hypercharge (Y=-2)comprising

$$e_R, \mu_R, \tau_R$$

The charged parts are responsible for the electromagnetic Interactions while all the leptons can have weak interactions'

Quarks

The Baryons are fundamentally constituted out of quarks, There are Three Genaration of Quarks showing Left handed nature and in a simillar fashipn we also get their Right Handed partners.

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$u_R, d_R, s_R, c_R, t_R, b_R$$

1.2.1 Spin 0 Bosons

The recently discoverd Higgs Particle shows a mass of around 126 GeV.It is the coupling of these particles through the Yukawa interacts with the SM fermions via the Yukawa coupling.When the Higgs fiels aquires a vacuum expextation valuethe fermions depending on the strength of the capacity.Moreover the Higgs also gives mass to the Z^0, W^+, W^- Bosons. Let us introduce the scalar doublet as:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix}$$

The Vacuum Expectation Value of the Higgs particle is given as

$$< \Phi >= \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

where $v = \sqrt{\frac{-\mu^2}{\lambda}}$

The Lagrangian for the scaler field:

$$L_{scalar} = \partial_\mu \Phi \partial^\mu \Phi - V(\Phi \dagger \Phi)$$

where the potential is given as:

$$V(\Phi\Phi) = \mu^2 \Phi \dagger \Phi + \lambda(\Phi \dagger \Phi)^2$$

In order to maintain the Gauge invariance we can get the new covariant derivative here,

$$D_\mu = \partial_\mu + \frac{1}{2}\iota g \tau^\iota W_\mu^i + ig' \frac{Y}{2} B_\mu$$

It is through this mechanism that the gauge bosons become massive.

1.2.1 Spin 1 Bosons

Vector Gauge Bosons

The gauge bosons wich are the carriers of different interactions,for example

$$W_\mu^{+-}, Z_\mu$$

are the carriers for weak interaction,which are called the Goldstone Bosons,and the photon

$$A_\mu$$

is the carrier for the electro-magnetic interactions.

Thus the electro-weak Interactions as given by

$$SU(2) \otimes U(1)_Y \to U(1)_{em}$$

which is the Symmetry of the present Universe.

1.3 The Gauge group

The success of the Standard Model is well established by its Gauge Invariance, We see that the weak current, for a generic lepton l, is given by,
If we introduce the left–handed isospin doublet (T = 1/2),
where the $T_3 = \frac{1}{2}$ and $T_3 = -\frac{1}{2}$ components are the left handed parts of the neutrino and of the charged leptons respectively. [2] The charged weak current can be written in terms of leptonic isospin currents:

In order to obtain the right combination of fields that couples to the electromagnetic current, let us make the rotation in the neutral fields, defining the new fields A and Z by,

$$W_\mu^3 = \sin\theta_W A_\mu + cos\theta_W Z_\mu$$

$$B_\mu = \cos\theta_W A_\mu - sin\theta_W Z_\mu$$

where θ is the Weinberg Angle. So,the relation between coupling constants and the Weinberg angle are given by ,

$$\sin\theta_W = \frac{g^{'}}{\sqrt{g^{'2} + g^2}}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^{'2} + g^2}}$$

We easily identify the electromagnetic current coupled to the field A_μ and the electromagnetic charge as,

$$e = g\sin\theta_W = g^{'}\cos\theta_W$$

where the g's are the Gauge coupling constants [2]

1.4 The Standard Model Lagrangian

The Lgrangian comprises of the kinetic terms and the interaction terms in which the S.M particles interact via the Higgs Mechanism through Yukawa Interactions.
(i). Gauge-boson and Scalar interaction
(ii). Leptons and Yukawa interaction
(iii). Quarks and Yukawa interaction

1.5 The Drawbacks

The SM is successfull in explaing the fundamental forces and there Interacions in nature ,However there are certain aspects which calls for it's extension and beyond the standard model framework.

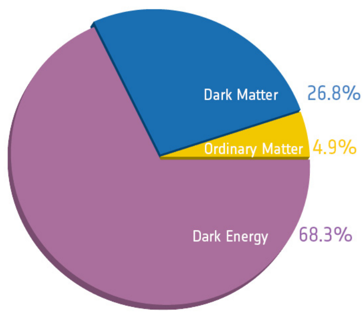
1.It doesn't explain the Non-Zero Nutrino mass predicted by oscillation experiments.
2.It doesn't explain the Dark Matter content of the universe which is now supported by the indirect evidences.

Thus in this project Thesis we try to explain the dark matter content of the Universe by using some extensions of the standard model.

1.6 Beyond The Standard Model

Evidence of Dark Matter existense

Today the universe according to PLANCK's and WMAP's recent papers is constituted by only about 4.9 % of real baryonic matter.But within the remaining part,there is about 26.8 % matter which accounts to the dark matter which cannot be explained within standard model. So,it is necessary to study the physics beyond the standard model.[9]



The most convincing and direct evidence for dark matter on galactic scales comes from the observations of the rotation curves of galaxies,

1.Rotation curve

The Rotational curve was first postulated by Oort in 1930.Rotation curves exhibit a characteristic flat behavior at large distances,even far beyond, the edge of the visible disks.But in Newtonian dynamics the circular velocity is expected to be,

$$v(r) = \frac{\sqrt{GM(r)}}{r}$$

$M(r) = 4\pi \int \rho(r) r^2 dr$
$\rho(r)$ is the constant mass density.This anomalous behavious can be explaned if we consider a invisible particle existing throughout space no matter how far we go from our origin. This is a fascinating proof for the existense of dark matter.

2. Gravitational Lensing

Dark Matter Candidates such as WIMP's being non-interactive ,there detection becomes almost impossible. But as a consequence of General Relativity we know light has a tendency to bend as it encounters mass on it's way reaching us.This difference from the usual non bending path shows the exixtence of our Dark Matter.

3. Collision of Galaxies

The Bullet cluster is the collision of two big galaxy clusters several billion years ago.In this collision the Galaxies are known to move at a faster rate leaving a cluster of highly condensed gaseous centre .This massive centre should give rise to a high gravitational field according to the existing physics but the results are opposite,making it inevitable for the presence of something unknown, which is our again our Dark Matter

Chapter 2

Standard Model of Cosmology

2.1 Introduction

the evolution of the universe is best understood in terms of FRW cosmology, which predicts cosmic microwave background as its relic.

2.2 The Robertson Walker Metric

Matter and radiation in the observable universe is homogeneous and isotropic. The universe is spatially homogeneous and isotropic on scales as large as the Hubble Volume. This is known as the Cosmological Principle. Here we introduce the symmetric Robertson-Walker metric,

$$ds^2 = dt^2 - R^2(t)(\frac{dr^2}{1-kr^2} + r^2 + d\theta^2 + r^2 \sin^2\theta d\phi^2)$$

Here k can be taken as +1,-1 and 0 for the surfaces with constant positive,negative and zero spatial curva- ture,respectively.The spatial part of the metric is denoted by,

$$\vec{dl}^2 = h_{ij} dx^i dx^j$$

General Relativity allows choices that are more natural and easier to work with. Comoving coordinates are an example of such a natural coordinate. They assign constant spatial coordinate values to observers who perceive the universe as isotropic. Such observers are called "Comoving" observers because they move along with the Hubble flow.

2.3 Kinematics

Kinematics of the RW Metric gives,

$$\frac{\lambda_1}{\lambda_0} = \frac{R(t1)}{R(t0)}$$

From the CMB radiation and the equation above we can conclude that the **Universe is Expanding**

2.4 Cosmology

In the Cosmology the Einstein equation from the General Theory of Relativity is:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

The relation between energy density and R can be summarized as, For a simple equation of state,$p = w\rho$,so then if we consider w is independent of time :

$$\rho \propto R^{-3(1+w)}$$

$$RADIAION p = \frac{1}{3}\rho then \rho \propto R^{-4}$$

$$MATTER p = 0 then \rho \propto R^{-3}$$

$$VACUUMENERGY P = -\rho then \rho = constant$$

2.5 Equilibrium Thermodynamics

At present the radiation and relativistic particles,in the universe is comprised of the 2.75K microwave photons,and the 3 cosmic seas of 1.96K relic neutrinos.
The Number Density

$$n = \frac{g}{(2\pi)^3}\int \vec{f}(\vec{p})d^3p$$

The Energy Density

$$\rho = \frac{g}{(2\pi)^3}\int E(p)\vec{f}(\vec{p})d^3p$$

The Pressure

$$p = \frac{g}{(2\pi)^3}\int \vec{f}(\vec{p})d^3p\frac{\vec{p}}{3E}$$

Where$f(\vec{p})$ is the familiar Fermi-Dirac and Bose-Einstein distribution function

2.6 Thermodynamics in The Expanding Universe and The Boltzmann Equation

To a good approximation,we can say that the most of the ingredients of the early universe are in thermal equilib- rium.But,some notable departures from the equilibrium conditions i.e,neutrino decoupling,decoupling of background radiation,Primordial nucleosynthesis,inflation,baryogenesis,Decoupling of relic WIMPs etc. The criterion of any species to be coupled or decoupled involves the comparison of the interaction rate of the particle Γ and the expansion rate of the Universe H

$$\Gamma \geq H \ (coupled)$$

$$\Gamma \leq H \ (decoupled)$$

Then the Boltzmann Equation can be written as:

$$\hat{L}[f] = C[f]$$

is the $\hat{L}$ Liouville operator,C is the collision operator. If we define the relic abundance as $Y = \frac{n_\psi}{s}$ and the relic abundance can be written as :

$$\frac{dn_\psi}{dt} + 3Hn_\psi = s\frac{dY}{dt}$$

2.7 Freez-out and the Cold Relics

If we consider the creation or annihilation of a process$\overline{\Psi}\Psi \rightarrow \overline{\chi}\chi$

Then considering the relativistic and non-relativistic form we have the Hot Relic and Cold Relic respectively. This project has been associated to the physics at the non-relativistic limit and hence only the Cold Relic will be considered

Freeze out occurs at values when ($x_f \geq 3$) . Y_{EQ} decreases exponentially with x. From the Boltzmann Equation we have : [4]

$$\frac{dY}{dt} = -\Lambda x^{-n-2}(Y^2 - Y_{EQ}^2)$$

where:

$$\lambda = 0.264\frac{g_s^*}{g^{\frac{1}{2}}}m_{pl} < \sigma|v| >$$

and

$$Y_{EQ} = 0.145\frac{g}{g_s^*}x^{3/2}e^{-x}$$

$$\Omega_\psi h^2 = 1.07 \times 10^9\frac{(n+1)x_f^{n+1}GeV^{-1}}{m_{pl}\frac{g_s^*}{g^{*\frac{1}{2}}} < \sigma|v| >}$$

For the calculation Relic abundance in the project I have used this final expression.

Chapter 3

The Singlet Fermionic Model

3.1 Introduction

The properties that we look into the Dark Matter candidate which allows us to chose a single nature :

1.It should be Massive
2.Should be electriclly neutral
3.It should be stable
4.It should interact weakly.

To determine the range of the mass in this model,we can proceed in this following way,which is followed by some steps are

1.At first,we determine some dark-matter particle self annihilation processes from the Lagrangian depicted in this model.
2.Then,we determine the cross-section of the all possible processes,and then sum them all up to determine the total cross-section of the all scattering processes.
3.Finally calculation of its relic Density [5]
4.The various constrains and Demerits of the model

3.2 About the Singlet Fermionic Model

It is by far the simplest fermionic Model of The DM candidates using only one parameter Λ and mass of the Dark matter candidates as **m** which alongwith the higgs coupling gives the interaction with the S.M particles

$$\mathcal{L} = m\overline{\Psi}\Psi + \overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi + \frac{\overline{\Psi}\Psi HH}{\Lambda}$$

The question now is weather this is complete for giving the correct Relic Abundace of the Dark Matter and what are the processes for its annihilation into the Standard Model Particles.

The Relic density of DM is well measured by WMAP and Plancks's experiments and the current value is :

$$\Omega_{DM}h^2 = 0.1199 \pm 0.0027$$

and the value of h is

$$h = 0.67 \pm 0.012$$

3.3 The Interaction terms

The interaction term can be expanded in the vacuum expection value of the higgs **H** coupling and the interaction of the D.M candidate Ψ with the S.M particles can be obtained as follow :

$$\frac{\overline{\Psi}\Psi HH}{\Lambda} \rightarrow \frac{\overline{\Psi}\Psi h^2}{2\Lambda} + \frac{v}{\Lambda}h\overline{\Psi}\Psi$$

1 . The First Term gives the tree level process in which the D.M candidates annihilate to higgs particles
2 . The second term gives the annihilation of the D.M candidate to the standard fermionic partiles (Three generation of Leptons and the Three generation of Quarks)

3.4 Calculation of the Cross-sections

3.4.1 $\overline{\Psi}\Psi \rightarrow hh$

The Feynmann Diagram

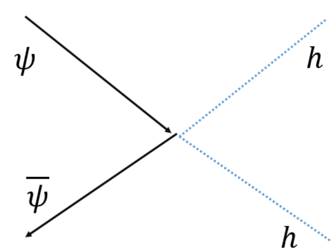


Fig:3.1 Feynmann digram for the process

The Fynmann Amplitude will be denoted by ιM

$$\iota M = \frac{m_h}{2\Lambda} * \overline{v(p')} * u(p)$$

The Amplitude modulus sqared:

$$|M|^2 = \frac{1}{2\Lambda^2} m_h^4 (s - 4m^2)$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac ALgebra:

$$< \sigma |v| >= \frac{4\pi}{(2\pi)^2} \frac{(m^2 - m_f^2)^{\frac{1}{2}} m_h^4 (s - 4m^2)}{32 s m \Lambda^2}$$

where the expression for s is given by

$$s = 4m^2 + 0.96m^2$$

3.4.2 $\overline{\Psi}\Psi \rightarrow \overline{f} f$

The Feynmann Diagram

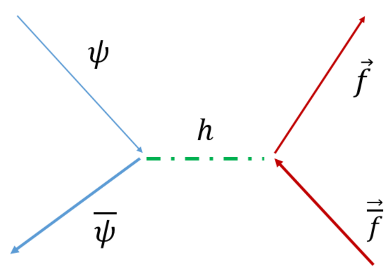


Fig:3.2 Feynmann digram for the process

The Fynmann Amplitude

$$\iota M = \overline{v_{s'}}(p') u_s(p) \frac{\iota}{q^2 - {m_h}^2 + \iota\epsilon} \overline{u_r}(k) v_{r'}(k')$$

The Amplitude modulus sqared:

$$|M|^2 = \frac{1}{(2\pi)^2} (\frac{m_f}{\Lambda})^2 \frac{(s - 2m^2 - 2m_f^2)(s - 4m^2)}{2 s m \Lambda^2 ((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac ALgebra:

$$< \sigma |v| >= \frac{4\pi}{(2\pi)^2} (\frac{m_f}{\Lambda})^2 \frac{(s - 2m^2 - 2m_f^2)(m^2 - {m_f}^2)^{1/2} (s - 4m^2)}{8 s m \Lambda^2 ((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

3.4.3 $\overline{\Psi}\Psi \rightarrow W^{-}W^{+}$

The Feynmann Diagram

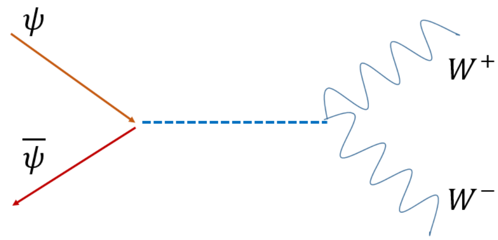


Fig:3.3 Feynmann digram for the process

The Fynmann Amplitude

$$\iota M = \frac{v}{\Lambda}\overline{v_{s'}}(p^{'})u_s(p)\frac{\iota}{q^2 - {m_h}^2 + \iota\epsilon}\epsilon^*_\mu(k)\epsilon_\nu(k^{'})$$

The Amplitude modulus sqared:

$$|M|^2 = (\frac{m_W}{\Lambda})^2 \frac{(s^2 + 4{m_W}^2 + 8 - 4s{m_W}^2)(s - 4m^2)}{}$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac ALgebra:

$$< \sigma|v| >= \frac{4\pi}{(2\pi)^2}(\frac{1}{m_W\Lambda})^2 \frac{(m^2 - {m_W}^2)^{1/2}(s - 4m^2)(s^2 + 4{m_W}^2 + 8 - 4s{m_W}^2)}{8sm((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

3.4.4 $\overline{\Psi}\Psi \rightarrow Z^{\mu}Z_{\mu}$

The Feynmann Diagram

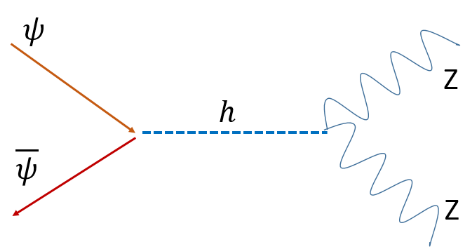


Fig:3.4 Feynmann digram for the process

The Fynmann Amplitude

$$\iota M = \frac{v}{\Lambda}\overline{v_{s'}}(p^{'})u_s(p)\frac{\iota}{q^2 - {m_h}^2 + \iota\epsilon}\epsilon^*_\mu(k)\epsilon_\nu(k^{'})$$

The Amplitude modulus sqared:

$$|M|^2 = (\frac{m_Z}{\Lambda((q^2 - {m_h}^2)^2 + \epsilon^2)})^2(s^2 + 4{m_Z}^2 + 8 - 4s{m_Z}^2)(s - 4m^2)$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac Agebra:

$$< \sigma|v| >= \frac{4\pi}{(2\pi)^2}(\frac{1}{m_Z\Lambda})^2\frac{(m^2 - {m_Z}^2)^{1/2}(s - 4m^2)(s^2 + 4{m_Z}^2 + 8 - 4s{m_Z}^2)}{32sm((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

3.4.5 $\overline{\Psi}\Psi \rightarrow hh$

The Feynmann Diagram

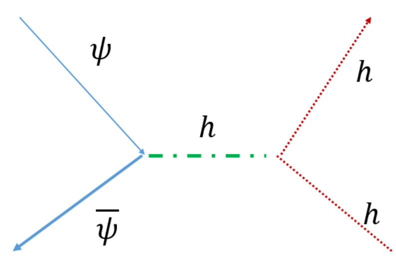


Fig:3.5 Feynmann digram for the process

The Fynmann Amplitude

$$\iota M = \frac{m_h^2}{2\Lambda}\overline{v_{s'}}(p^{'})u_s(p)\frac{\iota}{q^2 - {m_h}^2 + \iota\epsilon}$$

The Amplitude modulus sqared:

$$|M|^2 = \frac{m_h^4}{\Lambda^2((q^2 - {m_h}^2)^2 + \epsilon^2)}(s - 4m^2)$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac Agebra:

$$< \sigma|v| >= \frac{4\pi}{(2\pi)^2}{m_h}^4\frac{(m^2 - {m_h}^2)^{1/2}(s - 4m^2)}{8sm\Lambda^2((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

$$\epsilon = m_h \Gamma(h)$$

3.5 Results and Discussions

The variation of the cross-section$< \sigma|v| >$ is plotted against the mass of the Singlet Dark Matter (m) at three different values of the coupling constant Λ

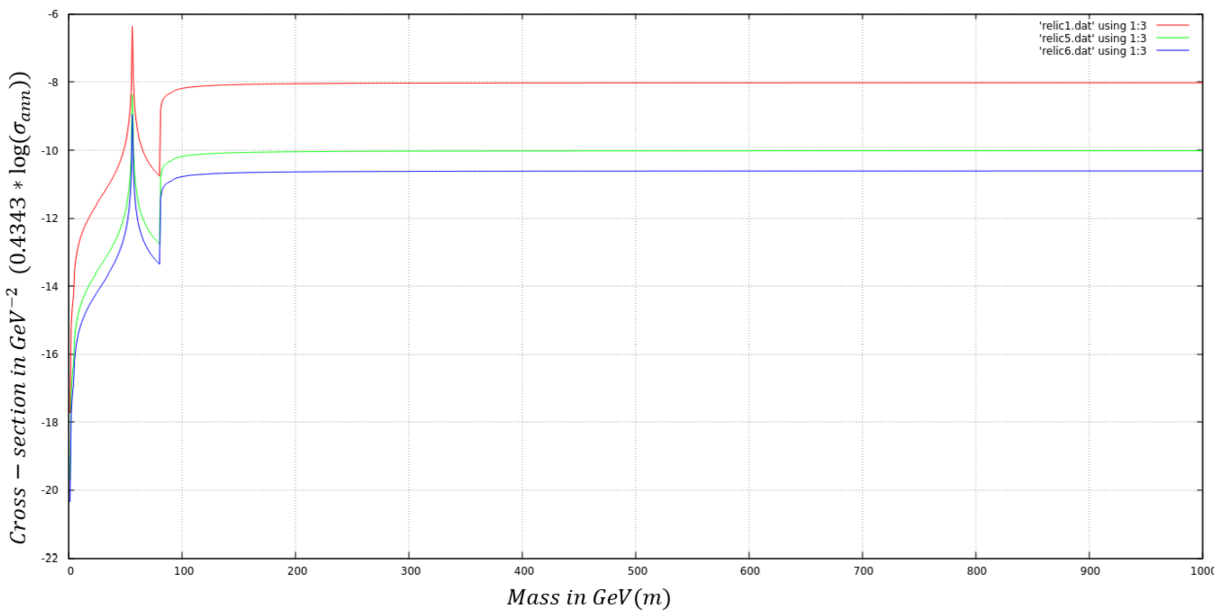


Fig:3.6 Variation of m with themally averaged cross-section at $\Lambda = 10,000; 2000; 1000$ in blue,green and red lines respectively

The variation of the Relic Density with mass at differnt values of Λ

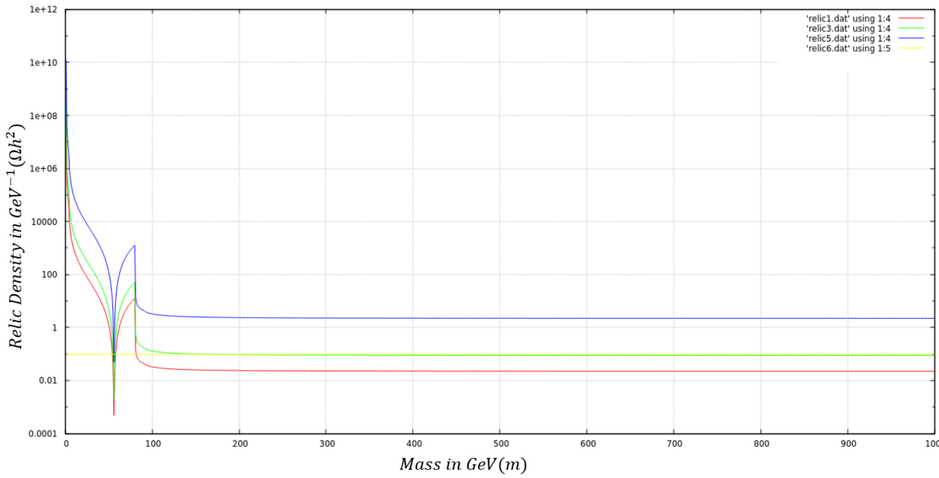


Fig:3.7 Variation of relic Abundance with mass with $\Lambda = 10000, 2000, 1000$ in blue,green and red lines respectivelyl

This shows the Relic Abundance becomes much in accordance to the observed value if λ is lower within 500 GeV to 1000 GeV.in the case o the singlet.

3.6 The Constraints on the Model

The two main unkhown parameters in the model being the mass(m) and the coupling constant (Λ) .We have to find the bounds over them that give a correct Relic Abundane in respet to the observed Data of Plank and WMAP.

From the Branching Ratio

firstly calculating the **Decay rate** Γ analytically for $h \to \overline{\Psi}\Psi$

$$\Gamma(h \to \overline{\Psi}\Psi) = \frac{v^2}{4\pi\Lambda^2}(1 - \frac{4m^2}{m_h^2})^{\frac{3}{2}} * m_h$$

The Decay width of h going to all possible particles

$$\Gamma(h \to All) \approx 4MeV$$

The Branching ration (B.R) may be defined as the ratio of the irst to the second expression above and which is eqivalent to[8]

$$B.R = \frac{\Gamma(h \to \overline{\Psi}\Psi)}{\Gamma(h \to All) + \Gamma(h \to \overline{\Psi}\Psi)} \leq 0.3$$

Which gives a lower bound on Λ as shown in the obtained plot

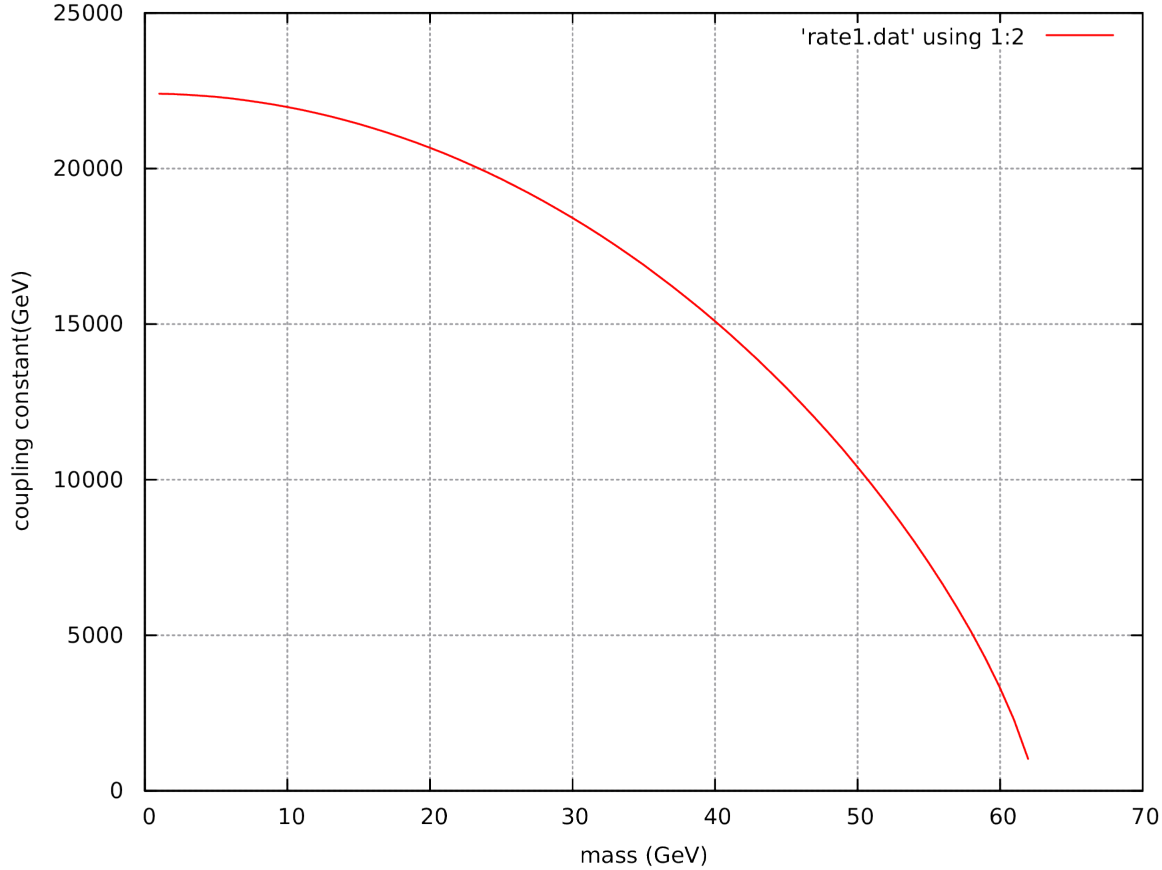


Fig:3.8 The variation of coupling constant Λ as obtained from the branching ratio

This shows that the maximum possibly alloed value of Lambda is 22,000 GeV wich is of very high magnitude. The graph terimnates at $m = 62GeV$ as that is the kinamatically favoured comdition for the Singlet Fermionic Dark Matter for invisible Higg's decay.

The Relic Density as observed by PLANCK

$$\Omega_{DM}h^2 = 0.1199 \pm 0.0027$$

If we vary the coupling parameter and mass of D.M particles parameterized by these two bounds of $\Omega_{DM}h^2$ we find

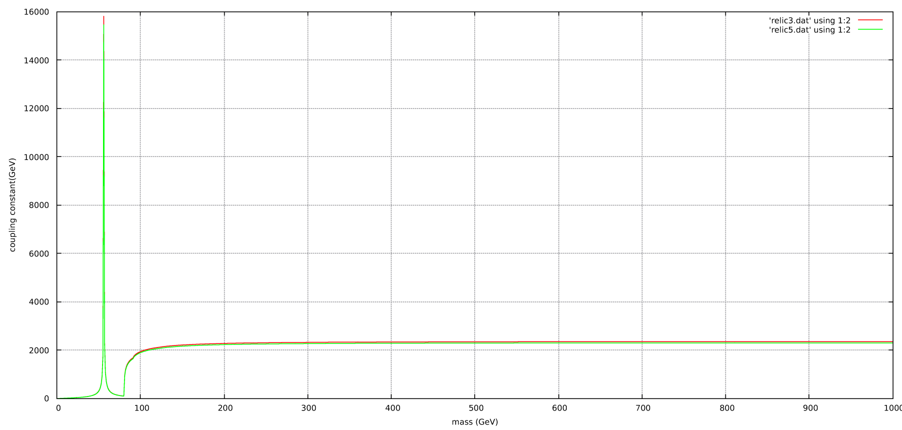


Fig 3.9 Variation of Λ with mass(m) at the two bounds of the Relic Density at 0.1172(lower red line) and 0.1226 (upper green line)

The variation of Λ for the two bounds on Ωh^2 with mass becomes appreciable only for $m \geq 100GeV$

Chapter 4

The Singlet-Doublet Fermionic Model

4.1 Introduction

Let us now consider a modifed version of the theory in which we take a simple extension of the Standard Model of Particle Physics and consider an $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry.We introduce a Doublet in the Lagrangian and hence study the [1] interaction and compare with the existing standard results provided by WMAP nad PLANCK.

4.2 The Lagrangian

$$\mathcal{L} = \iota\overline{\alpha}\not{D}\alpha + \iota\overline{\Psi}\not{\partial}\Psi - M_D\overline{\alpha}\alpha - M_S\overline{\Psi}\Psi - y\overline{\alpha}\tilde{H}\Psi - y\overline{\Psi}\tilde{H}^{\dagger}\alpha$$

$$\tilde{H} = \begin{pmatrix} H^o \\ -H^- \end{pmatrix} \quad \alpha = \begin{pmatrix} \alpha^o \\ \alpha^- \end{pmatrix}$$

and the Higgs Iso-doublet:

$$H = \begin{pmatrix} H^+ \\ H^o \end{pmatrix}$$

The Vacuum Expectation Value :

$$< H >= \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

The Lagrangian can be expressed in terms of the Mass Matrix in the (α, Ψ)

$$M = \begin{pmatrix} M_S & \frac{yv}{\sqrt{2}} \\ \frac{yv}{\sqrt{2}} & M_D \end{pmatrix}$$

The Matrix can be Diagonalized by the unitary transformation to

$$M_d = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

Where

$$M_1 = M_S - \frac{(yv)^2}{2(M_D - M_S)}$$

$$M_2 = M_D + \frac{(yv)^2}{2(M_D - M_S)}$$

And the mixing angle is given by :

$$\tan(\theta) = \frac{\sqrt{2}yv}{M_D - M_S}$$

The new particles are given by:

$$\chi_1 = \cos\theta\Psi + \sin\theta\alpha^o$$

$$\chi_2 = \cos\theta\alpha^o - \sin\theta\Psi$$

4.3 The Interaction Terms

The lagrangian can be written in the new basis as

4.3.1.The Yukawa interaction term

$$y\overline{\alpha}\tilde{H}\Psi - y\overline{\Psi}\tilde{H}^\dagger\alpha \to \frac{y}{\sqrt{2}}[(\overline{\chi_1}h\chi_2 - \overline{\chi_2}h\chi_1)\sin 2\theta + (\overline{\chi_2}h\chi_2 + \overline{\chi_1}h\chi_1)\cos 2\theta]$$

4.3.2. The Gauge interaction term

$$\frac{\iota g}{\sqrt{2}}\overline{\alpha^0}\gamma^\mu W_\mu^+\alpha^- h.c \to \frac{\iota g}{\sqrt{2}}[(\overline{\chi_1}\gamma^\mu W_\mu^+\alpha^-)sin2\theta + (\overline{\alpha_2}\gamma^\mu W_\mu^+\alpha^-)\cos\theta + h.c]$$

$$\frac{g}{2\cos\theta_W}\overline{\alpha^0}\gamma^\mu Z_\mu^+\alpha^0 \to \frac{g}{2\cos\theta_W}[(\overline{\chi_1}\gamma^\mu Z_\mu\chi_1)sin^2\theta + (\overline{\chi_2}\gamma^\mu Z_\mu\chi^2)\cos^2\theta + (\overline{\chi_2}\gamma^\mu Z_\mu\chi^2 + \overline{\chi_2}\gamma^\mu Z_\mu\chi^1)\cos\theta\sin$$

4.3.3. The Charged interaction

The Neutral part of the Doublet is not affected by the mixing of the Singlet and Doublet Complnents So we can ignore that

4.4 The Cross-Section evaluated

Out of all the possible interactions we see that as the limit of the mixing angle is kept very small ($\sin\theta \leq 0.05$) , else that would lead to unitarity violation.The yukawa coupling is also kept small (0.1-1.9)
Hence only a few terms in the annilation and co-annilation channels contribute to a large value that gives a Relic Density equal to the observed PLANK data

4.4.1 $\overline{\chi_1}\chi_1 \to hh$ through χ_1

The Feynmann Diagram

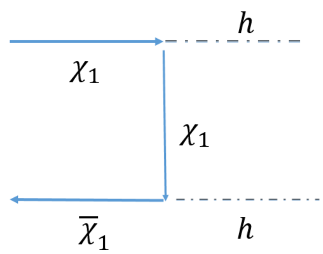


Fig:4.1 Fynmann diagram of the process

The Fynmann Amplitude

$$\iota M = \frac{\sin 4\theta}{2} v_{s'}(p^{'}) \frac{\iota(\not{p} - \not{k})}{\sqrt{t^2 - m_1^2 + \iota\epsilon}} u_s(p)$$

The Amplitude modulus sqared:

$$|M|^2 = \frac{\sin^2 4\theta}{(t^2 - m_1^2)^2 + \epsilon^2}[\frac{1}{2}(s - m_2^2 - m_1) - 2(s/4 + a\cos\phi) - m_1 m_2) m_1^2 + (s/4 - a\cos\phi)(m_1 m_2) m_1 + 2(s/4 + a$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac Algebra:

$$< \sigma|v| >= \frac{4\pi}{2\pi^2} \frac{\sin^2 4\theta}{16 m_2 m_1 s((m_W^2) - m\sqrt{s})^2}[\frac{1}{2}(s - m_2^2 - m_1)(m_1^2 - m_h^2) + (s/4 - s m_h^2 - m_1^2) m_1 m_2 - \frac{1}{2} s m_1^2 + \frac{1}{8} s^2$$

$$a = \frac{\sqrt{s - m_2^2 - m_1)^2 - 4m_1^2 m_2^2][(s - 2m_h)^2 - 4m_h^4]}}{4s}$$

4.4.2 $\overline{\chi_1}\chi_2 \to W^- W^+$ through α^-

The Feynmann Diagram

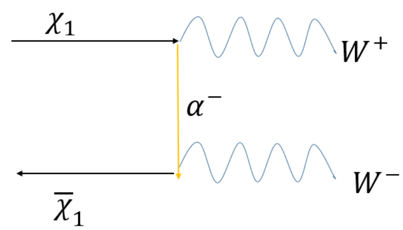


Fig:4.2 Feynmann digram for the process

The Fynmann Amplitude

$$\iota M = \frac{g^2 \sin 2\theta}{4} v_{s'}(p^{'}) \frac{i(\not p - \not k)}{\sqrt{t^2 - m_\alpha^2 + i\epsilon}} \not\epsilon_\mu(k) u_s p(p)$$

The Amplitude modulus sqared:

$$|M|^2 = \frac{g^4 \sin^2 4\theta}{4(t^2 - m_\alpha^2 + i\epsilon)}[(\frac{s}{4} - \frac{a\cos\phi}{4s})(\frac{s}{4} + \frac{a\cos\phi}{4s}) - m_W^2(m_1 m_1 + \frac{(s - m_1^2 - m_2^2)}{2})]$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac Algebra: [6]

$$< \sigma|v| >= \frac{4\pi}{(2\pi)^2} \frac{\sqrt{(s - 2m_W^2)^2 - 4m_W^2}}{16 s m_1 m_2}[\frac{s^2}{8} - \frac{a^2}{3} - m_W^2(m_1 m_1 + \frac{(s - m_1^2 - m_2^2)}{2})]$$

where:

$$a = \sqrt{s - m_2^2 - m_1)^2 - 4m_1^2 m_2^2][(s - 2m_W)^2 - 4m_W^4]}$$

4.4.3 $\overline{\chi_1}\chi_1 \to \overline{f}f$ through h

The Feynmann Diagram

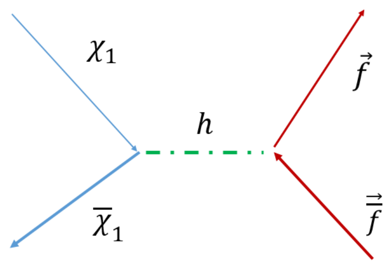


Fig:4.3 Feynmann diagram for the process

The Fynmann Amplitude

$$\iota M = \overline{v_{s'}}(p^{'})u_s(p)\frac{\iota}{q^2 - {m_h}^2 + \iota\epsilon}\overline{u_r}(k)v_{r'}(k^{'})$$

The Amplitude modulus sqared:

$$|M|^2 = \frac{1}{(2\pi)^2}(\frac{m_f}{\Lambda})^2\frac{(s - 2m^2 - 2m_f^2)(s - 4m^2)}{2sm\Lambda^2((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

The thermal averaged cross-section,in the nonrelativistic limit after using trace calculations and Dirac ALgebra:

$$< \sigma|v| >= \frac{4\pi}{(2\pi)^2}(\frac{m_f}{\Lambda})^2\frac{(s - 2m^2 - 2m_f^2)(m^2 - {m_f}^2)^{1/2}(s - 4m^2)}{8sm\Lambda^2((q^2 - {m_h}^2)^2 + \epsilon^2)}$$

4.5 Results and Discussions

We have kept the mixing anglein the small limit of $\sin\theta \leq 0.05$ to avoid unitarity violation. The yukawa coupling has also been kept in the small coupling range of $0.1 \leq y \leq 0.9$ .out of 16 possible annihilation channels and 16 possible co-annihilation channels we get dominant contribution fron the abouve mentioned processes.

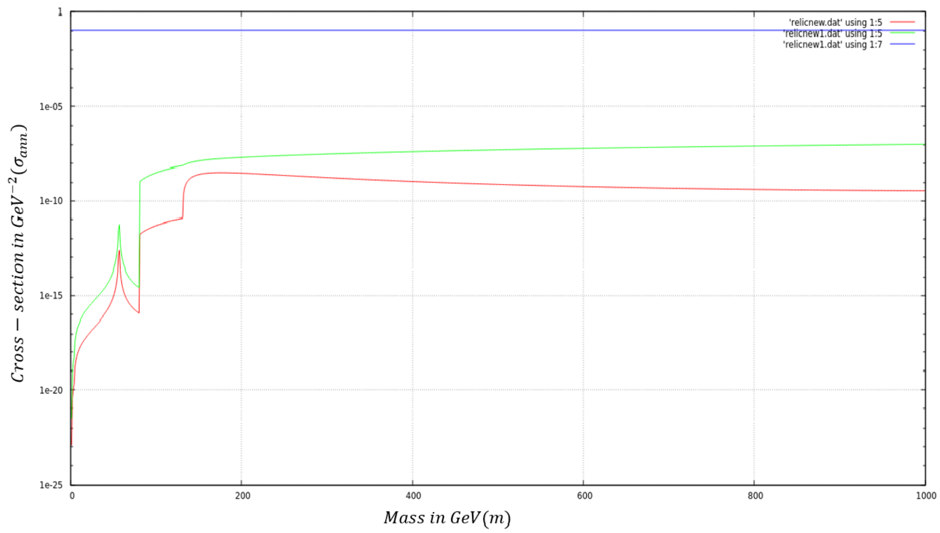


Fig: 4.4 Variation of cross-section and mass at two values of the coupling angles $\sin\theta = 0.001$(in red line) and $\sin\theta = 0.005$ (in green line) for coupling y=0.5

The variation of Relic Density

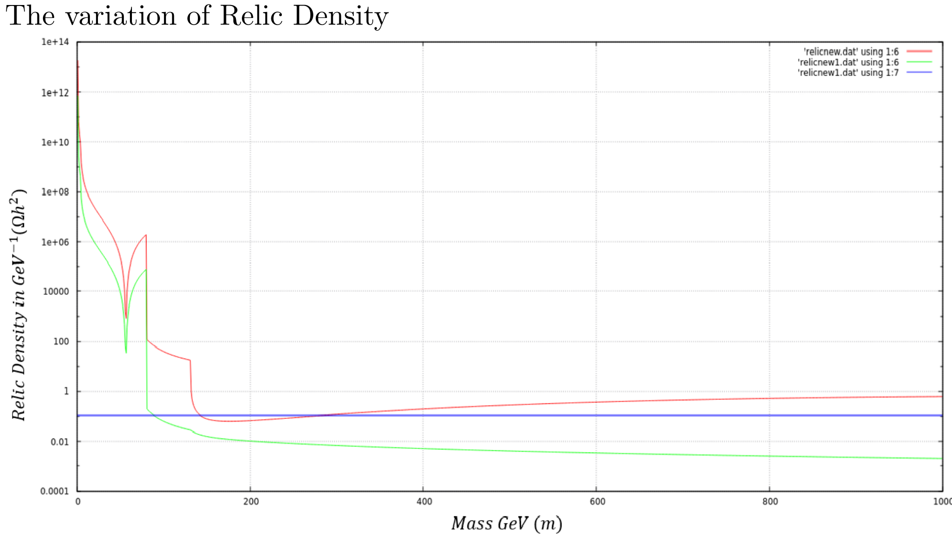


Fig:4.5 The plot for Variation of Relic Density and mass at two values of the coupling angles $\sin\theta = 0.001$(in red line) and $\sin\theta = 0.005$ (in green line) for coupling $y = 0.5$. The blue line shows $\Omega_{DM}h^2 = 0.11$

This shows at $\sin\theta = 0.005$ we get a much purfect fit of the relic density to the observed value that highlights the beauty of our extension.

Chapter 5

Conclusions

We present two viable solutions of the dark matter as an extension of the standard model of particle physics.

1.First we studied a singlet fermion and found the limitations on the coulping parameter (Λ). We found the parameter space in which we get the correct relic abundance.$\lambda \leq 22,000 GeV and mass (m) \geq 63 GeV for initiation of the interactions.$ The only drawback is that it cannot be experimentally verified. .Being the simplest model it results always dosn't exactly match the observed data.

2. In the second case we considered the mixing between the Singlet and a doublet as a minimal vector like structure having a Z_2 extension of the S.M. Here we only considered the annihilation channels in the calculations.The work was parameterized by three terms$m, y, \sin\theta$.
To avaiod Unitarity violation the mixing angle has been kept lower that an upper bound given by $\sin\theta \leq 0.01$ .The parameter space in which we get the correct relic abundance for the yukawa couplig in the small range of $0.1 \leq y \leq 0.9$ and a mass range of 0 to 1000 GeV. The dominant contribution gives a better matching to the observed values.This implies an extensive calculation of the model is a promising solution to give a more accurate matching with the experimental data.The main success in the model is that it involves an extra particle α_- which can give some signal at the detetor for testing it's validity.

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