The Standard Model and Electron Vertex Correction

Arunima Bhattcharya

A Dissertation Submitted to

Indian Institute of Technology Hyderabad

In Partial Fulfillment of the requirements for

the Degree of Master of Science

भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Department of Physics

April, 2016

Declaration

I declare that this report contains my ideas in my own words, and where others ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misinterpreted or falsified or fabricated any idea/data/fact/source in this report. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have not been properly cited, or from whom proper permission has not been taken when needed.

Arunima Bhattacharya

Signature

Arunima Bhattacharya

Student Name

PH14MSCST11004

Roll No.

Approval Sheet

This thesis entitled "The Standard Model and Electron vertex correction" by Arunima Bhattacharya is approved for the degree of Master of Science from IIT Hyderabad.

- Dr. Raghavendra Srikanth Hundi

- Advisor

- Examiner 1

- Examiner 2

- Examiner 3

- Examiner 4

- Examiner 5

Acknowledgement

I would like to thank my guide Dr. Raghavendra Srikanth Hundi for his constant guidance and supervision as well as for providing necessary information regarding the project and also for his support in completing the project . He had always provided me with new ideas of how to proceed and helped me to obtain an innate understanding of the subject.

I would also like to thank Joydev Khatua for supporting me as a project partner and for many useful discussion, and Chayan Majumdar and Supriya Senapati for their help in times of need and their guidance.

Abstract

In this thesis we have started by developing the theory for the electroweak Standard Model. A prerequisite for this purpose is a knowledge of gauge theory. For obtaining the Standard Model Lagrangian which describes the entire electroweak SM and the theory in the form of an equation, we need to develop ideas on spontaneous symmetry breaking and Higgs mechanism which will lead to the generation of masses for the gauge bosons and fermions. This is the first part of my thesis. In the second part, we have moved on to radiative corrections which acts as a technique for the verification of QED and the Standard Model. We have started by calculating the amplitude of a scattering process depicted by the Feynman diagram which led us to the calculation of g-factor for electron-scattering in a static vector potential. Then, we have calculated the one-loop contribution to the electron vertex function which has acted as a correction to the g-factor value calculated previously. While performing these calculations we have come across ultraviolet and infrared divergence. Although this thesis does not show the mathematical calculations leading to the removal of the divergence, we have discussed the solution to this problem in a theoretical manner. We have also discussed the precision tests of QED which have proved fundamental in the verification of the Standard Model over the last few years and the role of radiative corrections in such tests.

Contents

"*Is the purpose of theoretical physics to be no more than a cataloging of all the things that can happen when particles interact with each other and separate? Or is it to be an understanding at a deeper level in which there are things that are not directly observable (as the underlying quantized fields are) but in terms of which we shall have a more fundamental understanding?*"

— Julian Schwinger,

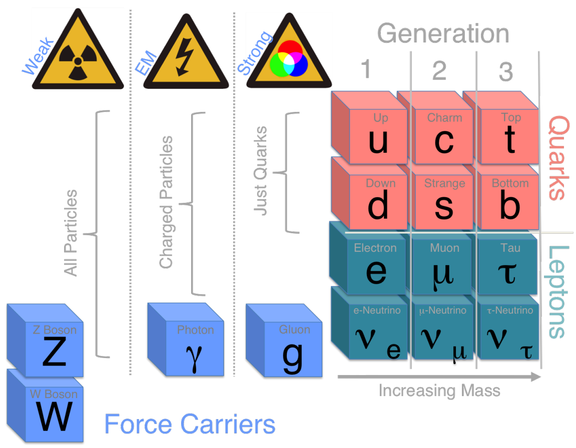
Quantum Mechanics: Symbolism of Atomic Measurements[1]

[1]https://www.goodreads.com/quotes/tag/particle-physics

Part I
The Standard Model

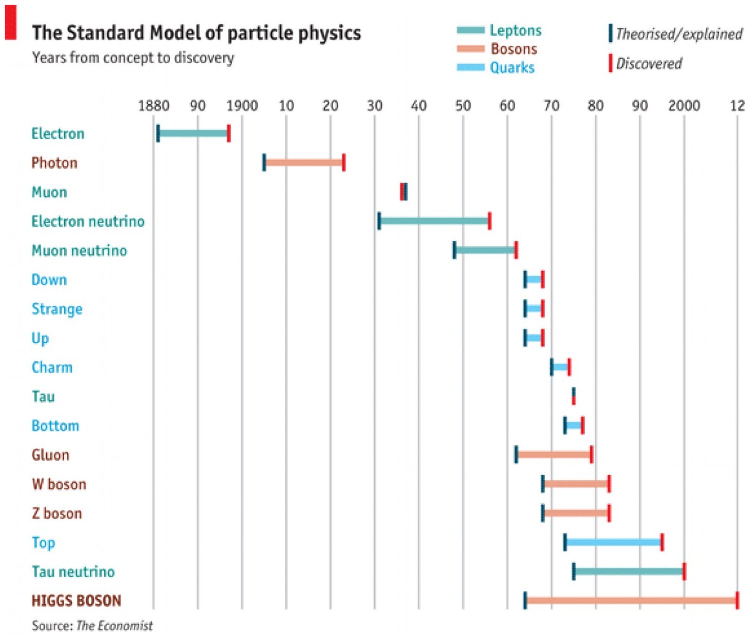
1 Introduction

J.J. Thomson's discovery of the electron set in motion a series of events that changed the face of modern physics. Other particles that were subsequently discovered were the protons and neutrons which had a more complex internal structure, unlike electron. So the questions that arose were: What are the fundamental constituents of matter? How do they interact? How are they categorized? A lot of experimental and theoretical efforts were put in to find the answers to these questions.The Standard Model of particle physics is the outcome of that effort. This describes our universe at the most fundamental level. [2]This model describes all fundamental particles and their interactions via three of the four fundamental forces - strong, electromagnetic and weak. These forces are mediated by the exchange of the corresponding spin-1 gauge fields: eight massless gluons, a massless photon and three massive bosons, respectively. It is a gauge theory described by the symmetry groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The Standard Model is one of the most successful achievements of modern physics because it is successful in explaining all known experimental facts with high precision. The model can be depicted graphically as below :



This model was tested many times and each time it came up with a satisfying theory. The discoveries that followed led to the confirmation of the theories predicted by the Standard Model and this increased our confidence in it.

[2]The Standard Model of Electroweak Interactions A. Pich IFIC, University of Val'encia – CSIC, Val'encia, Spain



However, we will be discussing the Standard model of electroweak interactions which obeys the $SU(2)_L \otimes U(1)_Y$ gauge symmetry. [3]In particle physics, the electroweak interaction is the unified description of two of the four fundamental interactions of nature: electromagnetism and the weak interaction. Although these two forces appear very different at everyday low energies, the theory models them as two different aspects of the same force. Above the unification energy, on the order of 100 GeV, they would merge into a single electroweak force. Glashow, Salam, and Weinberg were awarded the Nobel Prize in Physics in 1979[4] for their contributions to the unification of the weak and electromagnetic interactions. The existence of the electroweak interactions was experimentally established in two stages[5]: first being the discovery of neutral currents in neutrino scattering by the Gargamelle collaboration in 1973, and second in 1983 by the UA1 and the UA2 collaborations that involved the discovery of the W and Z gauge bosons in proton–antiproton collisions at the converted Super Proton Synchrotron.

In this model, the leptons and quarks are arranged in generations. The vector bosons, $W\pm$, Z_0 and γ, that mediate the interactions are introduced. The heart of the model is the scalar potential which is added to generate masses in a gauge invariant way, via the Higgs mechanism. We will now gradually develop and describe the theories that led to the final electroweak Standard Model Lagrangian.

[3]https://en.wikipedia.org/wiki/Electroweak_interaction
[4]https://en.wikipedia.org/wiki/Glashow-Salam-Weinberg_model
[5]https://en.wikipedia.org/wiki/Electroweak_interactions

2 Handedness of Fermions

Prior to 1956, “mirror symmetry” was taken for granted – the mirror image of any physical process was assumed to be a perfectly possible physical process. In 1956, Lee and Yang started looking for experimental proof of the fact. Not finding sufficient prove of parity conservation in weak decay, they proposed a test - the beta decay of Cobalt 60 - which was carried out by C.S. Wu and parity violation was observed for the first time in weak decay. The SU$(2)_L$ gauge group which is associated with weak decays was thus concluded to act differently on left and right-handed particles.

The helicity projections are

$$\psi_L = \frac{1-\gamma^5}{2}\psi \text{ and } \bar{\psi}_L = \frac{1+\gamma^5}{2}\bar{\psi}$$

$$\psi_R = \frac{1+\gamma^5}{2}\psi \text{ and } \bar{\psi}_R = \frac{1-\gamma^5}{2}\bar{\psi}$$

As all fermions have a spin, let us consider the Dirac equation and express it as a sum of left and right handed parts as below:

$$\psi = \psi_L + \psi_R$$

The Dirac Lagrangian is $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - \text{m})\psi$ = i$\bar{\psi}\not{\partial}\psi$ - m$\bar{\psi}\psi$

The mass term gets modified as m$\bar{\psi}\psi$ = m$(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$.

The left-handed components form doublets under SU$(2)_L$whereas the right-handed components are singlets. So, this equation breaks gauge invariance. To avoid this, the fermionic mass must be made zero according to this approach. But experiments show that fermions have a finite mass. Then the theory of spontaneous symmetry breaking was brought into the picture. This indicates that left-handed fermions participate in charged-current weak interactions i.e. the W-bosons couple to only the left-handed components.

3 Choice of Gauge Theories for constructing the Model

[6]Glashow in 1961 noticed that in order to accommodate both weak and electromagnetic interactions we should go beyond the SU(2) isospin structure. He suggested the gauge group SU(2) $\otimes$ U(1), where the U(1) was associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge through the analogous of the Gell-Mann-Nishijima formula ($Q = T^3 + \frac{Y}{2}$). The theory now requires four gauge bosons: a triplet (W_1,W_2,W_3) associated with the generators of SU(2) and a neutral field (B_μ) related to U(1). The charged weak bosons appear as a linear combination of W_1 and W_2, while the photon and a neutral weak boson Z_0 are both given by a mixture of W_3 and B_μ. The mass terms for $W^\pm$ and Z^0 were put "by hand". However, this procedure breaks the gauge invariance of the theory explicitly. In 1967, Weinberg and independently Salam in 1968, employed the idea of spontaneous symmetry breaking and the Higgs mechanism to give mass to the weak bosons and, at the same time, to preserve the gauge invariance, making the theory renormalizable. The Glashow–Weinberg–Salam model is known, at the moment, as the Standard Model of Electroweak Interactions, reflecting its impressive success.

The recipe to choose a gauge theory is as follows:

- Choose the gauge group G with N_G generators.
- Add N_G vector fields(gauge bosons) according to the gauge group representation.
- Add scalar fields to give masses to the vector bosons, if required.
- Define the covariant derivative and write the most general renormalizable Lagrangian, invariant under G, which couples all these fields.
- Shift the scalar fields so that the minimum of the potential is at zero.
- Apply quantum field theory to verify the theory and make predictions.
- Check with Nature if the model has anything to do with reality; If not, restart from the very beginning!

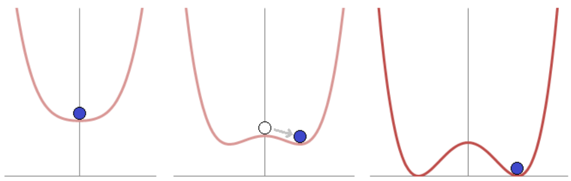
There were several attempts to construct a gauge theory for the electroweak interaction.

[6]Standard Model: An Introduction ∗ S. F. Novaes Instituto de F´ısica Te´orica Universidade Estadual Paulista Rua Pamplona 145, 01405–900, S˜ao Paulo Brazil

4 Spontaneous Symmetry Breaking

4.1 Introduction

Whenever the ground state is no longer invariant under a symmetry of the Lagrangian, we call it spontaneous symmetry breaking[7].



At higher energy states, the system is symmetric and the ball settles at the center. But as the energy decreases, the symmetric nature of the system gradually vanishes and eventually we have an asymmetric state resulting in the ball being anywhere at the bottom.

We know that both $SU(2)_L$ and $U(1)_Y$ are violated in weak interactions. Furthermore, the weak interactions are short ranged so that we would like the gauge bosons to be massive. Both these issues can be addressed simultaneously if the local symmetries are spontaneously broken by the Higgs phenomenon and this is what we discuss next.

4.2 The Higgs Mechanism

This can be best explained by the following example known as Einstein analogy : There are a number of physicists in a room chatting silently. Einstein suddenly enters the room which causes a disturbance. The people now start forming clusters around Einstein forming a massive object in the room.

Let us explain this mathematically using spontaneous symmetry breaking of the local $U(1)_Y$ symmetry.

Consider the scalar field Lagrangian $\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi)$ - $\mu^2 \phi^* \phi$ - $\lambda(\phi^*\phi)^2$.

To achieve local $U(1)_Y$ symmetry, we need to to use the corresponding transformations as below:

We introduce the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$
The gauge field transforms as $A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha$
The field transforms as $\phi \rightarrow e^{i\alpha(x)}\phi$

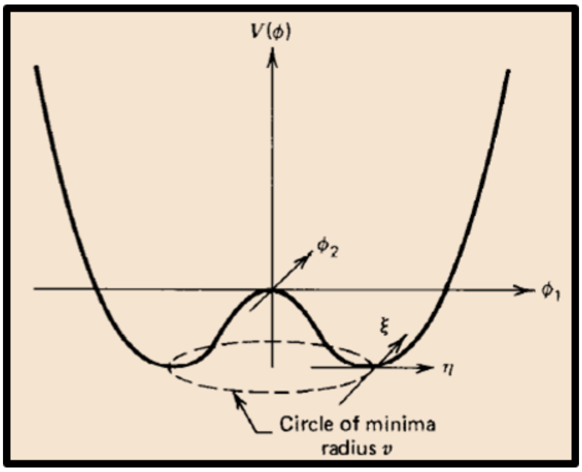
[7]See Appendix A for non-abelian gauge theory in spontaneous symmetry breaking.

Using this we get the gauge invariant Lagrangian to be

$$\mathcal{L}= (\partial_\mu - ieA_\mu)\phi^*(\partial_\mu - ieA_\mu)\phi \text{ - } \mu^2\phi^*\phi \text{ - } \lambda(\phi^*\phi)^2$$

For $\mu^2 > 0$, this is the QED Lagrangian for a charged scalar particle. So, we move to a new case where $\mu^2 < 0$.

First, we minimize the potential and determine the vacuum expectation value which is given by $\phi_1^2 + \phi_2^2 = \text{v}^2$ where $\text{v}^2 = -\frac{\mu^2}{\lambda}$.



Potential Energy plot

If the scalar transforms as $\phi(x) = \sqrt{\frac{1}{2}}(\text{v} + \eta(x) + i\varepsilon(x))$.

Using this in the Lagrangian we get

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu\xi)^2 + \frac{1}{2}(\partial_\mu\eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A^\mu A_\mu - evA_\mu\partial^\mu\xi \text{-} \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \text{ interaction terms}$$

Although we have generated masses for the gauge fields A_μ and η, we also see in this Lagrangian a massless ξ particle called the Nambu-Goldstone boson. The interaction term of this particle $A_\mu\partial^\mu\xi$ represents an unphysical process. A_μ has two degrees of freedom. The interaction signifies A_μ changing to ξ which has one degree of freedom (being a scalar particle). This is not possible. So, a particular form of gauge transformation was chosen to eliminate this ξ field.

This is given by

$$\phi(x) = \sqrt{\frac{1}{2}}(\text{v} + h(x))\text{e}^{\frac{i\theta(x)}{v}}$$

Using this form in the Lagrangian we get

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 \mathrm{v}^2 \mathrm{A}_\mu^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2 A_\mu^2 h^2 + \mathrm{ve}^2 \mathrm{A}_\mu^2 \mathrm{h}$$
$$- \frac{1}{4}F^{\mu\nu}F_{\mu\nu}.$$

So, we have successfully eliminated the Goldstone boson from the Lagrangian. This Lagrangian includes two interacting massive particles, a vector gauge boson A_μ and a massive scalar particle h. This is the Higgs particle, and it is said that by this Higgs mechanism via the Higgs particle the gauge boson absorbs the goldstone boson, thus eliminating it from the theory.

4.2.1 Masses of vector bosons

For this purpose, we first introduce a scalar field

$$\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \sqrt{1/2}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

where ϕ is an $\mathrm{SU}(2)_L$ doublet of complex scalar fields.

We next introduce the scalar field Lagrangian

$$\mathcal{L} = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$$

To achieve local $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetry of the Lagrangian we introduce the covariant derivative $\mathrm{D}_\mu = \partial_\mu + \mathrm{ig}\frac{\tau_a}{2}\mathrm{W}_\mu^a + \mathrm{ig}'\mathrm{B}_\mu\frac{Y}{2}$

We choose the vacuum expectation value of the Higgs field as

$$\phi_0 = \sqrt{\tfrac{1}{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \text{......................}(1.1)$$

The masses of the gauge bosons are determined by using the vacuum expectation value of the scalar field ϕ in the Lagrangian.

The relevant term in the Lagrangian is

$$| (-\text{ig}\tfrac{1}{2}\tau.W_\mu - i\tfrac{g\prime}{2}B_\mu)\phi|^2 =$$

$$\tfrac{g^2}{8}|\begin{pmatrix} gW_\mu^3 + g\prime B_\mu & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -gW_\mu^3 + g\prime B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}|^2$$

$$= \tfrac{g^2v^2}{8}\left[(W_\mu^1)^2 + \left(W_\mu^2\right)^2\right] + \tfrac{v^2}{8}(\text{g}\prime B_\mu - gW_\mu^3)(\text{g}\prime B_\mu - gW^{3\mu})$$

$$= (\tfrac{vg}{2})^2 W_\mu^+ W^{-\mu} + (\tfrac{v^2}{8})(W_\mu B_\mu)\begin{pmatrix} g^2 & -gg\prime \\ -gg\prime & g\prime^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \text{..................}(1.2)$$

where $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$

Comparing this with the boson mass term i.e. $\frac{1}{2}M^2B_\mu^2$ we get masses of the gauge bosons to be $M=\frac{1}{2}gv$.

The remaining off-diagonal terms of the matrix are

$$\tfrac{v^2}{8}\left[g^2\left(W_\mu^3\right)^2 - 2gg\prime W_\mu^3 B^\mu + g\prime^2 B_\mu^2\right] = \tfrac{v^2}{8}\left[gW_\mu^3 - g\prime B_\mu\right]^2 + 0\left[g\prime W_\mu^3 + gB_\mu\right]^2 \text{.....................}(1.3)$$

where we have introduced the fields as an orthogonal combination of each other and the zero introduced is an eigenvalue of the 2×2 matrix in equation (1.1).

To identify this with the mass form of $\frac{1}{2}M_Z{}^2Z_\mu{}^2$ and $\frac{1}{2}M_A{}^2A\mu^2$ we have to normalize the mass terms in the above equation (1.2) such that

$$A_\mu = \frac{g\prime W_\mu^3 + gB_\mu}{\sqrt{g^2+g\prime^2}} \text{with } M_A = 0.$$

$$Z_\mu = \frac{gW_\mu^3 - g\prime B_\mu}{\sqrt{g^2+g\prime^2}} \text{ with } M_z = \tfrac{v}{2}\sqrt{g^2 + g\prime^2}$$

So, by Higgs mechanism which is a consequence of spontaneous symmetry breaking, we are able to generate masses for the vector bosons.

4.2.2 Photon Mass

A specific choice of the vacuum state breaks the $SU(2)_L \otimes U(1)_Y$ symmetry.

For the vacuum state discussed above in equation (1.1) we have $\phi_0 = \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$.

This breaks the symmetry if $T = \frac{1}{2}$, $T^3 = -\frac{1}{2}$ and Y = 1 because simple matrix calculations show that

$$T^3\phi_0 \neq 0$$

$$Y\ \phi_0 \neq 0$$

According to the Gell-Mann-Nishijima formula, we have

$$Q = T^3 + \frac{Y}{2}$$

where Q is the electric charge, T^3 is the third component of isospin and Y is the hypercharge.

For the specific set of values given above, we get $Q\phi_0 = 0$.

The transformation is given by $\phi_0 \rightarrow e^{i\alpha(x)Q}\phi_0 = \phi_0$.

So, the desired symmetry breaking scheme used here is $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$.

As the vacuum state is invariant under the $U(1)_Q/U(1)_{em}$ transformation, we can say that $U(1)_{em}$ symmetry remains unbroken. Thus, the Higgs mechanism leads to the result that photon is massless.

4.2.3 Lepton Mass

Here we will show the case mass generation for an electron and it's neutrino. A similar method can be applied to determine the masses of the other leptons.

The $SU(2)_L$doublet is $\Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ and the $U(1)_Y$ singlet is e_R. We do not have a right-handed neutrino because being a massless particle it can have only one helicity state. i.e. ν_L.

To drive the process of spontaneous symmetry breaking, we have to introduce doublets of Higgs boson to make the interactions gauge invariant. These are then called Yukawa-type interactions.

To generate mass, we need an $SU(2)_L \otimes U(1)_Y$ symmetry invariant Lagrangian which is given by

$$\mathcal{L} = -\, G_e \left[\begin{pmatrix} \bar{\nu_L} & \bar{e_L} \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e_R} \begin{pmatrix} \phi^- & \bar{\phi^0} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right]$$

where G_e is a coupling constant.

Using the concept of spontaneous symmetry breaking, we again determine the vacuum expectation value i.e. $\phi = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

Substituting this in the Lagrangian and performing simple calculations we get

$$\mathcal{L} = - m_e(\bar{e_L} e_R + \bar{e_R} e_L) - \frac{m_e}{v}(\bar{e_L} e_R + \bar{e_R} e_L)h \text{ ....(1.4)}$$

where $m_e = \frac{G_e v}{\sqrt{2}}$ is a parametric form of the electron mass.

The second term represents an interaction between the electron and the scalar Higgs particle. So, again we can say that Higgs mechanism is successfully able to transfer mass to electrons.

4.2.4 Quark Mass

Quark masses can be generated by a similar method as adopted for generating lepton masses. However, in quarks, we have an $SU(2)_L$ doublet $\begin{pmatrix} u \\ d \end{pmatrix}_L$ and two singlet particles corresponding to $U(1)_Y$ - u_R and d_R.

So, we need a new Higgs doublet which is given by

$$\phi_c = - i\tau_2 \phi^* = \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix}$$

whose vacuum expectation state is given by $\sqrt{\frac{1}{2}} \begin{pmatrix} v + h \\ 0 \end{pmatrix}$

ϕ_c transforms under $SU(2)_L$ in the same way as ϕ but with an opposite weak hypercharge Y= -1.

The gauge invariant Lagrangian can now be constructed in the same way as before

:

$$\mathcal{L} = - G_d \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - G_u \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} u_R$$

$$- G_d \bar{d}_R \begin{pmatrix} \phi^- & \bar{\phi}^0 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_L - G_u \bar{u_R} \begin{pmatrix} -\phi^0 & \phi^+ \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_L$$

Substituting the vacuum expectation states of ϕ and ϕ_c in the Lagrangian and simply solving it will give

$$\mathcal{L} = - m_d \left(d_L d_R \bar{+} d_R \bar{d}_L \right) - \frac{m_d}{v} \left(\bar{d}_L d_R + \bar{d}_R d_L \right) h - m_u (\bar{u_L} u_R + \bar{u_R} u_L) - \frac{m_u}{v} (\bar{u_L} u_R + \bar{u_R} u_L) h \text{ ....(1.5)}$$

where $m_d = \frac{G_d v}{\sqrt{2}}$ and $m_u = \frac{G_u v}{\sqrt{2}}$.

So, we have terms representing masses of the up and down quarks. And the second and fourth terms account for the interaction of these quarks with the Higgs particle h resulting in these particles acquiring mass.

So, by using the concept of spontaneous symmetry breaking we have generated masses of the gauge bosons, the fermions, the quarks and rendered the photon massless which involves all members of the Standard model. Although the theory does not predict an exact value of the mass, it only indicates the masses in terms of certain parameters which cannot be exactly predicted.

5 The Standard Model Lagrangian

$$\mathcal{L} = - \tfrac{1}{4}\mathbf{W}_{\mu\nu}.\mathbf{W}^{\mu\nu} - \tfrac{1}{4}\mathbf{B}_{\mu\nu}.\mathbf{B}^{\mu\nu}$$

$$+\bar{L}\gamma^{\mu}\left(i\partial_{\mu} - g\tfrac{1}{2}\tau.\mathbf{W}_{\mu} - g\prime\tfrac{Y}{2}B_{\mu}\right) L + \bar{R}\gamma^{\mu}\left(i\partial_{\mu} - g\prime\tfrac{Y}{2}B_{\mu}\right) R$$

$$+\left| \left(i\partial_{\mu} - g\tfrac{1}{2}\tau.\mathbf{W}_{\mu} - g\prime\tfrac{Y}{2}B_{\mu}\right) \phi \right|^2 - \mathrm{V}(\phi)$$

$$- \left(G\bar{L}\phi R + G_2\bar{L}\phi_c R + \text{Hermitian conjugate}\right) \text{..................(1.6)}$$

The first two terms represent the kinetic energies and self-interactions of the W±, Z, γ.

The next two terms represent the kinetic energies and interactions of the leptons and quarks with the W±, Z and γ.

The next two terms represent the W±, Z, γ and Higgs masses and couplings.

The last set of terms represents the lepton and quark masses and their coupling the Higgs.

$W^a_{\mu\nu}$ represents the $SU(2)_L$ gauge field tensor, where 'a' runs from 1 to 3, corresponding to the three generators of the group. "g" represents the coupling. We can write this in terms of the gauge field $W^a{}_\mu$ as

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig\frac{(W_\mu W_\nu - W_\nu W_\mu)}{2}$$

$B_{\mu\nu}$ represents the gauge field tensor of the $U(1)_Y$ group; g' represents the coupling and the corresponding gauge field is B_μ such that

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

This entire Lagrangian obeys the $SU(2)_L \otimes U(1)_Y$ symmetry. Also, it can be shown explicitly that each term of the Lagrangian obeys this symmetry. This is what we will show in the next few sections.

5.1 Invariance of the Lagrangian

To check if the SM Lagrangian is invariant under the $SU(2)_L \otimes U(1)_Y$ symmetry transformation, we must know how each of the fields transform under this symmetry. And they transform as below:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \text{ ....(1.7)}$$

$$L' = e^{(i\alpha T + i\beta Y)} L \text{ ....(1.8)}$$

$$R' = e^{i\beta Y} R \text{ ....(1.9)}$$

$$\bar{L}' = e^{-(i\alpha T + i\beta Y)} \bar{L} \text{ ....(1.10)}$$

$$\bar{R}' = e^{-i\beta Y} \bar{R} \text{ ....(1.11)}$$

$$\phi' = e^{(i\alpha T + i\beta Y)} \phi \text{ ....(1.12)}$$

$$\phi_c' = e^{i\alpha_i T_i} \phi_c \text{ ....(1.13)}$$

$$B_\mu' = B_\mu - \frac{\partial_\mu \beta}{g'} \text{ ....(1.14)}$$

$$W_\mu' = G\left[W_\mu + \frac{i}{g} G^{-1} \left(\partial_\mu G\right)\right] G^{-1} \text{ .(1.15)}$$

5.1.1 Invariance of the gauge boson terms

We do not do a direct proof of this fact. Instead, we first see how the covariant derivative transforms under an $SU(2)_L$ transformation and then move on to see if it transforms in the same way under a transformation of the form $SU(2)_L \otimes U(1)_Y$.

We know the wavefunction transforms as

$$\Psi' = G\Psi \text{ ....(1.16)}$$

where G is the transformation. So, for the term to be invariant, the covariant derivative must transform in the same way.

Since we already know how the covariant derivative transforms under $SU(2)_L$ transformation, we first choose a form of the covariant derivative and see if it leads to the same result.

We choose the covariant derivative to be $D_\mu = (\partial_\mu + igW_\mu)$ ....(1.17)

So, we have $D'_\mu \Psi' = (\partial'_\mu + igW'_\mu)\Psi'$

Using equations 1.15, 1.16 and 1.17 and solving we finally get

$$D'_\mu \Psi' = G(D_\mu \Psi)$$

which is the desired result.

Now we move on to see if the covariant derivative for an $SU(2)_L \otimes U(1)_Y$ transforms in a similar way.

The covariant derivative we choose is $D_\mu = (\partial_\mu + i\frac{g}{2}W_\mu ig'\frac{Y}{2}B_\mu)$

Also, $\Psi' = Ge^{i\beta Y}\Psi$

Solving as explained above we get to see that even in this case we have

$$D'_\mu \Psi' = Ge^{i\beta Y}(D_\mu \Psi)$$

As it transforms in the same way as in $SU(2)_L$, we conclude that the first two terms are invariant under $SU(2)_L \otimes U(1)_Y$ symmetry of the electroweak theory.

5.1.2 Invariance of the next two set of terms

$$\bar{L}\gamma^{\mu}\left(i\partial_{\mu} - g\tfrac{1}{2}\tau.\mathbf{W}_{\mu} - g\prime\tfrac{Y}{2}B_{\mu}\right)L + \bar{R}\gamma^{\mu}\left(i\partial_{\mu} - g\prime\tfrac{Y}{2}B_{\mu}\right)R$$
$$+\left|\left(i\partial_{\mu} - g\tfrac{1}{2}\tau.\mathbf{W}_{\mu} - g\prime\tfrac{Y}{2}B_{\mu}\right)\phi\right|^{2} \text{ - V}(\phi)$$

The first term involves the covariant derivative for the SU$(2)_L\otimes$ U$(1)_Y$ transformation which as discussed above is inherently invariant under the transformation.

The second term involves the right-handed fermions which are singlets under SU$(2)_L$ transformation. So their covariant derivative used omits the SU$(2)_L$ gauge field term. Being the covariant derivative for U$(1)_Y$ transformation, this term is also invariant.

The third term can be written and explained in simpler terms as below:

$$|\ D_{\mu}\phi|^{2}\rightarrow|(D_{\mu}\phi)^{\dagger}G^{\dagger}G\,(D_{\mu}\phi)\ | = |(D_{\mu}\phi)^{\dagger}\,(D_{\mu}\phi)\ | = |\ D_{\mu}\phi|^{2}$$

The last term represents the potential energy and all potential energy terms involve $\phi^{\dagger}\phi$ which is invariant under SU$(2)_L\otimes$ U$(1)_Y$ transformation.

Thus, we have proved that each term of the Standard Model Lagrangian of particle physics is invariant under the symmetry transformation SU(2)$_L\otimes$ U(1)$_Y$. So, till now we have discussed the concept of Higgs mechanism which is responsible for lending mass to the massive particles and eventually we have formulated the Standard Model Lagrangian which forms the basis of particle physics.

6 Problems of the Standard Model

Although the Standard Model for electroweak interactions have been successful in explaining several experimental findings, still it is believed that it is not a full picture of nature, and physics exists beyond it in the energy range higher than the electroweak breaking range. Some of its drawbacks are:

KUNGL. VETENSKAPS-AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

English
English (pdf)

Swedish
Swedish (pdf)

Press Release

6 October 2015

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2015 to

Takaaki Kajita
Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan

and

Arthur B. McDonald
Sudbury Neutrino Observatory Collaboration
Queen's University, Kingston, Canada

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

- It has been proved recently that neutrinos have a finite mass. However, in the Standard Model for electroweak interactions neutrinos are assumed to be massless.

- It also cannot explain the requirement for dark matter.
- The three gauge couplings seem to converge to a unified value at higher energy scale. The Standard Model cannot account for this fact satisfactorily.

All aspects of the SM have not been tested sufficiently. Inspite of its drawbacks, this is one of the milestones of modern physics. Without delving deep into this, we stop here.

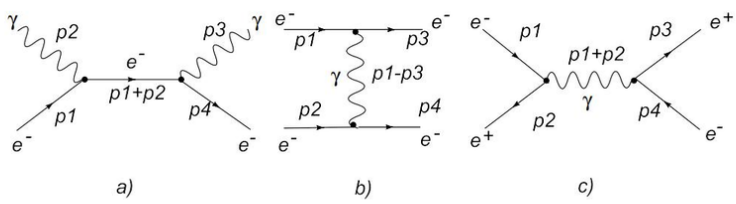
Next we will move on to radiative corrections which aims at calculating the scattering processes between the SM particles and also acts as a tool for the verification of the theory.

Part II
Electron Vertex Correction

1 Introduction to Radiative Corrections

[8]The formalism of quantum field theory is a generalization of quantum mechanics to an infinite dimensional space in which the number of particles is not a conserved quantity. This enables one to describe processes like scattering, annihilation, creation and decay of particles using a set of well-defined rules. We know that in QM the cross-section of a process is given by the square of the amplitude of the process calculated using the Feynman rules. Since exact calculations of the probability amplitude is not possible, we use perturbation theory to obtain the result in the form of a power series. The leading terms of this series represent the tree-level Feynman diagrams i.e. those without any loops. The loop diagrams represent the higher-order terms of the series.

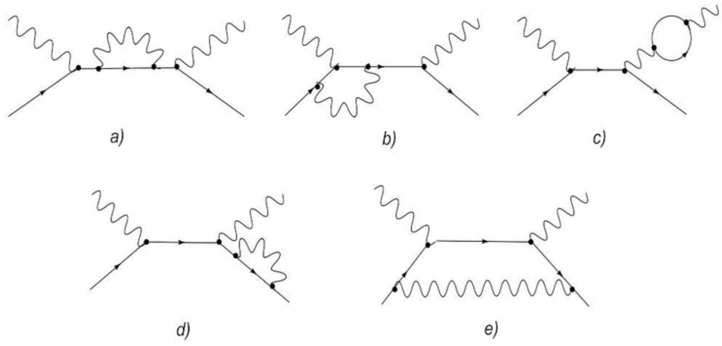
The contribution to the amplitude from all tree-level diagrams is proportional to the square of the coupling constant e^2. In tree-level diagrams, all external particles are physical and observable.



Tree-level Diagrams[9]

[8]arXiv:0901.2208v1 [hep-ph] 15 Jan 2009
[9]arXiv:0901.2208v1 [hep-ph] 15 Jan 2009

With higher order diagrams, the coupling constant raises to a higher degree depending on the number of vertices involved in the process. All these diagrams are proportional to the fourth power of the coupling constant e^4.[10]



Higher order diagrams[11]

Over the last few years, high energy experiments are being conducted with a very high precision and complexity. So, now testing a theory by a direct comparison of observed cross-sections and other calculated quantities is only an idealized picture.This is mainly due to experimental inefficiencies. This is the motivation of studying radiative corrections in the field of high-precision high-energy data analysis.

What are radiative corrections? As is evident, the name comprises of two parts - radiative and corrections. This name was given because in electrodynamics this resembles the emission and absorption of photons. This name is used in some other theories which use perturbative corrections.

These corrections yield results of very high precision. Let us consider "g" which is a proportionality constant connecting the magnetic moment of a particle to its angular momentum quantum number and a unit of magnetic moment i.e. $\mathbf{m} = \frac{ge\hbar}{2m_e c}\mathbf{S}$. According to Dirac's theory, the relativistic generalization of QM, we get g $\approx$ 2. However, Schwinger showed that QED radiative corrections lead to the more precise result of $\frac{g-2}{2} = \frac{\alpha}{2\pi}$.

In the next few sections we will gradually develop this theory starting with the basic tools required to calculate the amplitude and then move on to the one-loop correction.

[10]arXiv:0901.2208v1 [hep-ph] 15 Jan 2009
[11]arXiv:0901.2208v1 [hep-ph] 15 Jan 2009

1.1 Feynman Rules

In theoretical physics, Feynman diagrams is a pictorial way of depicting the processes that goes on at the subatomic level. It derives it's name from it inventor - Richard Feynman. Using this technique one can depict several subatomic processes very elegantly and in a much simpler manner making the representation more visually appealing. In theoretical physics, an important work involves calculating the probability amplitudes of processes which involve very large integrals and the integration also involves a large number of variables. However, these integrals have definite structure which is graphically depicted in the Feynman diagrams. There are a specific set of rules which are used to determine these integration structures. The rules are as follows:

vertex : -ieγ^μ

photon propagator : iD$_{F,\mu\nu}$= $-\frac{ig_{\mu\nu}}{k^2+i\epsilon}$

fermion propagator : iS$_F(p)$ = $\frac{i}{\not{p}-m+i\epsilon}$

initial, final electron : u(p), $\bar{u}(p)$

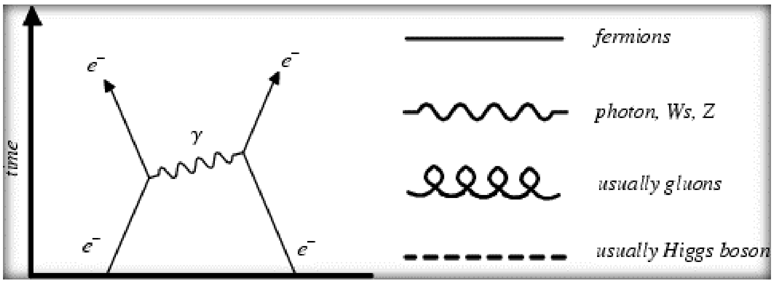
initial, final positron : $\bar{v}(p)$, v(p)

initial, final photon : $\varepsilon(k)$, $\varepsilon^*(k)$

In addition to this, we have to take care of a few more points while writing down the integral:

1. Momentum must be conserved at every vertex.
2. Integration over each undetermined loop momentum is $\int \frac{d^4k}{(2\pi)^4}$
3. For each closed fermion loop, we have to introduce a factor of $-\frac{1}{symmetry-factor}$ owing to the Pauli exclusion principle.
4. Only connected diagrams count.
5. Amputate external legs

A Feynman diagram looks like

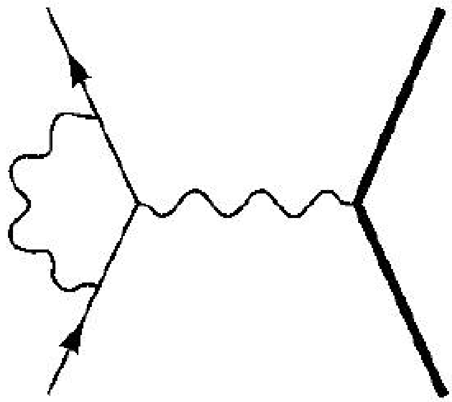


The lines do not depict the trajectory but is indicative of the progress of the process.

2 Electron vertex function

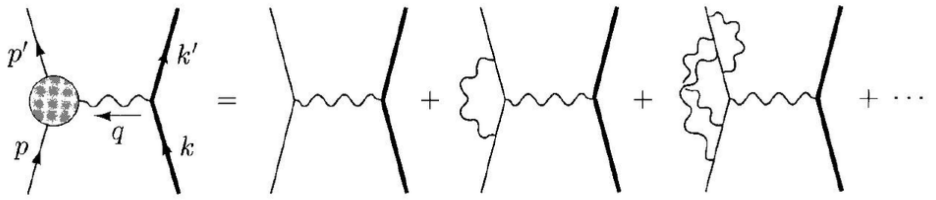
2.1 Introduction

Having discussed the basics of radiative corrections, we can now discuss the form of the vertex function of an electron scattering due to a virtual photon depicted as



Evaluating Feynman diagrams with loops is a tedious process. So, instead of jumping into correction calculations, let us first see what form we expect the outcome to be and interpret the possible terms. This will make our future calculations easier and more predictable.

Consider the set of diagrams



The gray circle called the blob represents the sum of lowest-order electron-photon vertex and all amputated loop corrections. This total vertex contribution is represented by $-ie\Gamma^{\mu}(p',\ p)$.

The diagrams on the right represent all possible diagrams for the scattering process, the first one being the tree-level diagram and the rest being the higher-order corrections. Now, using Feynman rules we can find the form of the amplitude for electron scattering from a heavy target. This is given by

$$i\mathcal{M} = \bar{u}(p')\,(-ie\Gamma^{\mu}(p',p))\,u(p)(-\tfrac{ig_{\mu\nu}}{q^2})\bar{u}(k')(-ie\gamma^{\nu})u(k)$$

$$= ie^2(\bar{u}(p')\Gamma^{\mu}(p',p)u(p))(\tfrac{1}{q^2})(\bar{u}(k')\gamma_{\mu}u(k))$$

By momentum conservation at the vertex we have, $\mathbf{q} = \mathbf{p}'$- $\mathbf{p}$. Γ^{μ} is a Lorentz vector and can be expressed as a linear combination of several other Lorentz vectors like $\gamma^{\mu},\ p^{\mu},\ p'^{\mu},\ \not{p},\ \not{p}',\ p^2,\ p'^2,\ g^{\mu\nu},\ \varepsilon^{\mu\nu\rho\sigma}$ and the list is exhaustive. But $\varepsilon^{\mu\nu\rho\sigma}$ has odd parity and thus, is not included in the expression for Γ^{μ}. Otherwise, it would lead to parity violation. Among the other variables, we can make any number of possible combinations we want (ensure the order of index is one every time). Now, by simple calculations, obeying momentum conservation law and using the relations $\not{p}u(p) = mu(p)$[12], $\bar{u}(p')\not{p}' = \bar{u}(p')m$[13], $g^{\mu\nu}p_{\mu} = p^{\nu},\ g^{\mu\nu}p'_{\mu} = p'^{\nu},\ g^{\mu\nu}\gamma_{\nu} = \gamma^{\mu}$ *and* $\{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu}$we will be able to reduce the terms to a much simpler form that looks like

$$\Gamma^{\mu} = A\gamma^{\mu} + p^{\mu}B' + p'^{\mu}C'.$$

[12] *See appendix B.1*
[13] *See Appendix B.2*

This can be written for our convenience as

$$\Gamma^{\mu} = A\gamma^{\mu} + (p^{\mu} + p'^{\mu})B + (p^{\mu} - p'^{\mu})C \text{ ....(2.1)}$$

Now we will use the Ward Identity $q_{\mu}\Gamma^{\mu} = 0$[14] to further simplify the expression. Although we will not discuss much about the Ward identity here, but it is essentially a statement of current conservation, which is a consequence of the gauge symmetry.

Substituting equation (2.1) in the Ward identity we get,

$$\not{q}A + q_{\mu}(p^{\mu} + p'^{\mu})B + q_{\mu}(p^{\mu} - p'^{\mu})C = 0$$

Using the momentum conservation relation and the relation $\bar{u}(p')\not{q}u(p)=0$, we see that the first two terms of the above equation vanish. As the third term does not vanish automatically, we set C = 0. So, we finally get,

$$\Gamma^{\mu} = A\gamma^{\mu} + (p^{\mu} + p'^{\mu})B \text{ ....(2.2)}$$

The coefficients can involve Dirac matrices dotted into vectors i.e. $\not{p}$ or $\not{p}'$. We can write this in terms of ordinary numbers without loss of generality using $\not{p}u(p) = mu(p)$ and $\bar{u}(p')\not{p}' = \bar{u}(p')m$. Since, $q^2 = 2m^2 - 2\mathbf{p.p'}$, the coefficients can be assumed to be functions of only q^2.

We can further simplify this relation using the Gordon identity,

$$\bar{u}(p')\gamma^{\mu}u(p) = \bar{u}(p')\left[\frac{p'^{\mu}+p^{\mu}}{2m} + \frac{i\Sigma^{\mu\nu}q_{\nu}}{2m}\right]u(p) \text{ where } \Sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^{\mu},\gamma^{\nu}\right]$$ [15]

Using this and expressing the coefficients as functions of q^2 and some constant, say m, we can write the final expression as

$$\Gamma^{\mu}(p',p) = \gamma^{\mu}F_1(q^2) + \frac{i\Sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2) \text{ ....(2.3)}$$

where $F_1(q^2)$ and $F_2(q^2)$ are called the form factors and their exact form is not determined.

To the lowest order, $F_1(q^2) = 1$ and $F_2(q^2) = 0$.

[14] *See Appendix D for details*

[15] For derivation, see Appendix B

2.2 Amplitude in non-zero electrostatic potential

In case of interaction, we add the perturbed Hamiltonian to the unperturbed one. The interaction Hamiltonian in QED is

$$\triangle H_{int} = \int d^3x e A_\mu^{cl} j^\mu$$

where $j^\mu = \bar{\psi}(x)\gamma^\mu\psi(x)$ is the electromagnetic current and A_μ^{cl} is a fixed classical potential.

The scattering amplitude is then given by

$$i\mathfrak{M}(2\pi)\delta(p^{0\prime} - p^0) = -ie\bar{u}(p')\gamma^\mu u(p).\tilde{A}_\mu^{cl}(p' - p)$$

Because of vertex correction, we have to modify this as

$$i\mathfrak{M}(2\pi)\delta(p^{0\prime} - p^0) = -ie\bar{u}(p')\Gamma^\mu u(p).\tilde{A}_\mu^{cl}(p' - p) \text{ ....(2.4)}$$

To compute the amplitude for coulomb scattering of a non-relativistic electron in a non-zero electrostatic potential, we set $A_\mu^{cl}(x) = (\phi(x), 0)$

Then $\tilde{A}_\mu^{cl}(q) = \Big(2\pi\delta(q^0)\tilde{\phi}(\mathbf{q}), \mathbf{0}\Big)$. Using this in equation (2.4), we get

$$i\mathfrak{M} = -ie\bar{u}(p')\Gamma^0(p', p)u(p)\tilde{\phi}(q)$$

Substituting equation (2.3) in the above equation, we can express it in terms of the form factors.

If the electrostatic field is very slowly varying over a large region, $\tilde{\phi}(q)$ will be concentrated about q = 0. So, in the limit q $\rightarrow$0, $F_2(q^2)$ does not contribute.

We know the Dirac spinors are given by

$$u(p) = \begin{pmatrix} -\frac{\sigma.\mathbf{p}}{|E|+m}\xi \\ \xi \end{pmatrix} \text{ ....(2.5)}$$

Using this in the non-relativistic limit, we get

$$\bar{u}(p')\gamma^0 u(p) = u^\dagger(p')u(p) \approx 2m\xi^\dagger\xi$$

So, the amplitude of scattering of an electron from an electric field is given by

$$\mathrm{i}\mathfrak{y} = \text{-ieF}_1(0)\tilde{\phi}(\mathbf{q}).2\mathrm{m}\xi^\dagger\xi$$

We can interpret $\mathfrak{y}$ as the scattering of an electron from a potential well. Using this Born approximation for scattering from a potential we get

$$\mathrm{V(x)} = \mathrm{eF}_1(0)\phi(x)$$

So, $F_1(0)$ is the electronic charge in units of e.

2.3 Electron scattering from a static vector potential

We can do the same analysis as above for electron scattering from a static vector potential. Here, we set $\boldsymbol{A}^{cl}_\mu\boldsymbol{(x)} = \left(0, A^{cl}_\mu(\mathbf{x})\right)$

So, the scattering amplitude is given by $\mathrm{i}\mathfrak{y} = \text{-ie}\bar{u}(p')\Gamma^i(p',p)\mathrm{u(p)}\tilde{A}^{cl}_i(q)$

Using the Gordon identity this is finally written as

$$\mathrm{i}\mathfrak{y} = \text{-ie}[\bar{u}(p')\left\{\gamma^i F_1 + \tfrac{i\Sigma^{i\nu}q_\nu}{2m}F_2\right\}\mathrm{u(p)}]\tilde{A}^{cl}_i(q) \text{ ....(2.6)}$$

Using the Dirac spinors from equation (2.5) and the relation
$\sigma^i\sigma^j = \delta^{ij} + i\varepsilon^{ijk}\sigma^k$ to solve the above equation we get,

$$\bar{u}(p')\gamma^i u(p) = 2\mathrm{m}\xi^\dagger\left[-\tfrac{i}{2m}\varepsilon^{ijk}q^j\sigma^k\right]\xi$$

$$\text{and } \bar{u}(p')\left\{\tfrac{i}{2m}\Sigma^{i\nu}q_\nu\right\}u(p) = 2m\xi^\dagger\left[-\tfrac{i}{2m}\varepsilon^{ijk}q^j\sigma^k\right]\xi$$

So, the complete equation appears to be

$$\bar{u}(p')\left\{\gamma^i F_1 + \tfrac{i\Sigma^{i\nu}q_\nu}{2m}F_2\right\}\mathrm{u(p)} \approx 2\mathrm{m}\xi^\dagger\left(-\tfrac{i}{2m}\varepsilon^{ijk}q^j\sigma^k\left[F_1(0)+F_2(0)\right]\right)\xi$$

Inserting this in equation (2.6), we get the final form of the amplitude to be

$$\mathrm{i}\mathfrak{y} = \text{-i(2m)e}\xi^\dagger\left(-\tfrac{1}{2m}\sigma^k\left[F_1(0)+F_2(0)\right]\right)\xi\tilde{B}^k(\mathbf{q}) \text{ ....(2.7)}$$

where $\tilde{B}^k(\mathbf{q}) = -i\varepsilon^{ikj}q^i\tilde{A}^{cl}_j(\mathbf{q})$ is the fourier transform of the magnetic field produced by $\mathbf{A}^{cl}(\mathbf{x})$.

Again using the Born approximation for scattering from a potential we get V($\mathbf{x}$) = - $<\mu>$.$\mathbf{B}(\mathbf{x})$ and comparing this with equation (2.7) we get

$$<\mu> = \frac{e}{2m} 2\left[F_1(0) + F_2(0)\right] \xi^\dagger \frac{\sigma}{2} \xi$$

where $\xi^\dagger \frac{\sigma}{2} \xi$ is the spin operator.

Comparing this with the known relation μ= g$\left(\frac{e}{2m}\right) S$, we get

$$\mathrm{g} = 2\left[F_1(0) + F_2(0)\right] = 2 + 2\mathrm{F}_2(0)$$

In the lowest order, since F_2 is zero, we get g $\approx$ 2.

From this we can conclude that if we include higher-order corrections where F_2 will not be zero; g will be modified, and we will get a small but finite difference of the electron's magnetic moment from the Dirac value.

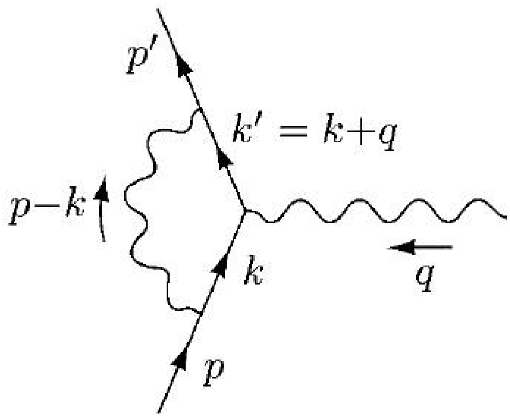
So, we now have an understanding of Feynman diagrams and the calculation of probability amplitude from them. We will see that these corrective calculations enables us to produce certain results with more precision. Although the calculations involve some hectic algebra, but they eventually simplify leading to extraordinary results. This is one of the results of radiative corrections which was discussed previously. Now that we know what form the answer will take, we will move on to see how to calculate the exact form of a one-loop correction to the electron vertex function.

3 The Electron Vertex Function - Evaluation

3.1 Introduction

One-loop diagrams are the first step towards the correction of the vertex function. What comes next are higher order contributions.

Consider the following Feynman diagram with one-loop.



Since, here we will consider first-order correction to the vertex function, we can write the vertex correction as

$$\Gamma^{\mu} = \gamma^{\mu} + \delta\Gamma^{\mu} \ .$$

Here we express Γ^{μ} to be the sum of corrections of all the vertices. So, this is written as the sum of single-vertex correction and the first order correction.

So, we have $\bar{u}(p')\delta\Gamma^{\mu}u(p) =$

$$\int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\nu\rho}}{(k-p)^2+i\varepsilon} \bar{u}(p')(-ie\gamma^{\nu}) \frac{i\left(\not{k}'+m\right)}{\left(\not{k}'^2-m^2+i\varepsilon\right)} \gamma^{\mu} \frac{i\left(\not{k}+m\right)}{\left(\not{k}^2-m^2+i\varepsilon\right)} \left(-ie\gamma^{\rho}\right) u(p)$$

$$= 2\mathrm{ie}^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')\left[\not{k}\gamma^{\mu}\not{k}'+m^2\gamma^{\mu}-2m(k+k')^{\mu}\right]u(p)}{\left\{(k-p)^2+i\varepsilon\right\}(k'^2-m^2+i\varepsilon)(k^2-m^2+i\varepsilon)} \text{ ....(2.8)}$$

To obtain the second line use $\left[\gamma^{\nu}\gamma^{\mu}\gamma_{\nu} = -2\gamma^{\mu}\right]$ and simplify.

As we see, this integration is not at all easy. It is almost impossible to solve this with common integration tools. We would require a new set of computational tools known as the Feynman parameters. Before proceeding, we must have a clear idea about this method of integration.

3.2 Feynman parameters

Feynman parametrization is a technique to evaluate loop integrals which arise from Feynman diagrams containing one or more loops. It expresses the denominator of a fractional integral as the product of the terms i.e. we get a single polynomial in the denominator. However, we have to introduce some auxiliary parameters for the purpose.

For example,

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA+(1-x)B]^2} = \int_0^1 dxdy\delta\left(x+y-1\right)\frac{1}{[xA+yB]^2}$$

However, we have three terms in the required denominator of equation (2.8). So, we need a better identity. A more general identity can be obtained by induction and is given by

$$\frac{1}{A_1A_2...A_n} = \int_0^1 dx_1dx_2...dx_n\delta\left(\Sigma x_i - 1\right)\frac{(n-1)!}{[x_1A_1+...+x_nA_n]^n} \quad ....(2.9)$$

The variables x and y which help in this simplification are called the Feynman parameters.

Using this we will first simplify the denominator and then simplify the numerator separately. Let us do this step by step.

3.3 Simplification of the Denominator

Let us apply the above formula of equation (2.9) to the denominator in equation (2.8). We get

$$\frac{1}{\left\{(k-p)^2+i\varepsilon\right\}(k'^2-m^2+i\varepsilon)(k^2-m^2+i\varepsilon)} = \int_0^1 dxdydz\delta\left(x+y+z-1\right)\frac{2}{D^3}$$

$$\text{where D} = z\left(k-p\right)^2 + y\left(k'^2-m^2\right) + x(k^2-m^2) + \left(x+y+z\right)i\varepsilon$$

$$= x\left(k^2-m^2\right) + y\left(k'^2-m^2\right) + z\left(k-p\right)^2 + i\varepsilon$$

To obtain the second line we have used $k' = k+q$ and x+y+z=1.

Even this form of the denominator has a number of integrable variables which can eventually prove to be gruesome. So, we further try to simplify the form of the denominator.

Let $l = k + yq - zp$.

Using this in D and doing a bit of simple algebra will yield the result

$$D = l^2 - \triangle + i\varepsilon \text{ where } \triangle = -xyq^2 + \left(1-z\right)^2 m^2$$

Since $q^2 < 0$ for scattering process, $\triangle$ is positive and we can consider it to be the effective mass term.

Since D depends only on the magnitude of l, we have

$$\int \frac{d^4l}{(2\pi)^4}\frac{l^4}{D^3} = 0 \text{ ....(2.10)}$$

$$\int \frac{d^4l}{(2\pi)^4}\frac{l^\mu l^\nu}{D^3} = \int \frac{d^4l}{(2\pi)^4}\frac{\frac{1}{4}g^{\mu\nu}l^2}{D^3} \text{ ....(2.11)}$$

The first relation follows from symmetry.

The second relation vanishes if $\mu \neq \nu$. So, for Lorentz invariance, the integral must be made proportional to $g^{\mu\nu}$. The relation can be cross-checked by multiplying both sides by $g^{\mu\nu}$.

Next, we try to express the numerator in terms of l using the above relations.

3.4 Simplification of the Numerator

As shown in equation (2.8), the numerator is

$$\mathrm{N} = \bar{u}\left(p'\right)\left[\not{k}\gamma^\mu \not{k}' + m^2\gamma^\mu - 2m\left(k+k'\right)^\mu\right]u\left(p\right)$$

Substituting the values of $\not{k}$ and $\not{k}'$ in the above equation of the numerator we get,

$$\mathrm{N} = \bar{u}\left(p'\right)\left[\left(\not{l} - y\not{q} + z\not{p}\right)\gamma^\mu\left(\not{l} + \not{q} - y\not{q} + z\not{p}\right) + m^2\gamma^\mu - 2mq^\mu - 4m\left(l - yq + zp\right)^\mu\right]u(p)$$

All linear terms in l become zero on integration by virtue of equations (2.10) and (2.11) and hence, vanish eventually. So, we have

$$\mathrm{N} = \bar{u}(p')[\not{l}\gamma^\mu\not{l} + \left(-y\not{q} + z\not{p}\right)\gamma^\mu\left((1-y)\not{q} + z\not{p}\right) + m^2\gamma^\mu]u(p) \text{ ....(2.12)}$$

Now we will solve each term separately.

The first term can be simplified using clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$and $l^\mu l^\nu = \frac{1}{4}g^{\mu\nu}l^2$. We get,

$$\not{l}\gamma^\mu\not{l} = -\frac{1}{2}l^2\gamma^\mu \text{ ....(2.13)}$$

Moving on to the second term, this requires quite a bit of algebra. To simplify this term, we would require the following relations :

$$x + y + z = 1$$

$$\not{p}\gamma^\mu = 2p^\mu - \gamma^\mu\not{p}^{16}$$

$$\not{q} = \not{p}' - \not{p}$$

$$\not{p}u(p) = mu(p)$$

$$\bar{u}(p')\not{p}' = \bar{u}(p')m$$

Using this and gradually simplifying, we get

$$[(-y\not{q} + z\not{p})\,\gamma^\mu\,((1-y)\,\not{q} + z\not{p})] =$$
$$(1-x)(1-y)\gamma^\mu q^2 + 2mz(x-1)(m\gamma^\mu - p^\mu) + mz(1-y)(2q^\mu - \not{q}\gamma^\mu) + m^2z^2\gamma^\mu$$
..............(2.14)

Now we can express the numerator as obtained in equation (2.12) using the equations (2.13) and (2.14). What we will get is a set of three terms depending on γ^μ, p^μ and q^μ. We will get a more convenient form after some simple algebraic simplifications of each of the terms and finally using the equality $p^\mu = \frac{1}{2}\left\{(p^\mu + p'^\mu) + (p^\mu - p'^\mu)\right\} = \frac{1}{2}\left\{(p^\mu + p'^\mu) - q^\mu\right\}$.

We finally get ,

$$N = \bar{u}(p')[\gamma^\mu\left\{-\tfrac{1}{2}l^2 + (1-x)(1-y)q^2 + m^2(1-2z-z^2)\right\} +$$
$$mz(z-1)(p^\mu + p'^\mu) + m(z-2)(x-y)q^\mu]u(p) \text{ ....(2.15)}$$

So, we see that we have reached the desired form of

$$\Gamma^\mu = A\gamma^\mu + (p^\mu + p'^\mu)B + q^\mu C.$$

The coefficient of q^μ must vanish according o the Ward identity which was discussed before. Moreover, the denominator is symmetric under x↔y. The coefficient of q^μ is odd under x↔y and so it vanishes on integrating over x and y. Next, we use the Gordon identity to eliminate the form of $p^\mu + p'^\mu$. We finally get the numerator to be,

$$N =$$

$$\bar{u}(p')\left[\gamma^\mu\left\{-\tfrac{1}{2}l^2 + (1-x)(1-y)q^2 + (1-2z-z^2)m^2\right\} + \tfrac{i\Sigma^{\mu\nu}q_\nu}{2m}2m^2z(1-z)\right]u(p)$$

[16]See appendix A.2

3.5 Final calculations

Our complete expression for the first order contribution to the electron vertex is

$$\bar{u}(p')\delta\Gamma^{\mu}u(p) =$$

$$2ie^2 \int \frac{d^4l}{(2\pi)^4} \int_0^1 dxdydz\delta\left(x+y+z-1\right)\frac{2}{D^3}\times$$

$$\left[\gamma^{\mu}\left\{-\frac{1}{2}l^2+(1-x)(1-y)q^2+(1-2z-z^2)m^2\right\}+\frac{i\Sigma^{\mu\nu}q_{\nu}}{2m}2m^2z(1-z)\right]$$

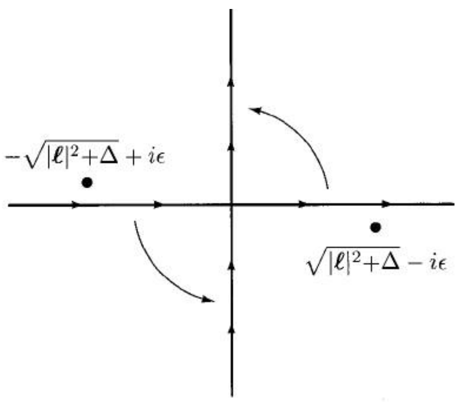
..............(2.16)

Now our aim is to evaluate this integral. There are two integrals which are to be evaluated and they can be generalized to express in the form as below:

$$\int \frac{d^4l}{(2\pi)^4}\frac{1}{(l^2-\triangle)^m}$$

and $\int \frac{d^4l}{(2\pi)^4}\frac{l^2}{(l^2-\triangle)^m}$

It is not difficult to evaluate these integrals using contour integral for the l^0 integration and then do the integration of the spatial part in spherical coordinates. We will use a trick called the Wick rotation. This technique enables us to find the solution of this problem in the Euclidean space by substituting an imaginary number variable for a real number variable. The integration of the spatial part also becomes much simpler in the Euclidean space.



The contour of the ℓ^0 integration can be rotated as shown.

The denominator can be expressed as D $= l_0^2-(|\ l\ |^2+\triangle)+i\varepsilon$

In the Euclidean space, the four-vector is expressed as $l_E \equiv (il_0, l_i)$. The change in variable is so chosen as to take into account the minus sign that comes in the Minkowski space. The location of the poles and the fact that the integrand falls off rapidly at large $| l^0 |$, allow us to rotate the contour counter-clockwise by 90^o. We can now evaluate the integral in four-dimensional spherical coordinates.

Let us first evaluate

$$\int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2-\triangle)^m} = \frac{i(-1)^m}{(2\pi)^4} \int \frac{d^4 l_E}{\left(l_E^2+\triangle\right)^m}$$

$$= \frac{i(-1)^m}{(2\pi)^4} \int d\Omega_E \int_0^\infty dl_E \frac{l_E^3}{\left(l_E^2+\triangle\right)^m}$$

Substituting $\alpha = l_E^2 + \Delta$ and then solving by integration by parts we finally get

$$\int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2-\triangle)^m} = \frac{i(-1)^m}{(4\pi)^2} \frac{1}{(m-1)(m-2)} \frac{1}{\triangle^{m-2}} \quad ....(2.17)$$

Next, we evaluate the second integral in the same way as above. The difference will be that in this integral we will have a factor of l_E^5 instead of l_E^3 because of the extra l^2term in this case. So, we finally have

$$\int \frac{d^4l}{(2\pi)^4} \frac{l^2}{(l^2-\triangle)^m} = \frac{i(-1)^{m-1}}{(4\pi)^2} \frac{2}{(m-1)(m-2)(m-3)} \frac{1}{\triangle^{m-3}} \quad ....(2.18)$$

This second integral is valid only for m $>$ 3. For m $=$ 3, the Wick rotation cannot be justified and the term becomes divergent in any event. But m $=$ 3 is the only condition we need to evaluate the vertex function. To render this integral finite, we will use a method known as Pauli-Villars regularization[17].

As an example, consider the modification of the original Feynman propagator as

$$\frac{1}{(k-p)^2+i\varepsilon} \to \frac{1}{(k-p)^2+i\varepsilon} - \frac{1}{(k-p)^2-\Lambda^2+i\varepsilon}$$

Here Λ is a very large mass. The second term in the modification can be thought of as the propagator of a fictitious heavy photon whose contribution is subtracted from that of the ordinary photon. So, now terms involving the heavy photon will be modified. The numerator will remain the same but the denominator will get modified as

$$\triangle \to \triangle_\Lambda = -xyq^2 + (1-z)^2 m^2 + z\Lambda^2$$

[17]See appendix E

Now we can modify the integral (2.18) by replacing with a convergent integral, which can be Wick rotated and evalated:

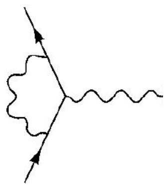
$$\int \frac{d^4l}{(2\pi)^4}\left[\frac{l^2}{\left(l^2-\triangle\right)^3}-\frac{l^2}{\left(l^2-\triangle_\Lambda\right)^3}\right]=\frac{i}{(4\pi)^2}\int_0^\infty dl_E^2\left[\frac{l_E^4}{\left(l_E^2+\triangle\right)^3}-\frac{l_E}{\left(l_E^2+\triangle_\Lambda\right)^3}\right]$$

$$=\frac{i}{(4\pi)^2}log\left(\frac{\triangle_\Lambda}{\triangle}\right)+\left(\Lambda^{-2}\right) \text{ ....(2.19)}$$

The convergent terms present previously are modified by terms of order Λ^{-2} which we ignore.

Now we have all the tools to evaluate the correction integral. We use the equations (2.17) and (2.19) to evaluate the parts of the integration involving $\int \frac{d^4l}{(2\pi)^4}$ and l terms.

After solving we get the vertex correction to be like



$$= \frac{\alpha}{2\pi}\int_0^1 dxdydz\delta\left(x+y+z-1\right)\times$$

$$\bar{u}(p')[\gamma^\mu\left\{log\left(\frac{z\Lambda^2}{\triangle}\right)+\frac{1}{\triangle}\left(\left(1-x\right)\left(1-y\right)q^2+\left(1+z^2-4z\right)m^2\right)\right\}$$

$$+\frac{i\Sigma^{\mu\nu}q_\nu}{2m}\left\{\frac{1}{\triangle}2m^2z(1-z)\right\}]u(p) \text{ ....(2.20)}$$

Comparing this with equation (2.3), we can determine $F_1(q^2)$ and $F_2(q^2)$.

As discussed in the previous section, we have seen that for the determination g-factor we need $F_2(q^2=0)$. So, we evaluate the integration (2.20) for $q^2= 0$.

Evaluating :

$$F_1\left(q^2=0\right)=\frac{\alpha}{2\pi}\int_0^1 dxdydz\delta\left(x+y+z-1\right)\bar{u}\left(p'\right)[log\left(\frac{z\Lambda^2}{\triangle}\right)+ \frac{1}{\triangle}\left(\left(1-x\right)\left(1-y\right)q^2+\left(1+z^2-4z\right)m^2\right)]$$

$$=\frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z}dy\int_0^{1-y-z}dx\delta\left(x+y+z-1\right)\bar{u}\left(p'\right)[log\left(\frac{z\Lambda^2}{\triangle}\right)+ \frac{1}{\triangle}\left(\left(1-x\right)\left(1-y\right)q^2+\left(1+z^2-4z\right)m^2\right)]$$

Solving this integration we will see that the first and last terms become divergent while the second term becomes zero. Therefore, $F_1\left(q^2=0\right)$ becomes divergent. This is the case of infrared divergence as the energy of the object contributing to this term is approaching zero.

Now evaluating again :

$$F_2(q^2=0) = \frac{\alpha}{2\pi}\int_0^1 dxdydz\delta\left(x+y+z-1\right)\bar{u}\left(p'\right)\frac{i\Sigma^{\mu\nu}q_\nu}{2m}\left\{\frac{1}{\triangle}2m^2z(1-z)\right\}u(p)$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z}dy\int_0^{1-y-z}dx\delta\left(x+y+z-1\right)\times$$

$$\bar{u}\left(p'\right)\frac{i\Sigma^{\mu\nu}q_\nu}{2m}\left\{\frac{1}{\triangle}2m^2z(1-z)\right\}u(p)$$

$$= \frac{\alpha}{2\pi}\int_0^1 dz\int_0^{1-z}dy\int_0^{1-y-z}dx\delta\left(x+y+z-1\right)\times$$

$$\bar{u}\left(p'\right)\frac{i\Sigma^{\mu\nu}q_\nu}{2m}\left\{\frac{2m^2z(1-z)}{m^2(1-z)^2}\right\}u(p)$$

Finally we get

$$F_2\left(q^2=0\right) = \frac{\alpha}{2\pi} = \frac{g-2}{2}$$

$$a_e = \frac{g-2}{2} \approx 0.0011614$$

This result was first obtained by Schwinger in 1948. Experiments give $a_e = 0.0011597$. Apparently, the theoretical value of a_e we calculated is also unambiguously correct upto higher orders of α.

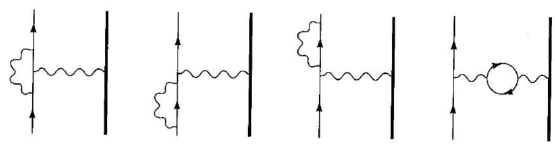
So, we have successfully calculated the one-loop correction to the electron vertex function. This has also led us to a more precise value of the g-factor. Calculations can be performed for higher order corrections to the vertex function and a sum of all possible corrections gives the true vertex function. Successive generations of physicists have developed more advanced techniques of determining this coefficient a_e with higher accuracy theoretically and experimentally. Now the coefficients of QED formula for a_e are known through order α^4. Calculation of higher order coefficients requires a systematic study of ultraviolet divergence.

3 A solution to the divergence problem

Throughout we have considered the scattering process of an electron from a very heavy particle. Assuming that the heavy particle accelerates less and thus, radiates less during the scattering process enables us to neglect the contribution of this vertex to the correction.

Previously we have come across infrared divergence. Ultraviolet divergence is also observed in the amplitude cross-section calculations when the loop integral diverges for $k \to \infty$. Throughout we have considered only one loop-diagram and performed the necessary calculations. However, there are many such loop diagrams which contribute to the correction of the vertex function.

So, after the tree-level diagram, the first order correction terms come from the following four loop diagrams:

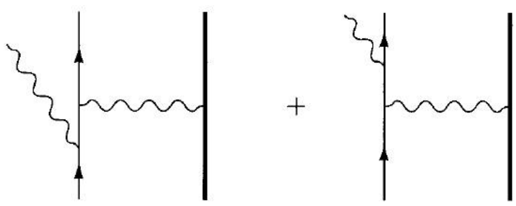


As discussed before, the first diagram, the vertex correction diagram, is the most intricate and gives the largest variety of new effects. For example, the anomalous magnetic moment of the electron which was evaluated in the previous sections.

The next two diagrams are the external leg corrections.

The final diagram is called vacuum polarization. This requires more advanced and complicated machinery for its evaluation and hence is not discussed in this thesis.

The first three diagrams gives ultraviolet divergence but the divergent parts of these integrals cancel out on being summed together for measurable quantities like cross-section. These diagrams also contain infrared divergences - divergence coming from the $k \to 0$ end of the loop-momentum integrals - which are canceled out on including the bremsstrahlung diagrams shown below :



These diagrams are divergent in the region where the momentum of the photon tends to zero. In this regime, the photon cannot be detected by any physical detector. So, the cross-sections from these diagrams must be added to the cross-section for scattering without radiation.

Adding the contributions of all these diagrams we get a completely finite, non-diverging value for a measurable quantity like cross-section.

4 Precision Tests of QED

Different atoms play different roles in the modern world. For example, a unit of time, the second, is defined via the hyperfine interval in the cesium atom, while the atomic mass unit and the Avogadro number are defined via the mass of a carbon atom. So, when studying a new system or new mechanics or new model, one tries to apply and validate the theory for the simplest systems available. So, modern physics started with the study of free particles and then simple atoms. QED proves successful for a broad range of problems from atomic spectra to scattering, from low energy, related to microwave radiation, to high energy phenomena with hard annihilation and bremsstrahlung, from nano- to giga- electronvolt. A remarkable outcome for QED of the hydrogen atom is that the anomalous magnetic moment of an electron was first discovered by Rabi and his colleagues as an anomaly in the hyperfine structure of hydrogen. Immediately that was interpreted as a possible anomaly related to a free electron and only afterwards was that confirmed by a direct experiment. We have proved this theoretically in the earlier sections and derived a more precise value of this proportionality constant. Often accuracy of theory and experiment are not compatible. However, there is a broad range of effects, for which theory and experiment approach the same high level of accuracy. The study of such effects forms a field called precision tests of QED[18].

The coefficients of QED formula for a_e are now known through order α^4. The calculation of order α^2 and higher orders require a systematic treatment of ultraviolet divergences. The most recent calculation of a_e was evaluated by Dehmelt and his collaborators by trapping a single electron in a system of electric and magnetic fields and exciting to a spin resonance. Today, the best theoretical and experimental values of a_e match to eight significant figures.

The Standard Model of electroweak theory is a renormalizable gauge theory. At the tree level, the SM has its properties and these properties have been extensively tested like the discovery of the neutral current. However, the genuine features of this theory as a renormalizable theory is proved by studying the small but finite quantum effects on physical observables, i.e. radiative corrections with the data obtained from precision experiments like LEP, CDF, etc. The precision tests of such finite radiative corrections to the electroweak parameters like gauge couplings and gauge boson masses are used to

[18]Precision physics of simple atoms: QED tests, nuclear structure and fundamental constants Savely G. Karshenboim D. I. Mendeleev Institute for Metrology, 190005 St. Petersburg, Russia Max-Planck-Institut f¨ur Quantenoptik, 85748 Garching, Germany

validate the SM. Higher order QED calculations have been carried out for several other quantities like transition energies in hydrogen and hydrogen-like atoms, anomalous magnetic moment of a muon, decay rates of singlet and triplet states positronium, etc[19]. The precision comparison between QED theory and experiments requires an extremely precise value of the fine-structure constantα which can be obtained from another QED precision experiment. So, each comparison of theory and experiment is assumed to be an independent determination of α. Each α is assigned an error because of the uncertainties between theory and experiments. The desired results are generally obtained by the fitting of experimental data with a theoretical expression containing α. Consider the table below displaying values of α^{-1} obtained from QED precision experiments of different processes:

Low-Energy QED:	
Electron $(g-2)$	137.035 992 35 (73)
Muon $(g-2)$	137.035 5 (1 1)
Muonium hyperfine splitting	137.035 994 (18)
Lamb shift	137.036 8 (7)
Hydrogen hyperfine splitting	137.036 0 (3)
$2\,^3S_1$–$1\,^3S_1$ splitting in positronium	137.034 (16)
$^1S_0$ positronium decay rate	137.00 (6)
$^3S_1$ positronium decay rate	136.971 (6)
Neutron compton wavelength	137.036 010 1 (5 4)
High-Energy QED:	
$\sigma(e^+e^- \to e^+e^-e^+e^-)$	136.5 (2.7)
$\sigma(e^+e^- \to e^+e^-\mu^+\mu^-)$	139.9 (1.2)
Condensed Matter:	
Quantum Hall effect	137.035 997 9 (3 2)
AC Josephson effect	137.035 977 0 (7 7)

Each value of α in the table is obtained by fitting an experimental measurement to a theoretical expression that contains α as a parameter. The numbers in parenthesis are the standard errors in the last displayed digits. Experimentally the value of a_e is determined which is then plugged in the corresponding theoretical expression and solved for α (the expression is different for different processes and also depends on the order of α being dealt with). We have performed the calculations for an electron vertex upto order 1. Higher order terms provide more precise results and are solved by the method of fitting.

[19]The Physics of the Standard Model and Beyond By T. Morii, C. S. Lim, S. N. Mukherjee

Higher-loop calculations are much more complicated because the number of diagrams increases very rapidly with the number of loops; at 4-loop order there are thousands of diagrams; a computer is needed just to count them! Also, at higher orders one has to include effects like strong and weak interactions because photons interact not just with electrons and other charged leptons, but also with hadrons and $W^{\pm}$ particles, which in turn interact with other hadrons, Z^0, Higgs, etc. Nevertheless, people have calculated the electron's and muon's g factors up to order α^4 back in the 1970s and more recent calculations are good up to α^5 order.

> *Considerable evidence for the general validity of QED is provided by the enormous variety of ordinary phenomena seen to be consistent with it. The superfluidity of helium and the superconductivity of metals having recently been explained, there are to my knowledge no phenomena occuring under known conditions, where quantum electrodynamics should provide an explanation, and where at least a qualitative explanation in these terms has not been found. The search for discrepancies has turned from looking for gross deviations in complex situations to looking either for large discrepancies at very high energies, or by looking for tiny deviations from the theory in very simple, but very accurately measured situations*[20].
>
> - Richard P. Feynman (Solvay Conference in 1961)

[20] *The Solvay Conferences on Physics: Aspects of the Development of Physics since 1911 by Jagdish Mehra*

Part III
Conclusion

This thesis started with the discussion of the electroweak Standard Model which forms the heart of particle physics. While studying the theories and tools necessary to obtain the corresponding Lagrangian we could see that writing down the Lagrangian obeying a specific symmetry is easy once we know the rules underlying the theory. Writing the Lagrangian requires a lot of prerequisites as shown in the first chapter. The discovery of parity violation set off a series of discoveries that led to the evolution of modern particle physics to a form as it is now. If no symmetry breaking was involved, we would not have been able to generate masses for the gauge bosons and fermions as was the case with photons. The Lagrangian obtained contains kinetic energy terms of the SM particles and also shows their interaction with each other. Since the formulation of the Standard model, several attempts have been made to verify the theory. An outcome of such attempts was Radiative Corrections. As in many realms of physics, a deeper understanding and an extensive "box of tricks" can render a seemingly unsolvable problem doable. In this thesis we demonstrated how we can manipulate the theory and use such tricks to extract information in clever ways[21].

In particle physics, quantum electrodynamics (QED) is the relativistic quantum field theory of electrodynamics. In technical terms, QED can be described as a perturbation theory of the electromagnetic quantum vacuum. Richard Feynman called it "the jewel of physics" for its extremely accurate predictions of quantities like the anomalous magnetic moment of the electron and the Lamb shift of the energy levels of hydrogen[22]. In quantum electrodynamics, the vertex function describes the coupling between a photon and an electron beyond the leading order of perturbation theory.

Radiative corrections in electrodynamic processes was first calculated by Schwinger for electron scattering in an external field and by Brown and Feynman for Compton effect. It was shown that the Standard Model is a renormalizable field theory. This means that when we go beyond the tree level (Born approximation) we are still able to make definite predictions for observables. The general procedure to evaluate these quantities at the quantum level is to collect and evaluate all the loop diagrams up to a certain level. This is what we have done in the next part of the thesis. We have evaluated the Feynman diagrams for an electron scattering process which has generated the form of the g-factor and a higher order calculation (one-loop correction) has provided us with a more precise value, more consistent with experimental results. We

[21] Calculating Massive One-Loop Amplitudes in QCD Ori Yudilevich, Institute of Theoretical Physics, Utrecht University Theory Group, Nikhef Supervised by: prof. dr. E.L.M.P. Laenen

[22] http://en.wikipedia.org/wiki/Quantum_electrodynamics

have actually explicitly calculated the anomalous magnetic moment of the electron and then moved on to calculate the one-loop vertex correction of an electron vertex emitting a virtual photon. The calculations gave us the value of 'g' correct upto one order of α. Using the above framework, later calculations leading higher order corrections were performed for electron vertex and several other processes as discussed previously. In the last 30 years, we have witnessed the striking success of a gauge theory for the electroweak interactions.

Great works are being carried out around the world which are proving the importance of radiative corrections. Although this thesis is very fundamental in this regard but it lays the foundation of more advanced theoretical calculations that are being performed in this field (more higher order calculations). Nowadays, computers and programming software's are being used for generating all possible Feynman diagrams, evaluation of scattering amplitudes, fitting of experimental and theoretical results, etc. The LHC detectors at CERN are measuring fundamental scattering reactions with unprecedented experimental precision and the interpretation of these high-quality demands an equally high precision in theoretical predictions. In order to connect the observed phenomena with the underlying theoretical models, one needs a precise understanding of the involved processes at the quantum level.

As a final personal remark, this thesis work was my first plunge into this fascinating world of particle physics, in a period which could be either the beginning of a new era in physics or a strong confirmation that we are on the right path, all depending on the results of the experiments going around worldwide. If you have reached this point (without skipping), I hope this thesis proved helpful to you in one way or another.

Part IV
Reference

1. Introduction to Elementary Particle by David J. Griffiths

2. Gauge Theories of the Strong, Weak and electromagnetic Interactions by Chris Quigg

3. Quarks and Leptons: An Introductory Course in Modern Particle Physics by Francis Halzen and Alan D. Martin

4. An Introduction to Quantum Field Theory by Michael E. Peskin and Daniel V. Schroeder

5. Lectures on Quantum Field Theory by Ashok Das

6. A first Book of Quantum Field Theory by Amitabha Lahiri and Palash B. Pal

7. Lie Algebras in Particle Physics by Howard Georgi

Appendix

A Field Transformation in a Non-abelian gauge theory

A gauge theory is a type of field theory in which the Lagrangian is invariant under a continuous group of local transformations. Historically, while trying to explain the quantum effects of electrodynamics, it was found QED could be explained by a U(1) abelian gauge theory. Yang and Mills showed that the gauge principle could be generalized from phase (U(1)) to isospin (SU(2)) transformations. The main difficulty associated with this extension is that the isospin transformations do not commute with one another, thus their theory is termed non-abelian gauge theory, in contrast with the abelian electromagnetism. These theories at first seemed unsuitable for describing fundamental interactions since they involved massless gauge bosons which had not been observed. It turns out this problem can be avoided in two ways: the bosons can become massive due to spontaneous symmetry breaking or the bosons can not be observed in the particle spectrum due to confinement.[23]

To ensure local gauge invariance of a theory we need a gauge covariant derivative degined as

$$D_\mu = I\partial_\mu + igB_\mu \text{ where I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This serves as a reminder that operators are 2×2 matrices in the isospin space and g is the strong interaction coupling constant. B_μ is a 2×2 matrix defined by

$$\mathrm{B}_\mu = \tfrac{1}{2}\tau.b_\mu = \tfrac{1}{2}\begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix}$$

where $\mathbf{b}_\mu$ is an isovector with three components.

[23]Spontaneous Symmetry Breaking in Non Abelian Gauge Theories Michael LeBlanc

If G(x) $= e^{\frac{i}{2}\tau.\alpha(x)}$ is the local gauge transformation, the field transforms as $\psi' = G\psi$.

So,

$$\partial_\mu \psi' = G\left(\partial_\mu \psi\right) + \left(\partial_\mu G\right)\psi$$

and

$$D_\mu \psi' = \left(\partial_\mu + igB'_\mu\right) G\psi = G(x) D_\mu \psi$$

Using the condition $igB'_\mu \left(G\psi\right) = igG\left(B_\mu \psi\right) - \left(\partial_\mu G\right)\psi$ we get

$$B'_\mu \psi'(x) = G\left(B_\mu \psi\left(x\right)\right) + \tfrac{i}{g}\left(\partial_\mu G\right)\psi$$

Multiplying both sides by G^{-1} and writing as an operator equation we get,

$$B'_\mu = G\left[B_\mu + \tfrac{i}{g} G^{-1}\left(\partial_\mu G\right)\right] G^{-1}$$

For local gauge transformation in electromagnetism, $G_{EM} = e^{iq\alpha(x)}$
Using this in the above trasformation equation we get,

$$A'_\mu = A_\mu - \partial_\mu \alpha$$

B Feynman Slash Notation

B.1 Dirac equation in slash notation

Since the Dirac field obeys the K-G equation, it can be written as a linear combination of plane waves as $\psi(x) = u(p)e^{-ipx}$

Plugging this into the Dirac equation$(i\gamma^{\mu}\partial_{\mu} - m)\,\psi(x) = 0$ and expanding as the time and space components we get,

$$(i\gamma^0\partial_0 + i\gamma^i\partial_i - m)\,u(p)e^{-ip\left(p_0x^0 - p_ix^i\right)} = 0$$

Doing the differentiation we get $\gamma^{\mu}p_{\mu}u(p) = mu(p)$

By definition $\not{A} = \gamma^{\mu}A_{\mu}$.

So we finally get the Dirac equation in slash notation as

$$\not{p}u(p) = mu(p)$$

Taking the conjugate of this equation and solving by simple algebra we will get

$$\bar{u}(p)\not{p} = \bar{u}(p)m$$

B.2 Formula

$$\not{p}\gamma^{\mu} = p_{\nu}\gamma^{\nu}\gamma^{\mu}$$

Using $[\gamma^{\mu}, \gamma^{\nu}] = 2g^{\mu\nu}$

$$= p_{\nu}\left[2g^{\mu\nu} - \gamma^{\mu}\gamma^{\nu}\right]$$

$$=2p^{\mu} - \gamma^{\mu}\gamma^{\nu}p_{\nu}$$

So, we finally have

$$\not{p}\gamma^{\mu} =2p^{\mu} - \gamma^{\mu}\not{p}$$

C Gordon Identity Proof

Consider the term

$$\bar{u}(p')\frac{i\Sigma^{\mu\nu}q_\nu}{2m}u(p)$$

$$= \frac{i}{2m}\frac{i}{2}\bar{u}(p')\left[\gamma^\mu, \gamma^\nu\right]q_\nu u(p) \text{ using } \Sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$$

$$= \text{-}\frac{1}{4m}\bar{u}(p')\left[\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)p'_\nu - \left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)p_\nu\right]u(p)$$

Using Clifford Algebra we get,

$$= \text{-}\frac{1}{4m}\bar{u}(p')\left[\left(2g^{\mu\nu} - 2\gamma^\nu\gamma^\mu\right)p'_\nu - \left(2\gamma^\mu\gamma^\nu - 2g^{\mu\nu}\right)p_\nu\right]u(p)$$

Using Dirac equation we get,

$$= \text{-}\frac{1}{4m}\bar{u}(p') \times 2 \times \left[\left(p'^\mu - m\gamma^\mu\right) - \left(m\gamma^\nu - p^\mu\right)\right]u(p)$$

$$= \text{-}\frac{1}{2m}\bar{u}(p')\left[\left(p'^\mu + p^\mu\right) - 2m\gamma^\mu\right]u(p)$$

Rearranging the terms we finally have

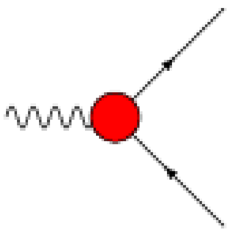
$$\bar{u}(p')\gamma^\mu u(p) = u(p')\left[\frac{p'^\mu + p^\mu}{2m} \bar{+} \frac{i\Sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

D Ward Identity

From the classical equations of motions, we know current density j^μ is conserved : $\partial_\mu j^\mu = 0$. Provided that this still holds in quantum theory, we can write

$$k_\mu M^\mu = 0$$

$$\text{where } M^\mu(k) = \int d^4x e^{ikx} < f \mid j^\mu(x) \mid i >$$



It is essentially a statement of current conservation which is a consequence of gauge symmetry. It describes physically possible scattering processes and thus have all their external particles on-shell. If $\mathrm{M}(k) = \varepsilon_\mu(k) M^\mu(k)$ is the amplitude of some QED process involving an external photon with momentum k, then this amplitude vanishes. To explain this, consider an arbitrary QED process involving an external photon with momentum k. Since the amplitude always contains $\varepsilon_\mu(k)$, we have extracted this factor and defined $M^\mu(k)$ to be the rest of the amplitude M(k).

E Pauli-Villars regularization

There are several equivalent methods to regularize a divergent integral which means an introduction of a cutoff which makes the integral finite. This is a technique that is used to separate divergent terms from the finite parts of a loop calculation in field theory and is named after it's inventors, Pauli and Villars, who invented the technique in 1949. This is based on the introduction of a set of additional heavy fields with a wrong sign of the kinetic term. These fields are not physical and are introduced essentially with the purpose of regularization of divergent integrals. The main trick is in the replacement[24]

$$\frac{1}{p^2-m^2} \to \frac{1}{p^2-m^2} - \frac{1}{p^2-M^2} = \int_{m^2}^{M^2} -\frac{1}{(p^2-z)^2} dz$$

where M $\to \infty$ is the mass of the Pauli-Villars fields. This allows us to simply square the propagator and add another Feynman-like parameter z. So the propagator for large momenta decreases faster, which ensures the convergence of the integrals.[25]

Since PV works by introducing massive particles to regulate UV divergence. Even though PV works for photon at 1-loop, it fails in more complicated scenarios like non-abelian gauge theories. PV is also impractical to implement in multi-loop diagrams where many PV fields have to be introduced[26].

[24] JOINT INSTITUTE FOR NUCLEAR RESEARCH Bogoliubov Laboratory of Theoretical Physics Radiative corrections divergences Regularization Renormalization Renormalization group and all that in examples in quantum field theory D.I.KAZAKOV

[25] http://isites.harvard.edu/fs/docs/icb.topic792163.files/15-regschemes.pdf

[26] https://www.quora.com/What-are-the-pros-and-cons-of-Pauli-Villars-regularization-and-that-of-dimensional-regularization