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$\mu \rightarrow e\gamma$ in a supersymmetric radiative neutrino mass model

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We consider a supersymmetric version of the inert Higgs doublet model, whose motivation is to explain smallness of neutrino masses and existence of dark matter. In this supersymmetric model, due to the presence of discrete symmetries, neutrinos acquire masses at loop level. After computing these neutrino masses, in order to fit the neutrino oscillation data, we show that by tuning some supersymmetry-breaking soft parameters of the model, neutrino Yukawa couplings can be unsuppressed. In the above-mentioned parameter space, we compute the branching ratio of the decay $\mu \rightarrow e\gamma$. To be consistent with the current experimental upper bound on $\text{Br}(\mu \rightarrow e\gamma)$, we obtain constraints on the right-handed neutrino mass of this model.

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I. INTRODUCTION

There are many indications for physics beyond the standard model (SM) [1]. One among them is the existence of nonzero neutrino masses [2]. Some of the indications for new physics can be successfully explained in supersymmetric models [3]. For this reason, neutrino masses have been addressed in supersymmetry. In a neutrino mass model, there is a possibility for lepton flavor violation (LFV) [4], for which there is no direct evidence. Experiments have put upper bounds on the branching ratios of these LFV processes [5–7]. Due to the Glashow-Iliopoulos-Maiani cancellation mechanism, these processes are highly suppressed in the SM and the above-mentioned upper bounds are obviously satisfied in it. However, a signal for any LFV process with an appreciable branching ratio gives a confirmation for new physics.

In this work, we study LFV processes of the form $\ell_i \rightarrow \ell_j \gamma$ in a supersymmetrized model for neutrino masses [8]. Here, $\ell_i, i = 1, 2, 3$, are charged leptons. The above-mentioned model arises after supersymmetrizing the inert Higgs doublet model [9,10]. The inert Higgs doublet model [9] offers an explanation for neutrino masses and dark matter. In this model [9], dark matter is stable due to an exact Z_2 symmetry and the neutrinos acquire masses at the one-loop level. This model has been extensively studied and some recent works on this can be seen in Ref. [11]. Supersymmetrizing this model could bring new features and this was done in Ref. [8]. In the supersymmetrization of the inert Higgs doublet model [8], the discrete symmetry is extended to $Z_2 \times Z'_2$. In this model, dark matter can be multipartite [12] due to the presence of R parity and the Z'_2 symmetry. Some variations of this model were also presented in Refs. [13,14]. In the model of Ref. [8], gauge coupling unification is possible by embedding it in a

supersymmetric $SU(5)$ structure [15]. The origin of the discrete symmetry $Z_2 \times Z'_2$, which is described above, is also explained by realizing it as a residual symmetry from a $U(1)$ gauged symmetry [16].

In this work we consider the model of Ref. [8] and present the expression for neutrino masses, which arises from two one-loop diagrams. We will demonstrate that neutrino masses are tiny in this model if either the neutrino Yukawa couplings are suppressed or some certain soft parameters of the scalar potential are fine-tuned. We consider the latter case, in which the neutrino Yukawa couplings can be $\mathcal{O}(1)$, and they can drive LFV processes such as $\mu \rightarrow e\gamma$. In our work we assume that the slepton mass matrices and the A -terms of sleptons are flavor diagonal. Hence, in our model, lepton flavor violation is happening due to nondiagonal Yukawa couplings. Under the above-mentioned scenario, we compute the branching ratio for the decays $\ell_i \rightarrow \ell_j \gamma$. Among these decays, we show that $\mu \rightarrow e\gamma$ can give stringent constraints on model parameters, especially on the right-handed neutrino mass. Early calculations on $\mu \rightarrow e\gamma$ in a lepton-number-violating supersymmetric model can be seen in Ref. [17].

In the model of Ref. [8], apart from $\mu \rightarrow e\gamma$ there can also be an LFV decay of $\mu \rightarrow 3e$. In a type-II seesaw mechanism for neutrino masses, the decay $\mu \rightarrow 3e$ can take place at tree level, due to the presence of a triplet Higgs boson. In our model [8], there are no triplet Higgses, and hence the decay $\mu \rightarrow 3e$ will take place at loop level. The current experimental upper limit on $\text{Br}(\mu \rightarrow 3e)$ is 1×10^{-12} [18], which is about 2 times larger than that of $\text{Br}(\mu \rightarrow e\gamma)$. So we can expect $\text{Br}(\mu \rightarrow e\gamma)$ to put somewhat tighter constraints on model parameters than that due to $\text{Br}(\mu \rightarrow 3e)$. Hence, in this work we focus on the computation of $\text{Br}(\mu \rightarrow e\gamma)$. It may happen that $\text{Br}(\mu \rightarrow 3e)$ and $\text{Br}(\mu \rightarrow e\gamma)$ may put some additional constraints on model parameters, but we study these in a separate work.

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86 This paper is organized as follows. In the next section,
87 we describe the model of Ref. [8]. In Sec. III, we present
88 the expressions for neutrino masses and branching ratios
89 for the decays $\ell_i \rightarrow \ell_j \gamma$. In Sec. IV, we give numerical
90 results on neutrino masses and $\mu \rightarrow e \gamma$. We conclude
91 in Sec. V.

92 II. THE MODEL

93 The model of Ref. [8] is an extension of the minimal
94 supersymmetric standard model (MSSM). The additional
95 superfields of this model are as follows: (i) three right-
96 handed neutrino fields, \hat{N}_i , $i = 1, 2, 3$; (ii) two electro-
97 weak doublets $\hat{\eta}_1 = (\hat{\eta}_1^0, \hat{\eta}_1^-)$, $\hat{\eta}_2 = (\hat{\eta}_2^+, \hat{\eta}_2^0)$; (iii) a singlet
98 field $\hat{\chi}$. Under the electroweak gauge group $SU(2)_L \times$
99 $U(1)_Y$, the charges of these additional superfields are
100 given in Table I. The model of Ref. [8] contains the
101 discrete symmetry $Z_2 \times Z'_2$, under which all the quark
102 and Higgs superfields can be taken to be even. The
103 leptons and the additional fields described above are
104 charged nontrivially under this discrete symmetry [8].
105 The purpose of this symmetry is to disallow the Yukawa
106 term $\hat{L}_i \hat{H}_u \hat{N}_j$ in the superpotential of the model, and as a
107 result the neutrino remains massless at tree level. Here,
108 $\hat{L}_i = (\hat{\nu}_i, \hat{\ell}_i)$, $i = 1, 2, 3$ are the lepton doublet super-
109 fields. The singlet charged lepton superfield is repre-
110 sented by \hat{E}_i^c , $i = 1, 2, 3$. We denote up- and down-type
111 Higgs superfields as \hat{H}_u and \hat{H}_d , respectively.

112 The superpotential of our model consisting of electro-
113 weak fields can be written as [8]

$$\begin{aligned}
 W = & (Y_E)_{ij} \hat{L}_i \hat{H}_d \hat{E}_j^c + (Y_\nu)_{ij} \hat{L}_i \hat{\eta}_2 \hat{N}_j + \lambda_1 \hat{H}_d \hat{\eta}_2 \hat{\chi} \\
 & + \lambda_2 \hat{H}_u \hat{\eta}_1 \hat{\chi} + \mu \hat{H}_u \hat{H}_d + \mu_\eta \hat{\eta}_2 \hat{\eta}_1 + \frac{1}{2} \mu_\chi \hat{\chi} \hat{\chi} \\
 & + \frac{1}{2} M_{ij} \hat{N}_i \hat{N}_j.
 \end{aligned} \tag{1}$$

114 Here, there is a summation over indices i, j which run
115 from 1 to 3. The first and second terms in the above
116 equation are Yukawa terms for charged leptons and
117 neutrinos, respectively. But, as described before, $\hat{\eta}_2$ is
118 odd under the discrete symmetry of the model and
119 hence the scalar component of it does not acquire a
120 vacuum expectation value [8]. So neutrinos are still
121 massless at tree level. Apart from the superpotential of
122 Eq. (1), we should consider the scalar potential. The
123 relevant terms in the scalar potential are given below:

TABLE I. Charge assignments of additional superfields of the model under the electroweak gauge group.

Field	\hat{N}_i	$\hat{\eta}_1$	$\hat{\eta}_2$	$\hat{\chi}$
$SU(2)_L \times U(1)_Y$	(1, 0)	(2, -1/2)	(2, 1/2)	(1, 0)

$$\begin{aligned}
 V = & (m_L^2)_{ij} \tilde{L}_i^\dagger \tilde{L}_j + m_\eta^2 \eta_1^\dagger \eta_1 + m_\eta^2 \eta_2^\dagger \eta_2 + m_\chi^2 \chi^* \chi \\
 & + (m_N^2)_{ij} \tilde{N}_i^\dagger \tilde{N}_j + \left[(AY_\nu)_{ij} \tilde{L}_i \eta_2 \tilde{N}_j + (A\lambda)_1 H_d \eta_2 \chi \right. \\
 & + (A\lambda)_2 H_u \eta_1 \chi + b_\eta \eta_2 \eta_1 + \frac{1}{2} b_\chi \chi \chi \\
 & \left. + \frac{1}{2} (b_M)_{ij} \tilde{N}_i \tilde{N}_j + \text{c.c.} \right].
 \end{aligned} \tag{2}$$

As we have explained before, our motivation is to study
LFV processes in the above-described model. The LFV
processes can be driven by charged sleptons. For instance,
the off-diagonal elements of soft parameters, $(m_L^2)_{ij}$, can
drive LFV processes. Similarly, we can write soft mass
terms for singlet charged sleptons, \tilde{E}_i , $i = 1, 2, 3$, in the
scalar potential. Also, there can exist A -terms connecting \tilde{L}_i
and \tilde{E}_j . The off-diagonal terms of the above-mentioned
soft terms can drive LFV processes, which actually exist in
the MSSM. Since our model [8] is an extension of the
MSSM, we are interested in LFV processes generated
by the additional fields of this model. Hence, we assume
that the off-diagonal terms of the soft terms (which are
described above) are zero.

For simplicity, we assume that the parameters of the
superpotential and scalar potential of our model are real.
Then, by an orthogonal transformation among the neutrino
superfields \hat{N}_i , we can make the following parameters
diagonal:

$$M_{ij} = M_i \delta_{ij}, \quad (m_N^2)_{ij} = (m_N^2)_i \delta_{ij}, \quad (b_M)_{ij} = (b_M)_i \delta_{ij}. \tag{3}$$

By going to an appropriate basis of \hat{L}_i and \hat{E}_j , we can get
the Yukawa couplings for charged leptons to be diagonal.
After doing this, we are left with no freedom and hence the
neutrino Yukawa couplings $(Y_\nu)_{ij}$ can be nondiagonal.
These nondiagonal Yukawa couplings can drive LFV
processes such as $\ell_i \rightarrow \ell_j \gamma$. These LFV processes are
driven at the one-loop level, which we describe in the next
section. As explained before, neutrinos also acquire masses
at the one-loop level in this model [8]. To calculate these
loop diagrams we need to know the mass eigenstates of the
scalar and fermionic partners of the fields shown in Table I,
since these fields enter into the loop processes. Expressions
for these mass eigenstates are given in Ref. [19]. However,
our notations and conventions are different from those of
Ref. [19]. Hence, for the sake of completeness we present
them below.

The charged components of $\hat{\eta}_1, \hat{\eta}_2$ can be fermionic and
scalar, which can be written as $(\tilde{\eta}_1^-, \tilde{\eta}_2^+)$ and (η_1^-, η_2^+) ,
respectively. The two charged fermions represent chargino-
type fields whose mass is μ_η , whereas the charged scalars,
in the basis $\Phi_+^T = (\eta_2^+, \eta_1^{*-})$, will have a mass matrix which
is given below:

166

$$\mathcal{L} \ni -\Phi_+^\dagger \begin{pmatrix} \mu_\eta^2 + m_{\eta_2}^2 + \frac{g^2 - g'^2}{4} v^2 \cos(2\beta) & b_\eta \\ & \mu_\eta^2 + m_{\eta_1}^2 - \frac{g^2 - g'^2}{4} v^2 \cos(2\beta) \end{pmatrix} \Phi_+. \quad (4)$$

167 Here, g, g' are the gauge couplings of $SU(2)_L$ and $U(1)_Y$,
 168 respectively. β is defined as $\tan \beta = \frac{v_2}{v_1} = \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle}$ and
 169 $v^2 = v_1^2 + v_2^2$. We can diagonalize the above mass matrix
 170 by taking Φ_+ as

$$\Phi_+ = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_{m_2}^+ \\ \eta_{m_1}^+ \end{pmatrix},$$

$$\tan 2\theta = \frac{2b_\eta}{m_{\eta_2}^2 - m_{\eta_1}^2 + (g^2 - g'^2)v^2 \cos(2\beta)/2}. \quad (5)$$

171 Here, $\eta_{m_1}^+$ and $\eta_{m_2}^+$ are mass eigenstates of the charged scalar
 172 fields and we denote their mass eigenvalues by m_{1+} and
 173 m_{2+} , respectively.

174 The neutral fermionic and scalar components of $\hat{\eta}_1, \hat{\eta}_2, \hat{\chi}$
 175 can be written as $\Psi^T = (\tilde{\eta}_1^0, \tilde{\eta}_2^0, \tilde{\chi})$ and $\Phi_0^T = (\eta_1^0, \eta_2^0, \chi)$,
 176 respectively. The neutral fermionic fields will have a
 177 mixing mass matrix, which is given below:

$$\mathcal{L} \ni -\frac{1}{2} \Psi^T M_\eta \Psi, \quad M_\eta = \begin{pmatrix} 0 & -\mu_\eta & -\lambda_2 v_2 \\ -\mu_\eta & 0 & \lambda_1 v_1 \\ -\lambda_2 v_2 & \lambda_1 v_1 & \mu_\chi \end{pmatrix}. \quad (6)$$

185

$$m_\eta^2(\epsilon) = \begin{pmatrix} m_{11}^2 & m_{12}^2 & m_{13}^2 \\ m_{12}^2 & m_{22}^2 & m_{23}^2 \\ m_{13}^2 & m_{23}^2 & m_{33}^2 \end{pmatrix}, \quad m_{11}^2 = \mu_\eta^2 + m_{\eta_1}^2 + \lambda_2^2 v_2^2 + \frac{g^2 + g'^2}{4} v^2 \cos(2\beta),$$

$$m_{22}^2 = \mu_\eta^2 + m_{\eta_2}^2 + \lambda_1^2 v_1^2 - \frac{g^2 + g'^2}{4} v^2 \cos(2\beta), \quad m_{33}^2 = \mu_\chi^2 + m_\chi^2 + \lambda_1^2 v_1^2 + \lambda_2^2 v_2^2 + \epsilon b_\chi,$$

$$m_{12}^2 = -\lambda_1 \lambda_2 v_1 v_2 - \epsilon b_\eta, \quad m_{13}^2 = -\lambda_1 v_1 \mu_\eta - \lambda_2 v_2 \mu_\chi - \epsilon[(A\lambda)_2 v_2 - \mu \lambda_2 v_1],$$

$$m_{23}^2 = \lambda_1 v_1 \mu_\chi + \lambda_2 v_2 \mu_\eta + \epsilon[(A\lambda)_1 v_1 - \mu \lambda_1 v_2]. \quad (10)$$

187 Here, ϵ can take +1 or -1. We have $m_{\eta_R}^2 = m_\eta^2(+1)$ and
 188 $m_{\eta_I}^2 = m_\eta^2(-1)$. These two mixing mass matrices can be
 189 diagonalized by orthogonal matrices U_R and U_I , which are
 190 defined below:

$$U_R^T m_{\eta_R}^2 U_R = \text{diag}(m_{\eta_{R1}}^2, m_{\eta_{R2}}^2, m_{\eta_{R3}}^2),$$

$$U_I^T m_{\eta_I}^2 U_I = \text{diag}(m_{\eta_{I1}}^2, m_{\eta_{I2}}^2, m_{\eta_{I3}}^2). \quad (11)$$

192 At last, the fermionic and scalar components of right-
 193 handed neutrino superfields, \hat{N}_i , can be denoted by N_i and
 194 \tilde{N}_i , respectively. The fermionic components have masses

The above mixing matrix can be diagonalized by an
 orthogonal matrix as

$$U_\eta^T M_\eta U_\eta = \text{diag}(m_{\tilde{\eta}_1}, m_{\tilde{\eta}_2}, m_{\tilde{\eta}_3}). \quad (7)$$

The neutral scalar fields of Φ_0 can be written as 180

$$\Phi_0 = \frac{1}{\sqrt{2}} \Phi_R + \frac{i}{\sqrt{2}} \Phi_I = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_{1R}^0 \\ \eta_{2R}^0 \\ \chi_R \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \eta_{1I}^0 \\ \eta_{2I}^0 \\ \chi_I \end{pmatrix}. \quad (8)$$

The mixing matrix among these fields can be written as 182

$$\mathcal{L} \ni -\frac{1}{2} \Phi_R^T m_{\eta_R}^2 \Phi_R - \frac{1}{2} \Phi_I^T m_{\eta_I}^2 \Phi_I. \quad (9)$$

Here, the mixing matrices $m_{\eta_R}^2, m_{\eta_I}^2$ can be obtained from
 the following matrix: 183
184

M_i . The scalar components can be decomposed into mass
 eigenstates as 195
196

$$\tilde{N}_i = \frac{1}{\sqrt{2}} (\tilde{N}_{Ri} + i\tilde{N}_{Ii}). \quad (12)$$

The masses squared of \tilde{N}_{Ri} and \tilde{N}_{Ii} , respectively, are 197

$$m_{Ri}^2 = M_i^2 + (m_N^2)_i + (b_M)_i,$$

$$m_{Ii}^2 = M_i^2 + (m_N^2)_i - (b_M)_i. \quad (13)$$

198

III. NEUTRINO MASSES AND LFV PROCESSES

As described before, in the model of Ref. [8] neutrinos are massless at tree level due to the presence of the discrete symmetry $Z_2 \times Z'_2$. However, in this model neutrinos acquire masses at the one-loop level, whose diagrams are shown in Fig. 1 [8]. After computing these one-loop diagrams, we find the following mass matrix for neutrinos:

$$(m_\nu)_{ij} = \sum_{k,l=1}^3 \frac{(Y_\nu)_{ik}(Y_\nu)_{jk}}{16\pi^2} M_k \left[[U_R(2,l)]^2 \frac{m_{\eta_{Rl}}^2}{m_{\eta_{Rl}}^2 - M_k^2} \ln \frac{m_{\eta_{Rl}}^2}{M_k^2} - [U_I(2,l)]^2 \frac{m_{\eta_{ll}}^2}{m_{\eta_{ll}}^2 - M_k^2} \ln \frac{m_{\eta_{ll}}^2}{M_k^2} \right] + \sum_{k,l=1}^3 \frac{(Y_\nu)_{ik}(Y_\nu)_{jk}}{16\pi^2} [U_\eta(2,l)]^2 m_{\tilde{\eta}_l} \left[\frac{m_{Rk}^2}{m_{Rk}^2 - m_{\tilde{\eta}_l}^2} \ln \frac{m_{Rk}^2}{m_{\tilde{\eta}_l}^2} - \frac{m_{lk}^2}{m_{lk}^2 - m_{\tilde{\eta}_l}^2} \ln \frac{m_{lk}^2}{m_{\tilde{\eta}_l}^2} \right]. \quad (14)$$

It is to be noticed that the first and second lines of the above equation arise from the left- and right-hand diagrams of Fig. 1.

In our work we assume supersymmetry breaking to be around 1 TeV. Hence, we can take all the supersymmetric (SUSY) particle masses to be around a few hundred GeV. With this assumption, we can estimate the neutrino Yukawa couplings by requiring the neutrino mass scale to be around 0.1 eV [2]. With this requirement, we find that $(Y_\nu)_{ij} \sim 10^{-5}$. Here there are six different Yukawa couplings, which need to be suppressed to $\mathcal{O}(10^{-5})$. This could be one possibility in this model in order to explain the correct magnitude for neutrino masses. However, in this case, since the Yukawa couplings are suppressed, LFV processes such as $\ell_i \rightarrow \ell_j \gamma$ would also be suppressed. These LFV processes will be searched in future experiments [20], and hence it is worth considering the case where these processes can have a substantial contribution in this model. In other words, we have to look for a parameter region where we can have $(Y_\nu)_{ij} \sim \mathcal{O}(1)$.

From Eq. (14), it can be observed that each diagram of Fig. 1 contributes positive and negative quantities to the neutrino mass matrix. Without suppressing Yukawa couplings, by fine-tuning the masses of SUSY particles we may achieve partial cancellation between the positive and negative contributions of Eq. (14) and end up with tiny masses for neutrinos. To demonstrate this explicitly, using Eq. (13) we can notice that in the limit $(b_M)_i \rightarrow 0$ we get $m_{Ri}^2 - m_{li}^2 \rightarrow 0$, and hence the second line of Eq. (14) would give a tiny contribution. The first line of Eq. (14) can give a very small value in the following limiting process: $U_R(2,l) - U_I(2,l) \rightarrow 0$ and $m_{\eta_{Rl}} - m_{\eta_{ll}} \rightarrow 0$. To achieve this limiting process we have to make sure that the elements

of the matrices $m_{\eta_{Rl}}^2$ and $m_{\eta_{ll}}^2$ are close to each other. From the discussion around Eq. (10), we can observe that the elements of $m_{\eta_{Rl}}^2$ and $m_{\eta_{ll}}^2$ can differ by quantities which are proportional to ϵ . These quantities depend on the following parameters: b_χ , b_η , $(A\lambda)_1$, and $(A\lambda)_2$. By taking the limit $(A\lambda)_1 - \lambda_1 \mu v_2 / v_1 \rightarrow 0$, $(A\lambda)_2 - \lambda_2 \mu v_1 / v_2 \rightarrow 0$, $b_\eta \rightarrow 0$, $b_\chi \rightarrow 0$ we can get a tiny contribution from the first line of Eq. (14). To sum up the above discussion, without suppressing the neutrino Yukawa couplings we can fine-tune the following seven parameters, in order to get very small neutrino masses in this model:

$$(b_M)_i, i = 1, 2, 3, \quad b_\eta, \quad b_\chi, \quad (A\lambda)_1, \quad (A\lambda)_2. \quad (15)$$

Apparently, the above parameters are SUSY-breaking soft parameters of the scalar potential of this model. A study of neutrino masses depending on SUSY-breaking soft parameters can be seen in Ref. [21].

In the previous paragraph we have argued that Majorana masses for neutrinos are vanishingly small when we fine-tune certain soft parameters of the model. We can understand these features from symmetry arguments. For instance, when lepton number is conserved, neutrinos cannot have Majorana masses. For lepton number, we can propose a group $U(1)_L$, under which the following fields are assigned the corresponding charges and the rest of the superfields are singlets:

$$\hat{L}_i \mapsto +1, \quad \hat{E}_i^c \mapsto -1, \quad \hat{N}_i \mapsto -1. \quad (16)$$

With the above-mentioned charges, we can see that the last terms in Eqs. (1) and (2) are forbidden. In fact, in the limit $M_i \rightarrow 0$ and $(b_M)_i \rightarrow 0$, the two diagrams of Fig. 1 give zero masses to neutrinos. Hence, in order to get Majorana masses for neutrinos, we have softly broken the lepton number symmetry. Now, even if we have $M_i \neq 0$, we have described in the previous paragraph that the left-hand diagram of Fig. 1 can still give vanishingly small masses by fine-tuning some soft parameters. This suggests that apart from $U(1)_L$ there can exist some additional

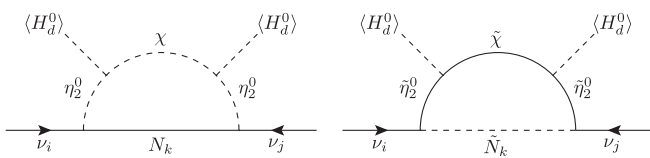


FIG. 1. Radiative masses for neutrinos.

272 symmetries. Suppose we set $(A\lambda)_1 v_1 - \lambda_1 \mu v_2 = 0$,
 273 $(A\lambda)_2 v_2 - \lambda_2 \mu v_1 = 0$. Then (as argued previously) the
 274 left-hand diagram of Fig. 1 gives zero neutrino masses
 275 for $b_\eta \rightarrow 0$ and $b_\chi \rightarrow 0$, even if $M_i \neq 0$. This case can be
 276 understood by proposing an additional symmetry $U(1)_\eta$
 277 under which the following fields have nontrivial charges
 278 and the rest of the fields are singlets:

$$\begin{aligned} \hat{L}_i &\mapsto +1, & \hat{E}_i^c &\mapsto -1, & \hat{\eta}_1 &\mapsto -1, \\ \hat{\eta}_2 &\mapsto -1, & \hat{\chi} &\mapsto +1. \end{aligned} \quad (17)$$

279 Using the above charges, we can notice that μ_η and μ_χ terms
 280 in Eq. (1) and b_η and b_χ terms in Eq. (2) are forbidden.
 281 Thus, the additional symmetry $U(1)_\eta$ can forbid the
 282 Majorana masses for neutrinos in the left-hand diagram
 283 of Fig. 1. Finally, one may ask how the relations
 284 $(A\lambda)_1 v_1 - \lambda_1 \mu v_2 = 0$, $(A\lambda)_2 v_2 - \lambda_2 \mu v_1 = 0$ can be satis-
 285 fied. In these two relations, SUSY-breaking soft masses are
 286 related to the SUSY-conserving mass μ . These relations
 287 may be achieved by proposing certain symmetries in the
 288 mechanism for SUSY breaking, which is beyond the reach
 289 of our present work.

290 Previously, we have motivated a parameter region where
 291 the neutrino Yukawa couplings can be $\mathcal{O}(1)$. For these
 292 values of the neutrino Yukawa couplings, LFV processes
 293 such as $\ell_i \rightarrow \ell_j \gamma$ can have substantial contributions in our
 294 model, and it is worth computing them. The Feynman
 295 diagrams for $\ell_i \rightarrow \ell_j \gamma$ are given in Fig. 2.

296 The general form of the amplitude for $\ell_i \rightarrow \ell_j \gamma$ is as
 297 follows:

$$\begin{aligned} \mathcal{M} = e e_\mu^*(q) \bar{u}_j(p-q) &\left[A_L^{(ij)} \frac{1-\gamma_5}{2} + A_R^{(ij)} \frac{1+\gamma_5}{2} \right] \\ &\times i\sigma^{\mu\nu} q_\nu u_i(p). \end{aligned} \quad (18)$$

298 It is to be noted that in the above equation, there is no
 299 summation over the indices i, j . The quantities $A_{L,R}^{(ij)}$ of the
 300 above equation can be found from the one-loop diagrams of
 301 Fig. 2, which we give below:

$$\begin{aligned} A_L^{(ij)} &= A^{(ij)} m_j, & A_R^{(ij)} &= A^{(ij)} m_i, \\ A^{(ij)} &= \sum_{k=1}^3 \frac{(Y_\nu)_{ik} (Y_\nu)_{jk}}{16\pi^2} \left\{ \frac{1}{4\mu_\eta^2} [f_2(x_{Rk}) + f_2(x_{Ik})] \right. \\ &\quad \left. - \left[\cos^2\theta \frac{f_2(x_{k2})}{2m_{2+}^2} + \sin^2\theta \frac{f_2(x_{k1})}{2m_{1+}^2} \right] \right\}, \\ x_{Rk} &= \frac{m_{Rk}^2}{\mu_\eta^2}, & x_{Ik} &= \frac{m_{Ik}^2}{\mu_\eta^2}, & x_{k2} &= \frac{M_k^2}{m_{2+}^2}, & x_{k1} &= \frac{M_k^2}{m_{1+}^2}, \\ f_2(x) &= \frac{1}{(1-x)^4} \left[\frac{1}{6} - x + \frac{1}{2}x^2 + \frac{1}{3}x^3 - x^2 \ln(x) \right]. \end{aligned} \quad (19)$$

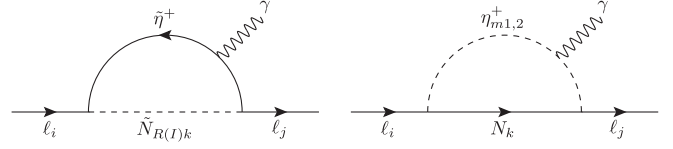


FIG. 2. Lepton-flavor-violating decays of the form $\ell_i \rightarrow \ell_j \gamma$. F2:1

302 From the above expressions, we can notice that in the curly
 303 brackets of $A^{(ij)}$, the first two and last two terms arise from
 304 the left- and right-hand diagrams of Fig. 2, respectively.
 305 Moreover, there is a relative minus sign in the contribution
 306 from these two diagrams.

307 Among the various decays of the form $\ell_i \rightarrow \ell_j \gamma$, the
 308 upper bound on the branching ratio of $\mu \rightarrow e\gamma$ is found to be
 309 stringent [5]. Moreover, we have $\text{Br}(\mu \rightarrow e\bar{\nu}_e\nu_\mu) \approx 100\%$.
 310 Using this and neglecting the electron mass, the branching
 311 ratio of $\mu \rightarrow e\gamma$ is found to be

$$\begin{aligned} \text{Br}(\mu \rightarrow e\gamma) &= \frac{3\alpha}{16\pi G_F^2} \left| \sum_{k=1}^3 (Y_\nu)_{1k} (Y_\nu)_{2k} \right. \\ &\quad \times \left\{ \frac{1}{4\mu_\eta^2} [f_2(x_{Rk}) + f_2(x_{Ik})] \right. \\ &\quad \left. \left. - \left[\cos^2\theta \frac{f_2(x_{k2})}{2m_{2+}^2} + \sin^2\theta \frac{f_2(x_{k1})}{2m_{1+}^2} \right] \right\} \right|^2. \end{aligned} \quad (20)$$

312 Here, $\alpha = \frac{e^2}{4\pi}$ and G_F is the Fermi constant.

313 Here we compare our work with that of Ref. [14]. The
 314 model in Ref. [14] is similar to that of Ref. [8]. But, in
 315 Ref. [14] a theory at a high scale with an anomalous $U(1)_X$
 316 symmetry was assumed. The $U(1)_X$ symmetry breaks into
 317 Z_2 symmetry at a low scale. Due to these differences, there
 318 exist three one-loop diagrams for neutrinos in Ref. [14],
 319 whereas only two diagrams generate neutrino masses in
 320 Ref. [8]. The diagrams for the LFV processes of $\ell_i \rightarrow \ell_j \gamma$
 321 in Ref. [14] are similar to the diagrams given in this paper
 322 (see Fig. 2). But the expression for $\text{Br}(\mu \rightarrow e\gamma)$, which is
 323 given in Eq. (20), is found to be different from that in
 324 Ref. [14]. We hope that these differences might have arisen
 325 because the model in Ref. [14] has a different origin than
 326 that of Ref. [8].

327 Although the main motivation of this paper is to study
 328 the correlation between neutrino masses and $\text{Br}(\mu \rightarrow e\gamma)$,
 329 below we mention muon $g-2$ in our model. It is known
 330 that the theoretical [22] and experimental [23] values of
 331 muon $g-2$ differ by about 3σ . However, there are hadronic
 332 uncertainties to muon $g-2$, which need to be improved
 333 [22]. Hence, the above-mentioned result is still an indica-
 334 tion for a new physics signal. In our model [8], muon $g-2$
 335 get contributions from MSSM fields [24] as well as from
 336 additional fields, which are shown in Table I. The con-
 337 tribution from MSSM fields can fit the 3σ discrepancy of

338 muon $g-2$.¹ Hence, in our model [8], it is interesting to
 339 know how large the contribution from the additional fields
 340 of this model would be. The contribution from these
 341 additional fields can be found from the amplitude of
 342 Eq. (18), which is

$$\Delta a_\mu = \frac{m_\mu^2}{16\pi^2} \sum_{k=1}^3 [(Y_\nu)_{2k}]^2 \left\{ \frac{1}{2\mu_\eta^2} [f_2(x_{Rk}) + f_2(x_{Ik})] - \left[\cos^2\theta \frac{f_2(x_{k2})}{m_{2+}^2} + \sin^2\theta \frac{f_2(x_{k1})}{m_{1+}^2} \right] \right\}. \quad (21)$$

343 Here, m_μ is mass of the muon.

344 IV. ANALYSIS AND RESULTS

345 As described in Sec. I, our motivation is to study the
 346 correlation between neutrino masses and $\text{Br}(\mu \rightarrow e\gamma)$. We
 347 have given the expression for neutrino masses in Eq. (14).
 348 We have explained in the previous section that to explain a
 349 neutrino mass scale of 0.1 eV, we can make the neutrino
 350 Yukawa couplings to be about $\mathcal{O}(1)$, but we need to fine-
 351 tune certain SUSY-breaking soft parameters which are
 352 given in Eq. (15). We consider this case, since for un-
 353 suppressed neutrino Yukawa couplings $\text{Br}(\mu \rightarrow e\gamma)$ can have
 354 maximum values. As mentioned before, experiments
 355 have found the following upper bound: $\text{Br}(\mu \rightarrow e\gamma) <$
 356 5.7×10^{-13} [5]. Hence, for the above-mentioned parameter
 357 space, where neutrino Yukawa couplings are unsuppressed,
 358 we compute $\text{Br}(\mu \rightarrow e\gamma)$ by fitting neutrino masses. We
 359 check if the computed values for $\text{Br}(\mu \rightarrow e\gamma)$ satisfy the
 360 experimental bound [5].

361 Before we compute $\text{Br}(\mu \rightarrow e\gamma)$, we first need to ensure
 362 that the neutrino Yukawa couplings can be unsuppressed in
 363 our model. We can calculate these Yukawa couplings from
 364 Eq. (14) by fitting to the neutrino oscillation data. The
 365 neutrino mass matrix of Eq. (14) is related to neutrino mass
 366 eigenvalues through the following relation:

$$m_\nu = U_{\text{PMNS}}^* \text{diag}(m_1, m_2, m_3) U_{\text{PMNS}}^\dagger. \quad (22)$$

367 Here, $m_{1,2,3}$ are the mass eigenvalues of neutrinos and
 368 U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata matrix.
 369 The matrix U_{PMNS} depends on three mixing angles
 370 ($\theta_{12}, \theta_{23}, \theta_{13}$) and Dirac CP -violating phase, δ_{CP} . In the
 371 above equation there is a possibility of Majorana phases,
 372 which we have taken to be zero, for simplicity. We have
 373 parametrized U_{PMNS} in terms of mixing angles and δ_{CP} as it
 374 is given in Ref. [7].

375 By fitting to various neutrino oscillation data, we know
 376 solar and atmospheric neutrino mass-squared differences
 377 and also about the neutrino mixing angles [26]. In the case

of normal hierarchy (NH) of neutrino masses, we have 378
 taken the mass-squared differences as 379

$$\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.6 \times 10^{-5} \text{ eV}^2, \quad (23)$$

$$|\Delta m_{31}^2| = |m_3^2 - m_1^2| = 2.48 \times 10^{-3} \text{ eV}^2.$$

In the case of inverted hierarchy (IH) of neutrino masses, 380
 the value of Δm_{21}^2 remains the same as mentioned above, 381
 but $|\Delta m_{31}^2| = 2.38 \times 10^{-3} \text{ eV}^2$. In this work, the neutrino 382
 mixing angles and CP -violating phase are chosen to be 383

$$\sin \theta_{12} = \frac{1}{\sqrt{3}}, \quad \sin \theta_{23} = \frac{1}{\sqrt{2}},$$

$$\sin \theta_{13} = 0.15, \quad \delta_{\text{CP}} = 0. \quad (24)$$

The above-mentioned neutrino mass-squared differences, 384
 mixing angles, and the CP -violating phase are consistent 385
 with the fitted values in Ref. [26]. From the mass-squared 386
 differences, we can estimate neutrino mass eigenvalues 387
 which are given below for the cases of NH and IH, 388
 respectively: 389

$$m_1 = 0, \quad m_2 = \sqrt{\Delta m_{21}^2}, \quad m_3 = \sqrt{|\Delta m_{31}^2|}, \quad (25)$$

$$m_3 = 0, \quad m_1 = \sqrt{|\Delta m_{31}^2|}, \quad m_2 = \sqrt{\Delta m_{21}^2 + m_1^2}. \quad (26)$$

In the previous paragraph, we mentioned numerical 391
 values of neutrino mass eigenvalues, mixing angles, and 392
 the CP -violating phase. By plugging these values into 393
 Eq. (22), we can compute the elements of the matrix m_ν , 394
 which are related to neutrino Yukawa couplings and SUSY 395
 parameters through Eq. (14). Using Eq. (14), we can 396
 calculate the neutrino Yukawa couplings in order to satisfy 397
 neutrino oscillation data. This calculation procedure would 398
 be simplified if we assume degenerate masses for right- 399
 handed neutrinos and right-handed sneutrinos. For $i = 1, 2,$ 400
 3, we assume the following: 401
 402

$$M_i = M, \quad (m_N^2)_i = m_N^2, \quad (b_M)_i = b_M. \quad (27)$$

Under the above assumption, all three right-handed neu- 403
 trinos have mass M . The corresponding sneutrinos have 404
 real and imaginary components [see Eq. (12)], whose 405
 masses would be 406

$$m_R^2 = M^2 + m_N^2 + b_M, \quad m_I^2 = M^2 + m_N^2 - b_M. \quad (28)$$

Under the above-mentioned assumption, the neutrino mass 407
 matrix of Eq. (14) will be simplified to 408

¹In Ref. [25], the discrepancy in muon $g-2$ was fitted in a supersymmetric model, where the contribution is actually from the MSSM fields.

$$(m_\nu)_{ij} = \frac{S_{ij}}{16\pi^2} \sum_{l=1}^3 \left\{ M \left[[U_R(2, l)]^2 \frac{m_{\eta_{RI}}^2}{m_{\eta_{RI}}^2 - M^2} \ln \frac{m_{\eta_{RI}}^2}{M^2} - [U_I(2, l)]^2 \frac{m_{\eta_{II}}^2}{m_{\eta_{II}}^2 - M^2} \ln \frac{m_{\eta_{II}}^2}{M^2} \right] + [U_\eta(2, l)]^2 m_{\tilde{\eta}_l} \right. \\ \left. \times \left[\frac{m_R^2}{m_R^2 - m_{\tilde{\eta}_l}^2} \ln \frac{m_R^2}{m_{\tilde{\eta}_l}^2} - \frac{m_I^2}{m_I^2 - m_{\tilde{\eta}_l}^2} \ln \frac{m_I^2}{m_{\tilde{\eta}_l}^2} \right] \right\}, \quad (29)$$

$$S_{ij} = \sum_{k=1}^3 (Y_\nu)_{ik} (Y_\nu)_{jk}. \quad (30)$$

The elements S_{ij} are quadratic in the neutrino Yukawa couplings. From the above relation we can see that for certain values of the SUSY parameters, S_{ij} can be calculated from $(m_\nu)_{ij}$. Using the above-mentioned assumption of degenerate masses for right-handed neutrinos and right-handed sneutrinos, we can see that Eqs. (20) and (21) would give us $\text{Br}(\mu \rightarrow e\gamma) \propto S_{21}^2$ and $\Delta a_\mu \propto S_{22}$.

In our model, there are plenty of SUSY parameters, and we need to fix some of them to simplify our analysis. In our analysis, we choose the following SUSY parameters:

$$\begin{aligned} \mu_\chi = 600 \text{ GeV}, \quad m_{\eta_1} = 400 \text{ GeV}, \quad m_{\eta_2} = 500 \text{ GeV}, \quad m_\chi = 600 \text{ GeV}, \\ m_N = 700 \text{ GeV}, \quad \lambda_1 = 0.5, \quad \lambda_2 = 0.6, \quad \tan\beta = 10. \end{aligned} \quad (31)$$

409

421

We freely vary the parameters μ_η and M . In the previous section, we explained that we need to fine-tune the parameters of Eq. (15) in order to get small neutrino masses. Among these parameters, we take $(A\lambda)_1 = \lambda_1 \mu v_2 / v_1$ and $(A\lambda)_2 = \lambda_2 \mu v_1 / v_2$. The other parameters of Eq. (15), without loss of generality, are taken to be degenerate:

$$b_M = b_\eta = b_\chi = b_{\text{susy}}. \quad (32)$$

We have explained before that we have assumed degenerate masses for right-handed neutrinos and right-handed sneutrinos. Under this assumption, information about the neutrino Yukawa couplings is contained in the quantities S_{ij} . Hence, it is worth plotting these quantities to understand the neutrino Yukawa couplings. In Fig. 3, for the case of NH, we plot S_{21} and S_{22} versus the right-handed neutrino mass M for $\mu_\eta = 1$ TeV. The plots of Fig. 3 indicate that S_{22} and S_{21} are around $\mathcal{O}(1)$. Since these quantities are the sum of the squares of neutrino Yukawa couplings [see Eq. (30)], we can expect that the neutrino Yukawa

couplings should be in the range of $\mathcal{O}(1)$. We do not plot the values of S_{11} , S_{31} , etc. in Fig. 3, but we have found that these will also be around $\mathcal{O}(1)$. We plot S_{21} and S_{22} in Fig. 3, since these two determine $\text{Br}(\mu \rightarrow e\gamma)$ and Δa_μ .

From the plots of Fig. 3, we can notice that the values of S_{22} are higher than those of S_{21} . This fact follows from Eq. (29), where we can see that S_{ij} are proportional to $(m_\nu)_{ij}$, which are determined by neutrino oscillation parameters. In the case of NH, we have seen that $(m_\nu)_{22}$ is greater than $(m_\nu)_{21}$ by a factor of 3.4, and hence S_{22} is always found to be larger than S_{21} . It is clear from the plots of Fig. 3 that by increasing b_{susy} , S_{21} and S_{22} would decrease. Again, this feature can be understood from Eq. (29). As explained in the previous section, the square brackets of Eq. (29) would tend to zero in the limit $b_{\text{susy}} \rightarrow 0$. So for a large value of b_{susy} there will be less partial cancellation in the square brackets, and hence S_{21} and S_{22} would decrease. In both plots of Fig. 3 it is found that the values of S_{21} and S_{22} initially decrease with M , go to a minima, and then increase. The shape of these

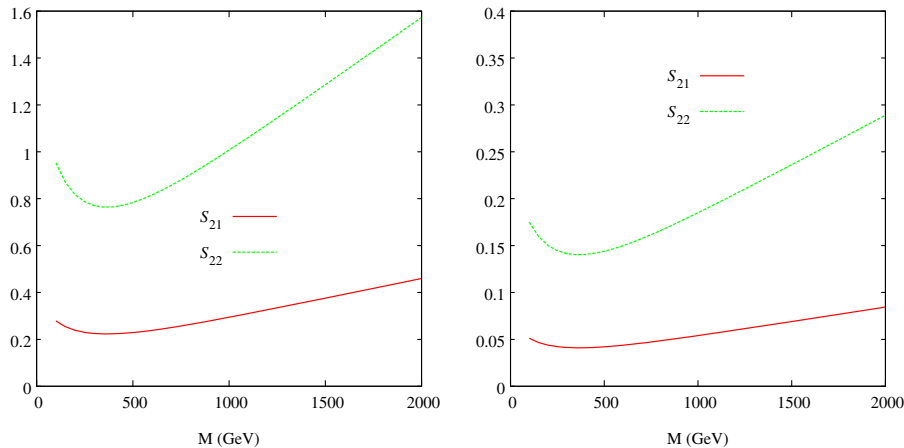


FIG. 3 (color online). The quantities S_{21} , S_{22} are plotted against the right-handed neutrino mass for $\mu_\eta = 1$ TeV, in the case of NH. In the left- and right-hand plots, b_{susy} is taken to be $(3 \times 10^{-2})^2 \text{ GeV}^2$ and $(7 \times 10^{-2})^2 \text{ GeV}^2$, respectively.

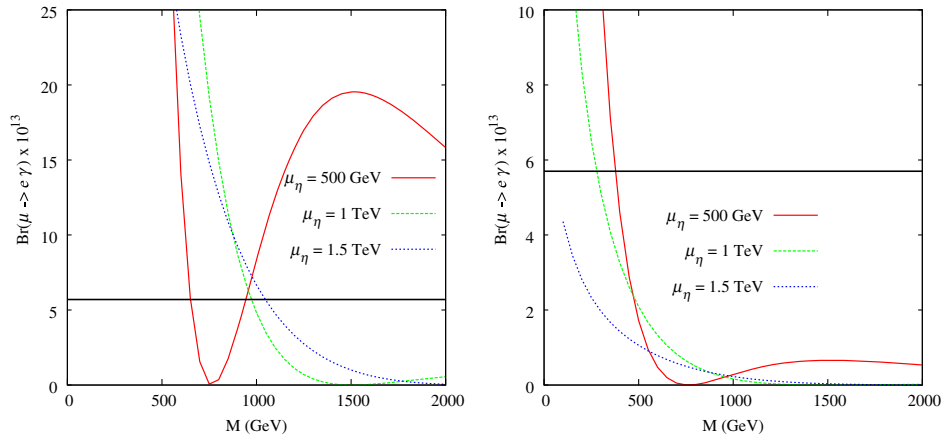


FIG. 4 (color online). $\text{Br}(\mu \rightarrow e\gamma)$ is plotted against the right-handed neutrino mass for different values of μ_η . In the left- and right-hand plots, b_{susy} is taken as $(3 \times 10^{-2})^2 \text{ GeV}^2$ and $(7 \times 10^{-2})^2 \text{ GeV}^2$, respectively. The horizontal line indicates the current upper bound on $\text{Br}(\mu \rightarrow e\gamma)$.

463 curves can be understood by applying the approximation of
 464 $\frac{b_{\text{susy}}}{M^2} \ll 1$ in Eq. (29). In the limit $b_{\text{susy}} \rightarrow 0$, we can take

$$m_{\eta_{Ri}}^2 = m_{\eta_i}^2(1 + \delta_{Ri}), \quad m_{\eta_{Li}}^2 = m_{\eta_i}^2(1 + \delta_{Li}),$$

$$U_R(2, l) \approx U_I(2, l) = U_0(2, l). \quad (33)$$

465 Here, $\delta_{Ri}, \delta_{Li} \ll 1$. Using the above-mentioned approxima-
 466 tions in Eq. (29), we get

$$(m_\nu)_{ij} = \frac{S_{ij}}{16\pi^2} \sum_{l=1}^3 \left\{ [U_0(2, l)]^2 (\delta_{Ri} - \delta_{Li}) M \frac{m_{\eta_i}^2}{m_{\eta_i}^2 - M^2} \right. \\ \times \left[1 - \frac{M^2}{m_{\eta_i}^2 - M^2} \ln \frac{m_{\eta_i}^2}{M^2} \right] \\ + [U_\eta(2, l)]^2 m_{\eta_i} \frac{2b_{\text{susy}}}{M^2 + m_N^2 - m_{\eta_i}^2} \\ \left. \times \left[1 - \frac{m_{\eta_i}^2}{M^2 + m_N^2 - m_{\eta_i}^2} \ln \frac{M^2 + m_N^2}{m_{\eta_i}^2} \right] \right\}. \quad (34)$$

467 In the summation of the above equation, the first and
 468 second lines arise due to the left- and right-hand diagrams
 469 of Fig. 1. From the above equation, we can understand that
 470 the contribution from the first line increases, reaches a
 471 maximum, and then decreases with M , whereas, the
 472 contribution from the second line of the above equation
 473 decreases monotonically with M . It is this functional
 474 dependence on M that determines the shape of the lines
 475 in Fig. 3. Physically, in the limit $b_{\text{susy}} \rightarrow 0$, the above
 476 description suggests that the right-hand diagram of Fig. 1 is
 477 significant only for very low values of M . For other values
 478 of M , the left-hand diagram of Fig. 1 gives the dominant
 479 contribution to neutrino masses. One remark about the plots
 480 in Fig. 3 is that we have fixed $\mu_\eta = 1 \text{ TeV}$ in these figures.
 481 We have varied μ_η from 500 GeV to 1.5 TeV and have
 482 found that the plots in Fig. 3 would change quantitatively,
 483 but qualitative features would remain same. Also, the plots
 484 in Fig. 3 are for the case of NH. Again, these plots can
 485 change quantitatively, if not qualitatively, for the case of IH.

For this reason, below we present our results on $\text{Br}(\mu \rightarrow e\gamma)$ and muon $g - 2$ for the case of NH only.

As described before, our motivation is to compute $\text{Br}(\mu \rightarrow e\gamma)$ in the model of Ref. [8]. In Fig. 3 we show that the neutrino Yukawa couplings in this model can be $\mathcal{O}(1)$, and for these values of Yukawa couplings $\text{Br}(\mu \rightarrow e\gamma)$ is unsuppressed. In the parameter space where the neutrino Yukawa couplings are unsuppressed, we plot $\text{Br}(\mu \rightarrow e\gamma)$ as a function of the right-handed neutrino mass. These plots are shown in Fig. 4, where we also vary μ_η from 500 GeV to 1.5 TeV. The horizontal line in these plots indicates the current upper bound of $\text{Br}(\mu \rightarrow e\gamma) < 5.7 \times 10^{-13}$. This upper bound would impose a lower bound on the right-handed neutrino mass, as can be seen in the plots of Fig. 4. In the left-hand plot of Fig. 4, for $\mu_\eta = 500 \text{ GeV}$, the right-handed neutrino mass is allowed to be between about 650 to 950 GeV. In the same plot, for $\mu_\eta = 1$ or 1.5 TeV, the right-handed neutrino mass has a lower bound of about 1 TeV. In the right-hand plot of Fig. 4, the lower bound on the right-handed neutrino mass is within 500 GeV, even for a low value of $\mu_\eta = 500 \text{ GeV}$.

The lower bounds on the right-handed neutrino mass M are severe in the left-hand plot of Fig. 4. The reason is that for a low value of b_{susy} , S_{21} would be high, and hence $\text{Br}(\mu \rightarrow e\gamma)$ would be large. From Fig. 4, we can observe that $\text{Br}(\mu \rightarrow e\gamma)$ initially decreases with M , goes to a minimum, and then increases. For instance, in the left-hand plot of Fig. 4, for $\mu_\eta = 500 \text{ GeV}$, $\text{Br}(\mu \rightarrow e\gamma)$ goes to a minimum around $M = 750 \text{ GeV}$, and then it has a local maxima around $M = 1.5 \text{ TeV}$. The reason that $\text{Br}(\mu \rightarrow e\gamma)$ initially decreases with M is due to the fact that the decay $\mu \rightarrow e\gamma$ is driven by right-handed neutrinos and right-handed sneutrinos, as given in Fig. 2. The masses of right-handed neutrinos and right-handed sneutrinos are proportional to M , and hence $\text{Br}(\mu \rightarrow e\gamma)$ would be suppressed with increasing M . After that, at a certain value of M , $\text{Br}(\mu \rightarrow e\gamma)$ would tend to become zero. The reason for this is that the sum of the two diagrams of Fig. 2 gives a relative minus sign to the contribution of $\text{Br}(\mu \rightarrow e\gamma)$, which is given in Eq. (20). Hence, for a particular value of

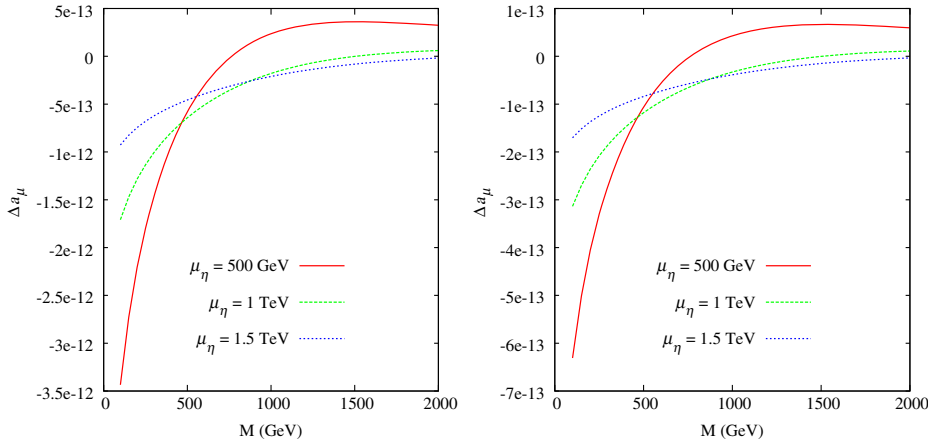


FIG. 5 (color online). Δa_μ is plotted against the right-handed neutrino mass for different values of μ_η . In the left- and right-hand plots, b_{susy} is taken as $(3 \times 10^{-2})^2 \text{ GeV}^2$ and $(7 \times 10^{-2})^2 \text{ GeV}^2$, respectively.

526 M , the contributions from both diagrams of Fig. 2 cancel
 527 out and give a minimum for $\text{Br}(\mu \rightarrow e\gamma)$. Also, $\text{Br}(\mu \rightarrow e\gamma)$
 528 can go to zero asymptotically when $M \rightarrow \infty$, since in this
 529 limit the masses of right-handed neutrinos and right-handed
 530 sneutrinos would become infinitely large and suppress
 531 $\text{Br}(\mu \rightarrow e\gamma)$. Hence, $\text{Br}(\mu \rightarrow e\gamma)$ has two zeros on the M
 532 axis. As $\text{Br}(\mu \rightarrow e\gamma)$ is a continuous function of M and is
 533 always a positive quantity, it has a local maxima between
 534 the two zeros on the M axis.

535 In the previous section we described muon $g-2$. In
 536 Eq. (21), we have given the contribution due to additional
 537 fields (see Table I) of our model to the muon $g-2$. Apart
 538 from this contribution, the MSSM fields of our model also
 539 contribute to muon $g-2$ [24], and it is known that this
 540 contribution fits the 3σ discrepancy of muon $g-2$. Hence,
 541 it is interesting to know if the additional contribution of
 542 Eq. (21) could be as large as that of the MSSM contribution
 543 to muon $g-2$. In Fig. 5, we plot the contribution of
 544 Eq. (21). In the plots of Fig. 5, we have chosen the
 545 parameter region such that the neutrino oscillation data is
 546 fitted. From the plots of Fig. 5, we can see that for low
 547 values of M , Δa_μ can be negative and it becomes positive
 548 after a certain large value of M . From these plots we can
 549 notice that the overall magnitude of Δa_μ is not more than
 550 about 10^{-12} . This contribution is at least 2 orders of
 551 magnitude smaller than the estimated discrepancy of muon
 552 $g-2$, which is $(29 \pm 9) \times 10^{-10}$ [22]. From this we can

conclude that the additional contribution to muon $g-2$ in
 our model [i.e., Eq. (21)] is insignificant compared to the
 MSSM contribution to muon $g-2$.

V. CONCLUSIONS

We have worked in a supersymmetric model where
 neutrino masses arise at the one-loop level [8]. We have
 computed these loop diagrams and obtained expressions
 for neutrino masses. We have identified a parameter
 region of this model, where the neutrino oscillation data
 can be fitted without the need for suppressing the
 neutrino Yukawa couplings. In our parameter region,
 the SUSY-breaking soft parameters [such as b_M , b_η ,
 b_χ , $(A\lambda)_1$, and $(A\lambda)_2$] need to be fine-tuned. In this
 parameter region, the branching fraction of $\mu \rightarrow e\gamma$ can
 be unsuppressed, and hence we have computed
 $\text{Br}(\mu \rightarrow e\gamma)$. We have shown that the current upper
 bound on $\text{Br}(\mu \rightarrow e\gamma)$ can put lower bounds on the mass
 of the right-handed neutrino field. Depending on the
 parametric choice, we have found that this lower bound
 can be about 1 TeV. We have also computed the
 contribution to muon $g-2$ arising from additional fields
 of this model, which are given in Table I. We have shown
 that, in the region where neutrino oscillation data is fitted,
 the above-mentioned contribution is 2 orders smaller than
 the discrepancy in muon $g-2$.

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