Non-linear consolidation with PVDs under ramp loading

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ABSTRACT

The paper presents a theory of non-linear consolidation for radial flow of a clay deposit treated with PVDs considering linear void ratio-log effective stress relationship, constant coefficient of consolidation but with ramp or construction loading. The governing equation of consolidation is solved numerically by the finite difference method. Parametric study quantifies the effects of magnitude of surcharge load and construction time on the variations of degree of settlement and degree of dissipation of excess pore pressures with time. The maximum induced excess pore water pressure is sensitive to the construction time for a given surcharge load. The variation of degree of settlement is dependent on both applied load intensity and the construction time. While the rate of building up of pore pressure with time during the construction is almost independent of stress increment ratio, the dissipation rate is shown to be dependent on stress increment ratio.

Keywords: consolidation, non-linear theory, construction time, degree of dissipation of pore pressure, ramp load, stress increment ratio

1 INTRODUCTION

Prefabricated Vertical Drains (PVDs) together with preloading are often used as ground improvement technique for compressible soil deposits. PVDs accelerate the rate of consolidation of soft ground by reorienting the flow horizontally and shortening the drainage path considerably. Barron (1948) presented a comprehensive analytical solution for the problem of radial consolidation of soft soils in terms of 'free' and 'equal' strain hypotheses. This classical theory of radial consolidation is based on linear void ratio-effective stress relation, constant coefficients of permeability, compressibility and consolidation, normally consolidated soil and instantaneous loading. However, during consolidation the coefficients of permeability and compressibility decrease and the void ratio is not linearly related to the effective stress for a large applied stress range. Moreover, the load is applied gradually with time. Hansbo (1979) modified the equations by Barron (1948) and presented a simple and equally accurate solution for radial consolidation with band shaped vertical drains by transforming the drain into an equivalent circular one. Many non-linear models considering variation of compressibility and permeability during consolidation are also developed for radial consolidation by Basak and Madhav (1978), Indraratna et al. (2005), Zhuang et al (2005), Conte and Troncone (2009) and Khan et al. (2010).

All the above theories assume that the surcharge load is applied instantaneously and kept constant thereafter. However, the loads are applied gradually with time and the consolidation occurs simultaneously with loading. As a result, the settlement rate and pore pressure dissipation are significantly affected by the load variations with time. Solutions for radial consolidation with vertical drain are presented by Olson (1977) and Tang and Onitsuka (2000). Zhu and Yin (2001) developed an analytical solution for consolidation with vertical drain considering both horizontal and vertical drainage under ramp loading by extending Barron's equation. Step and ramp loading cases are considered by Leo (2004) and closed-form analytical solutions of consolidation are developed considering the effects of both vertical and radial drainage in a fully coupled fashion. Geng (2008) presented a non-linear theory for sand drain consolidation under time dependent loading considering the variations of compressibility and permeability. Conte and Troncone (2009) presented an analytical solution for radial consolidation with vertical drains under general time- dependent loading by extending Barron's solution.

This paper presents a theory of non-linear consolidation for radial flow of a normally consolidated clay deposit treated with PVDs considering linear void ratio-log effective stress relation, constant coefficient of consolidation (the rate of decrease in permeability is proportional to the rate of decrease in compressibility) and ramp loading.

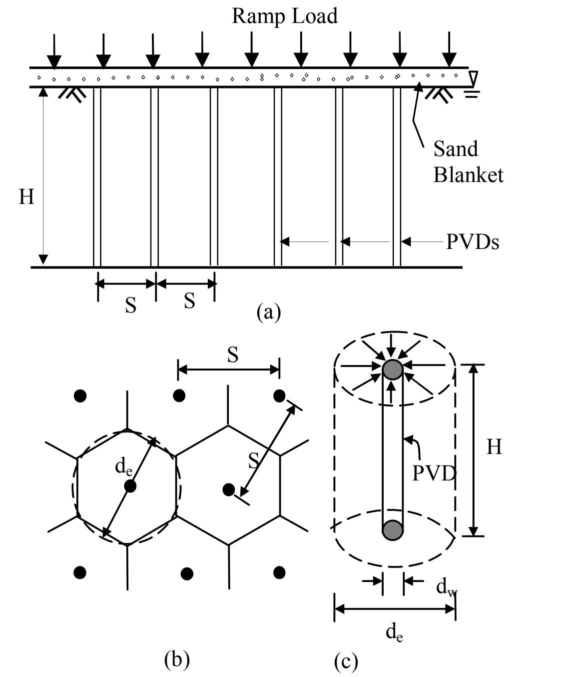

Fig. 1(a) Problem definition sketch, (b) Triangular arrangement of PVDs and (c) Flow in unit Cell

2 FORMULATION AND ANALYSIS

Normally consolidated clay layer treated with PVDs in triangular pattern and the unit cell considered are shown in Fig.1. Equivalent diameters of the zone of influence, d_e, and the drain, d_w are respectively 1.13S & 1.05S for square and triangular patterns and $2(a+b)/\pi$, where 'a' and 'b' are the width and thickness of the PVD. Void ratio, e is related to effective stress, σ' as

$$e = e_o - C_c \log_{10} \frac{\sigma'}{\sigma'_0} \tag{1}$$

where e_o is the initial void ratio corresponding to stress, σ_o', and C_c is compression index of the soil. Assuming the coefficient of consolidation, c_h constant, the governing equation of non-linear consolidation for radial flow is (Khan et al. 2010)

$$\frac{\partial u}{\partial t} = c_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{\sigma'}\left(\frac{\partial u}{\partial r}\right)^2 \right) \tag{2}$$

where u = excess pore water pressure at a radial distance, r and time, t. Eq. (2) derived for an instantaneous loading is extended to include the effect of ramp loading. The construction time, t_c is divided

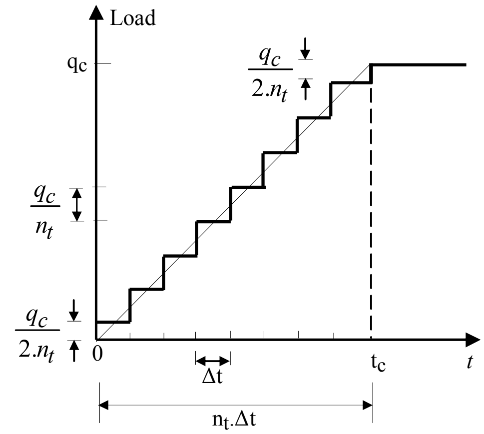

Fig. 2 Discretisation of ramp load

equally into n_t number of time intervals and the total ramp load (q_c) is considered to be applied in small increments (q_c/n_t) with time as shown in Fig. 2. At the end of each interval of time, Δt $(=t_c/n_t)$, the load increment is assumed to be applied instantaneously. However, the first and last increments are $0.5(q_c/n_t)$. After the time, t_c the magnitude of load remains constant as q_c. The initial and boundary conditions respectively are: At $t = 0$, $\Delta u = 0.5(q_c/n_t)$ for $r_w \le r \le r_e$; For $t > 0$, $u = 0$ at $r = r_w$ and $\partial u/\partial r = 0$ at $r = r_e$, where, $r_e = 0.5d_e$ and $r_w = 0.5d_w$. The finite difference method is used to solve Eq. (2) by discretizing the unit cell radially into m elements of equal thickness, Δr [$= (r_e - r_w)/m$]. The value of m is taken suitably and the numerical stability and convergence ensured. The finite difference form of Eq. (2) in non-dimensional form is

$$\begin{aligned} W_{i,T+\Delta T} &= W_{i,T} + \beta\left(W_{i-1,T} - 2.W_{i,T} + W_{i+1,T}\right) + \\ &\beta\left\{\frac{\Delta R}{R_i}\left(\frac{W_{i+1,T} - W_{i-1,T}}{2}\right) + f\left(\frac{W_{i+1,T} - W_{i-1,T}}{2}\right)^2\right\} \end{aligned} \tag{3}$$

$$W = u/q_c; \quad \Delta R = \Delta r/d_e; \quad R_i = r_i/d_e, \quad \beta = \frac{\Delta T}{\Delta R^2} \tag{4}$$

where i = a node in the discretized unit cell at a radius, r_i, Time factor, $T = \frac{c_h.t}{{d_e}^2}$ and $f = \frac{SIR}{1+SIR(1-W)}$ with SIR= q_c/σ'_o - the stress increment ratio. The normalized average residual pore pressure, W_{av}, is defined as

$$W_{av}(T) = \frac{U_{av}(T)}{q_c} = \frac{\int_{r_w}^{r_e} u\, 2\pi\, r\, dr}{\int_{r_w}^{r_e} q_c\, 2\pi\, r\, dr} \tag{5}$$

where $U_{av}(T)$ is the average residual pore pressure. The degree of settlement, U_s is defined as

$$U_s = \frac{\int_{r_w}^{r_e} (e_o - e) 2\pi\, r\, dr}{\int_{r_w}^{r_e} (e_o - e_f) 2\pi\, r\, dr} \tag{6}$$

$$\text{with } (e_o - e) = C_c \log\left[1 + SIR(1 - W)\right] \tag{7}$$

$$\text{and } (e_o - e_f) = C_c \log\left[1 + SIR\right] \tag{8}$$

3 RESULTS & DISCUSSION

Numerical experimentation is carried out for different values of n ($=r_e/r_w$), SIR and T_c ($=n_t.\Delta T$) and the variations of average excess pore water pressure and settlement with time are determined. Commonly used spacing of PVDs is 1.0 to 2.0 m (Indraratna et al., 2005). For this spacing, the value of n works out to be in the range of about 15 to 30. n is taken as 15 for parametric study in the present case. The effect of construction time, T_c on the variation of normalized residual average pore water pressure, W_{av} with time factor, T is shown in Fig. 3 for SIR of 1.0. The curve for T_c=0 represents instantaneous loading. Pore pressure rises with the loading gradually and reaches the maximum value at time, T=T_c. Thereafter, the pressure dissipates with time during the consolidation process. The residual maximum average pore pressure varies with T_c and decreases with increase in T_c as the pore pressure dissipates simultaneously during the application of load. Thus the construction time has significant effect on the maximum residual average pore pressure. The maximum pore pressure decreases from 100% for instant loading to 46% for ramp loading with T_c=0.5 for SIR=1. Surcharge loading can be applied slowly on soft soils to avoid building up of high pore pressure and consequent shear failure.

Fig. 4 presents the effect of T_c on variation of degree of settlement, U_s with time factor, T. The curve for T_c =0 represents instantaneous loading and is identical to Barron's theory. As the construction time, T_c increases, the degree of settlement appears to decrease at a given time factor as the time is reckoned from the instant of first increment of loading. The degree of settlement decreases from 86% for an instant loading to 62% at a time factor, T=0.5, T_c =0.5. Identical results are also reported by Tang and Onitsuka (2000) and Zhu and Yin (2001).

The effect of SIR on the residual average pore pressure is depicted in Fig. 5 for a ramp loading with T_c=0.1. While the rate of building up of pore pressure with time during construction is almost independent of SIR, the dissipation rate is dependent on SIR. The average residual pore pressure increases from 21% to 34% at T=0.5, for SIR increasing from 1 to 10 for T_c =0.1. The rate of dissipation of pore pressure decreases with increase in SIR. Thus the rate of dissipation of excess pore pressure is influenced by the stress increment ratio in the non-linear theory of consolidation in contrast to the load-independent dissipation of pore pressures in the conventional linear theory. This is in consonance with the results for non-linear theory for instantaneous loading where in the pore pressure dissipation rate decreases with increase in SIR (Davis and Raymond, 1965 for vertical flow; Khan et al., 2010 for radial flow). Similar results also reported by Conte and Troncone (2007) for vertical flow under ramp loading.

The non-linear theory of consolidation for thin layer of clay has already established that the degree of settlement under instantaneous loading is independent of SIR (Davis and Raymond, 1965 for vertical flow; Khan et al., 2010 for radial flow). However, the degree of settlement under ramp loading is dependent on SIR

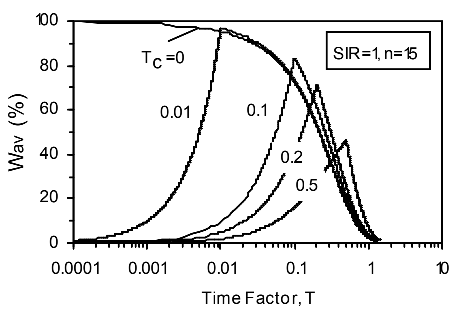

Fig. 3 Effect of T_c on W_{av}

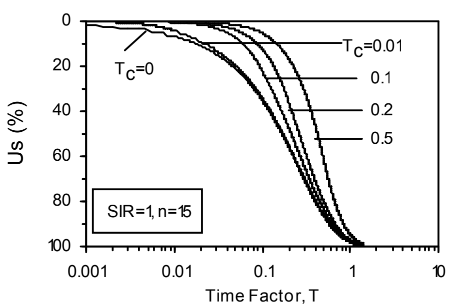

Fig. 4 Effect of T_c on U_s

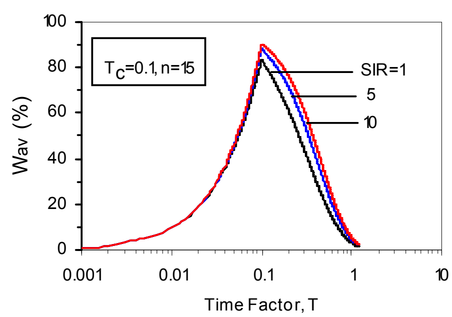

Fig. 5 Effect of SIR on W_{av}

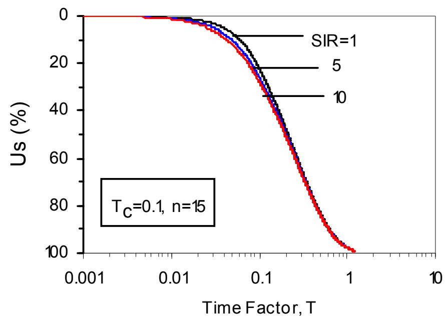

Fig. 6 Effect of SIR on U_s

to some extent in the initial stages of consolidation as shown in Fig. 6. Similar results are also reported by Conte and Troncone (2007) for consolidation with vertical flow under ramp loading.

Fig. 7 depicts the effect of T_c on T_{90}, the time factor for 90% degree of settlement. For small construction times up to about T_c =0.01, T_{90} is almost identical to that of instantaneous loading case as expected. However, T_{90} increases for T_c increasing beyond 0.01. T_{90} increases from 0.58 to 0.84 for T_c increasing from 0.01 to 0.5.

The degree of settlement, U_s and the degree of dissipation of average pore pressure, U_p (=100-U_{av}) at the end of construction time, T_c, are compared in Fig.8. The degree of dissipation of excess pore pressure is relatively smaller compared to the corresponding degree of settlement.

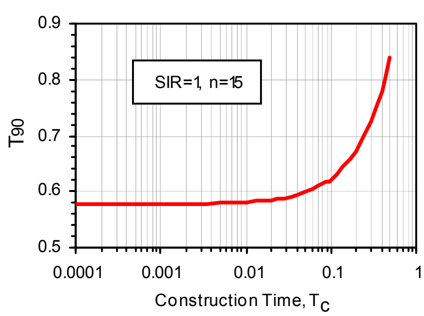

Fig. 7 Effect of T_c on T_{90}

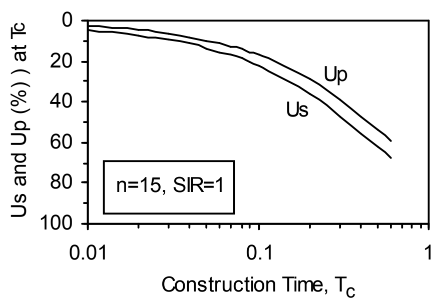

Fig.8 Variation of U_s and U_p at T_c

4 CONCLUSIONS

The residual maximum average pore pressure varies with construction time and decreases with increase in construction time. The rate of dissipation of pore pressure decreases with increase in SIR. Thus, the rate of dissipation of excess pore pressures with time is influenced not only by the magnitude of applied load intensity but also the construction time. The variation of degree of settlement is dependent on both applied load intensity and the construction time. The time for 90% degree of settlement is not significantly affected by small construction times. The degree of dissipation of excess pore pressure is relatively smaller compared to the corresponding degree of settlement at the end of construction time.

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