Fourth generation effect on Λ_b decays

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The rare decays of the Λ_b baryon governed by the quark level transitions $b \to s$ are investigated in the fourth quark generation model popularly known as SM4. Recently it has been shown that SM4, which is a very simple extension of the standard model, can successfully explain several anomalies observed in the CP violation parameters of B and B_s mesons. We find that in this model due to the additional contributions coming from the heavy t' quark in the loop, the branching ratios and other observables in rare Λ_b decays deviate significantly from their standard model values. Some of these modes are within the reach of the LHCb experiment and search for such channels is strongly argued.

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I. INTRODUCTION

The rare decays of B mesons involving flavor changing neutral current (FCNC) transitions are of great interest to look for possible hints of new physics beyond the standard model (SM). In the SM, the FCNC transitions arise only at one-loop level, thus providing an excellent testing ground to look for new physics. Therefore, it is very important to study FCNC processes, both theoretically and experimentally, as these decays can provide a sensitive test for the investigation of the gauge structure of the SM at the loop level. Huge experimental data on both exclusive and inclusive B meson decays [1] involving $b \to s$ transitions have been accumulated at the e^+e^- asymmetric B factories operating at $\Upsilon(4S)$, which motivated extensive theoretical studies on these mesonic decay modes.

Unlike the mesonic decays, the experimental results on FCNC mediated Λ_b baryon decays e.g., $\Lambda_b \to \Lambda\pi$, $\Lambda_b \to pK^-$, $\Lambda_b \to \Lambda\gamma$, and $\Lambda_b \to \Lambda l^+ l^-$ are rather limited. At present we have only upper limits on some of these decay modes [2]. Heavy baryons containing a heavy b quark will be copiously produced at the LHC. Their weak decays may provide important clues on flavor changing currents beyond the SM in a complementary fashion to the B decays. A particular advantage of the bottom baryon decays over the B mesons is that these decays are self-tagging processes which should make their experimental reconstructions easier.

Another important aspect is that, in the past few years, we have seen some kind of deviations from the SM results in the CP violating observables of B and B_s meson decays involving $b \to s$ transitions [1,3–6]. Several new physics scenarios are proposed in the literature to account for these deviations [7]. Therefore, it is quite natural to expect that if there is some new physics present in the $b \to s$ transitions of B meson decays it must also affect the corresponding Λ_b transitions. Therefore, the study of the rare Λ_b decays is of utmost importance to obtain an unambiguous signal of new physics.

In this paper we would like to study the rare Λ_b decays in a model with an extra generation of quarks, usually known as SM4 [8]. SM4 is a simple extension of the standard model with three generations (SM3) with the additional up-type (t') and down-type (d') quarks. The model retains all the properties of SM3. The t' quark like the other up-type quarks contributes to the $b \to s$ transition at the loop level. Because of the additional fourth generation there will be mixing between the b' quark, the three down-type quarks of the standard model, and the resulting mixing matrix will become a 4×4 matrix ($V_{\rm CKM4}$). The parametrization of this unitary matrix requires six mixing angles and three phases. The existence of the two extra phases provides the possibilities of an extra source of CP violation. Another advantage of this model is that the heavier quarks and leptons in this family can play a crucial role in dynamical electroweak symmetry breaking as an economical way to address the hierarchy problem [9]. The effects of the fourth generation of quarks in various B decays are extensively studied in the literature [10]. In Refs. [11,12], it has been shown that this model can easily explain the observed anomalies in the B meson sector.

The paper is organized as follows. In Sec. II we discuss the nonleptonic decay of the Λ_b baryon. The radiative decay process $\Lambda_b \to \Lambda\gamma$ is discussed in Sec. III. The results on semileptonic decays are presented in Sec. IV. Section V contains the summary and conclusion

II. DECAY WIDTH OF $\Lambda_b \to \Lambda\pi^0$ AND $\Lambda_b \to pK^-$ MODES

In this section we will discuss the nonleptonic rare Λ_b decay modes $\Lambda_b \to \Lambda\pi$ and $\Lambda_b \to pK^-$ induced by the quark level transition $b \to sq\bar{q}$ ($q = u, d$). The effective Hamiltonian describing these processes is given by [13]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{us}^* \sum_{i=1,2} C_i(\mu)O_i - V_{tb}V_{ts}^* \sum_{i=3}^{10} C_i(\mu)O_i\Big], \quad (1)$$

where $C_i(\mu)$'s are the Wilson coefficients evaluated at the renormalization scale μ, $O_{1,2}$ are the tree level current-current operators, $O_{3\text{–}6}$ are the QCD and $O_{7\text{–}10}$ are the electroweak penguin operators.

Let us first consider the decay process $\Lambda_b \to \Lambda\pi$. In the SM this mode receives contributions from the color-suppressed tree and the electroweak penguin diagrams and the amplitude for this process in the factorization approximation is given as [14]

$$\begin{aligned}\mathcal{A}(\Lambda_b(p) \to \Lambda(p')\pi^0(q)) &= \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\left(\frac{3}{2}(a_9 - a_7)\right)\right] \\ &\times \langle\Lambda(p')|(\bar{s}\gamma^\mu(1-\gamma_5)b|\Lambda_b(p)\rangle \\ &\times \langle\pi^0(q)|\bar{u}\gamma_\mu(1-\gamma_5)u|0\rangle, \end{aligned} \tag{2}$$

where $a_i = C_i + C_{i+1}/N(C_i + C_{i-1}/N)$ for $i =$ odd (even). In order to evaluate the matrix elements we use the following form factors and decay constants. The matrix elements of the various hadronic currents between initial Λ_b and the final Λ baryon are parametrized in terms of various form factors [15] as

$$\begin{aligned}\langle\Lambda(p')|\bar{s}\gamma_\mu b|\Lambda_b(p)\rangle &= \bar{u}_\Lambda(p')[g_1(q^2)\gamma_\mu + ig_2(q^2)\sigma_{\mu\nu}q^\nu \\ &+ g_3(q^2)q_\mu]u_{\Lambda_b}(p), \\ \langle\Lambda(p')|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(p)\rangle &= \bar{u}_\Lambda(p')[G_1(q^2)\gamma_\mu + iG_2(q^2)\sigma_{\mu\nu}q^\nu \\ &+ G_3(q^2)q_\mu]\gamma_5 u_{\Lambda_b}(p), \end{aligned} \tag{3}$$

where g_i (G_i)'s are the vector (axial vector) form factors and q is the momentum transfer i.e., $q = p - p'$. The matrix element $\langle\pi(q)|\bar{u}\gamma_\mu\gamma_5 u|0\rangle$ is related to the pion decay constant f_π as

$$\langle\pi^0(q)|\bar{u}\gamma^\mu\gamma_5 u|0\rangle = if_\pi q^\mu/\sqrt{2}. \tag{4}$$

With these values one can write the transition amplitude for $\Lambda_b \to \Lambda\pi$ as

$$\begin{aligned}&\mathcal{A}(\Lambda_b \to \Lambda\pi^0) \\ &= i\frac{G_F}{2}f_\pi\left(V_{ub}V_{us}^* a_2 - \frac{3}{2}V_{tb}V_{ts}^*(a_9 - a_7)\right) \\ &\quad\times \bar{u}_\Lambda(p')[(g_1(q^2)(m_{\Lambda_b} - m_\Lambda) + g_3(q^2)m_\pi^2) \\ &\quad+ (G_1(q^2)(m_{\Lambda_b} + m_\Lambda) - G_3(q^2)m_\pi^2)\gamma_5]u_{\Lambda_b}(p). \end{aligned} \tag{5}$$

The above amplitude can be symbolically written as

$$\mathcal{A}\,(\Lambda_b(p') \to \Lambda(p)\pi^0(q)) = i\bar{u}_\Lambda(p')(A + B\gamma_5)u_{\Lambda_b}(p), \tag{6}$$

where A and B are given as

$$\begin{aligned} A &= \frac{G_F}{2}f_\pi\left(V_{ub}V_{us}^* a_2 - \frac{3}{2}V_{tb}V_{ts}^*(a_9 - a_7)\right) \\ &\quad\times (g_1(q^2)(m_{\Lambda_b} - m_\Lambda) + g_3(q^2)m_\pi^2), \\ B &= \frac{G_F}{2}f_\pi\left(V_{ub}V_{us}^* a_2 - \frac{3}{2}V_{tb}V_{ts}^*(a_9 - a_7)\right) \\ &\quad\times (G_1(q^2)(m_{\Lambda_b} + m_\Lambda) - G_3(q^2)m_\pi^2). \end{aligned} \tag{7}$$

Thus, one can obtain the decay width for this process as [16]

$$\begin{aligned}\Gamma = \frac{p_{\rm c.m.}}{8\pi}\bigg[&\frac{(m_{\Lambda_b} + m_\Lambda)^2 - m_\pi^2}{m_{\Lambda_b}^2}|A|^2 \\ &+ \frac{(m_{\Lambda_b} - m_\Lambda)^2 - m_\pi^2}{m_{\Lambda_b}^2}|B|^2\bigg], \end{aligned} \tag{8}$$

where $p_{\rm c.m.}$ is the magnitude of the center-of-mass momentum of the outgoing particles.

For numerical analysis we use the following input parameters. The masses of the particles, the decay constant of the pion, and the lifetime of the Λ_b baryon are taken from [2]. The values of the effective Wilson coefficients are taken from [14]. The values of the Cabibbo-Kobayashi-Maskawa (CKM) elements used are $|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}$, $|V_{us}| = (0.2255 \pm 0.0019)$, $|V_{tb}| = 0.999$, $|V_{ts}| = (38.7 \pm 2.3) \times 10^{-3}$ [2], and the weak phase $\gamma = (70^{+14}_{-21})^\circ$ [17].

To evaluate the branching ratio for $\Lambda_b \to \Lambda\pi$ decay we need to specify the form factors describing $\Lambda_b \to \Lambda$ transition. In this analysis we use the values of the factors from [15] which are evaluated using the light-cone sum rules. In this approach, the dependence of form factors on the momentum transfer can be parametrized as

$$\xi_i(q^2) = \frac{\xi_i(0)}{1 - a_1(q^2/m_{\Lambda_b}^2) + a_2(q^4/m_{\Lambda_b}^4)}, \tag{9}$$

where ξ denotes the form factors g_1 and g_2. The values of the parameters $\xi_i(0)$, a_1, and a_2 have been presented in Table I. The other form factors can be related to these two as

$$g_1 = G_1, \qquad g_2 = G_2 = g_3 = G_3. \tag{10}$$

TABLE I. Numerical values of the form factors g_1 and g_2 and the parameters a_1 and a_2 involved in the double fit (9).

Parameter	Twist 3	Up to twist 6
$g_1(0)$	$0.14^{+0.02}_{-0.01}$	$0.15^{+0.02}_{-0.02}$
a_1	$2.91^{+0.10}_{-0.07}$	$2.94^{+0.11}_{-0.06}$
a_2	$2.26^{+0.13}_{-0.08}$	$2.31^{+0.14}_{-0.10}$
$g_2(0)(10^{-2}$ GeV$^{-1})$	$-0.47^{+0.06}_{-0.06}$	$1.3^{+0.2}_{-0.4}$
a_1	$3.40^{+0.06}_{-0.05}$	$2.91^{+0.12}_{-0.09}$
a_2	$2.98^{+0.09}_{-0.08}$	$2.24^{+0.17}_{-0.13}$

Thus, we obtain the branching ratio for the $\Lambda_b \to \Lambda\pi$ mode in the SM as

$$\mathrm{Br}(\Lambda_b \to \Lambda\pi) = (6.4 \pm 2.0)\times 10^{-8} \quad \text{(twist 3)},$$
$$\mathrm{Br}(\Lambda_b \to \Lambda\pi) = (7.4 \pm 2.3)\times 10^{-8} \quad \text{(up to twist 6)}, \tag{11}$$

where we have assumed 50% uncertainties due to nonfactorizable contributions. It should be noted that these values are beyond the reach of the currently running experiments and hence, observation of this mode will be a clear signal of new physics.

In the presence of a fourth generation of quarks, there will be an additional contribution due to the t' quark in the electroweak penguin loops. Furthermore, it should be noted that due to the presence of the t' quark the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$, where $\lambda_q = V_{qb}V_{qs}^*$.

Thus, in the presence of the fourth generation of quarks the amplitude for $\Lambda_b \to \Lambda\pi$ will become

$$\begin{aligned}\mathcal{A}(\Lambda_b \to \Lambda\pi^0) &= i\Big(\lambda_u a_2 - \frac{3}{2}\lambda_t(a_9 - a_7) - \frac{3}{2}\lambda_{t'}(a_9' - a_7')\Big)\\ &\quad \times \bar{u}_\Lambda(p')(X + Y\gamma_5)u_{\Lambda_b}(p),\end{aligned} \tag{12}$$

where X and Y are given as

$$\begin{aligned}X &= \frac{G_F}{2} f_\pi (g_1(q^2)(m_{\Lambda_b} - m_\Lambda) + g_3(q^2)m_\pi^2),\\ Y &= \frac{G_F}{2} f_\pi (G_1(q^2)(m_{\Lambda_b} + m_\Lambda) - G_3(q^2)m_\pi^2).\end{aligned} \tag{13}$$

The above amplitude can be represented in a more general way

$$\begin{aligned}\mathcal{A}(\Lambda_b(p') \to \Lambda(p)\pi^0(q)) &= i[\bar{u}_\Lambda(X + Y\gamma_5)u_{\Lambda_b}]\lambda_u a_2(1 + ra\exp(i(\delta + \gamma))\\ &\quad - br'\exp(i(\delta' + \phi_s + \gamma))),\end{aligned} \tag{14}$$

where the parameters a, b, r, and r' and the strong phases δ and δ' are defined as

$$\begin{aligned} a &= |\lambda_t/\lambda_u|, & b &= |\lambda_t'/\lambda_u|,\\ r &= \frac{3}{2}\left|\frac{a_9 - a_7}{a_2}\right|, & r' &= \frac{3}{2}\left|\frac{a_9' - a_7'}{a_2}\right|,\\ \delta &= \arg\Big(\frac{a_9 - a_7}{a_2}\Big), & \delta' &= \arg\Big(\frac{a_9' - a_7'}{a_2}\Big).\end{aligned} \tag{15}$$

The weak phases of the CKM elements are used as follows: $(-\gamma)$ is the phase of V_{ub}, π is the phase of V_{ts}, and ϕ_s is the phase of $\lambda_{t'}$. The decay width for this process can be given by

$$\begin{aligned}\Gamma &= \frac{p_{cm}}{8\pi}|\lambda_u a_2|^2\Big[\frac{(m_{\Lambda_b} + m_\Lambda)^2 - m_\pi^2}{m_{\Lambda_b}^2}|X|^2\\ &\quad + \frac{(m_{\Lambda_b} - m_\Lambda)^2 - m_\pi^2}{m_\Lambda^2}|Y|^2\Big][1 + a^2r^2 + b^2r'^2\\ &\quad + 2ar\cos(\delta + \gamma) - 2br'\cos(\phi_s + \gamma + \delta')\\ &\quad - 2abrr'\cos(\phi_s + \delta' - \delta)].\end{aligned} \tag{16}$$

For numerical evaluation of the branching ratio we need to know the values of the new parameters of this model. We use the allowed range for the new CKM elements as $|\lambda_{t'}| = (0.08 \to 1.4)\times 10^{-2}$ and $\phi_s = (0 \to 80)^\circ$ for $m_{t'} = 400$ GeV, extracted using the available observables which are mediated through $b \to s$ transitions [11]. To find out the values of the QCD parameters a_9' and a_7', we need to evaluate the new Wilson coefficients C_{7-10}' due to the virtual t' quark exchange in the loop. The values of these coefficients at the M_W scale can be obtained from the corresponding contribution due to t-quark exchange by replacing the mass of the t quark in the Inami-Lim functions [18] by $m_{t'}$. These values can then be evolved to the m_b scale using the renormalization group equation as discussed in [19]. The values of these coefficients for a representative t' mass $m_{t'} = 400$ GeV are listed in Table II.

With these inputs the variation of the branching ratio for the $\Lambda_b \to \Lambda\pi$ with $|\lambda_t'|$ is shown in Fig. 1. From the figure it can be seen that the branching ratio is significantly enhanced from its corresponding SM value and it could be easily accessible in the currently running LHCb experiment.

Now we will discuss the Λ_b decay mode $\Lambda_b \to pK^-$, mediated through $b \to s$ transition. In the SM, it receives contributions from the color allowed tree, QCD as well as electroweak penguins. Its amplitude in the SM is given as [14]

$$\begin{aligned}\mathcal{A}(\Lambda_b \to pK^-) &= i\frac{G_F}{\sqrt{2}} f_K \bar{u}_p(p')[(\lambda_u a_1 - \lambda_t(a_4 + a_{10}\\ &\quad + (a_6 + a_8)R_1)(g_1(m_K^2)(m_{\Lambda_b} - m_\Lambda)\\ &\quad + g_3(m_K^2)m_K^2) + (\lambda_u a_1 - \lambda_t(a_4 + a_{10}\\ &\quad - (a_6 + a_8)R_2)(G_1(m_K^2)(m_{\Lambda_b} + m_\Lambda)\\ &\quad - G_3(m_K^2)m_K^2)\gamma_5]u_{\Lambda_b}(p),\end{aligned} \tag{17}$$

TABLE II. Numerical values of the Wilson coefficients C_i' for $m_{t'} = 400$ GeV.

C_3'	C_4'	C_5'	C_6'
2.06×10^{-2}	-3.85×10^{-2}	1.02×10^{-2}	-4.43×10^{-2}
C_7'	C_8'	C_9'	C_{10}'
4.453×10^{-3}	2.115×10^{-3}	-0.029	0.006

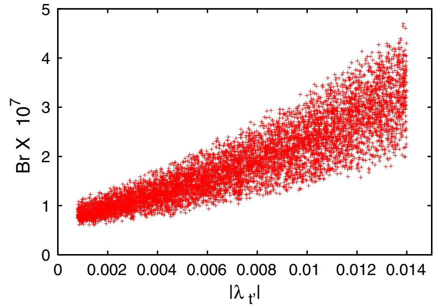

FIG. 1 (color online). The branching ratio versus $|\lambda_t'|$ for the process $\Lambda_b \to \Lambda\pi$.

where

$$R_1 = \frac{2m_K^2}{(m_b - m_u)(m_s + m_u)}, \quad R_2 = \frac{2m_K^2}{(m_b + m_u)(m_s + m_u)}. \tag{18}$$

From the above amplitude one can obtain the branching ratio using Eq. (8). Using the input parameters as discussed earlier in this section and assuming 50% uncertainties due to nonfactorizable contributions, we obtain the branching ratio in the SM

$$\mathrm{Br}(\Lambda_b \to pK^-) = 3.5 \times 10^{-6}, \tag{19}$$

which is lower than the present experimental value $\mathrm{Br}(\Lambda_b \to pK^-) = (5.6 \pm 0.8 \pm 1.5) \times 10^{-6}$ [20]. Here we have used the form factors for $\Lambda_b \to p$ transitions from [21], which are evaluated in the light-front quark model. The q^2 dependence of the form factors is given by the following three parameters fit as

$$\xi_i(q^2) = \frac{\xi_i(0)}{(1 - q^2/m_{\Lambda_b}^2)(1 - a_1(q^2/m_{\Lambda_b}^2) + a_2(q^4/m_{\Lambda_b}^4))}, \tag{20}$$

where the values of the different fit parameters are listed in Table III.

As discussed earlier in the presence of a fourth generation of quarks the amplitude (17) will receive additional contributions due to the heavy t' quark in the loop. The modified amplitude becomes

TABLE III. Numerical values of the form factors g_1 and g_2 and the parameters a_1 and a_2 for $\Lambda_b \to p$ transition (20).

ξ	$\xi(0)$	a	b
g_1	0.1131	1.70	1.60
g_3	0.0356	2.5	2.57
G_1	0.1112	1.65	1.60
G_3	0.0097	2.8	2.7

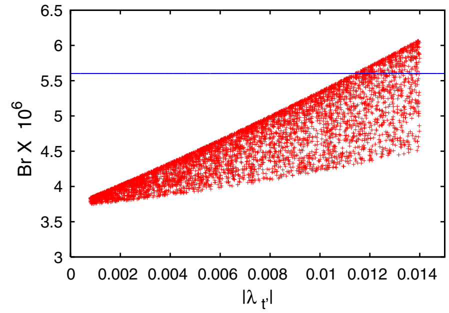

FIG. 2 (color online). The branching ratio versus $|\lambda_t'|$ for the process $\Lambda_b \to pK^-$, where the horizontal line represents the experimental central value.

$$\begin{aligned}\mathcal{A}(\Lambda_b \to pK^-) = {} & i\frac{G_F}{\sqrt{2}} f_K \bar{u}_p[(\lambda_u a_1 - \lambda_t(a_4 + a_{10} \\ & + (a_6 + a_8)R_1) - \lambda_t'(a_4' + a_{10}' \\ & + (a_6' + a_8')R_1))(g_1(m_K^2)(m_{\Lambda_b} - m_\Lambda) \\ & + g_3(m_K^2)m_K^2) + (\lambda_u a_1 - \lambda_t(a_4 + a_{10} \\ & - (a_6 + a_8)R_2 - \lambda_t'(a_4' + a_{10}' \\ & - (a_6' + a_8')R_2))(G_1(m_K^2)(m_{\Lambda_b} + m_\Lambda) \\ & - G_3(m_K^2)m_K^2)\gamma_5]u_{\Lambda_b}.\end{aligned} \tag{21}$$

Now using the values of the new Wilson coefficients C_{3-10}' from Table II and varying the new CKM elements between $0.0008 \le |\lambda_t'| \le 0.014$ and $(0 \le \phi_s \le 80)°$, we present in Fig. 2 the variation of $\mathrm{Br}(\Lambda_b \to pK^-)$ with $|\lambda_t'|$. From the figure it can be seen that the measured branching ratio can be easily accommodated in this model.

III. $\Lambda_b \to \Lambda\gamma$ DECAY WIDTH

In this section we will consider the rare radiative decay $\Lambda_b \to \Lambda\gamma$ which is induced by the quark level transition $b \to s\gamma$. The effective Hamiltonian describing $\Lambda_b \to \Lambda\gamma$ is given as

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\lambda_t C_7(m_b)O_7, \tag{22}$$

where C_7 is the Wilson coefficient and O_7 is the electromagnetic dipole operator given as

$$O_7 = \frac{e}{32\pi^2}F_{\mu\nu}[m_b\bar{s}\sigma^{\mu\nu}(1+\gamma_5)b + m_s\bar{s}\sigma^{\mu\nu}(1-\gamma_5)b]. \tag{23}$$

The expression for calculating the Wilson coefficient $C_7(\mu)$ is given in [22]. The matrix elements of the various hadronic currents between initial Λ_b and the final Λ baryon, which are parametrized in terms of various form factors as

$$\langle\Lambda|\bar{s}i\sigma_{\mu\nu}q^\nu b|\Lambda_b\rangle = \bar{u}_\Lambda[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3 q_\mu]u_{\Lambda_b},$$
$$\langle\Lambda|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\Lambda_b\rangle = \bar{u}_\Lambda[F_1\gamma_\mu\gamma_5 + iF_2\sigma_{\mu\nu}\gamma_5 q^\nu + F_3\gamma_5 q_\mu]u_{\Lambda_b}. \quad (24)$$

These form factors are related to the previously defined g_1 and g_2 through [15]

$$F_1(q^2) = f_1(q^2) = q^2 g_2(q^2) = q^2 G_2(q^2),$$
$$F_2(q^2) = f_2(q^2) = g_1(q^2) = G_1(q^2). \quad (25)$$

Thus, one can obtain the decay width of $\Lambda_b \to \Lambda\gamma$ in the SM as

$$\Gamma(\Lambda_b \to \Lambda\gamma) = \frac{\alpha G_F^2}{32 m_{\Lambda_b}^3 \pi^4}|V_{tb}V_{ts}^*|^2|C_7|^2(1-x^2)^3 \times (m_b^2 + m_s^2)[f_2(0)]^2, \quad (26)$$

where $x = m_\Lambda/m_{\Lambda_b}$. Using the input parameters as discussed in Sec. II we obtain the branching ratio in the SM as

$$\mathrm{Br}(\Lambda_b \to \Lambda\gamma) = (7.93 \pm 2.31)\times 10^{-6}, \quad (27)$$

which is well below the present experimental upper limit $\mathrm{Br}(\Lambda_b \to \Lambda\gamma) < 1.3\times 10^{-3}$ [2]. Now we would like to see the effect of the fourth quark generation on the branching ratio of $\Lambda_b \to \Lambda\gamma$. In the presence of the fourth quark generation of quarks, the Wilson coefficient C_7 will be modified due to the t' contribution in the loop. Thus the modified parameter can be given as

$$C_7^{\mathrm{tot}}(\mu) = C_7(\mu) + \frac{V_{t'b}V_{t's}^*}{V_{tb}V_{ts}^*}C_7'(\mu), \quad (28)$$

where C_7' can be obtained from the expression of C_7 by replacing the mass of the t quark by $m_{t'}$. The value of C_7' for $m_{t'} = 400$ GeV is found to be $C_7' = -0.375$.

Thus, in SM4 the branching ratio can be given by Eq. (25) by replacing C_7 by C_7^{tot}. Now varying $\lambda_{t'}$ between $0.0008 \le |\lambda_t'| \le 0.0014$ and ϕ_s between (0°–80°) we show in Fig. 3 the corresponding branching ratio, where we have included 30% uncertainties due to hadronic form factors. From the figure it can be seen that the branching ratio in SM4 has been significantly enhanced from its SM value and it could be easily accessible in the currently running experiments.

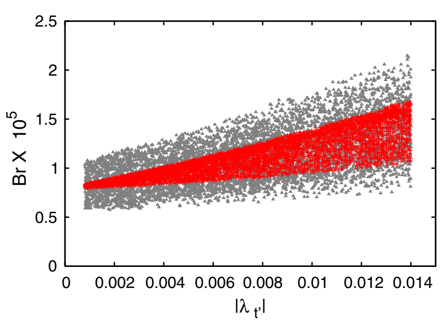

FIG. 3 (color online). The branching ratio versus $|\lambda_t'|$ for the process $\Lambda_b \to \Lambda\gamma$. The gray bands are due to the 30% uncertainties in the hadronic form factors.

IV. $\Lambda_b \to \Lambda l^+ l^-$ DECAYS

The decay process $\Lambda_b \to \Lambda l^+ l^-$ is described by the quark level transition $b \to s l^+ l^-$. These processes are extensively studied in the literature [23] in various beyond the standard model scenarios. The effective Hamiltonian describing these processes can be given as [19]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{ts}^*\Big[C_9^{\mathrm{eff}}(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma^\mu l) + C_{10}(\bar{s}\gamma_\mu Lb) \times (\bar{l}\gamma^\mu\gamma_5 l) - 2C_7^{\mathrm{eff}}m_b\Big(\bar{s}i\sigma_{\mu\nu}\frac{q^\mu}{q^2}Rb\Big)(\bar{l}\gamma^\mu l)\Big], \quad (29)$$

where q is the momentum transferred to the lepton pair, given as $q = p_- + p_+$, where p_- and p_+ are the momenta of the leptons l^- and l^+, respectively. $L, R = (1 \pm \gamma_5)/2$ and C_i's are the Wilson coefficients evaluated at the b quark mass scale. The values of these coefficients in next-leading-logarithmic (NLL) order are $C_7^{\mathrm{eff}} = -0.31$, $C_9 = 4.154$, and $C_{10} = -4.261$ [24].

The coefficient C_9^{eff} has a perturbative part and a resonance part which comes from the long distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair l^+l^-. Therefore, one can write it as

$$C_9^{\mathrm{eff}} = C_9 + Y(s) + C_9^{\mathrm{res}}, \quad (30)$$

where $s = q^2$ and the function $Y(s)$ denotes the perturbative part coming from one-loop matrix elements of the four quark operators and is given by [19]

$$Y(s) = g(m_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) - \tfrac{1}{2}g(0,s)(C_3 + 3C_4) - \tfrac{1}{2}g(m_b,s)(4C_3 + 4C_4 + 3C_5 + C_6) + \tfrac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \quad (31)$$

where

$$g(m_i, s) = -\frac{8}{9}\ln(m_i/m_b^{\mathrm{pole}}) + \frac{8}{27} + \frac{4}{9}y_i - \frac{2}{9}(2+y_i) \times\sqrt{|1-y_i|}\Big\{\Theta(1-y_i)\Big[\ln\Big(\frac{1+\sqrt{1-y_i}}{1-\sqrt{1-y_i}}\Big) - i\pi\Big] + \Theta(y_i - 1)2\arctan\frac{1}{\sqrt{y_i - 1}}\Big\}, \quad (32)$$

with $y_i = 4m_i^2/s$. The values of the coefficients C_i's in NLL order are taken from [24].

The long distance resonance effect is given as [25]

$$C_9^{\rm res} = \frac{3\pi}{\alpha^2}(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \times \sum_{V_i=\psi(1S),\ldots,\psi(6S)} \kappa_{V_i} \frac{m_{V_i}\Gamma(V_i \to l^+l^-)}{m_{V_i}^2 - s - im_{V_i}\Gamma_{V_i}}. \quad (33)$$

The phenomenological parameter κ is taken to be 2.3, so as to reproduce the correct branching ratio of $\mathrm{Br}(B \to J/\psi K^* l^+ l^-) = \mathrm{Br}(B \to J/\psi K^*)\mathrm{Br}(J/\psi \to l^+l^-)$.

The matrix elements of the various hadronic currents in (29) between initial Λ_b and the final Λ baryon are parametrized in terms of various form factors as defined in Eqs. (3) and (24). Thus, using these matrix elements, the transition amplitude can be written as

$$\begin{aligned}\mathcal{M}(\Lambda_b \to \Lambda l^+ l^-) &= \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^*[\bar{l}\gamma_\mu l\{\bar{u}_\Lambda(\gamma^\mu(A_1P_R + B_1P_L) \\ &+ i\sigma^{\mu\nu}q_\nu(A_2P_R + B_2P_L))u_{\Lambda_b}\} \\ &+ \bar{l}\gamma_\mu\gamma_5 l\{\bar{u}_\Lambda(\gamma^\mu(D_1P_R + E_1P_L) \\ &+ i\sigma^{\mu\nu}q_\nu(D_2P_R + E_2P_L) \\ &+ q^\mu(D_3P_R + E_3P_L))u_{\Lambda_b}\}], \end{aligned} \quad (34)$$

where the various parameters A_i, B_i and D_j, E_j ($i = 1, 2$ and $j = 1, 2, 3$) are defined as

$$\begin{aligned} A_i &= \frac{1}{2}C_9^{\rm eff}(g_i - G_i) - \frac{C_7m_b}{q^2}(f_i + F_i), \\ B_i &= \frac{1}{2}C_9^{\rm eff}(g_i + G_i) - \frac{C_7m_b}{q^2}(f_i - F_i), \\ D_j &= \frac{1}{2}C_{10}(g_j - G_j), \\ E_j &= \frac{1}{2}C_{10}(g_j + G_j). \end{aligned} \quad (35)$$

We will consider here the case when the final Λ baryon is unpolarized. The physical observables in this case are the differential decay rate and the forward-backward (FB) rate asymmetries. From the transition amplitude (34), one can obtain the double differential decay rate [26] as

$$\frac{d^2\Gamma}{d\hat{s}dz} = \frac{G_F^2\alpha^2}{2^{12}\pi^5}|V_{tb}V_{ts}^*|^2 m_{\Lambda_b} v_l \lambda^{1/2}(1, r, \hat{s})\mathcal{K}(s, z), \quad (36)$$

where $\hat{s} = s/m_{\Lambda_b}^2$, $z = \cos\theta$, the angle between p_{Λ_b} and p_+ in the center-of-mass frame of the l^+l^- pair, $v_l = \sqrt{1 - 4m_l^2/s}$, and $\lambda(a, b, c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)}$ is the usual triangle function. The function $\mathcal{K}(s, z)$ is given as

$$\mathcal{K}(s, z) = \mathcal{K}_0(s) + z\mathcal{K}_1(s) + z^2\mathcal{K}_2(s), \quad (37)$$

with

$$\begin{aligned} \mathcal{K}_0(s) &= 32m_l^2m_{\Lambda_b}^2\hat{s}(1 + r - \hat{s})(|D_3|^2 + |E_3|^2) + 64m_l^2m_{\Lambda_b}^3(1 - r - \hat{s})\,\mathrm{Re}(D_1^*E_3 + D_3E_1^*) \\ &+ 64m_{\Lambda_b}^2\sqrt{r}(6m_l^2 - \hat{s}m_{\Lambda_b}^2)\,\mathrm{Re}(D_1^*E_1) + 64m_l^2m_{\Lambda_b}^3\sqrt{r}(2m_{\Lambda_b}\hat{s}\,\mathrm{Re}(D_3^*E_3) + (1 - r + \hat{s})\,\mathrm{Re}(D_1^*D_3 + E_1^*E_3)) \\ &+ 32m_{\Lambda_b}^2(2m_l^2 + m_{\Lambda_b}^2\hat{s})((1 - r + \hat{s})m_{\Lambda_b}\sqrt{r}\,\mathrm{Re}(A_1^*A_2 + B_1^*B_2) - m_{\Lambda_b}(1 - r - \hat{s})\,\mathrm{Re}(A_1^*B_2 + A_2^*B_1) \\ &- 2\sqrt{r}[\mathrm{Re}(A_1^*B_1) + m_{\Lambda_b}^2\hat{s}\,\mathrm{Re}(A_2^*B_2)]) + 8m_{\Lambda_b}^2(4m_l^2(1 + r - \hat{s}) + m_{\Lambda_b}^2[(1 - r)^2 - \hat{s}^2])(|A_1|^2 + |B_1|^2) \\ &+ 8m_{\Lambda_b}^4(4m_l^2[\lambda + (1 + r - \hat{s})\hat{s}] + m_{\Lambda_b}^2\hat{s}[(1 - r)^2 - \hat{s}^2])(|A_2|^2 + |B_2|^2) - 8m_{\Lambda_b}^2(4m_l^2(1 + r - \hat{s}) \\ &- m_{\Lambda_b}^2[(1 - r)^2 - \hat{s}^2])(|D_1|^2 + |E_1|^2) + 8m_{\Lambda_b}^5\hat{s}v_l^2(-8m_{\Lambda_b}\hat{s}\sqrt{r}\,\mathrm{Re}(D_2^*E_2) + 4(1 - r + \hat{s}) \\ &\times \sqrt{r}\,\mathrm{Re}(D_1^*D_2 + E_1^*E_2) - 4(1 - r - \hat{s})\,\mathrm{Re}(D_1^*E_2 + D_2^*E_1) + m_{\Lambda_b}[(1 - r)^2 - \hat{s}^2][|D_2|^2 + |E_2|^2]), \end{aligned} \quad (38)$$

$$\begin{aligned} \mathcal{K}_1(s) &= -16m_{\Lambda_b}^4\hat{s}v_l\sqrt{\lambda}\{2\,\mathrm{Re}(A_1^*D_1) - 2\,\mathrm{Re}(B_1^*E_1) \\ &+ 2m_{\Lambda_b}\,\mathrm{Re}(B_1^*D_2 - B_2^*D_1 + A_2^*E_1 - A_1^*E_2)\} \\ &+ 32m_{\Lambda_b}^5\hat{s}v_l\sqrt{\lambda}\{m_{\Lambda_b}(1 - r)\,\mathrm{Re}(A_2^*D_2 - B_2^*E_2) \\ &+ \sqrt{r}\,\mathrm{Re}(A_2^*D_1 + A_1^*D_2 - B_2^*E_1 - B_1^*E_2)\}, \end{aligned} \quad (39)$$

and

$$\begin{aligned} \mathcal{K}_2(s) &= 8m_{\Lambda_b}^6v_l^2\lambda\hat{s}((|A_2|^2 + |B_2|^2 + |D_2|^2 + |E_2|^2) \\ &- 8m_{\Lambda_b}^4v_l^2\lambda(|A_1|^2 + |B_1|^2 + |D_1|^2 + |E_1|^2). \end{aligned} \quad (40)$$

The dilepton mass spectrum can be obtained from (36) by integrating out the angular dependent parameter z which yields

$$\left(\frac{d\Gamma}{ds}\right)_0 = \frac{G_F^2\alpha^2}{2^{11}\pi^5 m_{\Lambda_b}}|V_{tb}V_{ts}^*|^2 v_l\sqrt{\lambda}\left[\mathcal{K}_0(s) + \frac{1}{3}\mathcal{K}_2(s)\right], \quad (41)$$

where λ is the shorthand notation for $\lambda(1, r, \hat{s})$. The limits for s are

$$4m_l^2 \le s \le (m_{\Lambda_b} - m_\Lambda)^2. \quad (42)$$

Apart from the branching ratio in semileptonic decay, there are also other observables which are sensitive to new physics contributions in $b \to s$ transition. One such observable is the forward-backward asymmetry ($A_{\rm FB}$) of leptons which is also a very powerful tool for looking for new physics. The normalized forward-backward asymmetry is obtained by integrating the double differential decay width ($d^2\Gamma/d\hat{s}dz$) with respect to the angular variable z

$$A_{\rm FB}(s) = \frac{\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} dz - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz} dz}{\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} dz + \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz} dz}. \tag{43}$$

Thus one obtains from (36)

$$A_{\rm FB}(s) = \frac{\mathcal{K}_1(s)}{\mathcal{K}_0(s) + \mathcal{K}_2(s)/3}. \tag{44}$$

The FB asymmetry becomes zero for a particular value of dilepton invariant mass. Within the SM, the zero of $A_{\rm FB}(s)$ appears in the low q^2 region, sufficiently away from the charm resonance region and hence can be predicted precisely. The position of the zero value of $A_{\rm FB}$ is very sensitive to the presence of new physics.

For numerical evaluation we use the input parameters as presented in the previous sections. The quark masses (in GeV) used are $m_b = 4.6$, $m_c = 1.5$, and $\alpha = 1/128$ and the weak mixing angle $\sin^2\theta_W = 0.23$. The variation of differential branching ratios (41) and the forward-backward asymmetries (44) for the processes $\Lambda_b \to \Lambda\mu^+\mu^-$ and $\Lambda_b \to \Lambda\tau^+\tau^-$ in the standard model are shown in Figs. 4 and 5, respectively.

As discussed earlier in the presence of the fourth generation, the Wilson coefficients $C_{7,9,10}$ will be modified due to the new contributions arising from the virtual t' quark in the loop. Thus, these coefficients will be modified as

$$\begin{aligned} C_7^{\rm tot}(\mu) &= C_7(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_7'(\mu), \\ C_9^{\rm tot}(\mu) &= C_9(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_9'(\mu), \\ C_{10}^{\rm tot}(\mu) &= C_{10}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_{10}'(\mu). \end{aligned} \tag{45}$$

The new coefficients $C'_{7,9,10}$ can be calculated at the M_W scale by replacing the t-quark mass by m'_t in the loop functions. These coefficients are then to be evolved to the b scale using the renormalization group equation as discussed in [19]. The values of the new Wilson coefficients at the m_b scale for $m_{t'} = 400$ GeV are given by $C'_7(m_b) = -0.355$, $C'_9(m_b) = 5.831$, and $C'_{10} = -17.358$.

Thus, one can obtain the differential branching ratio and the forward-backward asymmetry in SM4 by replacing $C_{7,9,10}$ in Eqs. (41) and (44) by $C^{\rm tot}_{7,9,10}$. Using the values of the $|\lambda'_t|$ and ϕ_s for $m_{t'} = 400$ GeV, differential branching ratio and the forward-backward asymmetry for

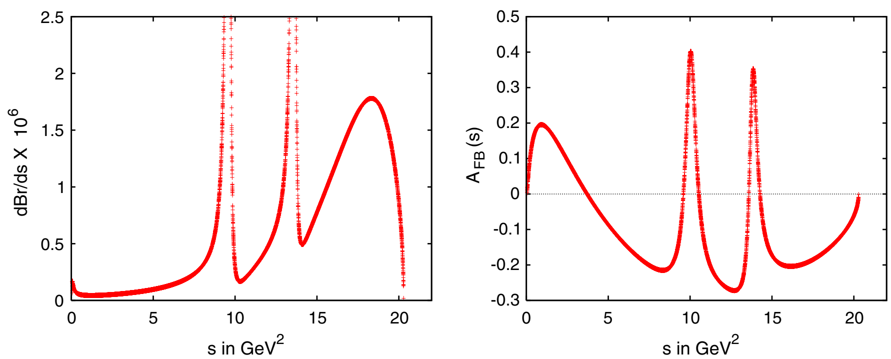

FIG. 4 (color online). The differential branching ratio $d\text{Br}/ds$ versus s (left panel) and the forward-backward asymmetry [$A_{\rm FB}(s)$] versus s (right panel) for the process $\Lambda_b \to \Lambda\mu^+\mu^-$.

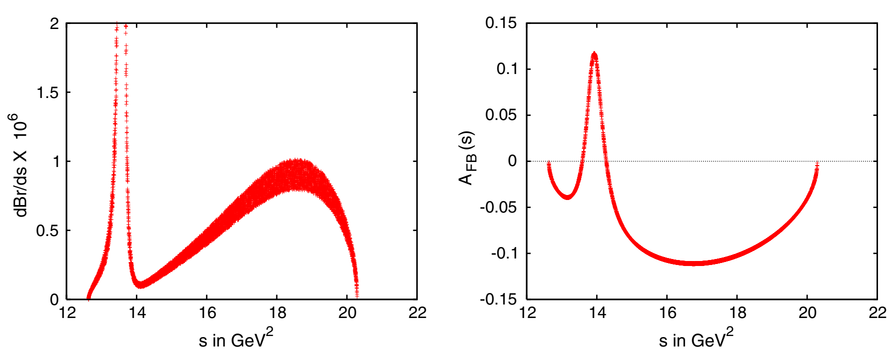

FIG. 5 (color online). Same as Fig. 4 for the process $\Lambda_b \to \Lambda\tau^+\tau^-$.

$\Lambda_b \to \Lambda\mu^+\mu^-$ are presented in Fig. 6, where we have not considered the contributions from intermediate charmonium resonances. From the figure it can be seen that the differential branching ratio of this mode is significantly enhanced from its corresponding SM value, whereas the forward-backward asymmetry is slightly reduced with respect to its SM value. However, the zero position of the FB asymmetry remains unchanged in the fourth quark generation model. Similarly for the process $\Lambda_b \to \Lambda\tau^+\tau^-$ as seen from Fig. 7, the branching ratio is significantly enhanced from its SM value, whereas the FB asymmetry remains almost unaffected in the SM4.

We now proceed to calculate the total decay rates for $\Lambda_b \to \Lambda l^+ l^-$ for which it is necessary to eliminate the backgrounds coming from the resonance regions. This can be done by using the following veto windows so that the backgrounds coming from the dominant resonances $\Lambda_b \to \Lambda J/\psi(\psi')$ with $J/\psi(\psi') \to l^+ l^-$ can be eliminated,

$$\begin{aligned} \Lambda_b \to \Lambda\mu^+\mu^-: &\quad m_{J/\psi} - 0.02 < m_{\mu^+\mu^-} < m_{J/\psi} + 0.02, \\ : &\quad m_{\psi'} - 0.02 < m_{\mu^+\mu^-} < m_{\psi'} + 0.02, \\ \Lambda_b \to \Lambda\tau^+\tau^-: &\quad m_{\psi'} - 0.02 < m_{\tau^+\tau^-} < m_{\psi'} + 0.02. \end{aligned}$$

TABLE IV. The branching ratios (in units of 10^{-6}) for various decay processes.

Decay modes	$\mathrm{Br}^{\mathrm{SM}}$	$\mathrm{Br}^{\mathrm{SM4}}$
$\Lambda_b \to \Lambda\mu^+\mu^-$	13.25	(14.7 → 53.5)
$\Lambda_b \to \Lambda\tau^+\tau^-$	3.83	(4.3 → 16.0)

Using these veto windows we obtain the branching ratios for semileptonic rare Λ_b decays which are presented in Table IV. It is seen from the table that the branching ratios obtained in the fourth quark generation model are reasonably enhanced from the corresponding SM values and could be observed in the LHCb experiment.

V. CONCLUSION

In this paper we have studied several rare decays of Λ_b baryon, i.e., $\Lambda_b \to \Lambda\pi$, $\Lambda_b \to pK^-$, $\Lambda_b \to \Lambda\gamma$, and $\Lambda_b \to \Lambda l^+ l^-$ in the fourth quark generation model. This model is a very simple extension of the standard model with three generations and it provides a simple explanation for several indications of new physics that have been

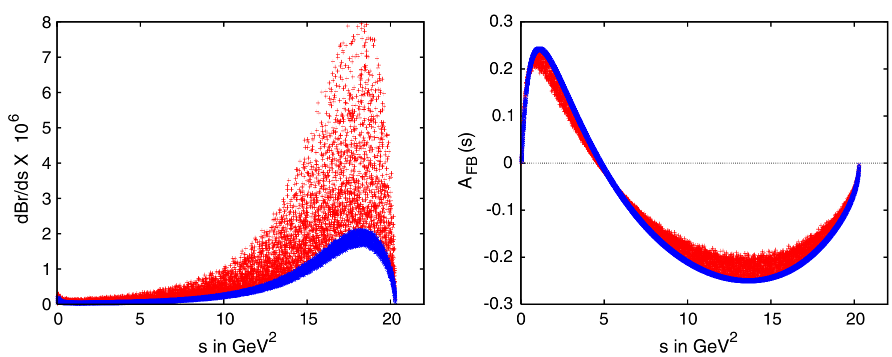

FIG. 6 (color online). Variation of the differential branching ratio (left panel) and the forward-backward asymmetry (right panel) with respect to the momentum transfer s for the process $\Lambda_b \to \Lambda\mu^+\mu^-$, in the fourth quark generation model [red (gray) regions] whereas the corresponding SM values are shown by blue (dark gray) regions.

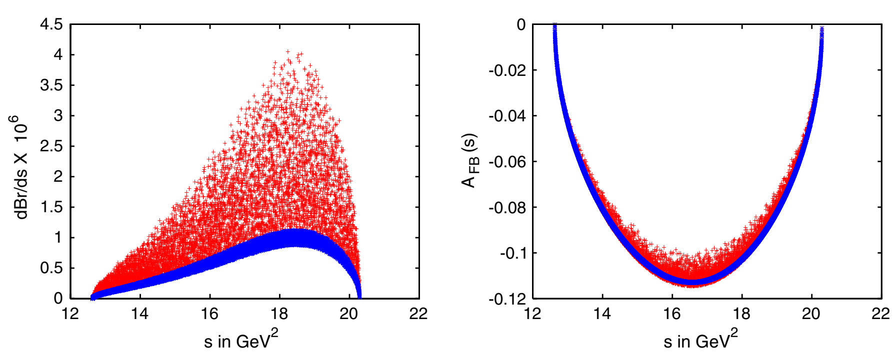

FIG. 7 (color online). Same as Fig. 6 for the process $\Lambda_b \to \Lambda\tau^+\tau^-$.

observed involving CP asymmetries in the B, B_s decays for m_t' in the range of (400–600) GeV. We found that in this model the branching ratios of the various decay modes considered here ($\Lambda_b \to \Lambda\pi$, $\Lambda_b \to pK^-$, $\Lambda_b \to \Lambda\gamma$, and $\Lambda_b \to \Lambda l^+ l^-$) are significantly enhanced from their corresponding SM values. However the forward-backward asymmetries in the $\Lambda_b \to \Lambda l^+ l^-$ processes do not differ much from those of the SM expectations. The zero point of the F_{AB} for $\Lambda_b \to \Lambda l^+ l^-$ process is also found to be unaffected in this model.

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