Electrostatic Forces in Fixed-Fixed Microbeams under Direct and Fringing Field Effects

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Abstract— **We propose simple approximate expressions for capacitance and electrostatic force for fixed-fixed beam based MEMS/NEMS devices subjected to direct electrostatic and fringing field effects. The configuration that are considered for study are fixed-fixed beam and bottom electrode, fixed-fixed beam and side electrode, and a combination of beam, bottom electrode and side electrode. The expressions are evaluated based on the numerical result obtained using FEA analysis in COMSOL software. The accuracy of the proposed formulas is compared with available literature. The formulas proposed in this paper are valid for large operating range and they can also be used for array applications.**

Index Terms—**Capacitance, Electrostatic force, fringing fields.**

I. INTRODUCTION

Electrostatic actuation is widely used method for driving MEMS/NEMS devices. It can be found under the assumptions of parallel plate capacitance. When voltage difference exists between two electrodes, it induces electrostatic field which can be used to find the driving force between plates. The net electrostatic field is due to the direct field and fringing field effects. Much effort has been made to evaluate the expression for capacitance (which can be used to find electrostatic force) considering the effect of fringing electrostatic field by numerical analysis [1]-[3]. Palmer [1] and Chang [2] used Schwartz-Christoffel conformal mapping transformation to derive approximate formulas for parallel-plate capacitance per unit length, however, they look complicated. Sakurai [3] proposed simple approximate analytical expressions for capacitances through numerical analysis for thin wires over the ground in VLSI circuits, but the operating range of the formula is limited. In this paper, we propose the simple formulas for finding capacitance and electrostatic force for different configurations of fixed-fixed MEMS/NEMS beam under the direct and fringing electrostatic fields. The configurations considered for study are fixed-fixed beam and bottom electrode, fixed-fixed beam and side electrode, and a combination of fixed-fixed beam, bottom electrode and side electrode. The formulas are evaluated based on the numerical solution obtained from FEA analysis using COMSOL software. After comparing the new formula with the given results, we also validate its effectiveness over the wider operating range. In the subsequent section, we present the modeling and procedure of finding the formula for simple configuration. Finally, we use the formula to compute the capacitance and force in complex geometries.

II. BEAM AND BOTTOM ELECTRODE

In this section, we evaluate the approximate expression for capacitance and electrostatic force for fixed-fixed beam and bottom electrode configuration subjected to direct and fringing field effects. To find the expression, we first compute the numerical values of capacitance in COMSOL. The front view and the top view of fixed-fixed beam and bottom electrode are shown in Figs.1 (a) and (b). In order to capture the correct fringing field effects from the side and top surfaces of the beam, we extend the side boundaries as well as the top boundary by more than three times the width of the beam. In this configuration, the width of the bottom electrode is taken as seven times the width of the beam. Figure 1(c) shows the 3D image of a beam, bottom electrode and the outer boundaries in COMSOL.
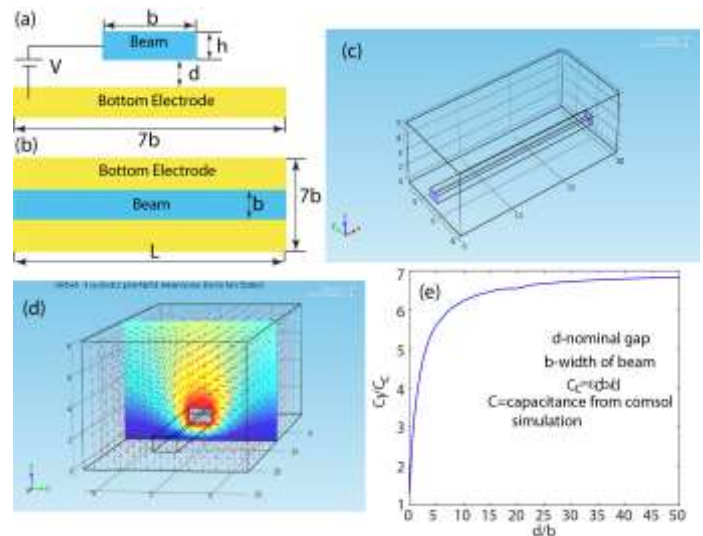

Fig. 1. (a) Front view of fixed-fixed beam and bottom electrode with respective notations for dimensions. (b) Top view of fixed-fixed beam and bottom electrode. (c) COMSOL model. (d) Direct and fringing electrostatic field distribution. (e) Normalized capacitance versus normalized nominal distance.

Figure 1(d) shows the distribution of direct electrostatic and fringing field lines subjected the potential difference between the beam and the bottom electrode. Figure 1(e)

shows the variation of non-dimensional capacitance with the non-dimensional gap between the beam and the bottom electrode. Subsequently, we obtain an empirical relation between the normalized capacitance versus normalized nominal gap based on the numerical results as shown in Fig. 1(e). An approximate expression for capacitance per unit length for this configuration is found to be

$$C_1 = \left[-0.0208 \left(\frac{d}{b}\right)^4 + 0.25 \left(\frac{d}{b}\right)^3 - 1.2 \left(\frac{d}{b}\right)^2 + 3.2 \left(\frac{d}{b}\right) + 1.1 \right] C_C$$
$$\text{for} \quad 0.03 \le d/b \le 4.5$$

$$C_1 = \left[0.00034 \left(\frac{d}{b}\right)^3 - 0.0182 \left(\frac{d}{b}\right)^2 + 0.34 \left(\frac{d}{b}\right) + 4.32 \right] C_C \quad (1)$$
$$\text{for} \quad 4.5 \le d/b \le 20$$

$$C_1 = \left[5.46 \times 10^{-6} \left(\frac{d}{b}\right)^3 - 0.00078 \left(\frac{d}{b}\right)^2 + 0.041 \left(\frac{d}{b}\right) + 6.048 \right] C_C$$
$$\text{for} \quad 20 \le d/b \le 50$$

where $C_c = \varepsilon_0 b / d$ is the capacitance of parallel plates without fringing effects, and ε_0 is the permittivity of free space. The corresponding electrostatic forces can also be derived by differentiating the energy stored in capacitor associated with the capacitances mentioned in Equation 1. The approximate expression for the electrostatic force per unit length can also be found as

$$F_1 = \frac{1}{2} \frac{V^2 \varepsilon_0}{b^3} \left| \frac{-0.0624(d-w)^4 + 0.5b(d-w)^3 - 1.2b^2(d-w)^2 - 1.1b^4}{(d-w)^2} \right|$$
$$\text{for} \quad 0.03 \le d/b \le 4.5$$

$$F_1 = \frac{1}{2} \frac{V^2 \varepsilon_0}{b^2} \left| \frac{0.00068(d-w)^3 - 0.0182(d-w)^2 - 4.32b^3}{(d-w)^2} \right| \quad (2)$$
$$\text{for} \quad 4.5 \le d/b \le 20$$

$$F_1 = \frac{1}{2} \frac{V^2 \varepsilon_0}{b^2} \left| \frac{1.092 \times 10^{-5}(d-w)^3 - 0.00078b(d-w)^2 - 4.32b^3}{(d-w)^2} \right|$$
$$\text{for} \quad 20 \le d/b \le 50$$

Figure 2(a) shows the comparison of capacitance/unit length verses uniform deflection of beam between the proposed formula (Eq. 1), that given by Sakurai [3], numerical solution obtained by COMSOL and with the expression without fringing effects. Also, Figure 2(b) shows the comparison of electrostatic force per unit length verses uniform deflection obtained from the proposed model from Equation 2 , that from Sakurai [3] , Dumitru [4] and the expression without fringing effects. Under the valid operating range, all of them vary similar to each other.
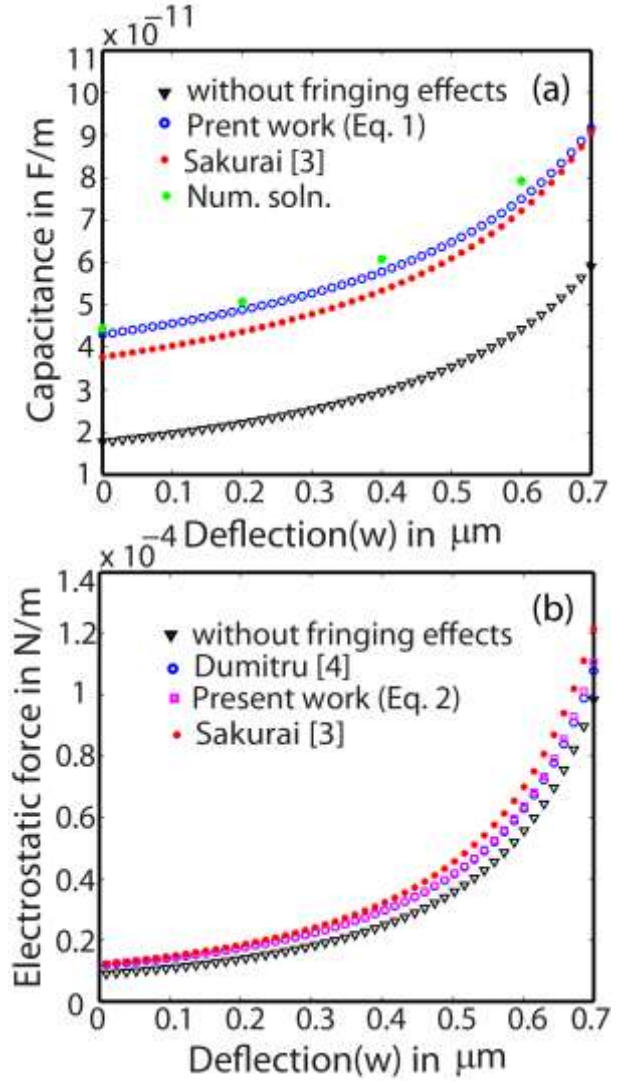

Fig. 2. (a) Comparison of capacitance/unit length verses uniform deflection of the beam between present Eq. 1, Sakurai [3], Num. soln.(COMSOL) and the expression without fringing effects. (b) Comparison of electrostatic force/unit length verses uniform deflection between present work Eq. 2, Sakurai [3], Dumitru [4] and the expression without fringing effects. For b=2µm, h=0.2µm, d=1µm, ε_0 =8.854×10^{-12}.

III. BEAM AND SIDE ELECTRODE

In this section we evaluate the capacitance and electrostatic force for configuration consisting of fixed-fixed beam and a side electrode subjected to the direct electrostatic and fringing field effects. The front view and top view of the configuration consisting of beam and electrodes are shown in Figs. 3(a) and 3(b). Figures 3(c) and 3(d) show the COMSOL model and electrostatic field distribution for given potential difference. Finally, we obtain Figure 3(e) shows the variation of normalized capacitance versus normalized nominal gap.
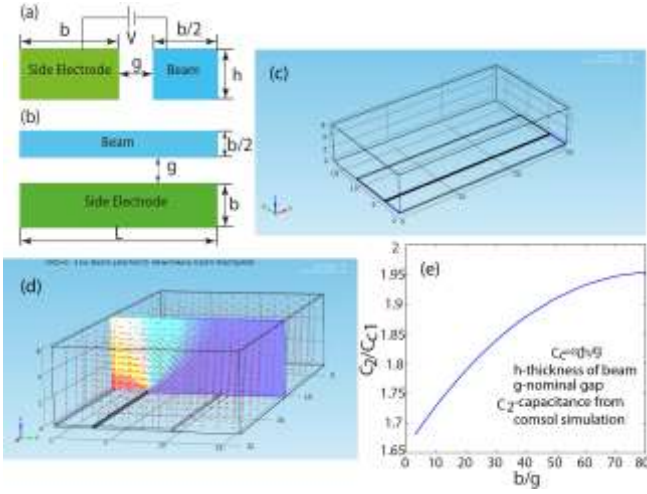
Fig. 3. (a) Front view of beam and side electrode. (b) Top view of beam and side electrode. (c) COMSOL model used for simulation. (d) Direct and fringing electrostatic field distribution. (e) Normalized capacitance Vs. normalized nominal gap

A numerical fit is found for the graph shown in Figure 3(e) and capacitance per unit length is evaluated and is given as

$$C_2 = \left[7.9 \times 10^{-8} \left(\frac{b}{g} \right)^3 - 5.5 \times 10^{-5} \left(\frac{b}{g} \right)^2 + 0.0076 \left(\frac{b}{g} \right) + 1.658 \right] C_{C1}$$

$$\text{for} \quad 1 \le b/g \le 70 \qquad (3)$$

where, $C_{C1} = \varepsilon_0 h / g$. By differentiating the energy stored in capacitor associated with capacitance given by Equation 3, the electrostatic force per unit length is evaluated and is given as

$$F_2 = \frac{1}{2} V^2 \varepsilon_0 h \left| \frac{\begin{pmatrix} -3.16 \times 10^{-7} b^3 + 16.5 \times 10^{-5} b^2 (g-w) \\ -0.0152 b (g-w)^2 - 1.658 (g-w)^3 \end{pmatrix}}{(g-w)^5} \right|$$

$$\text{for} \quad 1 \le b/g \le 70 \qquad (4)$$

Figure 4(a) shows the comparison of capacitance/unit length verses deflection of beam between present Equation 3, Sakurai [3], numerical solution obtained by using COMSOL and the expression without fringing effects. Figure 4(b) shows the comparison of electrostatic force\unit length verses deflection between present Equation 4, the expression by Sakurai [3] and the expression without fringing effects.
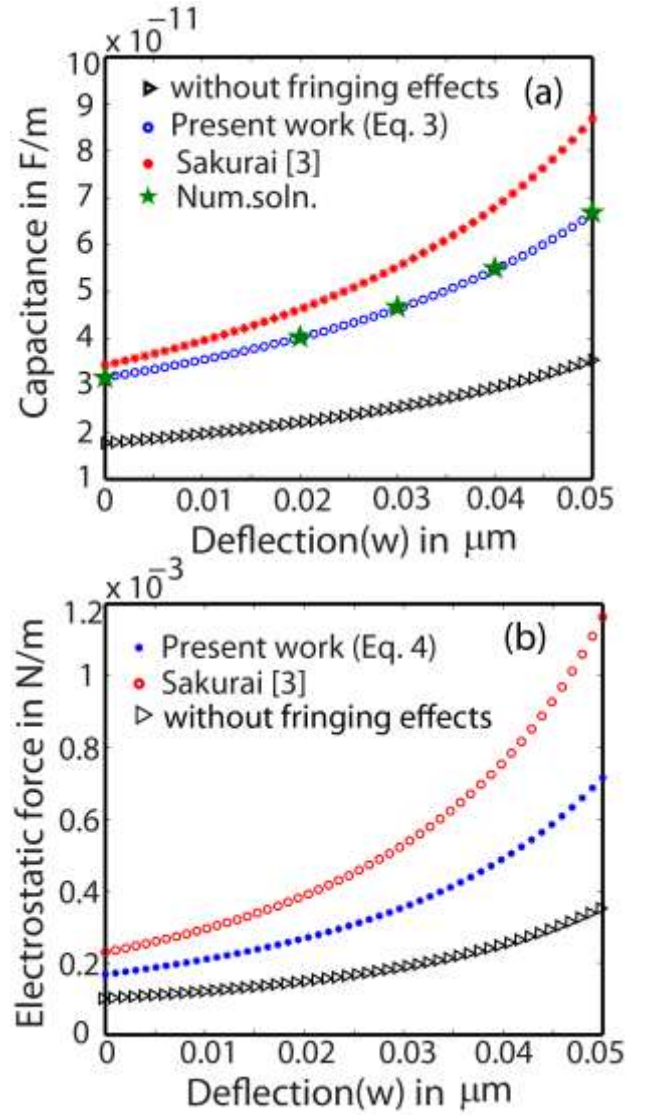

Fig. 4. (a) Comparison of capacitance/unit length verses deflection of beam between present Eq. 3, Sakurai [3], Num. soln. (COMSOL) and the expression without fringing effects. (b) Comparison of electrostatic force/unit length verses deflection between present work Eq. 4 , Sakurai [3], and the expression without fringing effects. For b=2μm, h=0.2μm, d=1μm, g=0.1μm, ε_0 =8.854×10^{-12}.

IV. COMBINATION OF BEAM, BOTTOM ELECTRODE AND SIDE ELECTRODE

In this section, we evaluate the capacitance and electrostatic expression for the configuration consisting of a beam, bottom electrode and a side electrode. Figures 5 (a), (b), and (c) shows front view, top view, COMSOL model and field distribution. It can be seen that it is a combination of the models discussed in section II and section III.
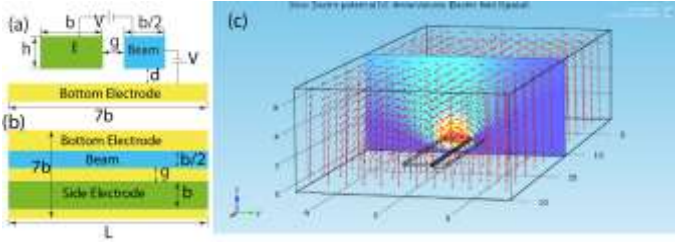
Fig. 5. (a) Front view of combination of beam, bottom electrode and side electrode. (b) Top view of configuration. (c) Comsol model and electrostatic field distribution.

Total capacitance per unit length for this configuration is summation of capacitance of configurations shown in Figures 1 and 3. Therefore the total capacitance per unit length and electrostatic force per unit length are given as

$$C_3 = C_1 + C_2$$
$$\text{for} \quad \begin{array}{l} 0.03 \le d/b \le 50 \\ 1 \le b/g \le 70 \end{array} \quad (5)$$

$$F_3 = F_1 + F_2$$
$$\text{for} \quad \begin{array}{l} 0.03 \le d/b \le 50 \\ 1 \le b/g \le 70 \end{array} \quad (6)$$

Figure 6(a) shows the comparison of capacitance/unit length verses deflection of beam between present Equation 5, that given by Sakurai [3], numerical solutions and the expression without fringing effects. Figure 6(b) shows the comparison of electrostatic force\unit length verses deflection between present Equation 6, expression by Sakurai [3] and the expression without fringing forces. It is noticed from the Figure 6(a) that relative error of Equation 5 with numerical solution is less than 5%. Therefore, Equations 5 and 6 can be used efficiently with relative error less than 5%.

V. CONCLUSION

All the expressions given in this work are validated with the Sakurai [3], Dumitru [4], and numerical solutions. All of them vary similarly, which indicates that our formulations are valid. The expression used by Dumitru [4] is based on first order approximation and which cannot be used for array applications. Sakurai [4] evaluated expressions for capacitance numerically which considers the coupled effect of bottom and thin wires in VLSI circuit and he did not evaluate the expressions which considers individual effects. The formulas given by Sakurai are defined for range $0.3 \le b/d \le 10$, $0.3 \le h/d \le 10$ and $0.5 \le g/d \le 10$. Whereas, in this work we first evaluated the expressions for capacitance and force considering individual effects of bottom and side electrodes separately and then we used these expressions to study the combined effects. And also the formulas are defined for wider range, thus, enhancing the sensing effect and utilizing the fringing field effectively. We have also extended our study for arrays which shows good results which are not found in literature.
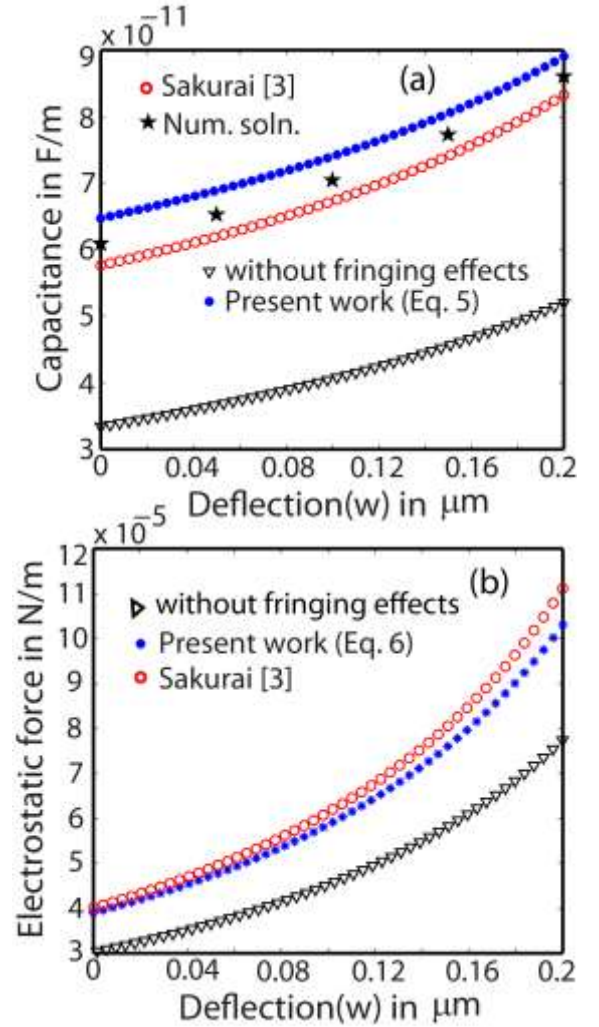

Fig. 6. (a) Comparison of capacitance/unit length verses deflection of beam between present Eq. 5, Sakurai [3], Num. soln. (COMSOL) and the expression without fringing effects. (b) Comparison of electrostatic force/unit length verses deflection between present work Eq. 6 , Sakurai [3] and the expression without fringing effects. For b=2μm, h=0.2μm, d=0.6μm, g=0.4μm, ε_0 =8.854×10^{-12}.

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