# Vibro-acoustic analysis of the Veena

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A Thesis Submitted to

Indian Institute of Technology Hyderabad

In Partial Fulfillment of the Requirements for

The Degree of Master of Technology

भारतीय प्रौद्योगिकी संस्थान हैदराबाद

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July 2015

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# Acknowledgements

I would like to thank my adviser Dr. B Venkatesham for his continuous support, motivation and valuable guidance. This work could not have been achieved if it was not for his encouragement and solutions to many problems that I faced. I thank my fellow lab members of Dynamics and Acoustic Lab, Mr. Mayur, Mr. Pravin, Mr. Sachin, Mr. Nagaraja and Mr. Tapan, for their constant support and friendly environment in the lab. I would also like to thank Ms. Raghavi and Ravi Music who helped me to understand the Veena as a musical instrument. I would also thank Dr. Surya from manufacturing department and Mr. K Sathyanarayana from workshop letting me use the workshop facilities. I would also like to thank entire workshop staff for their valuable assistance. As a friend my special thanks to Mr. Yagnik for his support and advice he given at every step. Also the help and support provided by my class mates and friends at IIT Hyderabad was exceptional.

# Dedication

*Dedicated to*

*My Loving Parents and Brother*

# Abstract

Sarasvati Veena is an ancient musical instrument present in South India. A good amount of literature is available on musical studies and practices but very limited knowledge on construction and systematic mechanical analysis with modern computational tools. Veena is a complex mechanical system and very good example of structural-acoustic coupled problem. The present work is an attempt to understand dynamical behaviour of Veena with advanced computational tools and experimental methods. Dynamic behaviour formulated by natural frequencies, mode shapes and damping factor. And it is determined by its components like wooden resonator, acoustic cavity, top-plate with the bridge and strings with twenty four fret supports placed in the logarithmic manner over the fret board. Along with the direct sound radiated by plucked string, energy gets transfer through a bridge to a top-plate to acoustical cavity and wooden resonator. Acoustic cavity and wooden resonator plays an important role in the sound radiation which makes it two significant entities to study and analyse as a whole structural-acoustic coupled problem. Veena covers the overall frequency range of 90 to 6000 Hz. This research aimed to establish a methodology to study and analyse the structural-acoustic coupled problem in Veena. Methodology includes experimental and numerical analysis of vibration behaviour.

Modal analysis is considered as a first step in any vibration analysis of the system. It is a process of determining the dynamic characteristics of the system in the form of natural frequencies, mode shapes and damping factor. These parameters are used to formulate the mathematical model for system's dynamic behaviour. Acoustic cavity with structural resonator has been considered as a whole structural - acoustic coupled system for experimental modal analysis. Geometry creation is done in the form of wire frame model. Coordinate measuring machine (CMM) has been used to get the exact coordinates of every point with local frame of reference and later coordinates transformation has been used to transform it into one global frame of reference. Roving hammer method has been used with a tri axial sensor on the top plate and a uni axial sensor on the resonator. Responses of the 160 points over the resonator and top plate were recorded at an average of three hits at every point. Numerical analysis has been done considering three different cases structural, acoustical and structural-acoustic coupled modal analysis. Boundary impedance considered as zero for acoustic analysis and clamped boundary conditions for structural analysis. The flexible structural surface area is linked with acoustic surface area in the coupled analysis. Modal Assurance Criteria (MAC) number has been calculated to study and analyse the spatial match between the experimental and numerical results. A comparative analysis of the three numerical cases done to understand the contribution of acoustic and structural modes in the coupled analysis. It will help to us find the a critical structural components in further acoustic analysis.

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# Chapter 1

# Introduction

## 1.1 Motivation

Sarasvati Veena is an ancient musical instrument present in South India. A good amount of literature is available on musical studies and practices but very limited knowledge on construction and systematic mechanical analysis with modern computational tools. Veena is a complex mechanical system and very good example of structural-acoustic coupled problem. Dynamic behaviour/properties of the Veena formulated by natural frequency, mode shapes and damping factor. These parameters are not yet studied extensively using FEA computational tools and modern experimental techniques.

Veena consist of components like wooden resonator, acoustic cavity, top-plate with the bridge and strings with twenty four fret supports placed in the logarithmic manner over the fret board. Along with the direct sound radiated by plucked string, energy gets transfer through the bridge to top-plate to acoustical cavity and wooden resonator. Acoustic cavity and wooden resonator plays an important role in the sound radiation. Acoustic air cavity adds additional stiffness to overall system and it possess its own independent dynamic properties. When Acoustic impedance of the air cavity is closer to the structural impedance, the acoustic wave excites the structure. The vibration of structure will induce acoustic pressure inside the air cavity. These phenomena will continue under coupling conditions. Coupling conditions depends on the acoustic and structural natural frequencies and spatial match of the mode shapes. This is termed as structural-acoustic coupling. Hence wooden resonator and acoustic cavity are two significant entities to be studied and analyzed as a whole structural-acoustic coupled problem.

Closed form solutions for structural acoustic coupled problems are available for simple geometries like simply supported plate, rectangular duct etc. FEA approached has been developed to study and analyze complex and irregular geometries. In particular, calculation of coupling coefficient and transfer factor for coupled system needs to be studied for complex and irregular geometry.

## 1.2 Literature Review

Musical instruments are always areas of research for many design engineers. Significant research is done on various types of musical instruments like string, bowled, wind, and percussion instruments. Specific to string instruments major research is done on the different types of guitars and violins. Veena is an ancient musical instrument found in south India. Its geometric complexity and sound radiation properties differentiate itself very uniquely. Veena covers an overall frequency range of 80 to 6500 Hz [1] spanning over about three and half octaves. An octave is a range over which the frequency doubles. In the year of 2001, P P Rao [2] studied the musical notes produced on Veena which are based on the physical and mathematical formulations associated with the vibrations of stretched strings. C V Raman [3] studied the geometric significance and effect of bridge on sound radiation.

Modal analysis is considered as a first step in any dynamic analysis of the system. System dynamic properties like natural frequencies, mode shapes and damping factor found by experimental and numerical analysis. Complete study of the resonator of Veena must include not only the wooden structure but also the air cavity. M. J. Elejabarrieta [4] studied the cavity modes of the guitar box using numerical computation. He found that the air-structure coupling lowers the natural frequency, especially for those modes in which the compressions and expansions imply volume changes. A. Ezcurra [5] presented a comparative study of the guitar box in which the interior gas is changed both experimentally and numerically. Modal patterns, natural frequencies and quality factors are determined when the box is full of helium, air and krypton. respectively. This allowed to characterize the soundboard back plate coupling via the cavity fluid, stressing the role of the structural and acoustic uncoupled modes. He concluded that the six lowest acoustic modes of the guitar box present the same pattern, independently of the type of fluid, for the three studied gases, and the pressure distribution inside the box is the same for the three gases in all the modes except the Helmholtz resonance. Hossein Mansour [6] studied the measurements of a Persian Setar, compared using an impulse hammer or a handheld shaker as excitation and an accelerometer or Laser Doppler Vibrometer (LDV) to record the response. Measurements were made with the Setar both suspended, to produce a free-free boundary condition, and clamped at its neck. Natural frequencies and mode shapes are extracted for the first 12 structural modes. He found that both the accelerometer and shaker dramatically affect the structure and thus, depending on the context, they are probably best avoided if possible for the case of the Setar and similar instruments. On the other hand, the modal map of the free-free Setar was in close agreement with the clamped condition. Therefore, measurements on the Setar and similar instruments can be performed on a clamped instrument unless the accurate damping properties are of interest. M. J. Elejabarrietaa [7] studied the coupled modes of the resonance box of the guitar where the numerical model was developed progressively, starting with the soundboard and back, then the assembled box and the inside air separately, and finally the whole box; that is, the wood structure and the air together. In this way, mode evolution is tracked, establishing the influence of each component on the final box. Comparison of the modal patterns and frequencies with the modal analysis results corresponding to a real guitar box confirms the quality of the model. Mariana R [8] studied the acoustic-structural coupling of the automotive compartment. He found that strong coupling between the thin-walled structure and the acoustic enclosure exists in the vicinity of any acoustic resonance. Also it was found that "combined" acoustic-structure modes of vibration exist in the vicinity of an acoustic resonance, which means that the coupled system manifests a new type of energy exchange.

Closed form solutions are available for simple and regular acoustic and structural geometries. However no analytical solutions are available for irregular geometries. FEM are used widely to study and analyze these geometries. Formulating the structural-acoustic coupled closed form solutions are not yet established very widely. J. Missaoui and L. Cheng [9] is presented an integro-modal approach in his paper for computing the acoustic properties of irregular-shaped cavities. The method consists of discretizing the whole cavity into a series of sub cavities, whose acoustic pressure is decomposed either over a modal basis of regular sub cavities or over that of the bounding cavities in the case of irregular-shaped boundaries. An integral formulation is then established to ensure continuity of both pressure and velocity between adjacent sub cavities using a membrane with zero mass and stiffness. M. R. Karamooz Ravari [10] studied frequency equations for the in-plane vibration of the orthotropic circular annular plate for general boundary conditions. To obtain the frequency equation, first the equation of motion for the circular annular plate in the cylindrical coordinate is derived by using the stress-strain displacement expressions. Helmholtz decomposition is used to uncouple the equations of motion. The wave equation is obtained by assumption a harmonic solution for the uncoupled equations. Finally, boundary conditions are exerted and the natural frequency is derived for general boundary conditions. The obtained results are validated by comparing with the previously reported and those from finite element analysis.The presented results were validated with the previous reports and the finite element model. This verification showed that the proposed method is accurate for calculating the natural in-plane frequencies of circular annular plates. C.J. Nederveen [11] studied the resonating air column in a thin-walled metal organ pipe. Effects became audible when a wall resonance frequency was nearly the same as that of the air column. He proposed a 2D analytical model for the same. He qualitatively explains the observed changes in resonance behaviour of the air column. It allows identification and verification of the parameters governing the interaction. The results suggest that similar effects might occur in other wind instruments such as saxophones, bassoons etc.

R. Benjamin Davis [12] in his PhD thesis proposed a methodology called component mode synthesis of structural acoustic coupled problem of complex and irregular geometries. A CMS approach offers the potential for acoustoelastic analyses that are more computationally efficient and robust than FE-FE models. Additionally, CMS techniques permit simpler FE models and promote increased insight into the underlying physics of the system. The extent to which the CMS approach accords these advantages ultimately depends upon the capabilities of the FE-FE (or FE-BE) code that would be used in its place. His study compares the capabilities of a CMS approach to the FE-FE capabilities of ANSYS.

## 1.3 Overview of the thesis

Chapter 2 discusses the history, types, dimensional details, music and mechanism of sound transfer.

Chapter 3 dedicated to theoretical formulation of coupled problem.

Chapter 4 discusses the methodology followed for the experimental and numerical modal analysis.

Chapter 5 discusses the development, validation and implementation of the Component Mode Synthesis (CMS) method.

Chapter 6 discusses the results of the chapter 4 and chapter 5.

Chapter 7 dedicated to conclusion of the thesis and future work.

# Chapter 2

# The Veena

## 2.1 History

Veena is an ancient musical instrument from south India and it is Indias national instrument. The Veena has a recorded history that dates back to the approximately 1500 BCE. It is mainly used in Carnatic and Hisdustani classical music. Veena is a Sanskrit word referred from the Rugveda, it is considered as a generic term for any string instruments in Indian history so far. The classifications of the musical instruments can be done based on the way the sound is produced. This was according to Bharata (500 B.C.) from India and also by Erich Von Hornbostel and Curt Sachss publication from 1914. Chordophones, Aerophones, Membranophones and Idiophones are the four classification and the Veena falls under the Chordophones category. Three approached has been followed to explain the construction of the Veena. The very first one is spiritualistic and mythological, the Hindu goddess of knowledge and wisdom, Saraswati plays the Veena found in many religious references. It is Hindu belief that lord Naradais the one who brought the music to the earth. Second one is symbolic approach where it is related to human body. Its big resonator (Kudam) is a human head, Its fret board with 24 frets connected to curved dragon or yali is compared to human spinal column. Its frets are vertebras of the spinal column and principles of the Yoga.

## 2.2 Classification of Veena

Veena basically classified into three major families

- **Veena with Frets**

    1. **Rudra Veena:** Plucked string instrument used in Hindustani music. Construction is based on bamboo fret board. It has two large resonators with a fret board with 24 frets supported in-between.
    2. **Saraswati Veena:** also known as Tanjore Veena. Its a plucked string instrument from Carnatic music. It has one bigger resonator with a small resonator and fret board with 24 frets in between.

- **Veena without frets**
  1. Vichitra Veena
  2. Chitra Veena

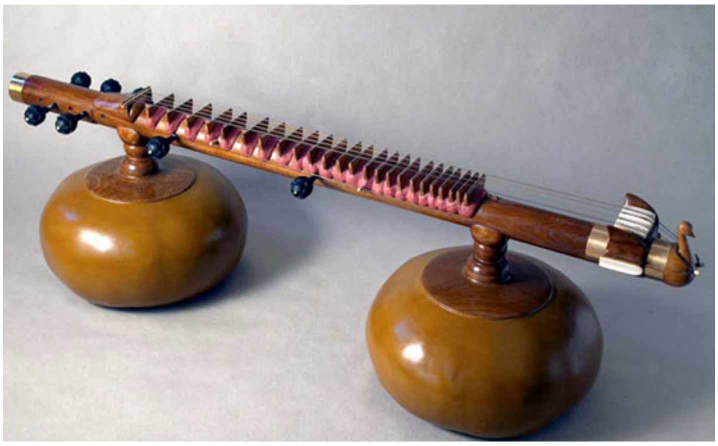


Figure 2.1: *Rudra Veena with two large resonator*

Figure 2.2: *Tanjore Veena 9(Saraswati Veena)*

## 2.3 Veena and Its parts

For our research, we have selected Saraswati Veena which is commonly used in India. Please refer fig no. 2.4 for part identification.

Saraswati Veena divided into major parts such as big resonator, fret board with twenty four frets, a small resonator and seven strings. Fret board connects these two resonators. In total it has

Figure 2.3: *Vichitra Veena*


Figure 2.4: *Veena and its parts*

seven strings out of which four passes over the fret board, they are called as main strings and three minor stretching along the side of the Veena. These three strings are without any support inbetween.

**The Resonator:** ***Kudam*** Its a dome shaped spherical part, hand crafted from the jack-fruit wood. Resonator is divided into two parts, the spherical wooden cavity, and the top plate. Top plate carries a bridge over which 4 major strings held stretched along its length. This has a unique design, a brass strip is attached at the top which has very small curvature, and it makes strings to pass on it tangentially. Top plate carries a small hole through the cavity near the neck of the instrument. Thickness of the wooden resonator cavity and the top-plate is maintained at 1/5 inch.

**Following are the detail dimensions:**

- Radius of the spherical wooden dome: 20.5 cm
- Radius of the top-plate: 19 cm
- Radius of the sound hole: 2 cm
- Height of the wooden dome: 27.5 cm

**The Fretboard:** Its a 60 cm long piece of wood connecting the resonator to other part of the instrument. *Ekandam* Veena is the class where entire instrument, from resonator to the dragon faced end carved in the single piece of the jack-fruit trunk. However in Tanjore Veena (current model considered for study as shown in fig. no. 2.4) resonator and fretboard are joint by nails and glue. 24 frets which are made up of brass are placed on a thick lining of the wax at a logarithmic scale.

**Support Resonator:** Support resonator is hollow and smaller in size. It should not be necessarily made up of the same wood. It can be of other material as its only purpose to provide the supports.

**Strings:** Veena has seven different strings each with the specific material and diameter. It has 4 major strings which passes over the fret board and three minor strings which are stretched at the side. Four major strings has more or less equal length, however three side strings has different lengths. Please refer below table for string material and diameter.

Table 2.1: String dimensional details

| **Sr. No.** | **String** | **Material** | **Gauge** | **Length cm** |
|---|---|---|---|---|
| 1 | String 1 | Brass | Gauge 21 | 87 |
| 2 | String 2 | Brass | Gauge 24 | 87 |
| 3 | String 3 | Steel | Gauge 27 | 87 |
| 4 | String 4 | Steel | Gauge 29 | 87 |
| 5 | String 5, 6, 7 | Steel | Gauge 31 | 70, 61.5, 52.5 |

**Musical notes:**
Veena covers the fundamental frequency range of 99 to 1056 Hz and considering the harmonics it covers the overall range from 90 to 6000 Hz. These ranges are covered by 4 strings and 24 frets, and divided into three and half octave. Octave is an interval where frequencies doubles in value. Here each octave consist of 12 distinct frequencies called notes, from Sa to next upper Sa. Each note has unique relation with the length of the string and diameter, and it is produce by plucking the string near the bridge by maintaining the distance by pressing the string on corresponding fret. Twelve Shruti frequencies are used to tune this string instrument. Values of the length of the strings from $l_0$ to $l_{12}$ can be found out using simple mathematical relationship as stated below.

$$l_{n+1} = \frac{l_n}{2^{\frac{1}{N}}} = \frac{l_0}{2^{\frac{n+1}{N}}} \tag{2.1}$$

**Mechanism of sound transfer**

Sound radiated by any string instrument is not solely by strings, other part of the instruments plays an important role. Consider a string held between two concrete block and plucked, it would give very little sound. Fundamentally strings vibrates against boundary which has a large impedance difference making it difficult to transfer the sound energy. With so little of its energy transmitted, string would vibrate for a longer time radiating very little sound.

To accomplish the better sound quality and quantity, Veena has a bridge, top-plate and resonator coupled with vibrating string. As string is plucked it vibrates with the particular frequency, since impedance change between the string and bridge is not to significant maximum amount of wave energy gets transferred. As this get transferred to bridge, topplate and resonator causing vibration of much greater surface area. This in turn moves significant amount of air than a string alone, making vibration clearly audible.

# Chapter 3

# Theoretical Formulation

In this chapter analytical formulation of various aspects of Veena has been studied. mainly string tension, frequency and density relation. This theoretical study helps to understand the inside physics of sound generation of Veena.

## 3.1 String's natural frequency calculations.

Please refer fig. no. 3.1 which is fundamental mode of string vibration.

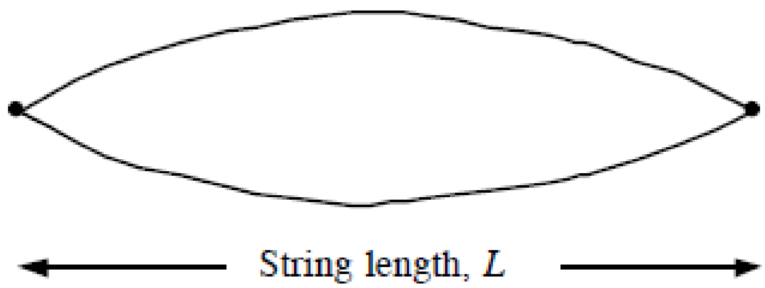



Figure 3.1: *Fundamental mode.*

Here length of the string is half the wavelength of the string.

$$L = \frac{1}{2}\lambda \Rightarrow \lambda = 2L \tag{3.1}$$

Hence frequency is given as

$$f_1 = \frac{v}{2L} \tag{3.2}$$

Wave speed of the string depends on the two main factors, tension of the string, and liner mass density. Linear mass density is defined as mass per unit length of the string. Mass of the string can be formulated in terms of density and volume [13].

$$v = \sqrt{\frac{T}{\mu}} \tag{3.3}$$

Hence,

$$f_1 = \frac{\sqrt{\frac{T}{\mu}}}{2L} Hz \tag{3.4}$$

Above equation gives three basic relationship with the fundamental frequency of the vibrating string.

Now consider the next possible mode of the string vibration. Two nodes at the ends, a node at the center of the string with two anti-nodes at the center of two subsequent nodes. Frequency can be formulated in the same way as that of the fundamental mode but the only difference is, wavelength is equal to the length of the string.

$$f_1 = \frac{v}{L} \Rightarrow \; f_1 = \frac{\sqrt{\frac{T}{\mu}}}{L} \tag{3.5}$$

Above frequency is exactly twice as that of fundamental frequency. Similarly frequencies of the subsequent modes can be formulated.

## 3.2 Structural-Acoustic Interaction

In real world structural-acoustic interaction is commonly observed in many applications. It is nothing but the dynamic formulation of coupling between acoustic pressure field and structural flexibility. In order to address the problem, it is necessary to formulate the two interacting system independently. Solving the acoustic problem considering coupled structure is perfectly rigid and solving structural problem assuming the system is in vacuum.

The present work focused on a structural- acoustic coupled system where acoustic fluid is enclosed in an irregular geometry. Resonator of the Veena is taken into consideration to study the mechanism of structural-acoustic interaction and mechanism of sound radiation at different musical notes.

### 3.2.1 Structural-Acoustic Interaction methods

Over the period many methodologies has been developed to define and solve the structural acoustic coupled problem. For a given problem it is not necessary that all the methods are appropriate. Each methods has limitations and advances, hence first and foremost task is to identify the classification of the problem and then apply the corresponding methodology.

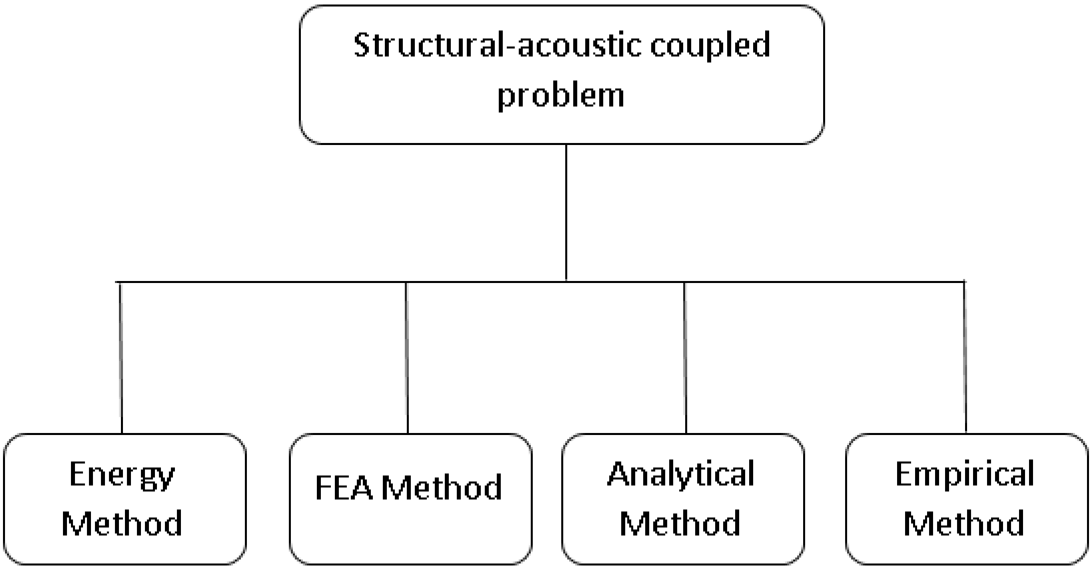



Figure 3.2: *Structural-acoustic methods.*

With energy methods such as Statistical Energy Analysis (SEA) [14, 15], the acoustic- structure system is divided into energy storage subsystems. A power balance between these subsystems is then enforced. The power balance can be formulated in various ways depending upon the system response parameters of interest. However, in the case of SEA, these system responses are calculated in an average sense and do not provide any information related to how the response may vary spatially throughout a subsystem. Energy methods are most accurate when considering the system response at high frequencies.

Discretization means the finite element/boundary element approach which is widely used to solve the structural acoustic coupled problem where acoustic and/ or acoustic domain has geometric complexity. General discretization methods involves interaction finite element model of acoustic with finite element model of structural (FEFE), and finite element model of structure with boundary element model of acoustic (FE-BE). FE-BE formulations are generally best suited to problems in which the structure is coupled to an infinite exterior acoustic domain. There are several commercially available software packages that facilitate FE-FE and/or FE-BE solutions. Two of the most widely used commercial FE packages, ANSYS and NASTRAN, refer to their respective FE-FE formulations as fluid-structure interaction (FSI) solutions.

Analytical approach is more precise, computationally reliable and applicable to simple and regular geometries like cylinder, rectangular duct etc. In the analytical model also two coupled system studied independently. B Venkatesham studied the structural acoustic coupling of the rectangular duct with one wall flexible based on impedance mobility approach. J. Missaoui studied the integro modal approach to formulate acoustic analysis in for regular geometries by dividing it into the some regular shapes. A disadvantage of purely analytical formulations is that they are most readily employable in systems with simple geometries. In the case of complex geometry, the analytical formulations must be discretized and solved numerically.

Certain complex acoustic-structure interaction problems can be approximated using empirical methods. While such empirical formulas can be accurate and easily applied, they are often limited in their scope of applicability and do not provide the analyst with any physical insight into the specific problem.
Current work focused on modeling and analysis of the structural-acoustic coupled problem in Veena, the flexible wooden resonator with enclosed acoustic cavity.

### 3.2.2 Structural-Acoustic equation of motion

Structural-acoustic coupled problem is of great interest for many researcher from last four decades. In 1960s, Warburton [16] was one of the first researcher who found that a cylindrical shell containing air as an acoustic fluid possesses natural frequencies which are close to either uncoupled structural natural frequencies or the uncoupled acoustic natural frequencies of the enclosed air. Dowell et al. [17] expanded upon this idea by developing a theoretical model that combines the uncoupled acoustic cavity modes and the uncoupled structural modes into a system of coupled ODEs. This model, which serves as the theoretical framework for much of the present work, has been used to investigate structural-acoustic interaction in a variety of systems of practical interest like rectangular enclosures, resonant modes of guitar bodies.

Please refer fig. no. an irregular acoustic cavity enclosed in structural interface.

General equation of inside cavity pressure $p$ at some location and the compliance wall vibration velocity $w$ at some location on flexible surface for the uncoupled cavity mode N and structural mode M are

$$P = \sum \psi_n a_n = \psi^T a \tag{3.6}$$

$$w = \sum \phi_m b_m = \phi^T b \tag{3.7}$$

$\psi_n$ is the uncoupled acoustic mode shape function, $a_n$ is the complex amplitude of the nth acoustic pressure mode, $\phi_m$ is the uncoupled vibration mode shape function, and $b_m$ is the complex amplitude of the mth vibration velocity.

Here normal modes satisfies the properties of orthogonality for uncoupled acoustic mode,

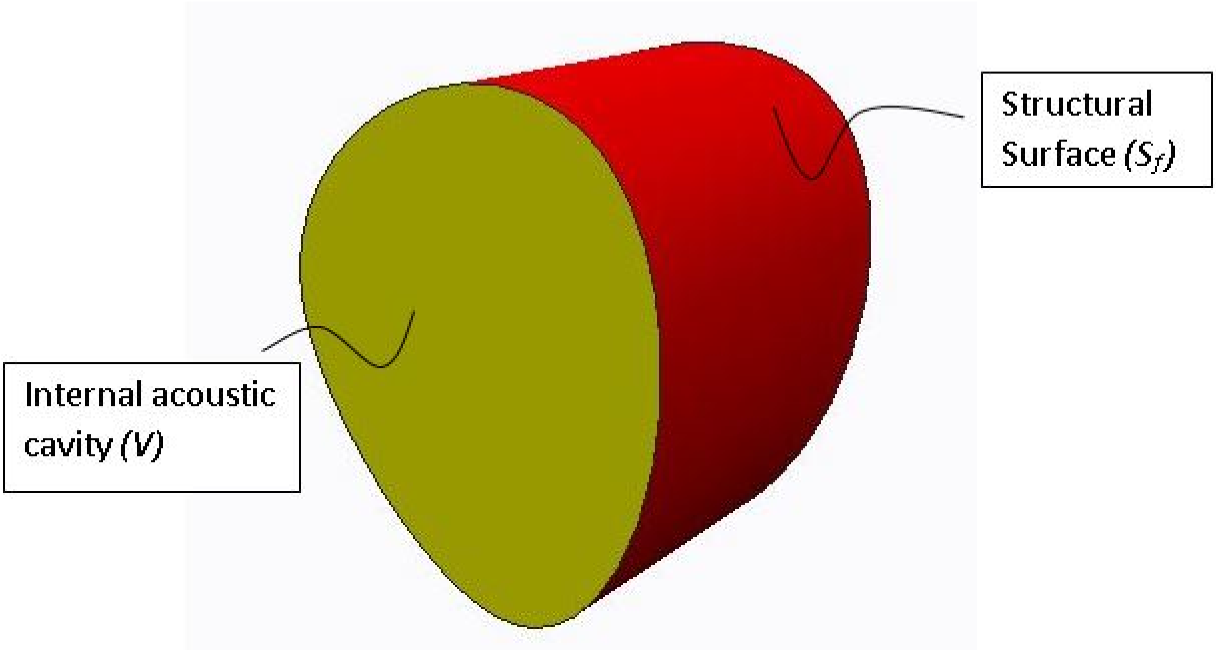



Figure 3.3: *structural-acoustic coupling*

$$\frac{1}{V}\sum \psi_i \psi_j = 0 \; for \; i \neq j \tag{3.8}$$

$$= M_n \; for \; i = J \tag{3.9}$$

Similarly for uncoupled structural mode,

$$\sum \phi_l \phi_k = 0 \; for \; l \neq k \tag{3.10}$$

$$= M_m \; for \; l = k \tag{3.11}$$

Complete equation for coupled structural-acoustic problem is given by

$$[M]\begin{Bmatrix} \ddot{a}_j \\ \ddot{q}_k \end{Bmatrix} + [G]\begin{Bmatrix} \dot{a}_j \\ \dot{q}_k \end{Bmatrix} + [k]\begin{Bmatrix} a_j \\ q_k \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.12}$$

where,

$$[M] = \begin{bmatrix} VM_n & 0 \\ 0 & M_m \end{bmatrix}, \quad [K] = \begin{bmatrix} VM_m\Omega_n^2 & 0 \\ 0 & M_m\Omega_m^2 \end{bmatrix}, \quad [G] = \begin{bmatrix} 0 & -S_f c_0^2[C_{m,n}] \\ S_f\rho_0[C_{m,n}]^T & 0 \end{bmatrix} \tag{3.13}$$

$[M]$ and $[K]$are mass and stiffness matrices and $[G]$ is the coupling matrix. Equation written for the structure in contact with the interior fluid.

Here $\Omega_n$ *and* $\Omega m$ are uncoupled acoustic and uncoupled structural natural frequencies. $S_f$ is flexible surface area, $c_0$ is velocity of sound in the fluid and $\rho_0$ is structural mass density.

$C_{m,n}$ is uncoupled structural-acoustic mode shape coefficient. It is relationship between the uncoupled structural and acoustic mode shape of vibration surface $S_f$ and its measure of spatial match between the panel and cavity mode. When value of coupling coefficient is unity then two modes are in exact spatial match and when value is zero no spatial match between two.

Hence $C_{m,n}$ is given as,

$$C_{m,n} = \frac{1}{S_f} \sum \psi_n \phi_m \quad \textit{over the area } S_f \tag{3.14}$$

Transfer factor is used to identify the well-coupled modes. The transfer factor for thin cavity flexible wall for nth acoustic and mth structural mode can be written as,

$$T_{m,n} = \left( 1 + \frac{\left(\Omega_m^2 - \Omega_n^2\right) \rho_s h S_f V}{4 \rho_0 c_0^2 C_{m,n}^2} \right)^{-1} \tag{3.15}$$

More general form of equation can be formulated considering viscous damping present in the system and application of external forcing function.

$$[M]\ddot{g}(t) + [C+G]\dot{g}(t) + [K]g(t) = Q \ , \tag{3.16}$$

Here $[C]$ is damping matrix and $[Q]$ is external forcing function.

# Chapter 4

# Modal Analysis

Modal analysis is a method of finding the dynamic characteristic of the system. These characteristics are natural frequencies, mode shapes and damping factor, which are used to formulate the mathematical model for system dynamic behavior.

The dynamics of a structure are physically decomposed by frequency and position. This is clearly evidenced by the analytical solution of partial differential equations of continuous systems such as beams and strings. Modal analysis is based upon the fact that the vibration response of a linear time-invariant dynamic system can be expressed as the linear combination of a set of simple harmonic motions called the natural modes of vibration. The natural modes of vibration are inherent to a dynamic system and are determined completely by its physical properties (mass, stiffness, damping) and their spatial distributions. Each mode is described in terms of its modal parameters: natural frequency, the modal damping factor and characteristic displacement pattern, namely mode shape. The mode shape may be real or complex. Each corresponds to a natural frequency.

Modal analysis can be done using numerical, analytical and experimental techniques. Application of the said techniques depends on the state of the problem. For a very simple geometry like plate, beam and strings etc. it is always advisable to use the analytical and experimental techniques. However, for complex and irregular geometries numerical techniques are well suited. The current study deals with experimental and numerical modal analysis of the Veena Resonator.

## 4.1 Numerical modal analysis of the Veena

Numerical modal analysis consist of discretization of the continuous system. Multi degree of freedom model used with the matrix formulation. Let us consider the equation of the motion for the undamped MDoF system for free vibrations.

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{4.1}$$

Above system leads to the following Eigen value problem.

$$([K] - \omega^2[M])\{\varphi\} = \{0\} \tag{4.2}$$

$[M]$ is mass matrix and $[K]$ is stiffness matrix, and both are symmetric in nature. The mass matrix is positive definite and stiffness matrix may become semi positive definite if system possess the rigid body modes.

Solution to above equation possess the n eigen values and n eigen vectors. Square root of the eigen values are the natural frequencies and eigen vectors are the simply mode shapes.

Following numerical models are considered for the modal analysis of Veena.

- Uncoupled structural modal model
- Uncoupled acoustic modal model
- Coupled structural-acoustic modal model

### 4.1.1 Geometric Details

The Veena as shown in fig. 3.1 is considered for experimental study. These dimensions have been measured with proper scale and multiple averaging methods. Figure 4.1 shows the CAD model of the Veena. Figure 4.2 and 4.3 shows the schematic diagram of the Veena with all the dimensions in meter.

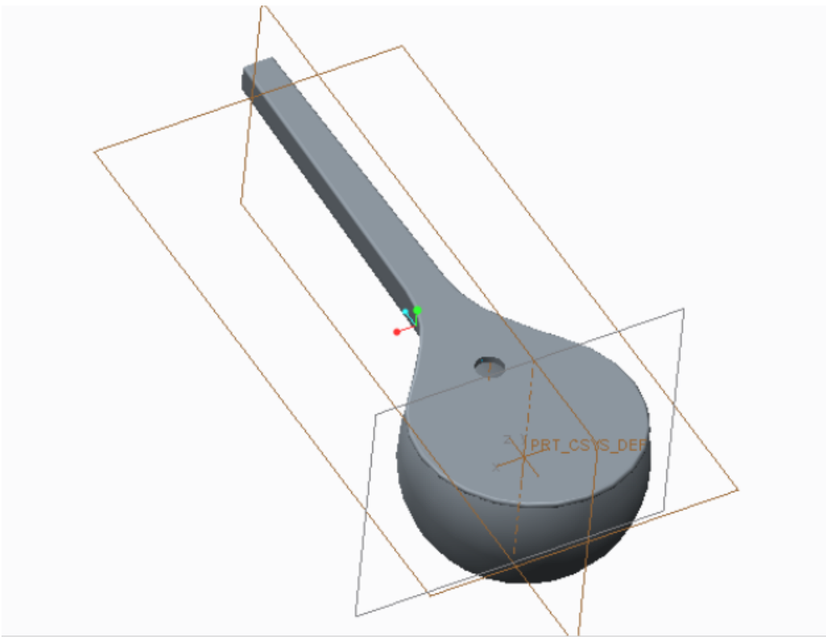


Figure 4.1: *CAD model of Veena*

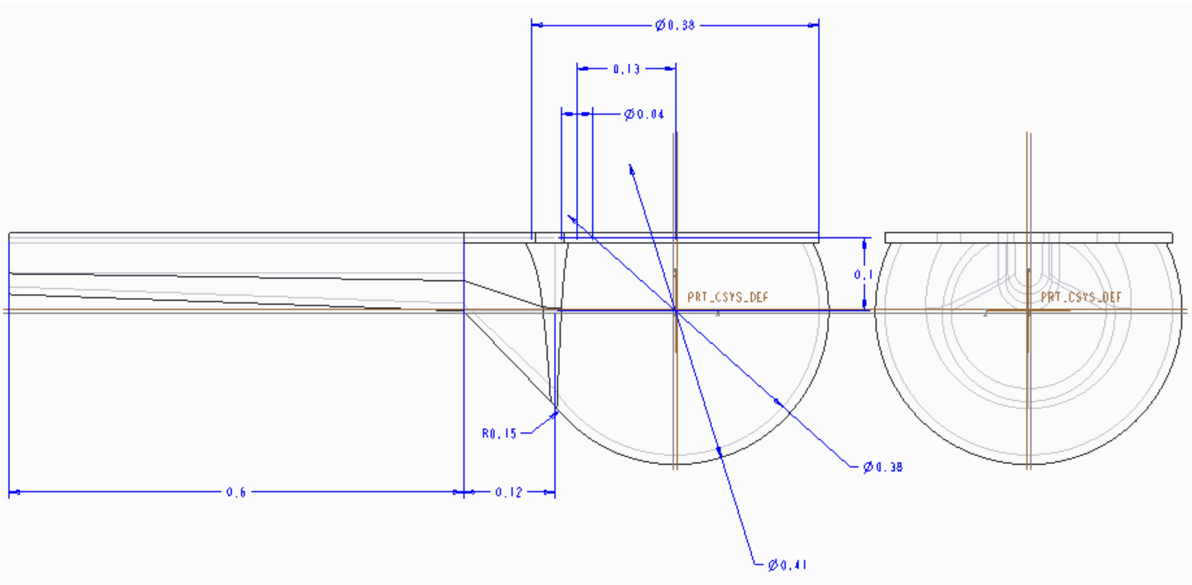



Figure 4.2: *Dimensional details of Veena*

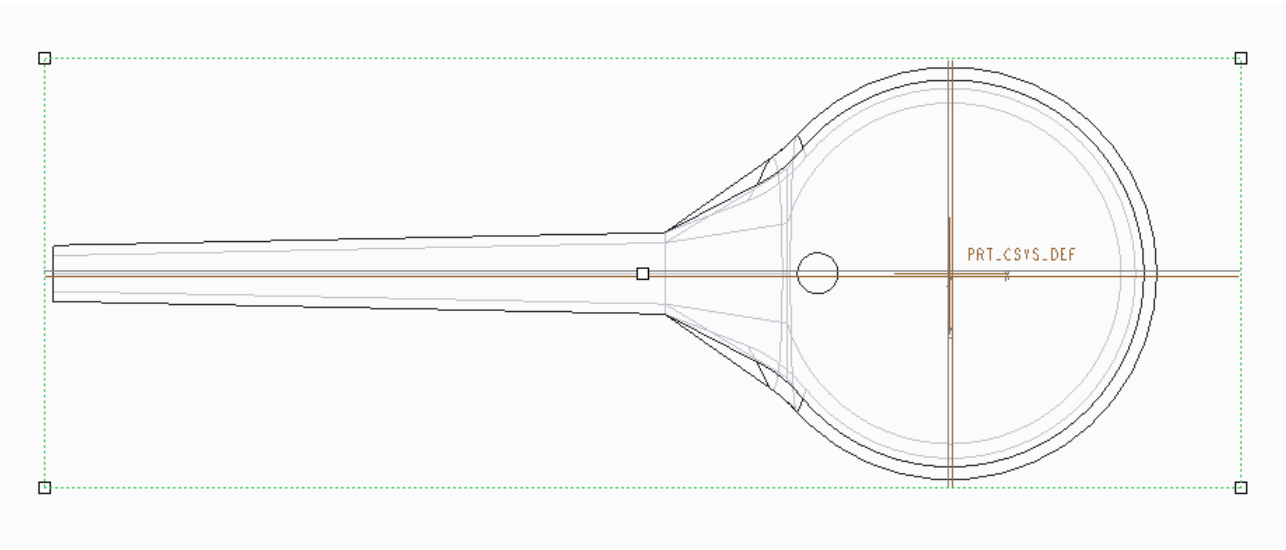



Figure 4.3: *Top view of the Veena*

### 4.1.2 Uncoupled structural modal model

It was previously mentioned that in order to study the coupled model study of uncoupled systems is necessary. Here structural model is studied in different section as follows,

1. Complete Veena model

    - Here complete structural part of the Veena has considered, right from resonator, top-plate and fretboard.

2. Resonator Veena model (partial model)

    - In this model, only the resonator and top-plate is considered as a single system for modal analysis.

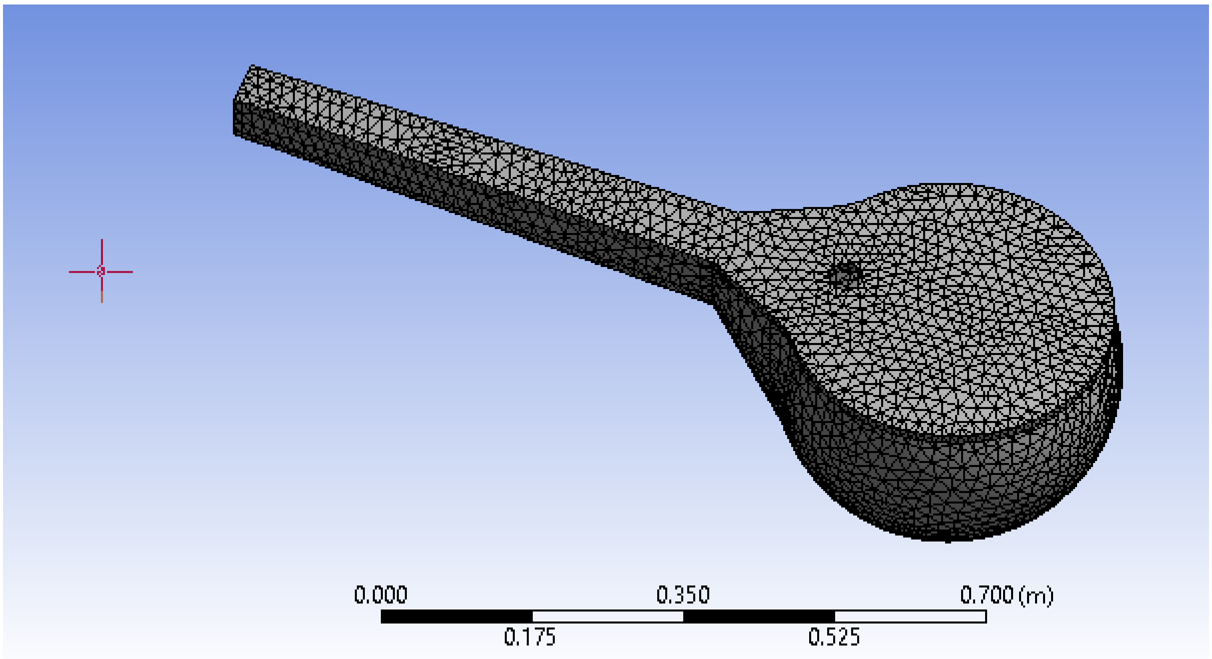



Figure 4.4: *Numerical modelfor complete Veena structure. structure.*

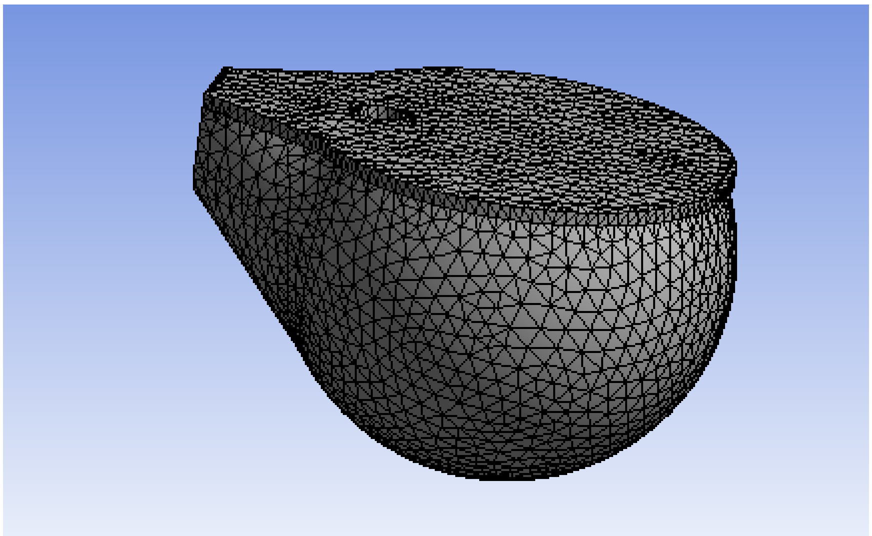


Figure 4.5: *Numerical model for partial Veena structure.*

**Numerical Details:** For structural analysis entire geometry is model and mesh with SOLID185 element. Element size of 0.01 is used with element number of 17372 for complete structural model and 12976 for partial acoustic model.

### 4.1.3 Uncoupled acoustic modal model

A complete acoustic cavity model and partial acoustic cavity model of Veena considered here for analysis in order to have comparison with structural models.

1. Complete acoustic cavity model
    - Acoustic cavity right from resonator to fret board considered here.
2. Partial acoustic cavity model (partial model)
    - Acoustic cavity of resonator and neck considered here.

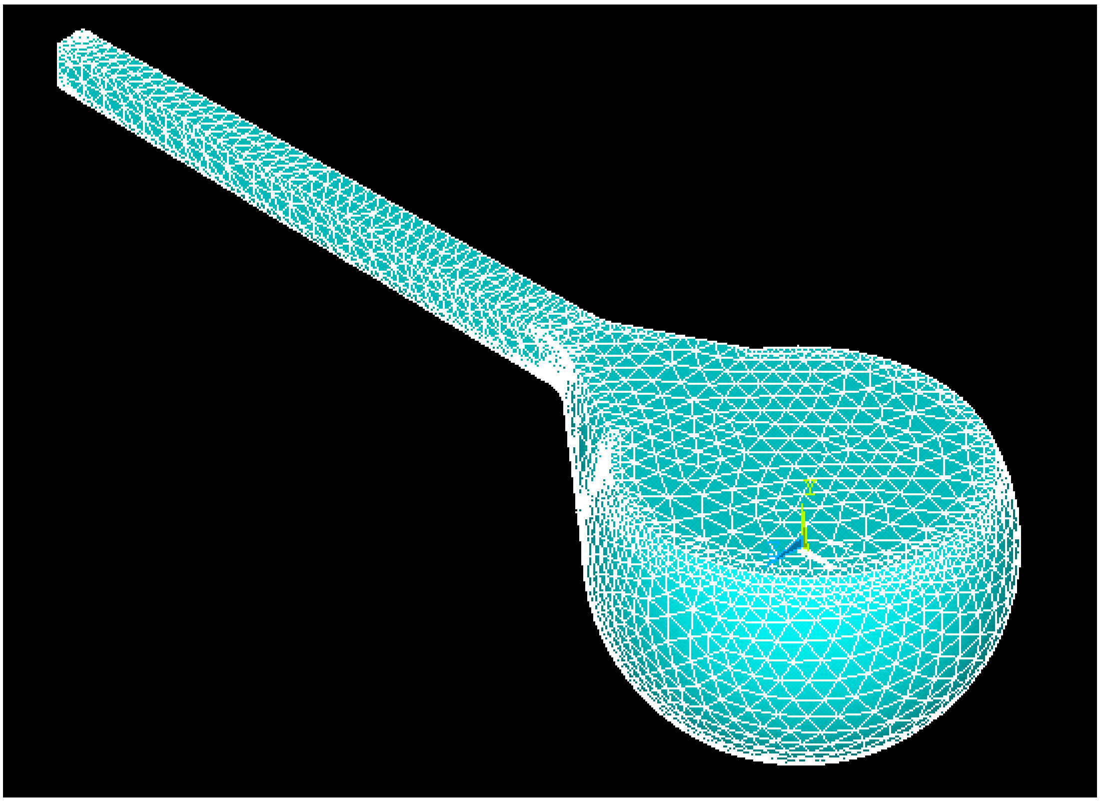


Figure 4.6: *Acoustic model complete Veena cavity.*

**Numerical Details:** FLUID220 element is used to model and mesh the uncoupled acoustic geometry. Element size of 0.01 is used with element number of 17372 for complete structural model and 12976 for partial acoustic model.

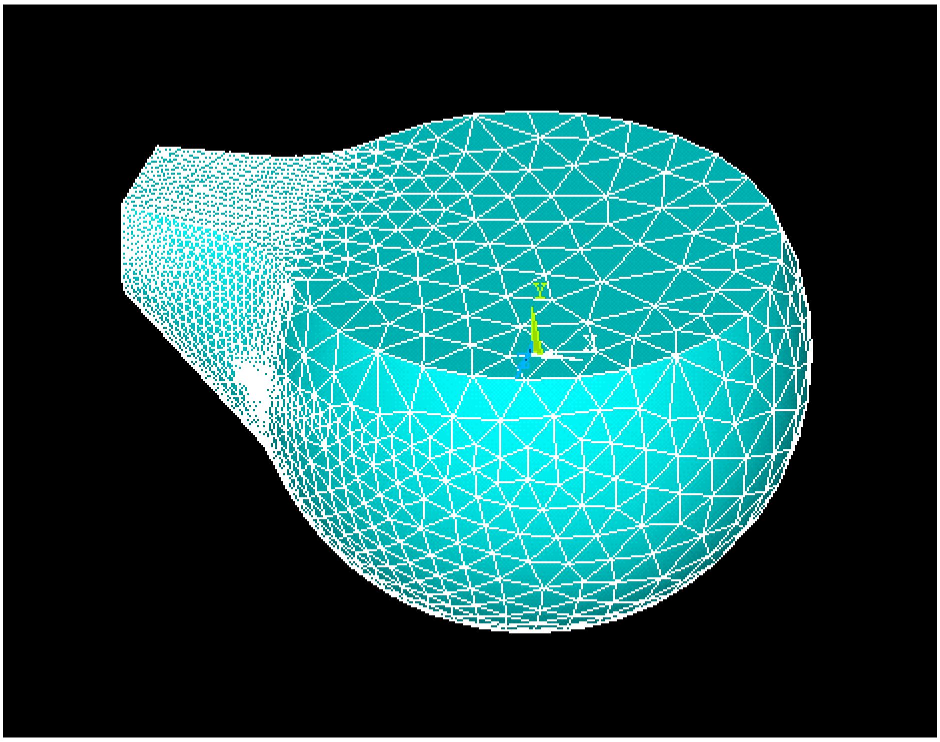


Figure 4.7: *Acoustic model of Veena resonator.*

### 4.1.4 Structural-acoustic coupled model

In order to have the comparison with uncoupled structural and acoustic model, two models are used here which are complete and partial models. FSI flag from ANSYS 2015 is used to apply coupling between structural and acoustic interface. SHELL281 (8 noded) with six degrees of freedom used for structural part and FLUID220 (3D Acoustic 20 noded) with four degrees of freedon (X,Y,Z and Pressure) used for acoustic part.Orthotropic properties of wood and acoustic properties of air used in material model. Unsymmetric solver used to solve the unsymmetric matrices arises out of coupling of two systems.

## 4.2 Experimental Modal Analysis

This part of chapter describes the complete experimental set up for determining the dynamics characteristics of Veena subjected to two different boundary conditions. Various measurement procedures to determine structural and acoustic parameters have been reported.

### 4.2.1 Instrumentation and Softwares

Instrumentation used for experimental study are as follows:

- Data Acquisition System (DAQ)
- Accelerometer

- SO analyser
- Force sensor (Hammer)

**Data Acquisition System (DAQ):** Data acquisition is the process of sampling signals that measure real world physical conditions and converting the resulting samples into digital numeric values that can be manipulated by a computer. SO analyzer is used to record the pressure and acceleration data while conducting vibration measurements.

**SO analyser:** m+p SO Analyzer is a fully integrated solution for dynamic signal measurement, analysis and advanced reporting of all noise and vibration, acoustics and general dynamic signal applications. Comprehensive time and frequency analysis is available with both online and offline data processing. Complete with advanced application wizards the m+p dynamic signal analyzer makes light work of gathering data, displaying results, performing specialized analysis and generating customer ready reports all within one familiar Microsoft Office style user interface. m+p SO Analyzer is designed for noise and vibration applications in the field and in the lab.

**Accelerometer:** Vibration measurements are done by using accelerometer for estimating structural parameters such as wall vibration velocity and wall displacement. Accelerometer used is from Dytran instruments, INC with model number 3055B3 and serial number 142B2.The sensitivity of the accelerometer is 503.96 mV/g. Figure shows the accelerometer.

**Force sensor (Impact Hammer):** It is designed to excite and measure impact forces on small to medium structures such as engine blocks, car frames and automotive components. An accelerometer (or laser velocity transducer) is used to measure the response of the structure. It uses for Impact-force measurements on small to medium structures, measurement of frequency response functions using impact excitation techniques and as part of a dynamic structural testing system for modal analysis and the prediction of structural response.

### 4.2.2 Experimental Setup

Fig. no.4.8 shows the experimental setup of the modal analysis. Support arrangement is as shown in the fig. here two high sensitivity accelerometers are used to pickup the vibration response one is triaxial fixed at the top-plate and a uniaxial at the end of resonator. Sensitivity values of accelerometer provided my manufacturer. Position of the same determined by pre FEM analysis. Mounting methods of these accelerometers are important because they affect usable frequency range. There are different mounting conditions are possible like stud mounting, wax mounting, holding by hand. In this analysis, wax mounting is used for mounting the accelerometer to beam because it is an easy and fast way of mounting. Care has to be taken on thickness of wax mounting used because it affects readings as stiffness changes.

Impact hammer is used for inducing vibrations in the structure. Impact hammer mainly consists of two parts, tip of the hammer and a force transducer connected to the tip. Tip of the hammer is decided based on the frequency range of interest which is decided by FEM analysis. A major factor which controls the frequency range of interest is hardness of hammer tip and compliance of impact


Figure 4.8: *Experimental setup*

surface. Minor factors are weight of the hammer and impact velocity. One more critical factor which controls the tip and weight of hammer is double impact. Double impact should be avoided while taking measurements as it increases noise in the measurements. The light weight hammer is useful for avoiding double impact.

m+p international SO Analyzer Rev. 4.1 hardware and associated software is used for data acquisition and processing purpose. Force transducer from impact hammer is connected to first input channel which is excitation input for software and the accelerometer is connected as the second and third inputs. Then this time domain data is converted to frequency domain data using Fast Fourier Transform (FFT) in the software. Then software creates Frequency Response Function (FRF) between two inputs. As the roving hammer method is used for analysis, software treats the excitation signal as a response signal and transducer (accelerometer) signal as a reference signal to calculate FRF. Average of three measurements for every point is considered.

Entire geometry is model using the wire frame mesh. Total of 160 points are considered on partial model of the Veena. Co-ordinate Measuring Machine (CMM) is used to measure the exact co-ordinates of each point.

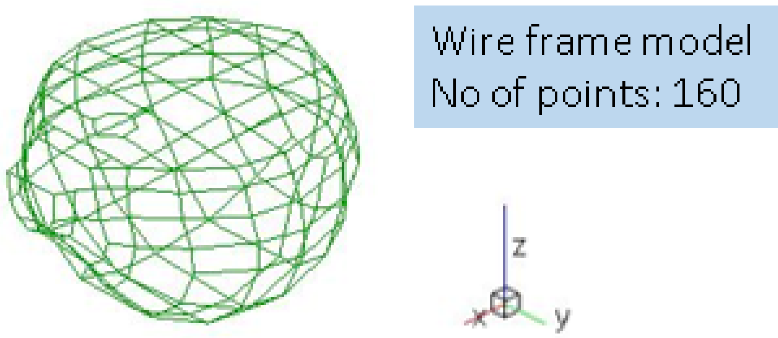



Figure 4.9: *Wireframe model of partial Veena*

# Chapter 5

# Component Mode Synthesis Method

This chapter discusses the implementation of a component mode synthesis (CMS) technique that avoids the use of a FE-based unsymmetric eigensolver and thus circumvents a variety of practical difficulties. This method useful to find out the transfer factor for complex and irregular geometries. The technique validated here using the simple rectangular duct structure, but it may be extended for use in more general cases. The following sections discuss the technique in greater detail and present the results of a benchmark test designed to directly compare the performance of the CMS technique to that of a modern, commercially-available, FE-FE code. Finally, the technique is used to analysis the structural-acoustic coupled problem in Veena.

### 5.0.3 CMS methodology

Figure 5.1 is a flowchart describing the CMS procedure which is implemented here.The first steps of the method involve building FE models to extract the natural frequencies and modes of the in uncoupled structure as well as those of the rigid wall fluid cavity(ies). The solutions to these uncoupled problems rely on symmetric formulations of the discretized equations of motion. ANSYS2015, a popular commercial FE package, is used in this study; however, any FE software package with a built-in eigensolver that supports both structural and fluid elements may be used. Next, the nodal displacements corresponding to each acoustic and structural mode are written to data files. These data files are then used as input for the matrix analysis software MATLAB. The extent to which the individual acoustic and structural modes couple to each other is calculated by means of numerical integration over the nodal displacements at the fluid-structure interface. Finally, the complete problem is written as a set of coupled ordinary differential equations according to the formulation presented in Chapter 3.

The CMS method thus requires the solution of two or more relatively large symmetric eigenvalue problems in addition to one relatively small unsymmetric problem. This is in contrast to the ANSYS FE-FE formulations which require the solution of one very large unsymmetric problem.

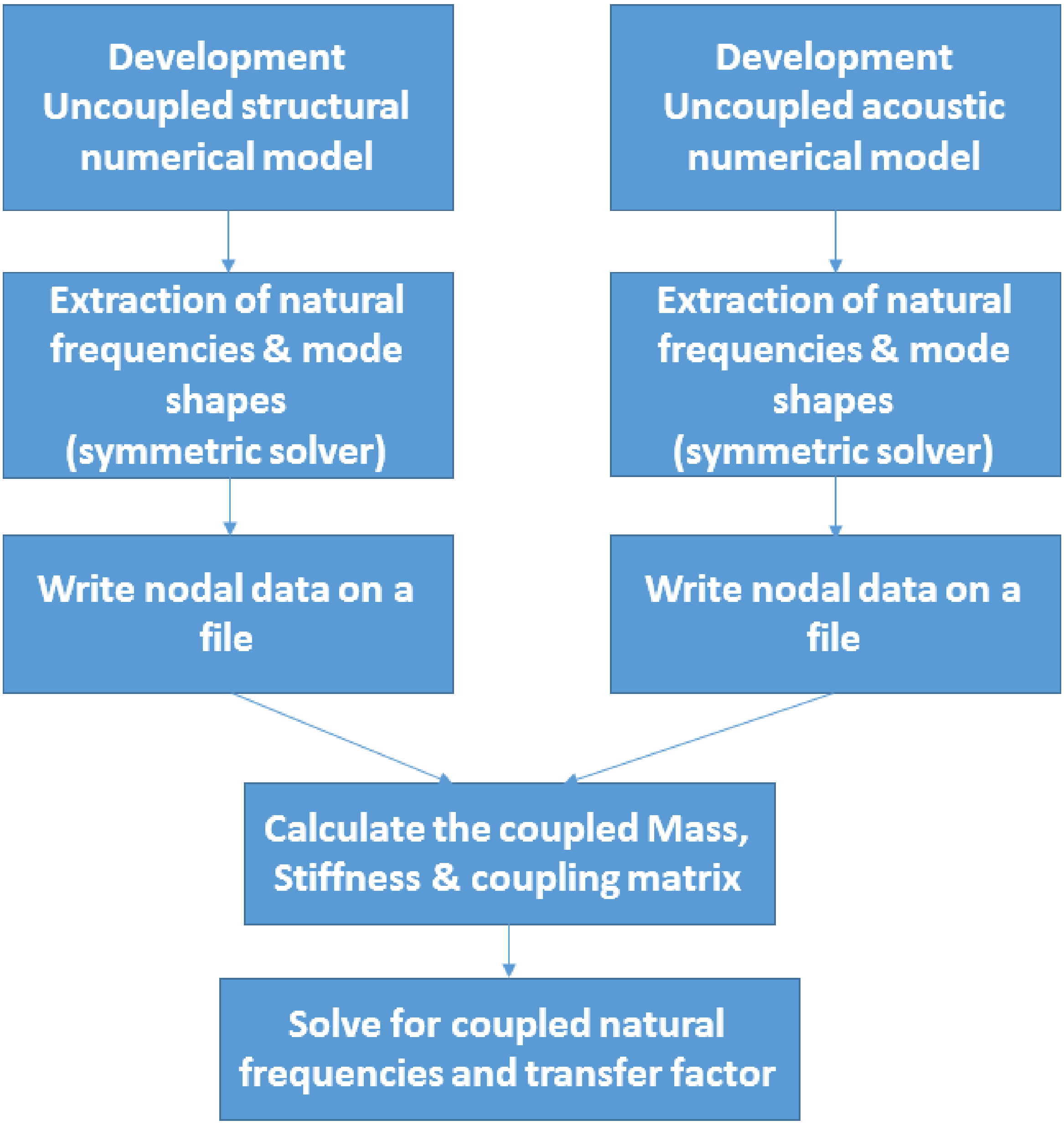



Figure 5.1: *CMS methodology Veena*

### 5.0.4 Implementation of CMS methodology

The primary effort involved with the implementation of the CMS method is writing the matrix analysis code to post-process the output of the uncoupled FE solutions. This code determines the modal coupling coefficients and transfer factor. The unforced coupled problem is then assembled in the following form.

$$[M]\left\{\frac{\ddot{a}_j}{\ddot{q}_k}\right\}+[G]\left\{\frac{\dot{a}_j}{\dot{q}_k}\right\}+[k]\left\{\frac{a_j}{q_k}\right\}=\left\{\frac{0}{0}\right\} \tag{5.1}$$

$$[M]=\begin{bmatrix}VM_n & 0\\ 0 & M_m\end{bmatrix},\quad [K]=\begin{bmatrix}VM_m\Omega_n^2 & 0\\ 0 & M_m\Omega_m^2\end{bmatrix},\quad [G]=\begin{bmatrix}0 & -S_f c_0^2[C_{m,n}]\\ S_f\rho_0[C_{m,n}]^T & 0\end{bmatrix} \tag{5.2}$$

The coupling coefficient $C_{m,n}$ , however, are calculated using the following expression.

$$C_{m,n}=\frac{1}{S_f}\sum \psi_n\phi_m \quad \textit{over the area } S_f \tag{5.3}$$

Transfer factor with gives the relation between the uncoupled acoustic and structural modes for spacial and energy match is given by equation no.5.4 If transfer factor is close to unity implies two modes are strongly coupled however transfer factor zero means weakly coupled system.

$$T_{m,n}=\left(1+\frac{\left(\Omega_m^2-\Omega_n^2\right)\rho_s h S_f V}{4\rho_0 c_0^2 C_{m,n}^2}\right)^{-1} \tag{5.4}$$

### 5.0.5 Limitation of the CMS methodology

While implementing the CMS method following limitations are taken care,

- Number of nodes on uncoupled structural and acoustic interface should me equal and must have spatial match.
- While developing the structural and acoustic geometry In FEA or CAD model the global frame of reference for coordinate system should remain same.

Above limitations can be eliminated in future work of the project by developing the more generalized code for CMS methodology.

### 5.0.6 Validation of the CMS methodology

A Rectangular duct with one wall flexible is considered to validate the code written for CMS methodology. Simply supported boundary condition is maintained. Fig. shows the schematic drawing of the duct which is of 0.3m x 0.4m x 1.5m dimension with 5 mm thickness. Its made up of steel of density 2770 kg/m3, young's modulus of 71e9 N/m2 and poisson's ratio of 0.29.

FEA analysis of uncoupled acoustic and structural is done is ANSYS15 Academic package and post processing is done in ANSYS as well as MATLAB. Total of 1621 nodes are considered on interface surface area of structural-acoustic coupling and total acoustic nodes 32169. Results of the CMS method compared with analytical and numerical results.

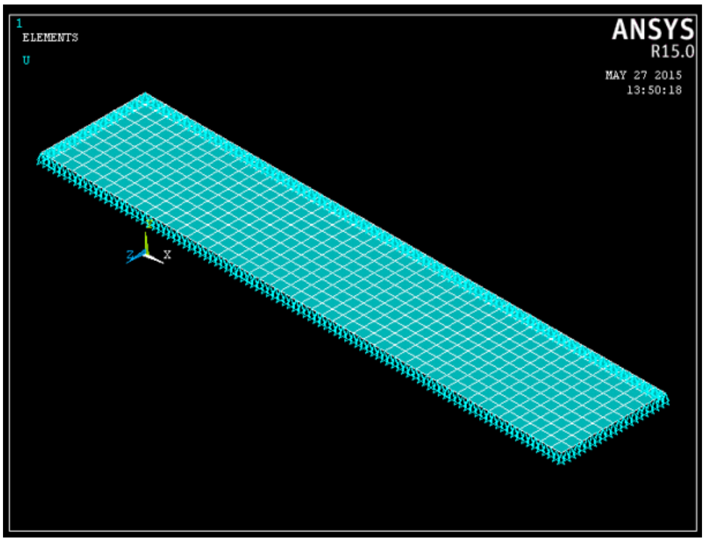



Figure 5.2: *Simply supported FEA model of flexible wall of duct*

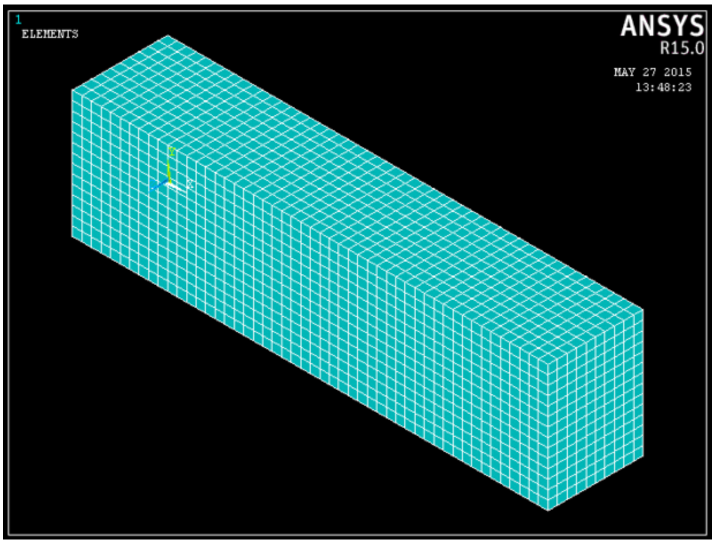



Figure 5.3: *FEA model of acoustic cavity of rectangular duct*

### 5.0.7 Implementation of the CMS methodology on

Here CMS methodology is applied to a irregular geometry of Veena. Mapped mesh technique is used to divided the geometry in different parts in order to have same interface nodes. Fig 5.4 and Fig 5.5 shows the similar mapped mesh sections on structural and acoustic geometry. Simply supported boundary condition is applied to the structure.

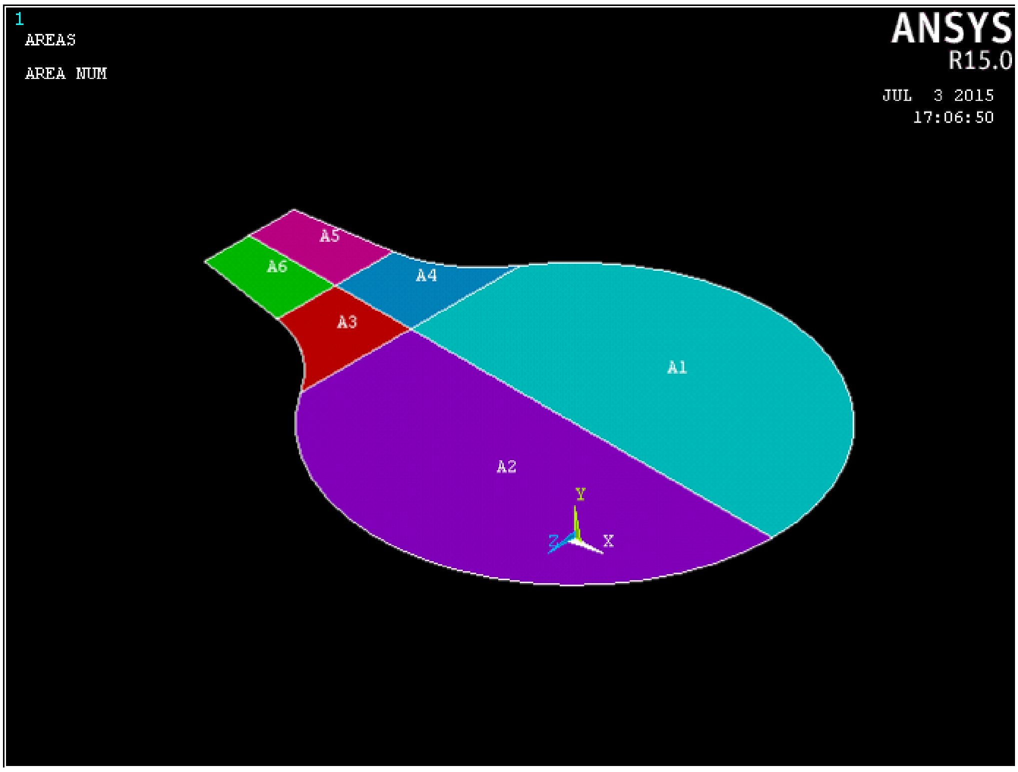



Figure 5.4: *Mapped geometry of structural top-plate*

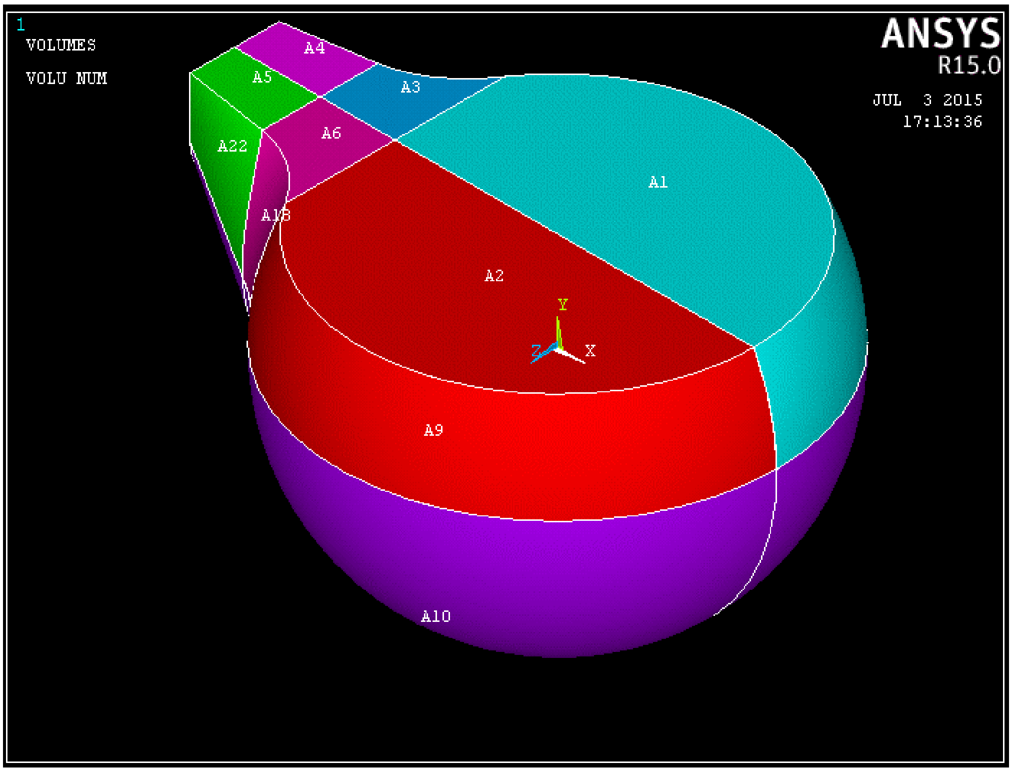



Figure 5.5: *Mapped geometry of acoustic air cavity*

# Chapter 6

# Results and discussion

This chapter is dedicated only for results of the study done in previous chapters. Comparison of experimental and numerical analysis of Veena with different partial and complete model of Veena. Results of the component mode synthesis method also discussed with two different cases of validation and implementation.

## 6.1 Modal Analysis

From chapter 4, it is understood that modal analysis is done for different cases under the uncoupled structural model two models are studied, complete and partial model. Similarly with uncoupled acoustic model. A separate case studied for structural-coupled problem. Results from experimental modal analysis considered as reference model.

### 6.1.1 Mode shape nomenclature

In order to denote the modes of the circular plate a nomenclature system is followed. It helps to understand the spacial distribution in numbers. Its defines as $(d, r)$ where $d$ is defined as number of nodal diameter. It is the complete diametric lines particular mode is having and $r$ is number of nodal circle concentric to the center.

Look at the fundamental mode (0,1) where no diametric line is present hence value of $d$ is zero where as a single concentric circle is present hence value of $r$ is 1. In (1,1) mode, a single diametric line is present and a nodal circle.

### 6.1.2 Experimental modal analysis

After recording time domain data of impact force and accelerometer, software itself will convert that data into frequency domain data and results will be displayed in form of FRF. After getting all FRFs, data need to be analyzed to get the correct information about modal parameters. Driving point is the point where the accelerometer is placed and impact also given. That is why driving point reading is very important while recording the data. Though FRF shows peaks clearly it doesn't

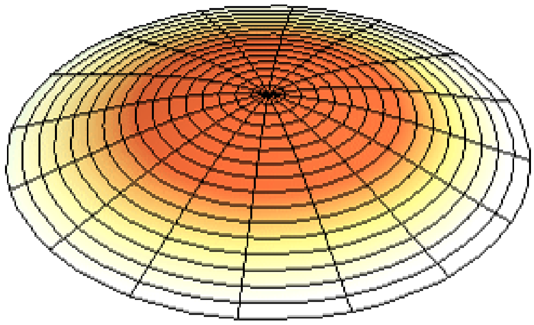


Figure 6.1: *(0,1) Mode*

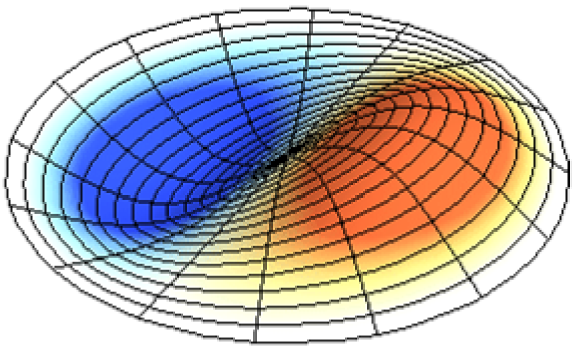


Figure 6.2: *(1,1) Mode*

mean that it has to be related with mode of system because it can be operating deflection shape also.

Please refer to fig 6.4 and fig. 6.5, X-axis is frequency range and Y axis is an amplitude of relative displacement. here peaks are clearly observed at 288.8 Hz, 478 Hz and 539.38 Hz. 288.8 Hz is a fundamental mode of vibration which is associated to top-plate of the resonator and which is (0,1) mode of vibration ( refer to figure ).

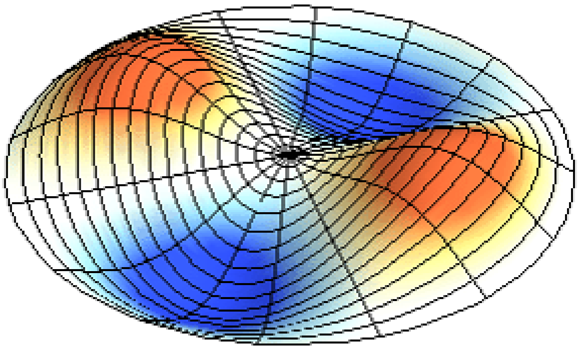
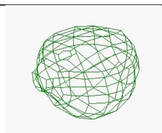
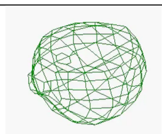
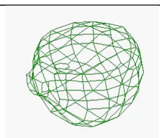


Figure 6.3: *(2,1) Mode*

Please refer tabulated data in table 6.1 for first three modes of Veena.

Table 6.1: Experimental results

| Mode No | **Mode name** | **Frequency (Hz)** | **Mode shape** | **damping** |
|---|---|---|---|---|
| **1** | (0,1) | 288.8 |  | 1.25% |
| **2** | (1,1) | 478 |  | 0.99% |
| **3** | (2,1) | 539.38 |  | 1.8% |

### 6.1.3 Numerical analysis: Partial Veena model

From chapter 4 it is clear that two different numerical models considered to compare and analyze our results with experimental results. One of it is partial Veena model.

Referring to table 6.4 were comparison is done based on uncoupled structural, uncoupled acoustic and coupled partial model of Veena.

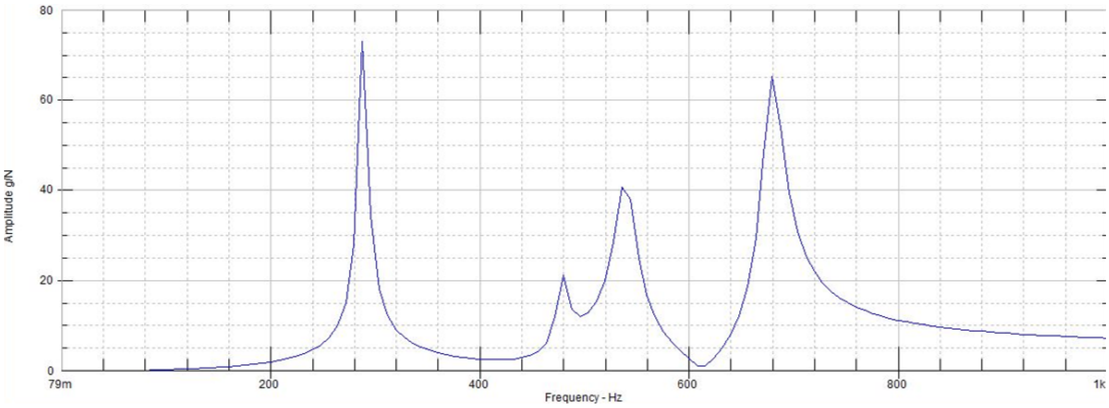



Figure 6.4: *FRF data at point no. 33*

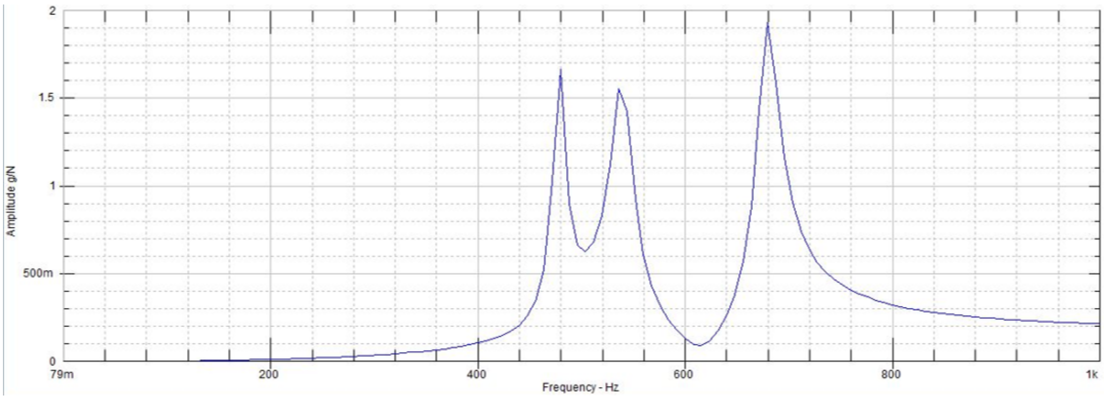



Figure 6.5: *FRF data at point no. 48*

- first column of the table refer to the experimental results which are three modes of the partial veena model. second column refers to the frequencies of the uncoupled structural analysis. Here 'T' refers to the mode associated with only top=plate, 'R' for only resonator of veena and (T+R)refers to the top-plate and resonator both. Third and fourth column for uncoupled acoustic and coupled modes, respectively. Last column refer to the appearance of the uncoupled modes in coupled analysis.

- Mode 288.8 Hz (0,1) is in close accordance with uncoupled structural mode of 252.58 Hz and coupled 281.8 Hz of coupled analysis. Uncoupled acoustic frequencies starting from 497 Hz itself, hence no influence on coupled frequency.

- Mode 478 Hz (1,1) is deviated from coupled mode of 485 Hz and 504 Hz with marginal frequency. This mode is reflected in uncoupled structural analysis of 429.98 Hz. Hence 429.98 Hz mode of uncoupled structural model happened to be stiffer due to acoustic coupling and reflected in coupled analysis.

- Mode 539.38 hz (1,1) which an symmetric mode to previous mode is close to coupled mode of 485.79 Hz and 502.26 hz.

Table 6.2: Comparison of partial Veena model with experimental results

| **Exp. Frequency (Hz)** | **Uncoupled Structural Resonator (Hz)** | **Uncoupled Acoustic Resonator (Hz)** | **Coupled resonator (Hz)** | **Type** |
|---|---|---|---|---|
| **288.8 (0,1)** | 102(R) | 497 | 117.14 (R) | Structural |
| **478 (1,1)** | 195.03 (R) | 580.7 | **281.8(T) (0,1)** | Structural |
| **539.38 (1,1)** | **252.58 (T) (0,1)** | | 330.07(R) | Acoustic |
| | 285.81(R) | | 423.59(R) | Structural |
| | 310.55(R) | | 427.47(T+R) | Structural |
| | 381.37(R) | | 442.82(R) | Acoustic |
| | 399.25(R) | | **485.79(T+R) (1,1)** | Structural |
| | 429.98(T) (1,1) | | 504.26(T+R) (1,1) | **structural** |
| | **483.5(R)** | | **576.49(T+R)** | **Structural** |
| | **510.62(R)** | | **587.46(T+R)** | **Structural** |
| | 525.94(R) | | | |
| | 547.98(R) | | | |
| | **564.3(T+R) (1,1)** | | | |
| | **584.09(T) (1,1)** | | | |

In order to quantify the spatial match between the experimental and numerical coupled model correlation analysis is done. Modal Assurance Criteria (MAC)number quantifies the spatial match between two model over the scale of 0 to 1. Here value close to 1 refer two the very close spatial match between two models and in the other hand zero stands for no spatial match.

- refer fig.6.6, X-axis specifies the coupled modes of the numerical model and Y-axis represents modes of the experimental modes.
- MAC number of 0.7 is observed at fundamental mode of vibration. however every low MAC number observed for rest of the modes.
- Hence mass contribution of the total Veena model has to be consider for analysis which will maintain the same condition as that of experimental model.

### 6.1.4 Numerical analysis: Complete Veena model

In this model, complete geometry means the complete mass of the Veena is considered for analysis. fixed-fixed boundary condition is maintained as mentioned in chapter 3. Three different analysis are done same as the previous case and comparison is done.

- Here first column is same as that of previous one which is of experimental results. Second, third and forth are of uncoupled structural, uncoupled acoustic and coupled modes, respectively. here the term 'B' is used additional to previous section which states the modes associated with beam of the Veena.

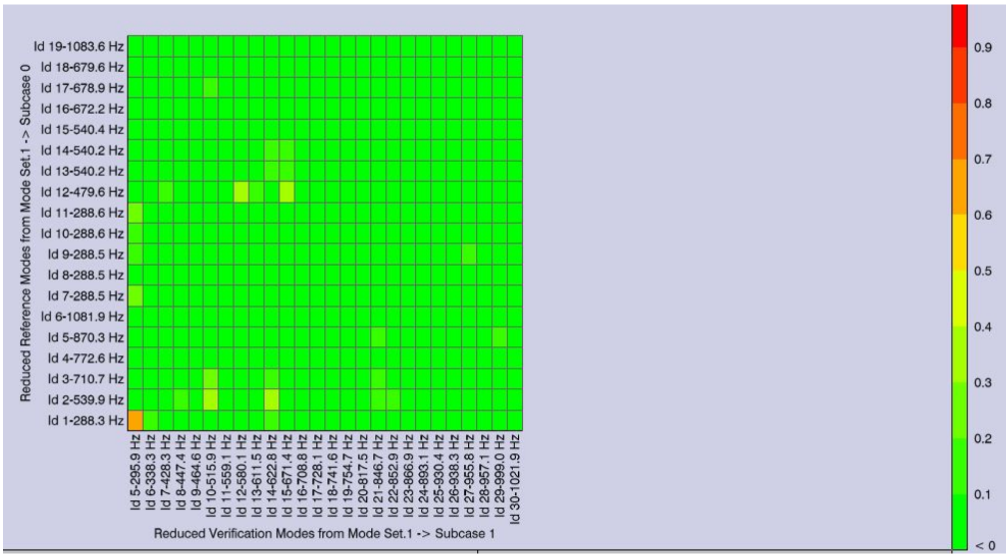



Figure 6.6: *Correlation Analysis: MAC values for partial Veena model*

- Mode 288 Hz (0,1)is very close with 273.44 Hz and 275.58 Hz. MAC number of 0.9 is observed between these modes. Please refer to fig. 6.7.
- MAC number of 0.7 is observed between second mode from experimental which is 478Hz (1,1) and 441 Hz from coupled numerical model.
- MAC number of 0.6 is observed between third mode of 539 Hz (1,1) from experimental results and 518 Hz of the coupled numerical model.

Table 6.3: Comparison table: complete veena model

| **Exp. Frequency (Hz)** | **Uncoupled Structural (Hz)** | **uncoupled Acoustic (Hz)** | **Coupled (Hz)** | **Type** |
|---|---|---|---|---|
| **288.8 (0,1)** | **98.4 (T)** | 154.5(B) | **98.104(T) (0,1)** | Structural |
| **478 (1,1)** | 184.42(B) | 400(B) | 128.9(R) | Acoustic |
| **539.38 (1,1)** | 195.18(R) | 537(B) | 222.06(B) | Structural |
| | 253.51(T) (0,1) | 590.05(R) | 252.22(B) | structural |
| | 286.78 (R) | 673.9(B) | **273.44(T) (0,1)** | Structural |
| | 315.26(R) | 733.92(B+R) | **275.58(T) (0,1)** | structural |
| | 368.12(B) | 850.56(B+R) | 366.89(R) | acoustic |
| | 375.97(B) | 954.6(R) | **441.63(T) (1,1)** | structural |
| | 383.76(R) | 964(B) | **456.07(T) (1,1)** | structural |
| | 403.88(R) | | 486.26(R) | acoustic |
| | 429.98(T) (1,1) | | 490.16(B) | Structural |
| | 461.78(B) | | **518(T+R) (1,1)** | Structural |

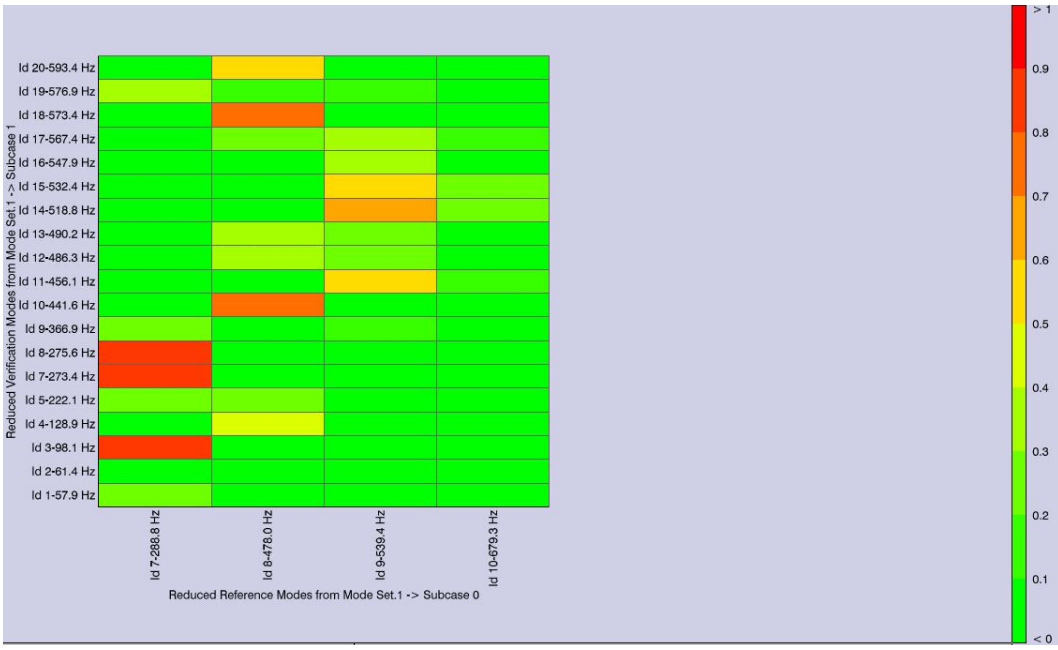



Figure 6.7: *Correlation Analysis: MAC table for complete Veena*

## 6.2 Component mode synthesis method

In chapter 5 complete methodology of the CMS method is discussed. Code for the same is written in steps using two different commercial softwares one is ANSYS15 which is used to solve the FEA solutions and other one is MATLAB of post processing and finding desired results. Code is first validated on simple geometry and implemented on irregular geometry of Veena.

### 6.2.1 Validation of CMS methodology

For validation a simple geometry of rectangular duct is selected with one wall flexible and simply supported boundary condition. Coupled natural frequencies and transfer factor has been found and compared with the analytical results. Table 6.4 shows the comparison for coupled natural frequencies and table 6.5 shows comparison between transfer factors.

- From the table 6.4 it is observed that results from CMS method is in good accordance with that of analytical results.
- maximum frequency difference of 3.8 Hz observed which is of 2.7 percentage variation with respect to analytical.
- From the transfer factor comparison, it is also very clear that CMS results are validated.

Table 6.4: CMS: comparison of coupled natural frequencies

| Sr. No | Coupled Natural frequency from Analytical solution (Hz) | Coupled natural frequency from CMS code (Hz) | Difference | Percentage variation |
|---|---|---|---|---|
| 1 | 111.5 | 112.4 | -0.9 | 0.84 |
| 2 | 141.7 | 137.9 | 3.8 | 2.7 |
| 3 | 158.9 | 156.1 | 2.8 | 1.7 |
| 4 | 182.3 | 179.1 | 3.2 | 1.7 |
| 5 | 221.3 | 217.1 | 4.1 | 1.8 |
| 6 | 228.5 | 230.2 | -1.6 | 0.7 |
| 7 | 270.5 | 265.3 | 5.2 | 1.9 |
| 8 | 328.8 | 322.8 | 5.9 | 1.8 |
| 9 | 341.8 | 344.3 | -2.5 | 0.7 |
| 10 | 399.9 | 392.2 | 7.6 | 1.9 |

Table 6.5: Transfer factor for rectangular duct with one wall flexible by CMS metod. Here first rows represents the uncoupled structural frequencies and first column represents uncoupled acoustic frequencies.

| | **138.44** | **154.17** | **180.49** | **217.45** | **265.07** | **323.37** | **392.33** |
|---|---|---|---|---|---|---|---|
| **114.33** | 0 | 0.4688 | 0 | 0.0205 | 0 | 0.0021 | 0 |
| **228.67** | 0.0411 | 0 | 0.3281 | 0 | 0.1181 | 0 | 0.0028 |
| **343** | 0 | 0.0139 | 0 | 0.0614 | 0 | 0.3853 | 0 |

Table 6.6: Transfer factor for rectangular duct with one wall flexible by CMS metod. here first rows represents the uncoupled structural frequencies and first column represents uncoupled acoustic frequencies.

| | **138.44** | **154.17** | **180.49** | **217.45** | **265.07** | **323.37** | **392.33** |
|---|---|---|---|---|---|---|---|
| **114.33** | 0 | 0.4424 | 0 | 0.02 | 0 | 0.0021 | 0 |
| **228.67** | 0.0479 | 0 | 0.3965 | 0 | 0.0913 | 0 | 0.0027 |
| **343** | 0 | 0.0158 | 0 | 0.0726 | 0 | 0.6085 | |

### 6.2.2 Implementation of CMS methodology

From validation of CMS code, it very clear that results are close to previously proven analytical methods. Hence same code is implemented on irregular geometry of partial model of the veena. Model consist of the top-plate which is simply supported and the acoustic resonator cavity of veena. Mapped mesh technique is used to generate the structured mesh of same dimensions on structural and acoustic geometry to match nodal data.

Table 6.7: Transfer factor for partial model of Veena by numerical method. here first rows represents the uncoupled structural frequencies and first column represents uncoupled acoustic frequencies.

| | **237.1** | **538.8** | **539.8** | **620.8** | **788.3** | **904.1** | **956.5** |
|---|---|---|---|---|---|---|---|
| **497.0** | 0 | 0.014 | 0 | 0.277 | 0.055 | 0 | 0 |
| **580.7** | 0 | 0.838 | 0.844 | 0.010 | 0.020 | 0 | 0 |
| **715.8** | 0.032 | 0 | 0 | 0.072 | 0.597 | 0.014 | 0.019 |
| **798.4** | 0.017 | 0 | 0 | 0.029 | 0.983 | 0.035 | 0 |
| **942.4** | 0 | 0 | 0 | 0 | 0.085 | 0.737 | 0.967 |

Table 6.8: Comparison of CMS and numerical model.

| Sr.No. | Uncoupled Structural freq. (Hz) | Uncoupled acoustic freq. (Hz) | Coupled freq. (Hz) Numerical | Coupled freq. (Hz) CMS method |
|---|---|---|---|---|
| 1 | 237.1 | 497.02 | 230 | 236.3 |
| 2 | 538.8 | 580.7 | 492.2 | 493.5 |
| 3 | 539.8 | 715.8 | 532.5 | 521.2 |
| 4 | 620.8 | 798.5 | 566.5 | 539.3 |
| 5 | 788.4 | 942.5 | 588.5 | 600.3 |
| 6 | 904.1 | | 716.0 | 623.3 |
| 7 | 956.5 | | 756.5 | 711.0 |
| 8 | | | 777.8 | 779.7 |
| 9 | | | 792.9 | 816.5 |
| 10 | | | 836.7 | 898.4 |
| 11 | | | 911.9 | 935.4 |
| 12 | | | 938.0 | 972.5 |

# Chapter 7

# Conclusion

This chapter is dedicated for conclusion of the entire work done in this thesis. Modal analysis results have been discussed previously for different numerical models. Based on it conclusions are drawn which generic applicable to all structural-acoustic coupled problem and some are very specific to Veena. Results of CMS method were discussed in previous chapter, it is observed that validation of the code justified well with supporting results from analytical solution. Transfer factor for irregular geometry is aslo found by CMS method.

## 7.1 Modal Analysis

In Modal analysis, studied different numerical model and results are compared with that of experimental modal analysis. Following are the conclusions:

- MAC number for partial model shows that only fundamental mode matches with moderately value.
- Mass addition to numerical system to be considered. Even analysis focus on the particular part of the geometry but mass contribution can not be ignored completely.
- Complete model shows good match with experimental modal analysis with MAC no of 0.9 for fundamental mode which shows that complete mass of the Veena plays an important role.
- It found that results which are obtained for the free-free analysis is in close accordance with that of fixed-fixed condition. Hence for the similar geometric condition it is very clear to have fixed-fixed condition.
- First few modes are mostly associated with top-plate rather than spherical dome. Hence in vibration transfer mechanism top-late plays an important role.
- Few natural frequencies are very close to the musical notes of the Veena.

## 7.2 CMS method

CMS method developed to understand the participation of uncoupled structural and acoustic mode in energy exchange and spatial match. Validation of CMS code is done on simple rectangular duct with one wall flexible. Code implemented on complex and irregular geometries.

Following are the conclusions from validation and implementation of the CMS method:

- CMS code validated for simple geometry with maximum error of 1.2 percent. Hence code is satisfying the underlying physics of the problem.
- Transfer factor which plays an important role in any coupled analysis which tells about the mechanism of energy transfer and spatial match is validated accurately.
- From the implementation of the code, it is found that not all the frequencies are in good accordance. Hence extensive studied related to mapped/structured mesh techniques need to be done.
- However, transfer factor gives the explanation overall physics of coupled problem analysis.

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