Cross Coupled Quadruped Robot Bounding with Leg Mass

B.Vikram

A Thesis Submitted to
Indian Institute of Technology Hyderabad
In Partial Fulfillment of the Requirements for
The Degree of Master of Technology

Department of Mechanical and Aerospace Engineering

July 2015

Declaration

I declare that this written submission represents my ideas in my own words, and where others' ideas or words have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

(Signature)

B.Vikram

(B Vikram)

ME13M1006

(Roll No)

Approval Sheet

This thesis entitled Cross Coupled Quadruped Robot Bounding with Leg mass by B.Vikram is approved for the degree of Master of Technology from IIT Hyderabad.

Dr. S Sireesh
Department of Civil Engineering
External Examiner

Dr. Viswanath Chinthapenta
Department of Mechanical and Aerospace Engineering
Internal Examiner

R. Prasanth Kumar

Dr. R Prasanth Kumar
Department of Mechanical and Aerospace Engineering
Adviser

Dr. C P Vyasarayani
Department of Mechanical and Aerospace Engineering
Chairman

Acknowledgements

I would like to extend my sincere gratitude to my thesis guide Dr.R.Prasanth Kumar for his constant support and excellent guidance throughout my thesis work.

I would like to thank Prof.U.B.Desai, Director, IIT Hyderabad and Prof.Vinayak Eswaran, HoD, Department of Mechanical and Aerospace Engineering, IIT Hyderabad for their direct and indirect support to my thesis.

I would also like to thank my examiners Dr.Chandrika Prakash Vyasarayani, Dr. Viswanath Chinthapenta, and Dr.S.Sireesh for their valuable suggestions and appreciations for all the reviews.

I would like to heartily thank my parents for their support and encouragement throughout my life.Special thanks to my roommate Vijendra for constantly inspiring me through his hardwork and dedication.

Finally, thanks to my lab mates Ratikant, Deepak, Nikhil, Santosh, Janardhan, Sivanand and all my M.Tech batch mates for their support.

B.Vikram

Dedicated to

My Parents and All My Teachers

Abstract

Quadruped robots find application in military for load carrying, inspection of nuclear power plants and submarine, planetary explorations.The range and duration of these missions depend on the capability of the robot to be dynamically stable and run for many cycles.Moreover, dynamically stable robots, unlike statically stable robots can tolerate departures of the centre of mass from the support polygon formed by the legs in contact with the ground .To achieve dynamic stability, the observation of control laws based on symmetry conditions led to the idea of physical cross coupling between legs.

In this thesis a numerical model of quadrupedal running via bounding gait, which features non-trivial leg mass and cross coupling between front and back legs is studied at various initial conditions and tested for stability.

Contents

Chapter 1

Introduction and Literature Review

1.1 Introduction

Robotics constitutes a relatively young branch of science and technology, which is devoted to studying and developing machines that have the ability to interact with their environment. The goal of robotics is to construct machines that can replace human beings in the execution of a task, as regards both physical activity and decision-making. Most of the mobile robots that have been designed and built use wheels for locomotion. This is a consequence of the inherent static stability and power efficiency of wheeled mobile robots, which made them an attractive first step for practical applications [1]. However, wheels and tracks have limitations when it comes to negotiating uneven terrain or climbing steps.

Mobility is one of the most important reasons for exploring the use of legs in locomotion.Wheels and tracks excel on prepared surfaces such as rails and roads, but most places have not yet been paved.The most important difference between wheeled and legged platforms lies in the fact that wheeled vehicles require a continuous path of support. This is in contrast with machines that use legs for locomotion, which can propel using series of isolated footholds allowing them to traverse irregular terrains. Legs also provide an active suspension that decouples the path of the body from the path of the feet. Thus the performance of legged vehicles can be independent of the detailed ground profile.

Two of the key points in designing reliable legged robots are stability and power efficiency. Trying to improve stability, many researchers develop legged machines that are statically stable, having at least three legs on the ground at the same time, while maintaining their centre of mass in the tripod formed by these legs. Moreover, static stability requires velocities and accelerations to be sufficiently small such that inertia effects do not disturb motion's stability.

Statically stable legged robots usually have a high number of legs and use many actuators per leg. This fact significantly limits the number of behaviours, increases weight, deteriorates energy efficiency and finally, it can result in low speeds, poor reliability and high costs.Unlike statically stable robots, dynamically stable robots can tolerate departures of the centre of mass from the support polygon formed by the legs in contact with the ground [1]. The ability of an actively balanced system to depart from static equilibrium relaxes the rules on how legs can be used for support, a fact that significantly improves the mobility of the robot.

Most creatures on earth use legs for locomotion on solid ground.Legs provide a unique trade off between efficient locomotion on level ground, and the ability to traverse uneven or difficult terrain.Moreover, the robot design has been inspired from the way animals adapt in rough terrain. One of the reasons for animal adaptability is their possession of elasticity in muscles and tendons which can be replicated in robots by providing compliant elements like springs and dampers in torso and legs instead of rigid links. Usage of compliant elements also offers other benefits like power reduction, stability and autonomous behavior.

1.2 Motivation

The realization of dynamic gaits resulting in smoother and more natural motions, higher mobility and higher speeds than those achieved in static gaits, while at the same time it requires less power. Moreover, static gaits usually require complex and computationally expensive control algorithms to regulate the foot placement based on static stability.Neverthelesss, dynamically stable legged locomotion provides a unique alternative when animal-like mobility and speed are required.

Inspired by the highly agile and efficient way animals move, we focus on investigating the main properties of dynamic legged locomotion by studying Scout II, a quadruped robot using only one actuator per leg. This is in striking contrast to the majority of legged machines. Keeping the number of the actuators to a minimum, leads to increased power efficiency, which in turn allows the robot to have a longer operational range. Moreover, low number of actuators also reduces the complexity of the mechanical and electronic design, thus keeping failures to a minimum, while increasing the reliability and decreasing the cost. It must be mentioned here that using a small number of actuators significantly complicates the associated control problem Indeed, Scout II is a highly under-actuated and nonlinear, dynamical system.

The practical motivation for studying the passive dynamics is threefold, [2]

1. First, if the system remains close to its passive behavior, then the actuators have less work to do to maintain the motion, and energy efficiency, a very important issue in mobile robots
2. Second, if there are operating regimes where the system is passively stable, then active stabilization is not required, or else will require less control effort and sensing.
3. Finally, passive dynamics can be used as a design tool to specify the desirable behaviour of complex, underactuated dynamical systems, where reference trajectory tracking is not possible

These facts has motivated me to search for a simple control law that enables the robot move more on its natural dynamics with simple controller or in other words make it more autonomous. Stated in simpler words, the control action aims at trying to help the robot move in the way it wishes to move, by exciting its passive dynamics i.e. the unforced response of the system.

1.3 Literature Review

The desire to build legged machines has been driving research efforts for many years. However, it is only in the past few decades with the advancement of technology that this goal became achievable. A large number of machines that use legs for locomotion have been built. These can be divided into statically stable and dynamically stable machines. In this thesis we are investigating the properties of dynamically stable legged robots.

1.3.1 History of Legged Machines

In the early 80's Raibert was the first to successfully build an actively balanced legged machine. He and his team built a pneumatically actuated monopod that was able to run with speed of 1 m/s, [3] see Fig.1.1. To control the forward speed of the monopod, the control system places the toe at a desired position with respect to the center of mass during flight. To control the pitch attitude of the body, the controller utilises the hip torques during the stance phase.Based on the same principles Raibert and his team built a 3D hopper that was able to run without being constrained on the sagittal plane, see Fig.1.1

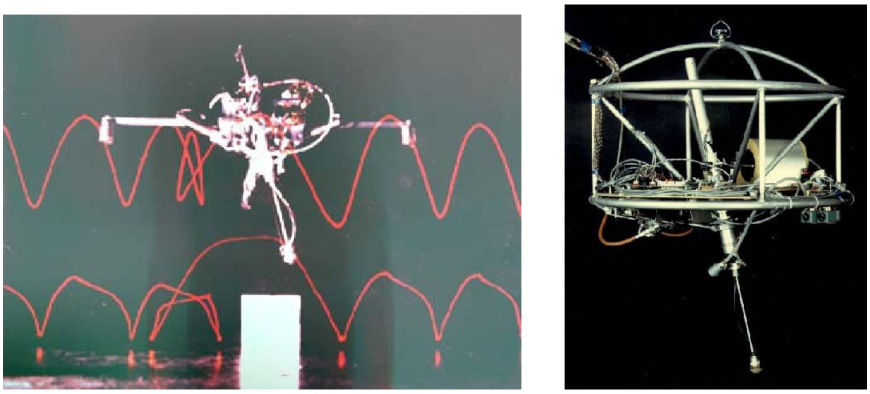

Figure 1.1: The first actively balanced legged robots built by Raibert 2D and 3D hoppers

The success of those simple algorithms in the control of an apparently complex task such as running, led Raibert to build biped and quadruped versions of the above robots and to apply the same basic ideas, see Fig.1.2.Raibert extended the control algorithms developed for monopods to quadruped robots He investigated quadrupedal running gaits that use the legs in pairs: the trot (diagonal legs in pairs), the pace (lateral legs in pair) and the bound (front and rear pairs).

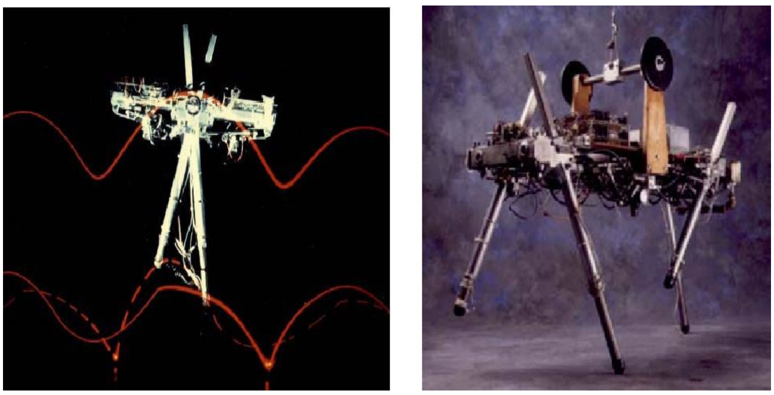

Figure 1.2: The MIT leg lab's biped (left) and quadruped (right) robots

In order to simplify the control problem, he used the virtual leg approach according to which legs that operate in pairs can be substituted by an equivalent virtual leg.Raibert's approach separates the control problem into two parts. The first part is a high level controller, based on the three-part algorithm developed for the monopod, that produces the commands needed to control the body motions and it results to the desired gaits. The second part is a low level controller that ensures that the conditions for the virtual leg approach are met, [1]. Again hydraulic actuators were used and each leg had three actuated DOF: two at the hip for moving the leg in the sagittal and in the frontal plane and one for changing the leg length.

1.3.2 Models for Legged Locomotion

Two of the most common patterns of locomotion are walking and running. At first glance, the difference between walking and running would appear obvious. In running all feet are in the air at some point in the gait cycle, whereas in walking there is always one foot on the ground. This distinction is appropriate for most animals, however there are cases when it fails. A better criterion for distinguishing walking and running is that in walking the centre of mass is at its highest point at midstance, while in running is at its lowest point. In walking, the center of mass vaults over a rigid leg, analogous to an inverted pendulum, see Fig.1.3.Like a pendulum, the kinetic and gravitational potential energies of the body are exchanged cyclically. Kinetic energy in the first half of the stance phase is transformed into gravitational potential energy, which is recovered as the body falls forward and downward in the second half of the stance phase.

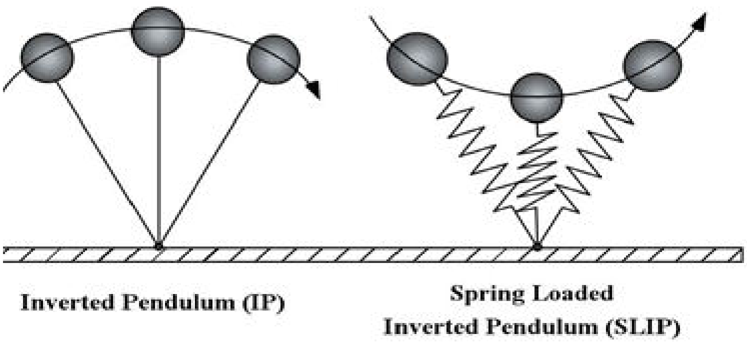

Figure 1.3: Models for walking and running

In running, the leg acts as a spring compressing during the breaking phase and decompressing during the propulsive phase. In SLIP model, the kinetic and gravitational potential energies is stored as elastic energy in the spring at the breaking phase and recovered in the propulsive phase. In running, higher speeds can be achieved because the compression of the spring diminishes the centrifugal effect, so that the leg remains in contact with the ground through midstance. Raibert used the SLIP model to derive controllers that managed the total energy of the centre of mass, to stabilise his legged robots.

1.3.3 Dynamic Stability Analysis

As legged systems exhibit highly non-linear dynamics, the equations of motion for a legged robot are a function of the legs on the ground, and different dynamics apply at different phases of the gait.The mathematical foundations of determining the dynamic stability of a running legged robot are based on methods drawn from nonlinear dynamics.An important conceptual tool for understanding the stability of periodic orbits is the Poincaré map, [5, 6]. It replaces an n^{th} order continuous-time autonomous system by an $(n-1)^{th}$ order discrete-time system. The problem of studying the stability properties of a periodic solution of a continuous-time system is thus reduced to the problem of studying the stability of the periodic points of the Poincaré map. In order to define the return map for a legged system a reference point in the cyclic motion must be selected and then the dynamic equations must be integrated starting from that point until the next cycle.

1.3.4 Passive Dynamics

The term "passive dynamics" means the unforced response of a system [2]. In general, characterising the properties and conditions of the passive behaviour and identifying the model parameters where the system can passively stabilise itself, can lead to designing controllers, which are not entirely based on continuous state-feedback like computed-torque controllers.Raibert and Hodgins stated, "We believe that the mechanical system has a mind of its own, governed by the physical structure and laws of physics. Rather than issuing commands, the nervous system can only make suggestions, which are reconciled with the physics of the system and task " [7].Simulation and analysis suggest that suitably designed legged machines will be able to run passively i.e., without actuation and control.

Passive dynamic bounding gait in quadruped robots was first reported in [8].Passive dynamic bounding gaits are periodic gaits and can begin at stable or unstable fixed points.Stable gaits do not require any control input and can tolerate disturbances.Unstbale gaits can be stabilized by the application of appropriate control inputs.Whether a periodic gait is stable or unstable is determined by the eigen values of the Poincare map.While self-stabilizing gaits are quite attractive to implement, the region of initial conditions(fixed points) where they exist is limited.Controller for stabilizing gaits starting from unstable fixed points is an active area of research [11]

1.4 Objective

The main objective of my thesis is to,

1. Formulate a numerical model of cross- coupled quadruped robot bounding with leg mass.
2. Test the stability of the robot by varying various parameters like forward speed, apex height, pitch rate of torso etc.,

1.5 Outline of Thesis

The thesis is organised as follows,

In Chapter 1, the history and background of development of legged robots, some template models proposed to study the dynamic stability of the quadrupeds are discussed.

In Chapter 2, the modeling aspects and assumptions made while designing the numerical model of quadruped robot with massless legs are explained .Equations of motion in various phases of bounding gait are derived and necessary conditions for phase transitions is discussed.

In Chapter 3, the numerical model is studied by using properties of fixed points and based on the analysis of properties of fixed points, a control strategy is proposed.

In Chapter 4, the concept of cross coupling between front and back legs is added to the quadruped model with leg mass and the stability is tested by also considering the leg mass.

In chapter 5, results of the cross coupled quadruped robot model are discussed.

In chapter 6, conclusion of the work done on proposed quadruped robot model with possible future work is explained.

Chapter 2

Modeling Of Quadruped Robot with Massless Legs

2.1 Physical Model of Quadruped Robot

The robot modeled has a rigid torso with compliant massless legs. The upper and lower legs have springs in between as shown in Fig.2.1. It has motor as actuator at each of the hip revolute joints.Each leg assembly consists of a lower and an upper leg, connected via a spring to form a compliant prismatic joint.Thus, each leg has two degrees of freedom, the hip DOF (actuated) and the linear compliant DOF (passive).

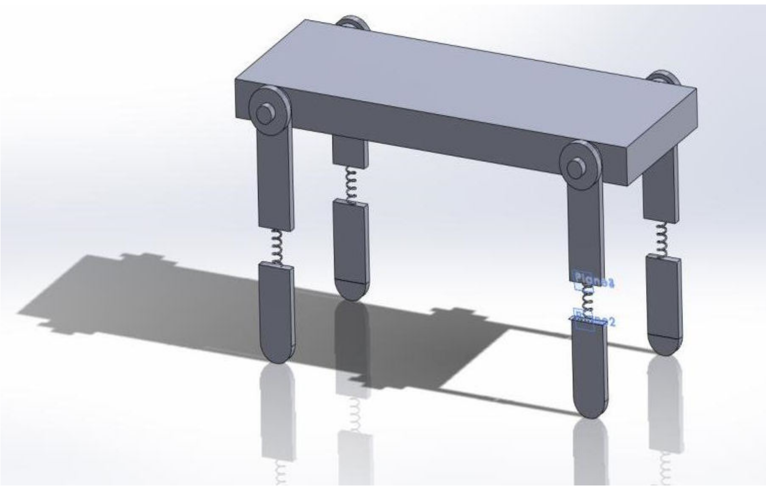

Figure 2.1: Schematic Representation of Quadruped Robot with Massless Legs using Solidworks Software

2.1.1 Physical Parameters

The physical parameters like body mass, leg mass, spring stiffness, inertia etc are taken from Scout II and are shown below

Parameter	Value	Units
Body mass, m	20.865	kg
Body Inertia, I	1.3	kgm2
Spring Constant, k	3520	N/m
Hip-separation, 2L	0.552	m
Leg rest length, l_0	0.323	m

Table 2.1: Physical Parameters taken from SCOUT II

2.1.2 Virtual Leg concept

In planar motion, Scout II can be considered as a three-body chain composed of the torso and the front and back leg pairs, also called the virtual legs as shown in Fig.2.2.The front and back virtual legs represent the pair of the front and back physical legs of the robot. The virtual legs and the original pair of physical legs both exert the same forces and moments on the robot's body so they both result in the same behavior.

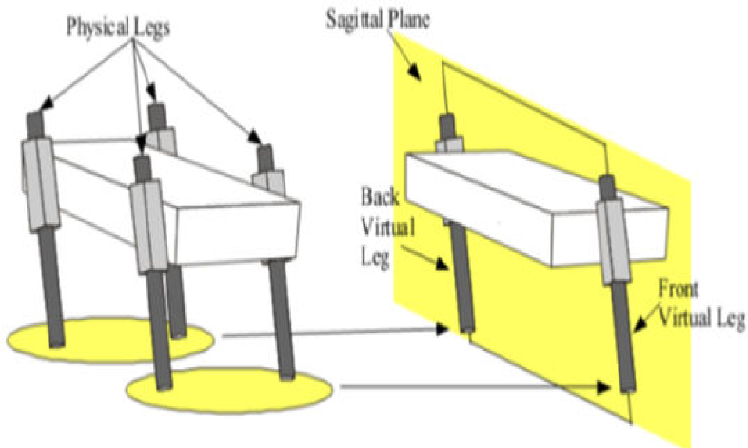

Figure 2.2: Virtual Leg Conccept

For the assumption of the virtual legs to be valid the following conditions have to be true for the bounding gait [10]:

1. The torque delivered at the hips of the physical legs should be equal to half the torque delivered at the hip of the virtual leg

2. The axial force exerted by the springs of each of the physical legs has to be half the force exerted by the spring of the corresponding virtual leg.
3. The feet of the physical legs forming a virtual leg should strike the ground in unison and leave the ground in unison.
4. The forward position of the feet of the virtual leg with respect to the hip has to be the same with the forward position of the feet of the physical legs.

2.2 Bounding Gait

Gait is the pattern of movement of the limbs of animals, including humans, during locomotion over a solid substrate. Most animals use a variety of gaits, selecting gait based on speed, terrain, the need to maneuver, and energetic efficiency. Most gaits are symmetric and periodic in nature. Some of the gait mechanisms are trotting, pacing, galloping, bounding, pronking etc.In bounding gait the front legs and the back legs are in unison which means both the front legs touch and leave the ground at the same time ,similarly back legs also touch and leave the ground at the same time.

Since bounding gait is a planar gait, the model of quadruped robot considered here is planar with body and two "mass-less" legs with identical springs on them. The mass-less legs are connected to the robot at the hip through revolute joints. The distribution of mass in the robot body is assumed to be uniform so that the center of mass is the geometric center.Figure.2.3 shows the model of quadruped robot,

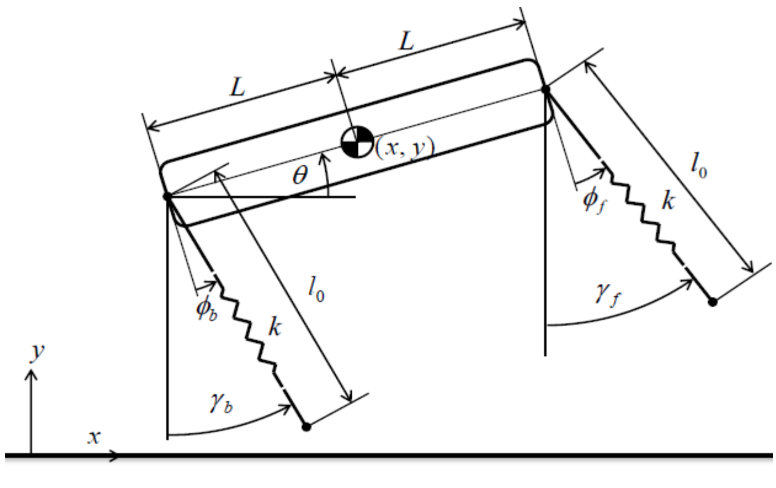

Figure 2.3: Quadruped Robot Model with Massless Legs

where,

k - spring constant

l_0 - initial length of back or front leg

L – half hip spacing of torso.

γ_b - angle of back leg with respect to vertical

γ_f - angle of front leg with respect to vertical

ϕ_b - angle of back leg with respect to perpendicular to torso

ϕ_f - angle of front leg with respect to perpendicular to torso

Bounding has the following three phases during its motion as shown in Fig.2.4

1. Flight phase : During flight phase, robot will not be in contact with the ground.
2. Back leg stance : In this phase ,only back leg will be in contact with the ground
3. Front leg stance : In this phase ,only front leg will be in contact with the ground

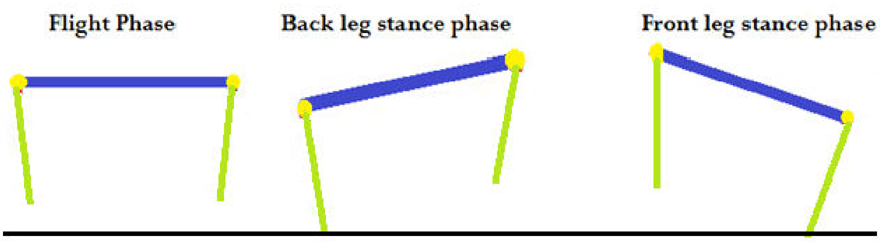

Figure 2.4: Various phases in bounding gait

The blue link indicate torso whereas the green links indicate springs.During flight, the controller adjusts the leg to a desired touchdown hip angle and then, during stance, it sweeps the leg hip backwards with constant commanded torque until a sweep limit angle is reached.The robot starts from flight phase at apex ,after backleg touch down it reaches backleg stance phase, after backleg lift off it reaches flight phase ,after front leg touch down it reaches frontleg stance phase, finally it again reaches to flight phase at apex after front leg lift off.

In the flight phase prior to the back-leg support phase, the back leg is controlled such that at the time of touching the ground it makes a back-leg touch down angle γ_b^{td} with vertical.During back-leg support phase the back-leg spring compresses and decompresses. As soon as the length of the leg equals to the free length l_0, back support phase ends and the robot lifts off the ground at lift-off angle γ_b^{lo} to flight phase.Similarly, during the flight phase prior to the front-leg support phase, the front leg is controlled such that at the time of touching the ground, it makes a front leg touchdown angle of γ_f^{td} with the vertical. Again when it lifts off the ground, it does so at a liftoff angle γ_f^{lo}. Since the legs are assumed to be massless, control action for touchdown does not influence the robot dynamics [11]

2.2.1 Equations of motion

During flight phase,the equations of motions are,

$$m\ddot{x} = 0 \tag{2.1}$$

$$m\ddot{y} = -mg \tag{2.2}$$

$$I_G\ddot{\theta} = 0 \tag{2.3}$$

where x and y are the coordinates of the center of mass of the robot body, and θ is the angle made by the longitudinal axis of the body with the horizontal.

During back-leg or front-leg support phase, the equations of motion are,

$$m\ddot{x} = F_x \tag{2.4}$$

$$m\ddot{y} = -mg + F_y \tag{2.5}$$

$$I_G\ddot{\theta} = r_xF_y - r_yF_x \tag{2.6}$$

where, F_x and F_y are the forces exerted by the back-leg or front-leg on the robot body at the hip joint, and r_x and r_y are the coordinates of the back or front hip joint with respect to the body center of mass. The forces F_x and F_y are calculated from the compression of the spring. If l is the length of the leg, then the spring force is given by k(l_0 - l). The direction of this force is along the leg where F_x and F_y are the components of this force along x and y-axes respectively [11]

While the stiffness k and free length l_0 are constants, the actual length l is calculated as follows:

$$l = \sqrt{(x_{tip} - x + L\cos\theta)^2 + (y - L\sin\theta)^2} \tag{2.7}$$

here x_{tip} is point on the ground where the tip of the back or front leg is in contact.

2.2.2 Touch down and Liftoff Events

The transition between phases occur at the touchdown and the liftoff events. There are two touch-down events (back leg touchdown and front leg touchdown) and two liftoff events (back leg liftoff and front leg liftoff). Conditions for event detection of back and front leg touchdown events respectively are given below:

$$y - L\sin\theta - l_0\cos\gamma_b^{td} = 0 \tag{2.8}$$

$$y + L\sin\theta - l_0\cos\gamma_f^{td} = 0 \tag{2.9}$$

Similarly, the conditions for event detection of back and front leg liftoff events respectively are given below:

$$l_0 - \sqrt{(x_{btip} - x + L\cos\theta)^2 + (y - L\sin\theta)^2} = 0 \tag{2.10}$$

$$l_0 - \sqrt{(x_{ftip} - x - L\cos\theta)^2 + (y + L\sin\theta)^2} = 0 \tag{2.11}$$

where x_{btip} and x_{ftip} are the back and front tip contact points during the back and front leg support phases.

2.3 Conclusions

The modeling aspects and assumptions made while designing the numerical model of quadruped robot with massless legs are explained .Equations of motion in various phases of bounding gait are derived and necessary conditions for phase transitions is discussed.

Chapter 3

Stability Analysis through Fixed Points

3.1 Fixed points and stability

Legged robots are hybrid systems with discrete transformations governing transitions from one phase to another phase of motion [8].Poincare map can be used to determine the stability of the robots as it replaces an n^{th} order continuous-time autonomous system by an $(n-1)^{th}$ order discrete-time system. The problem of studying the stability properties of a periodic solution of a continuous-time system is thus reduced to the problem of studying the stability of the periodic points of the Poincaré map. In order to define the return map for a legged system a reference point in the cyclic motion must be selected and then the dynamic equations must be integrated starting from that point until the next cycle.Here we take apex as initial condition i.e., $\dot{y_2}= 0$ .As x here is monotonically increasing, it is irrelevant to a periodic trajectory so we are left with only four variables y, $\theta, \dot{x}, \dot{\theta}$.

3.2 Finding Fixed points

Fixed points are the points in the function's domain which maps into themselves by function, in other words the value of function will remain equal to its initial values every time. In this context value of state variables of robot will remain same after every cycle with respect to particular point of reference in cycle.A poincare map can be defined mapping initial and final states :

$$X_{n+1} = P(X_n) \tag{3.1}$$

The above equation can be rearranged to define a function ,

$$F(X) = X_{n+1} - P(X_n) = 0 \tag{3.2}$$

The roots of the above equation which satisfy periodicity are called fixed points .For a given back and front leg touch down angles, Newton-Raphson Method can be used to find roots of the equation with a proper initial guess . To find a solution iteratively, the convergence value is set to 10^{-5} and the absolute,relative tolerances are both taken as 10^{-9}.The initial guess for the fixed point is updated using the equation ,

$$X_n^{(k+1)} = X_n^{(k)} + (I - \Delta P(X_n^{(k)}))^{-1}(P(X_n^{(k)}) - X_n^{(k)}) \tag{3.3}$$

Where n corresponds to the n^{th} apex height, k corresponds to the number of iterations and the gradient matrix (Jacobian) of the return map is given by,

$$\Delta P = [\ \delta P/\delta y \ \ \delta P/\delta\theta \ \ \delta P/\delta\dot{x} \ \ \delta P/\delta\dot{\theta}\ ] \tag{3.4}$$

Stability of fixed points can be found by examining the eigen values of Jacobian matrix of return map P.One of the eigen values is always unity ,revealing the conservative nature of the system [2].Stability of fixed points depend on whether the remaining eigen values are within unit circle (stable) or outside unit circle (unstable).

3.3 Properties of Fixed points

All the fixed points whether stable or unstable are observed to follow some common properties :

1. pitch angle at apex is zero and

$$\theta_{atapex} = 0 \tag{3.5}$$

2. the touchdown angle of the front leg is equal to the negative of the back leg liftoff angle and back leg touch down angle is equal to the negative of front leg liftoff angle

$$\gamma_b^{td} = -\gamma_f^{lo} \tag{3.6}$$

$$\gamma_f^{td} = -\gamma_b^{lo} \tag{3.7}$$

A sample fixed point is found using initial guess of (y, θ, $\dot{x}$, $\dot{\theta}$) as (0.33m,0,1.3m/s,120^0) and touchdown angles of backleg and frontleg are taken as 16deg and 14deg, and the corresponding fixed point found after 4 iterations is (0.3237m, 0, 1.392m/s, 145.6^0/s).

3.4 Control Law

If the bounding gait starts from a stable fixed point, it will continue to run for infinite cycles.But if it starts with a unstable initial condition, because of any small error that grows rapidly with time, the gait fails.The passive dynamic bounding is assumed to be failed if any event does not happen within a reasonable time or double support phase occurs.It is possible to stabilize unstable fixed points by using control law reported in, [9].

The algorithm for the control law is as follows, [11] :

1. Start with apex initial conditions for y, θ, $\dot{x}$, $\dot{\theta}$
   - θ should be zero this is the property of fixed points
   - $\dot{\theta}$ is positive so that backleg touchdown happens first
2. End the flight phase with some back leg touchdown angle if this is the first gait cycle or with the negative of front leg liftoff angle of the previous iteration if this is not the first gait cycle
3. Measure and store the back leg liftoff angle after back leg stance phase
4. End the flight phase after the back leg stance phase with front leg touchdown angle taken as the negative of back leg liftoff angle measured in 3.
5. Measure and store the front leg liftoff angle after the front leg stance phase.
6. Go to 2

A sample fixed point (0.35, 0, 4, 161.2434deg/s) with touchdown angles as (21.056deg, 20.02deg) is tested for passive dynamic bounding .As it is a unstable fixed point, the robot failed after 30 cycles [9].But after applying the above control law ,the robot was stable for more than 200 cycles within the norm of 1e-6.

3.5 Body Fixed Touchdown angles

An additional property has been observed that the touchdown lift off angles at fixed points show symmetry not only in terms of absolute angles measured with respect to vertical but also with relative angles measured with respect to robot body.i.e.,

$$\phi_b^{td} = -\phi_f^{lo} \tag{3.8}$$

$$\phi_f^{td} = -\phi_b^{lo} \tag{3.9}$$

Instead of using touchdown angles measured with respect to absolute vertical ,relative touchdown angles have these advantages :

1. No need to measure body pitch angle to maintain touchdown angle.

2. No active control is required during flight phase in order to obtain the desired leg angle at touchdown.

Figure.3.1, shows the stability region with back leg relative touchdown angles angle versus pitch angular velocity at apex for forward speeds of 1m/s and 2m/s [11] . No stability region could be found at higher forward speeds of 3 and 4 m/s. However, the advantage of easy implementation of body fixed touchdown angles is attractive.

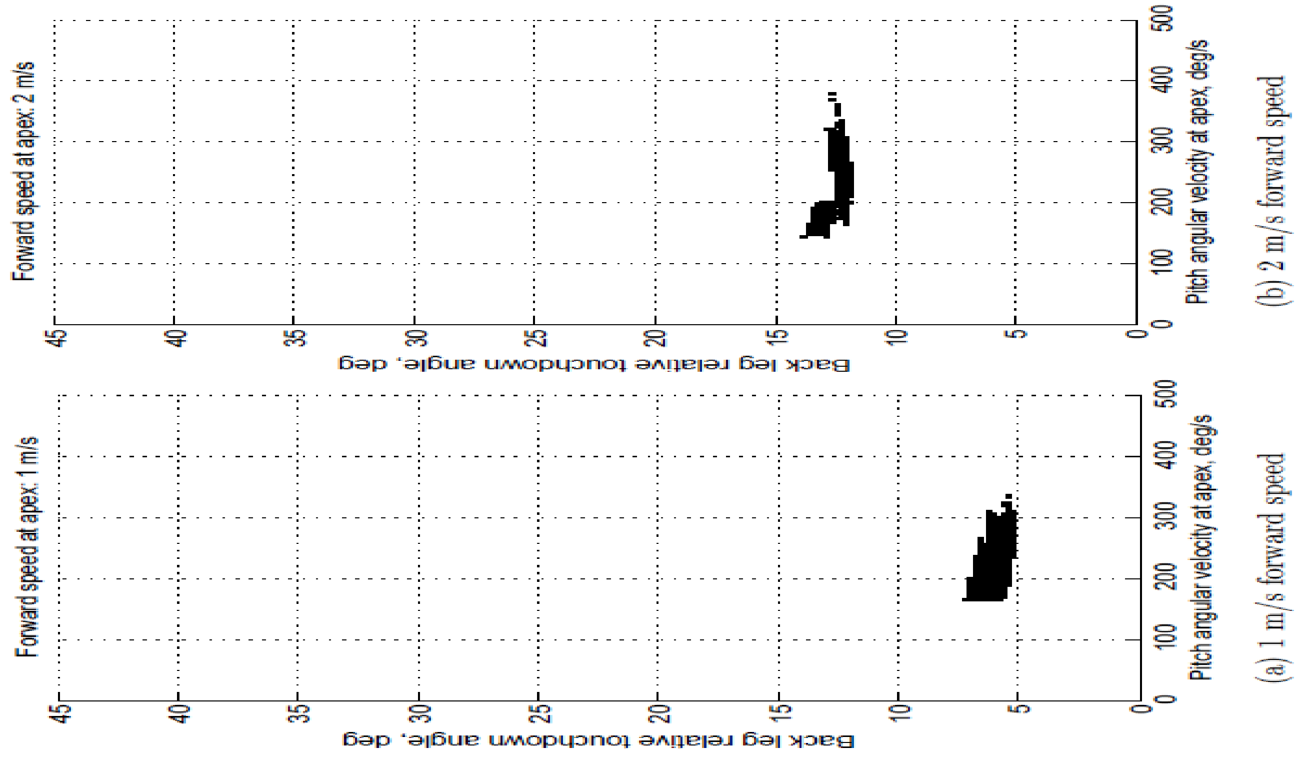

Figure 3.1: Stability region for backleg relative touchdown angle vs pitch angular velocity at apex height of 0.35m for forward velocities of 1m/s and 2m/s

3.6 Conclusions

The quadruped model with massless legs is studied by using properties of fixed points and based on the analysis of properties of fixed points, a control strategy is proposed.

Chapter 4

Cross Coupled Quadruped Robot Model with Leg Mass

4.1 Introduction

In Chapter 3, it is discussed that the touchdown ,lift off angles at fixed points show symmetry not only in terms of absolute angles measured with respect to vertical but also with relative angles measured with respect to robot body. This property led to the idea of cross coupling between front and back legs ,shown in Fig.4.1, because with cross coupling ,the property of backleg relative angle being equal to the negative of front leg relative angle is satisfied not only at the time of touch down and liftoff but it is satisfied throughout the bounding cycle.

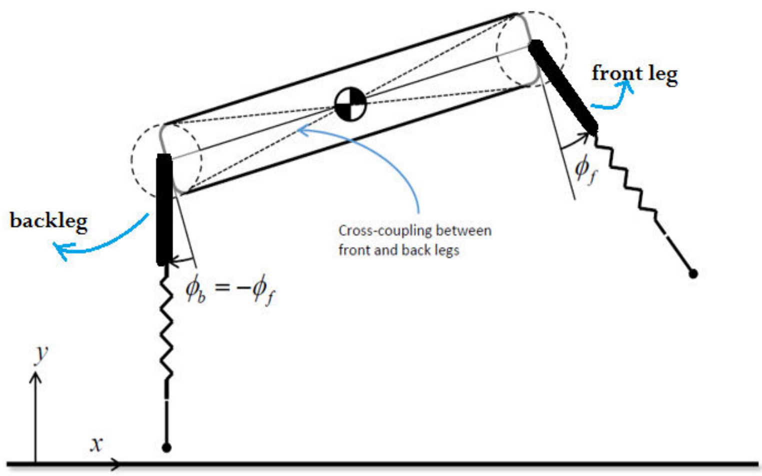

Figure 4.1: Cross Coupled Quadruped Robot Model with Leg Mass

There is a limitation introduced by the cross coupling of the legs. When both the legs are in contact with the ground, the robot body and the two legs form a four bar mechanism with the ground as a fixed link. The motion of the robot requires both the legs to rotate in the same direction about their respective contact points. This does not satisfy the symmetry condition. Hence, double support phase is not allowed when legs are physically cross coupled.The proposed quadruped model(in 3D) with cross coupling between front and back legs is shown below.

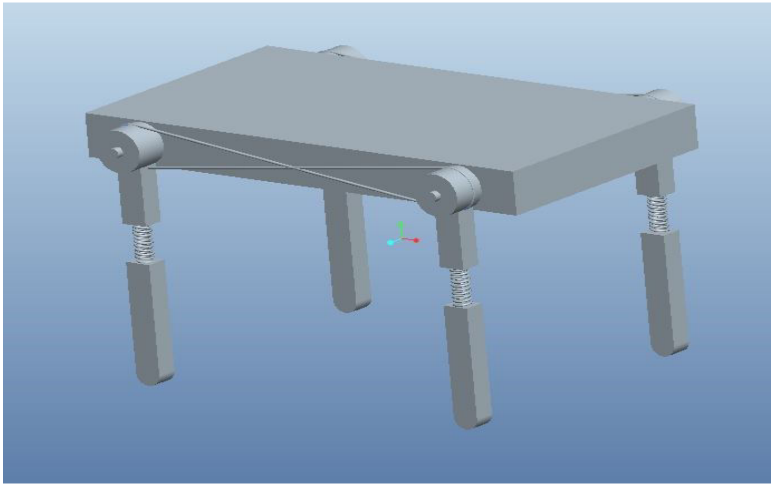

Figure 4.2: Schematic Representation of the Cross Coupled Model with Leg Mass using Pro-E software

The robot is modeled using 'Body-Coordinate Formulation' ,where we assign three coordinates(called body coordinates) x, y, θ for each rigid body [12].Here,the kinematic constraints are defined for each joint in terms of body coordinates. Though large number of equations are generated compared to other formulations ,this method is highly suitable for implementation on computer. In this proposed model, as we also consider leg mass,we have three links i.e.,torso, front leg and back leg which participate in the dynamics in every phase.So each link is assigned with three coordinates x ,y ,θ as shown in Fig.4.3.The torso is shown by blue line, and the front and back upper legs are shown by red lines and the green lines indicate the springs attached to the upper legs.

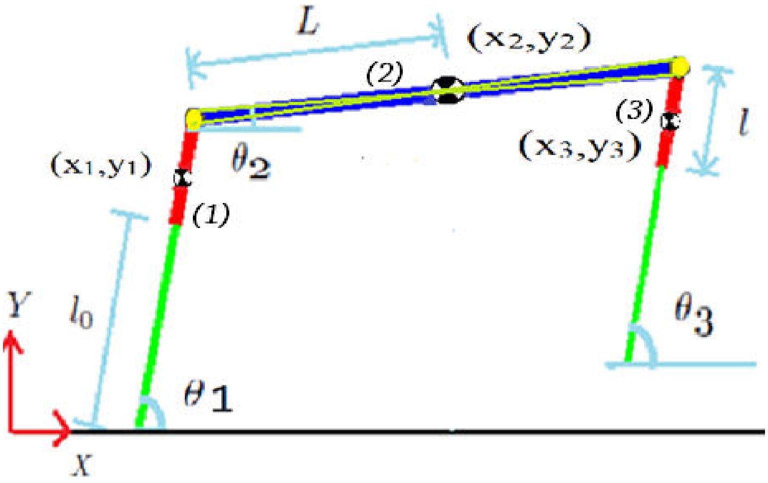

Figure 4.3: Quadruped Robot modeled using Body Coordinate System

While modeling the bounding gait numerically, as the robot goes from one phase to another, the end of one phase and start of another is detected mathematically by solving certain equations using Matlab through inbuilt event detection.

Physical Parameters of Cross Coupled Quadruped Robot:

Parameter	**Value**	**Units**
Body mass,m2	20.865	kg
Leg mass,m1 or m3	0.1	kg
Body Inertia,I	1.3	kgm2
Spring Constant,k	3520	N/m
Hip-separation,2L	0.552	m
length of uncompressed spring,l_0	0.216	m
length of upper legs,$l_1 or l_3$	0.216	m

Table 4.1: Physical Parameters used for simulation

In the case of massless legs, back and front legs can be set to required touch down and liftoff angles without much consumption of energy.But in this proposed model as leg mass is considered, setting legs to the required angle needs considerable amount of energy which deviates the model from the state of passive dynamics and makes energy inefficient.So, only the initial conditions of the model at apex is given and then its stability is tested.The model is considered failed if any event does not occur within a reasonable time or double support phase occurs.

4.2 Various Phases in Bounding

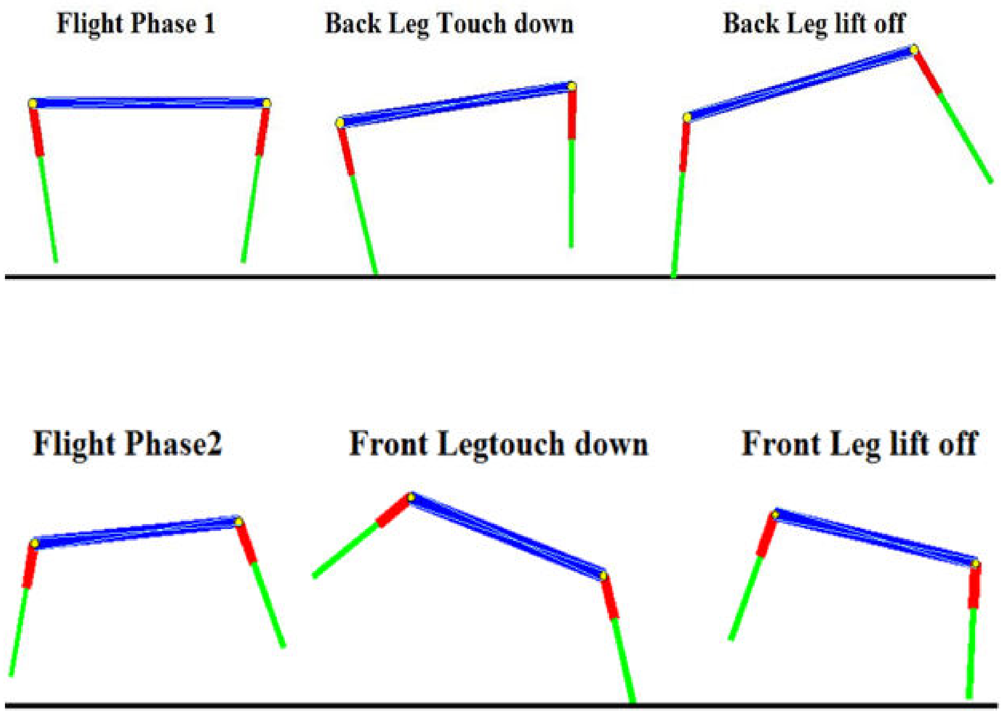

Figure 4.4: Various phases in cross coupled quadruped bounding

Various phases in the bounding gait are shown in Fig.4.4.The robot starts from flight phase 1 at apex ,after backleg touch down it reaches backleg stance phase, after backleg lift off it reaches flight phase 2,after front leg touch down it reaches frontleg stance phase, after front leg lift off it reaches flight phase 3 and finally apex is reached. During back leg or front leg stance phase, the spring compresses and decompresses .As soon as the length of spring equals to the free length of spring lo, support phase ends and the robot lifts off the ground and flight phase is reached.No slippage is assumed between the leg toe and the ground at touchdown, so that the contact point can be treated as a frictionless pin joint.

4.3 Equations of motion

The equations of motion in every phase of bounding gait are integrated numerically using ode45 solver in Matlab. By giving the proper initial conditions to the robot, it transcends from one phase to another to complete a cycle of bounding gait, where the transition between phases is detected using event detection.

1. Flight Phase :

   As none of the legs are in contact with the ground as shown in Fig.4.5, gravity and constraint forces are the only forces acting on the robot during flight phase. In flight phase we have the following constraints,

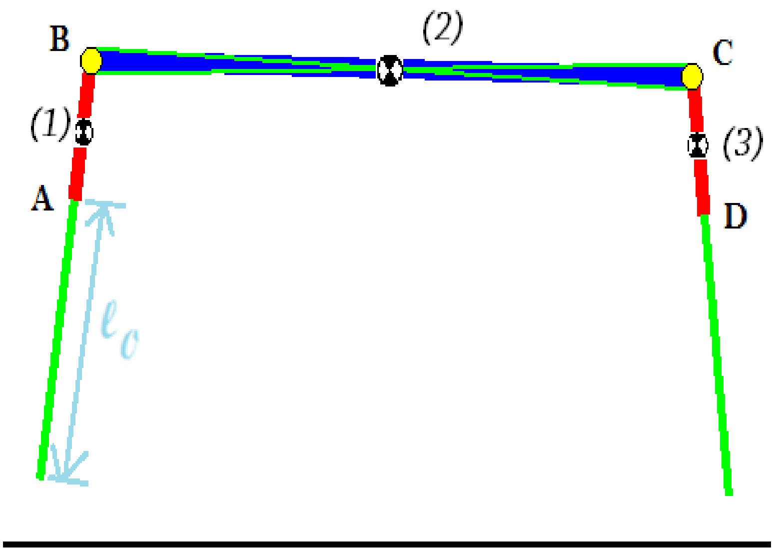

Figure 4.5: Quadruped model in flight phase

- two revolute constraints at the hip joints and
- a coupling constraint between front and back legs

The constraint matrix in flight phase is,

$$\Phi = \begin{pmatrix} r_2^B - r_1^B \\ r_3^C - r_2^C \\ \theta_1 - 2\theta_2 + \theta_3 - \pi \end{pmatrix}$$

The equations of motion in this phase is given by

$$M\ddot{c} = h + h^c \tag{4.1}$$

where,M is mass matrix given by,

$$M = \begin{pmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & J_3 \end{pmatrix}$$

m_1,m_3 = leg mass,
m_2 = torso mass,
c = coordinate matrix ,which contains co-ordinates of all the links,
h = gravity force matrix given by,

$$h = \begin{pmatrix} 0 & \text{-m}_1\text{g} & 0 & 0 & \text{-m}_2\text{g} & 0 & 0 & -\text{m}_3\text{g} & 0 \end{pmatrix}$$

h^c = constraint force matrix given by,

$$\text{h}^c = \begin{pmatrix} D'\lambda \end{pmatrix}$$

D = Jacobian of constraint matrix
λ = Lagrange multiplier

2. Back leg stance phase:

During the back leg stance phase, the spring compresses and decompresses to its original value,so the spring force also participate in the dynamics.We also have an additional translational constraint compared to flight phase.The quadruped model in back leg stance phase is shown below,

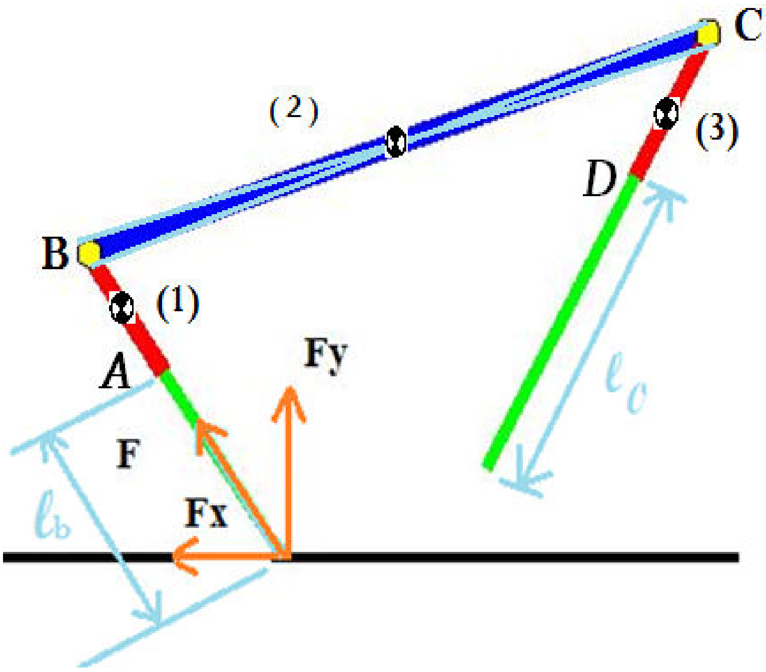

Figure 4.6: Quadruped model in backleg stance phase

In backleg stance phase,we have following constraints

- two revolute constraints at the hip joints and
- a coupling constraint between front and back legs
- a translational constraints between back upper leg and spring

The constraints matrix in back leg stance phase is,

$$\Phi = \begin{pmatrix} r_2^B - r_1^B \\ r_3^C - r_2^C \\ \theta_1\text{-}2\theta_2\text{+}\theta_3\text{-}\pi \\ u_1 r' d_1 \end{pmatrix}$$

here,u_1 is unit vector along back leg.The equations of motion in back leg stance phase is given by ,

$$M\ddot{c} = h + h^c + Springforce \tag{4.2}$$

the spring force is calculated as ,
F = k(l_0 - l_b)

the actual length of the spring l_b is calculated as follows,

$$l_b = -l_1 + \sqrt{(x_2 - L\cos\theta_2 - backtip)^2 + (y_2 - L\sin\theta_2)^2} \tag{4.3}$$

3. Front leg stance phase:

During the front leg stance phase also the spring force participate in the dynamics.The quadruped model in front leg stance phase is shown below,

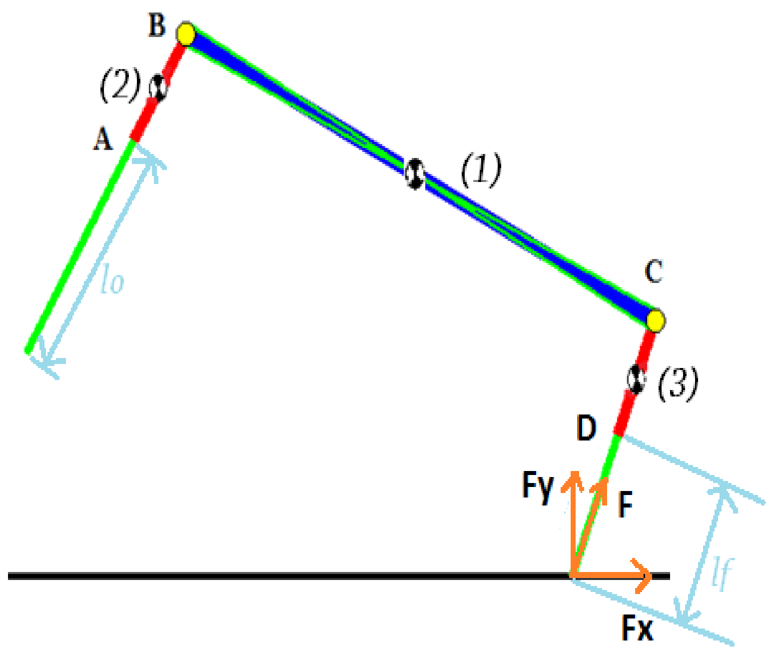

Figure 4.7: Quadruped model in frontleg stance phase

In frontleg stance phase,we have following constraints

- two revolute constraints at the hip joints and
- a coupling constraint between front and back legs
- a translational constraints between front upper leg and spring

The constraints matrix in front leg stance phase is,

$$\Phi = \begin{pmatrix} r_2^B - r_1^B \\ r_3^C - r_2^C \\ \theta_1\text{-}2\theta_2\text{+}\theta_3\text{-}\pi \\ u_3 r' d_3 \end{pmatrix}$$

here, u_3 is unit vector along front leg.

The equations of motion in front leg stance phase is given by

$$M\ddot{c} = h + h^c + Springforce \tag{4.4}$$

the spring force is calculated as ,
F = k(l_0 - l_f)
the actual length of the spring l_f is calculated as follows,

$$l_f = -l_3 + \sqrt{(x_2 + L\cos\theta_2 - fronttip)^2 + (y_2 + L\sin\theta_2)^2} \tag{4.5}$$

4.4 Phase Transition

The transition between phases occur at the touchdown and liftoff events.There are two touchdown events (back leg touch down and front leg touch down) and two liftoff events(back leg liftoff and front leg liftoff). The conditions for event detection is given below,

1. Back Leg Touch down event:

   Flight phase 1 ends with back leg touch down and backleg stance phase is reached.

$$y_2 - L\sin\theta_2 = (l_0 + l_1)\sin(\theta_1) \tag{4.6}$$

2. Back Leg liftoff event:

   Backleg stance phase ends with back leg lift off and flight phase 2 is reached.

$$l_b = l_0 \tag{4.7}$$

   the length of back leg is calculated as,

$$l_b = -l_1 + \sqrt{(x_2 - Lcos\theta_2 - backtip)^2 + (y_2 - Lsin\theta_2)^2} \tag{4.8}$$

3. Front leg touch down :

   Flight phase 2 ends with front leg touch down and front leg stance phase is reached.

$$y_2 + L\sin\theta_2 = (l_0 + l_3)\sin(\theta_3) \tag{4.9}$$

4. Front leg lift off event :

   Front leg stance phase ends with front leg lift off and flight phase 3 is reached.

$$l_f = l_0 \tag{4.10}$$

   the actual length of front leg is calculated as,

$$l_f = -l_3 + \sqrt{(x_2 + L\cos\theta_2 - fronttip)^2 + (y_2 + L\sin\theta_2)^2} \tag{4.11}$$

4.5 Impact Modeling

Due to the leg mass, at the end of flight phase when back or front leg touches the ground, an impact takes place.The impact model described in [13, 14] is used to find out the post impact velocities.After the impact the contact point is treated as an ideal pivot.The robots configuration remains unchanged during impact but there are instantaneous changes in the velocities.

Using the method of Lagrange,the dynamic model of the robot, [16] can be written as ,

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} + G(q) = \tau \tag{4.12}$$

where M,N,G and τ are mass matrix, Coriolis matrix, gravity vector and joint forces respectively.

When the spring comes in contact with the ground, velocity vector suddenly changes, [14].Using the conservation of linear and angular impulse and momentum, one can write the impulsive dynamic model as :

$$M(q)(\dot{q}^{+} - \dot{q}^{-}) = C(q)'\lambda \tag{4.13}$$

where, $\dot{q}^{+}$ and $\dot{q}^{-}$ are the velocities after and before the impact,C(q) is the jacobian of the constraint matrix and λ is a lagrangian parameter.

The lagrangian λ can be computed as,

$$\lambda = -(C(q)M(q)^{-1}C(q)'\dot{q}^{-}) \tag{4.14}$$

as $\dot{q}^{-}$ and λ are known beforehand, the post impact velocities are given by,

$$\dot{q}^{+} = \dot{q}^{-} + M(q)^{-1}C(q)'\lambda \tag{4.15}$$

4.6 Energy loss due to impact

Due to impact, there is energy loss in every cycle at backleg touch down and front leg touch down.A sample initial condition is taken where the robot has started bounding with an initial energy of 110.15237 J at apex in the first cycle and after 10 cycles the energy is 109.997 at apex.The energy loss in every cycle is shown in the table below,

Cycle	**Energy at apex**	**Energy loss**
1	110.15237	0
2	110.09654	0.0507
3	110.08551	0.0102
4	110.08521	0.003
5	110.084835	0.001
6	110.083735	0.007
7	110.082947	0.007
8	110.07985	0.0028
9	110.005	0.06
10	109.997	0.007

Table 4.2: Energy Loss per cycle

As energy is lost in every cycle, the robot will eventually fail as time progresses.So, to cope up this energy loss, a torque of -0.08 N is provided during the entire back-leg stance phase for every cycle, by the actuator at the hip joint between torso and backleg.

4.7 Error plot

A sample initial condition is taken as ,
q_0 = [-0.2667 0.2973 1.7453 0 0.3500 0 0.2667 0.2973 1.3963 1.3117 -1.5780 5.9167 1.0000 0 5.9167 1.3117 1.5780 5.9167] ,the relative and absolute tolerances were set to 10^{-12} and the constraint violation plot for the first cycle is shown in Fig.4.8,

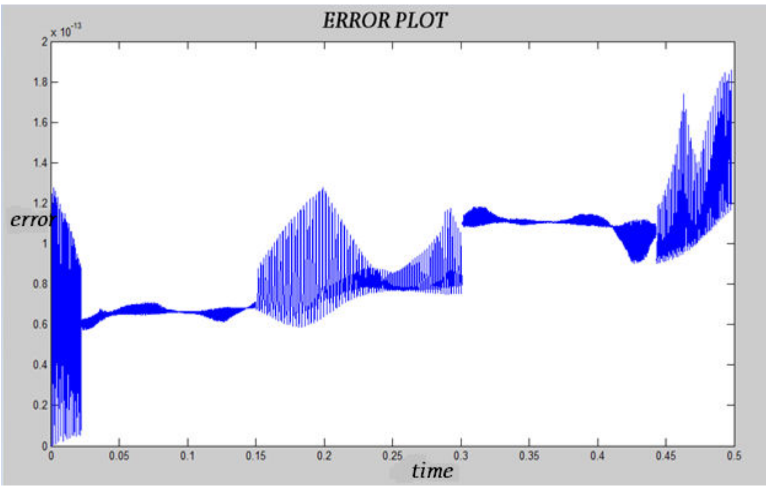

Figure 4.8: Constraint Error Plot

The constraint violation is observed to be very low in the range of 1e-13 ,which reveals that the constraints are satisfied during the entire bounding cycle.

4.8 Conclusions

The concept of cross coupling between front and back legs is added to the quadruped model with leg mass and the stability is tested successfully by changing various parameters.

Chapter 5

Results and discussions

5.1 Stability region with forward velocity vs pitch angular velocity at apex

Fig.5.1, shows the stability region where the robot was able to bound for more than 5 cycles and Fig.5.2, shows the stability region for bounding more than 10 cycles at various pitch rates and forward speeds when the initial apex height is 0.35 m.

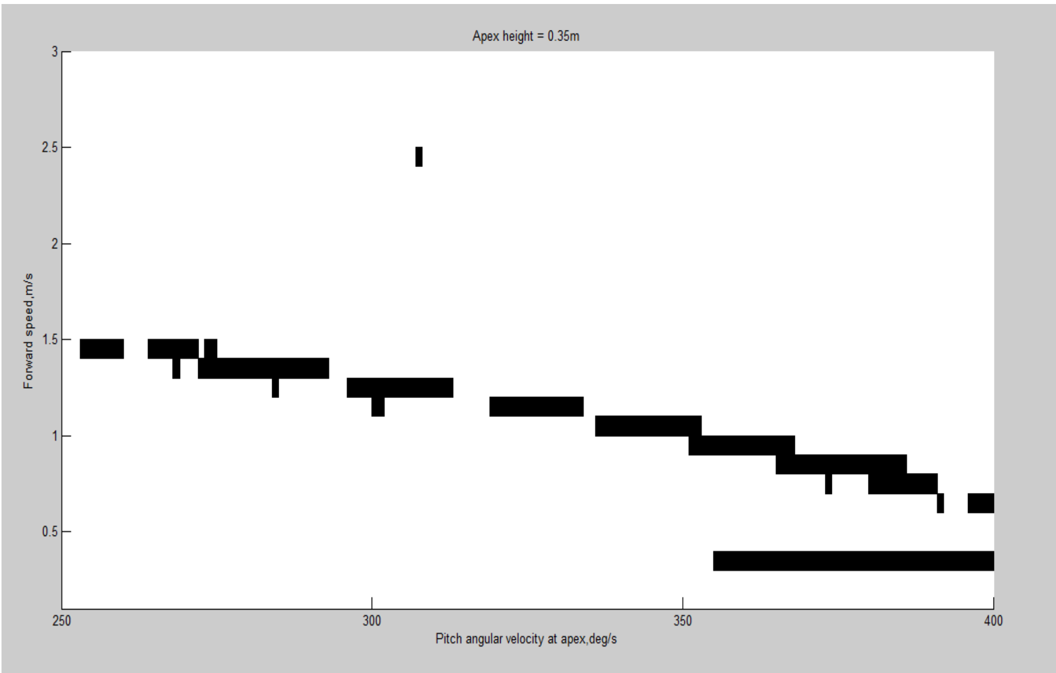

Figure 5.1: Stability region for bounding more than 5 cycles

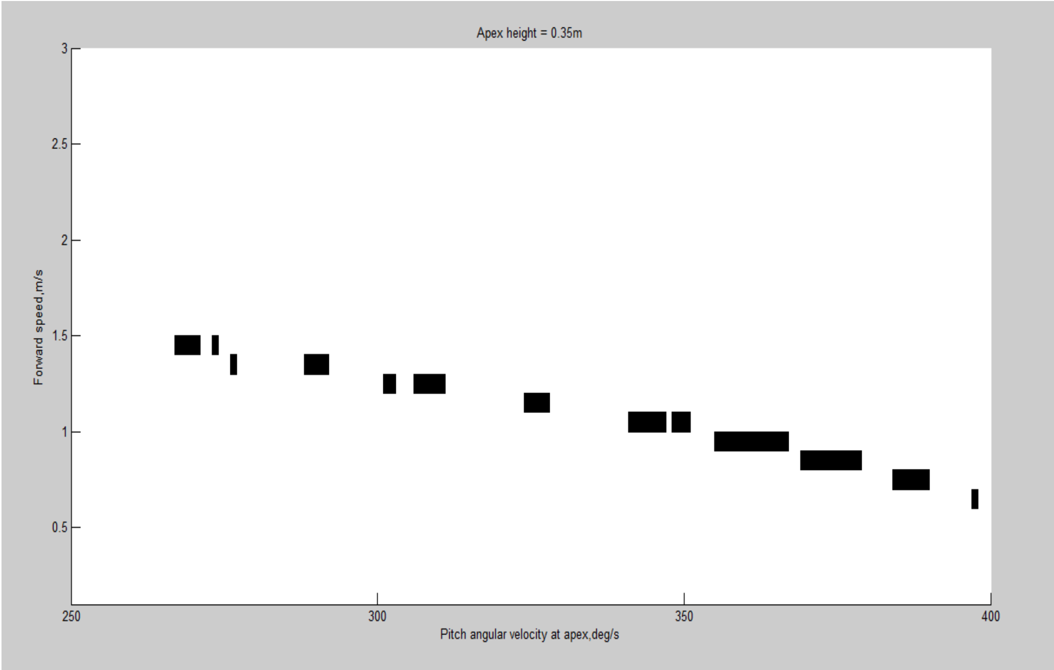

Figure 5.2: Stability region for bounding more than 10 cycles

From the figures it is clear that the cross coupled quadruped robot with leg mass is stable for lower forward speeds only upto 1.5m/s above which the robot is not even completing 10 cycles.

5.2 Stability region with initial back leg angle vs pitch angular velocity at apex

Fig.5.3, shows the stability region for more than 10 cycles at various pitch rates and initial back leg angle (keeping all other initial conditions unchanged)when the initial apex height is 0.35 m and 0.4 m.

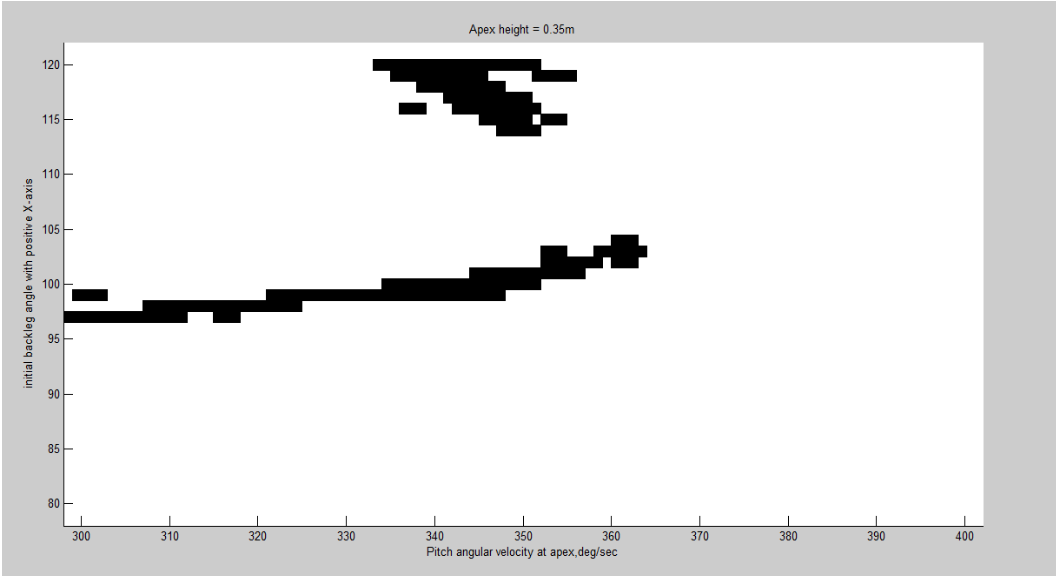

Figure 5.3: Stability region for bounding more than 5 cycles at apex height of 0.35m

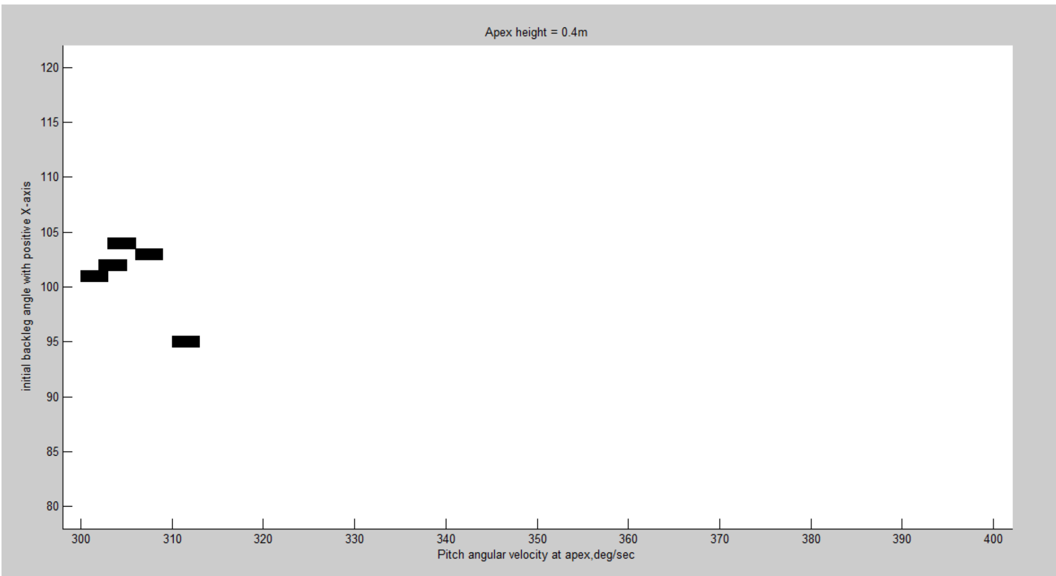

Figure 5.4: Stability region for bounding more than 5 cycles at apex height of 0.4m

From the Fig.5.4, we observe that by just changing the initial leg angle while keeping all other initial conditions unchanged ,the stability region is varying which clearly depict the sensitivity of the robot to the initial conditions.From the Fig.5.3 and Fig.5.4, it is clear that the stability region is more for apex height of 0.35 m than apex height of 0.4 m

5.3 Application of torque to increase stablity

A sample initial condition is taken as, q0 = [-0.2667 0.2973 1.7453 0 0.35 0 0.2667 0.2973 1.3963 1.3117 -1.5780 5.9167 1 0 5.9167 1.3117 1.5780 5.9167]

The model is tested for stability with this initial condition and it has failed after 7 cycles.

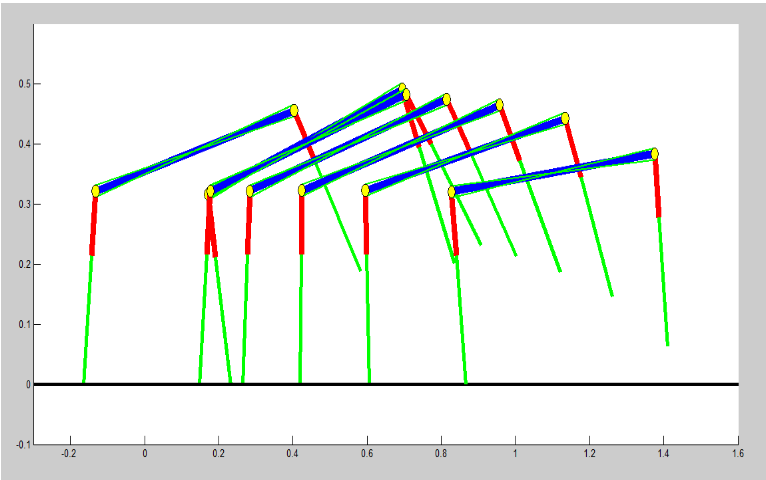

Figure 5.5: Snapshots taken at backleg liftoff event for 7 cycles

Figure.5.5, shows the position of the robot model at backleg liftoffevent in every cycle. From the Fig.5.5, we can observe that in every next cycle at back leg liftoff event, the torso tends to move clockwise and back leg moves anticlockwise .So a torque of -0.08 N-m is applied during the entire back leg stance phase in every cycle to bring the torso and back leg to its position at backleg liftoff event in previous cycle.Moreover, this energy supplied is also used to balance the energy loss due to impact in every cycle.After adding this torque,now the robot is able to bound for 9 cycles.

Figure.5.6, shows the position of the robot model at front leg touch down in every cycle. From the Fig.5.6, we can observe that in every next cycle at front leg touch down event, the torso tends to move anticlockwise and front leg moves clockwise.So

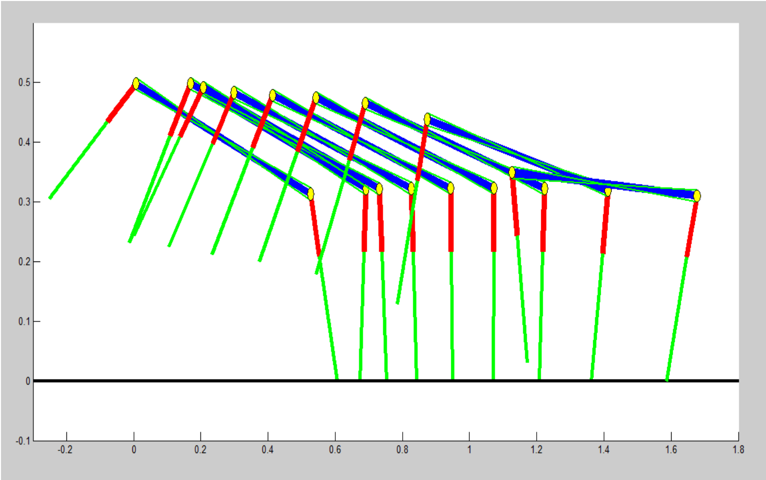

Figure 5.6: Snapshots taken at front leg touch down event for 9 cycles

a torque of -0.3 N-m is applied during the entire front leg stance phase in every cycle after 4 cycles, to bring the torso and front leg to its position at front leg touch down event in previous cycle.After adding this torque, now the robot is able to bound for 10 complete cycles.Using this simple control method of adjusting the torques, the robot model can be made to run for more number of cycles and the stability can be increased.

5.4 Using GUI

Graphical User Interaface or simply called as GUI is an inbuilt feature available in MATLAB.The main reason GUIs are used is because it makes things simple for the end-users of the program.If GUIs are not used ,people have to work from the command line interface, which can be extremely difficult and frustrating.GUI can be initialized by typing guide (graphical user interface development environment) in the command window.

Matlab GUI becomes very handy for the user, where we can use the slider component to choose a value in a range of values, to continuously vary the torque applied in any phase and then check whether the robot model is bounding further instead of entering the torque value manually which would be a hectic task for the user.

Sliders accept numeric input within a specific range by enabling the user to move a sliding bar.The location of the bar indicates a numeric value.

There are four properties that control the range and step size of the slider :

- Value – contains the current value of the slider
- Max – defines the maximum slider value
- Min – defines the minimum slider value

The value property contains the numeric value of the slider.The Max and Min properties specify the slider's range.The sliderstep property controls the amount of change in slider value when the slider is moved.Using the editText component, user can set the value by typing the required value in the edit box which will be sent to the sliders callback to obtain the value set by the user.If the user specifies a string or a value which is not in the range of the slider,then the slider value defaults to minimum slider value.

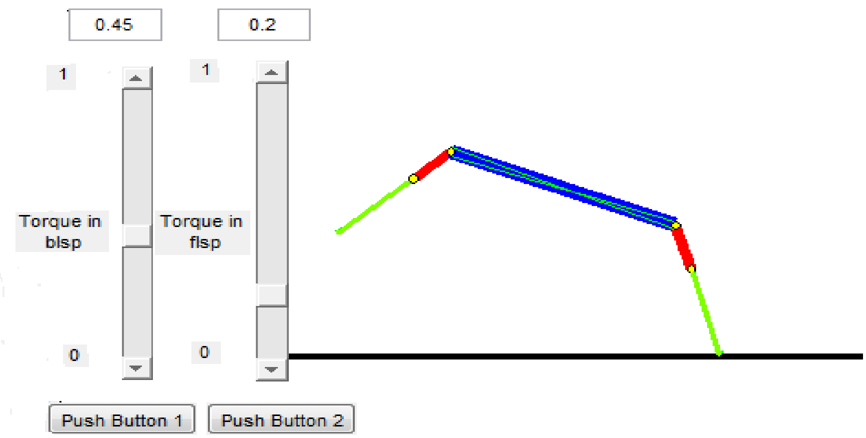

Figure 5.7: Using GUI for applying variable torque

Figure.5.7, shows the application of GUI, where two slider components are used with range from 0 to 1 Nm, to adjust the torque applied in stance phases. Slider 1 is used to control the torque applied in back leg stance phase, here it is set to 0.45 Nm and Slider 2 is used to control the torque applied in front leg stance phase, here it is set to 0.2 Nm.Using these sliders, user can easily adjust the torque applied by moving the slider or by directly entering the value in the edit box and then check the stability.This is a simple but tedious method that can be used to increase the stability of the quadruped model.So human intelligence plays a main role, as he has to observe the motion and continuously adjust the torque required in different phases of the quadruped model to make it stable.

Chapter 6

Conclusions and and Future work

6.1 Conclusions

- The stability region of cross coupled quadruped robot bounding with leg mass is less when compared to stability region with massless legs.
- The idea of cross coupling between legs is useful only at forward speeds less than 1.5 m/sec .
- The cross coupled quadruped model can be made to run for more cycles by continuously adjusting the torque applied in each phase using human intelligence

6.2 Future Work

- Build a physical quadruped model with cross coupling and verify the results obtained.
- Develop an algorithm which also includes human intelligence in the loop to continuously adjust the torque required in different phases of the quadruped model to make it bound further.

Chapter 7

References

1. Poulakakis Iaonnis,"On the Passie Dynamics of Quadrupedal Running",Masters dissertation,McGill University,Montreal,Canada,2002.

2. Poulakakis Ioannis, Evangelos Papadopoulos, and Martin Buehler. "On the stability of the passive dynamics of quadrupedal running with a bounding gait."The International Journal of Robotics Research (2006)

3. Raibert M. H., "Hopping in Legged Systems – Modeling and Simulation for the Two-Dimensional One-Legged Case", in IEEE Tr. On Systems, Man and Cybernetics, Vol. 14, No. 3, pp. 451-463, 1984.

4. Raibert M. H. and Brown H. B., "Experiments in Balance with a 2D one-legged hopping machine", in ASME J. Dynamic Systems, Measurement and Control, Vol. 106, pp. 75-81.

5. Guckenheimer, J. and Holmes P., Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1983.

6. Khalil H. K., Nonlinear Systems, Prentice Hall, 2nd ed., 1996.

7. Raibert M. H. and Hodgins J. A., "Legged Robots", in Biological Neural Networks in Invertebrate Neuroethology and Robotics, R. Beer, R. Ritzmann and T. McKenna (eds), Boston MA: Academic Press, pp. 319-354, 1993.

8. I. Poulakakis, E. Papadopoulos, and M. Buehler. On the stable passive dynamics of quadrupedal running. In IEEE International Conference on Robotics and Automation, vol-ume 1. IEEE, 2003 13681373.

9. C. Sivanand," Passive Dynamic Quadruped Robot Bounding with Front and Back Leg Coupling", Master's dissertation, Indian Institute of Technology Hyderabad, India 2014.

10. Raibert M. H., “Trotting, Pacing and Bounding by a Quadruped Robot”, in J. of Biomechanics, Vol. 23, Suppl. 1, pp. 79 – 98, 1990.

11. P.Murali Krishna,”Energetics and Passive Dynamics of Quadruped Robot Planar Running Gaits”,Doctorate’s dissertation, Indian Institute of Technology Hyderabad, India 2015

12. Parviz E.Nikravesh,”Planar Multibody Dynamics:Formulation,Programming and Applications”,2007,CRC Press.

13. Azevedo, Christine and Poignet, Philippe and Espiau, Bernard,”On line optimal control for biped robots”,Proceedings of the IFAC 15th World Congress,2002

14. Westervelt, Eric R and Grizzle, Jessy W and Chevallereau, Christine and Choi, Jun Ho and Morris, Benjamin,”Feedback control of dynamic bipedal robot locomotion”,volume28,2007,CRC Press

15. Spong, Mark W and Vidyasagar, Mathukumalli,”Robot Dynamics and Control”,2008

16. Wieber, Pierre-Brice,”Modelisation et commande d’un robot marcheur anthropomorphe”,2000