

# A DSM Framework for Optimal Scheduling of Residential Consumer Loads in Mitigating the Financial Risk of LSE

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Department of Electrical Engineering

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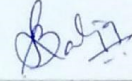
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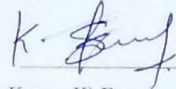
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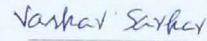
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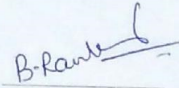
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# Dedication

*To the law of the land*

# Abstract

In this work, a centralized framework is developed for scheduling consumer loads over a distribution network as a part of the demand side management. Preparing financially bound schedules for consumers helps in exerting some natural control over the power consumptions towards system security as well as in improving the economic position of the load serving entity (LSE). Unlike existing approaches, the methodology proposed employs direct price quotes from the end consumers along with detailed information of load composition. In order to minimize the complexity of the process, a concept of price area is introduced. The load schedules are prepared by optimally distributing the net consumer load over different areas maximizing LSE's revenue. The total area load scheduled is distributed over individual consumers according to min-max fairness criterion. The LSEs benefit function with regard to the wholesale market participation is constructed by suitably mapping the net consumer load to the actual power drawal from the grid. The net consumer load to be served is determined from the cleared volume of grid power drawal by means of inverse mapping. Mapping functions are generated through curve fitting on sample evaluations. Detailed case studies are performed to demonstrate the effectiveness of the proposed methodology.

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# Nomenclature

$i$	Index for price-area.
$j$	Index for bus in the distribution network.
$k$	Index for type of load.
$p$	Index for phase.
$g$	Index for load group.
$h$	Index for house.
$l$	Index for branch in the distribution network.
$M$	Total number of price-areas.
$N$	Total number of 3-phase buses in the distribution network.
$\eta_k$	Total number of load groups of Type $k$ .
$H_i$	Total number of houses in Price-area $i$ .
$G_i$	Set of load groups associated with houses in Price-area $i$ .
$z$	LSE's revenue from the end consumers.
$n_{house,h,i,g,k}$	Variable representing the number of load elements that are committed for House $h$ from Load Group $g$ of Type $k$ .
$n_{house,max,h,i,g,k}$	Upper limit on $n_{house,h,i,g,k}$ .
$P_{sid,sp}$	Specified value of the net incremental consumer load.
$P_{grid}$	Total active power drawn at the grid bus.
$P_{min,g,k}$	Minimum power requirement for each element of Load Group $g$ of Type $k$ .
$P_{max,g,k}$	Maximum power requirement for each element of Load Group $g$ of Type $k$ .
$P_{house,h,i}$	Variable indicating the active power allocated to House $h$ of Price-area $i$ .
$P_{house,h,i,g,k}$	Variable indicating the active power scheduled for Load Group $g$ of Type $k$ in House $h$ .
$\mathbf{A}_{ld(k)}^{(p)}$	$M \times \eta_k$ matrix defining the incidence of load groups of Type $k$ on Phase $p$ of Price-area $i$ .

$\mathbf{A}_{area}$	$N \times M$ matrix defining the incidences of price-areas onto network buses.
$\mathbf{A}_{house,i}$	$3 \times H_i$ matrix defining the incidences of houses onto phases of Price-area $i$ .
$\mathbf{n}_{ld(k)}$	$\eta_k \times 1$ variable vector representing the number of committed load elements for different load groups of Type $k$ .
$\mathbf{n}_{ld(k),max}$	Upper limit on $\mathbf{n}_{ld(k)}$ .
$\mathbf{P}_{bld}^{(p)}$	$M \times 1$ vector of base load active power requirements of different price areas over Phase $p$ .
$\mathbf{P}_{sld(k)}$	$\eta_k \times 1$ variable vectors representing the incremental active power scheduled for load groups of Type $k$ .
$\mathbf{P}_{inj}^{(p)}$	$N \times 1$ vector of bus active load power injection variables for Phase $p$ .
$\mathbf{P}_{house,i}$	$H_i \times 1$ variable vector representing power allocations to houses of Price-area $i$ .
$\mathbf{P}_i^*$	$3 \times 1$ vector representing incremental active power scheduled over different phases of Price-area $i$ for a particular hour.
$\mathbf{S}_P^{(p)}$	$N \times 1$ slack active power injection vector corresponding to Phase $p$ .
$\boldsymbol{\pi}^T$	$1 \times M$ vector of price bids from different price-areas.
$\mathbf{1}_N^T$	$1 \times N$ vector of all ones.

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# Chapter 1

## Introduction

### 1.1 Background

The basic idea of demand side management (DSM) is to actively influence power consumption and thus have a certain degree of controllability on the demand side of the power system. Controlling and influencing energy demand can reduce the overall peak load demand, reshape the demand profile, and within the context of smart grid, promote the overall efficiency and security of the system [1]. The key to make demand side management more effective and the grid smarter is to fully and dynamically integrate consumers, their loads, and information about their usage into the operation of the grid [2]. To carry out the DSM functionality of the smart grid, the advanced metering infrastructure (AMI), including advanced metering, communication and control method, will help in realizing the interaction of consumers and power suppliers [3].

Mainly there are two load control strategies employed in demand side management [4]. One approach is direct load control (DLC). In DLC programs, based on an agreement between the LSE and the consumers, the LSE, can remotely control the energy consumption of certain appliances in a household. For example, it may control lighting, thermal comfort equipment (i.e., heating, ventilating, and air conditioning), refrigerators, and pumps. Although DLC is a simple and efficient approach for high power consumption facilities control, it is not suitable for load management in systems comprising a large number of appliances with relatively low power consumption. Again, when it comes to residential load control, users' privacy can be a major concern and even a barrier in implementing DLC programs. The control strategy suitable for residential load management is indirect load control. With this approach, the load control is handled locally by the consumer, whereas the utility has the opportunity to influence the consumers decision on power consumption by sending an appropriate signal in real-time according to the energy market, network load and other economic and technical factors. The price-based demand response programs which include dynamic pricing schemes such as time-of-use tariffs (ToU), critical-peak pricing (CPP), extreme day pricing (EDP) and realtime pricing (RTP) are implemented through indirect load control strategy.

It is a common observation that consumers participating in indirect load control programs employ a home energy management system (HEMS). HEMS is a device that monitors, controls and schedules the energy consumption in the home by taking input as a specification of consumer or a signal from the LSE. In literature, two paradigms are found to be reported for the LSE to carry out the demand

response programs. In one paradigm [4]–[13], the term price is estimated by the LSE and is informed to the end consumers through HEMS. The HEMS carries out the scheduling of loads for the sent price signal and returns the load schedule information to the LSE. Then, the LSE carries out a load flow analysis on different load schedules received and the net power drawal from the grid is bid as a self scheduled request in the wholesale market. This is an ex-ante approach in which the price is posted a priori to the HEMS. In another paradigm [14]–[16], the LSE estimates the demand elasticity of the individual end consumers in the form of individual benefit functions. Then an aggregated benefit function is generated by just aggregating the individual benefit functions. The resultant aggregated bid curve is submitted in the wholesale market and the cleared price is then sent as a signal to the HEMS of individual consumer. This is an ex-post approach in which the price signal is sent to the HEMS after the wholesale market participation. The two approaches given in literature suffer from two disadvantages. Firstly, the price or demand elasticity estimation is itself a very complex task. The LSE has to take into account several factors such as market scenario, seasonal conditions etc. to estimate the price or demand elasticity and if it is not done properly, the LSE may have to bear the financial risk or the consumers have to pay high energy prices due to volatility in the wholesale prices. The second disadvantage in both the paradigms is the non-accountal of network loss in the demand response model for which the consumers do not pay.

In our approach, we have done a centralized scheduling by directly accepting the explicit price bids from the end consumers. In contrast to the two paradigms presented in literature, rather than price, scheduled energy quantity is the input signal to the HEMS. In carrying out a centralized scheduling in a distribution network, the optimal power flow (OPF) based approach is quite difficult to implement. In [17]–[18], OPF is carried out considering the three phase distribution network as a balanced one. In [19], an effort has been made to carry out the unbalanced three phase optimal power flow. The approach we have followed in our work is a simple three phase load flow for distribution networks following the optimal scheduling. The technique used for three phase load flow analysis is given in [20]. In [21], three phase power flow for unbalanced distribution network is carried out. To reduce the complexity in our approach, we have evolved the concept of price areas in which the consumers are grouped into various areas based on their economic conditions. Our framework proposes to reduce the financial risk of LSE by generating the price sensitive bid without any price or demand forecast. The power allocation to houses in an area is carried out by min-max fair share scheme [22].

## 1.2 Objectives

The present work has the following objectives.

- Quantifying the aggregation of loads connected to various nodes in a large distribution network by an aggregate benefit curve.
- Generation of a price sensitive bid by optimal scheduling of the loads for submitting in the wholesale electricity market.
- Allocation of power to houses in a particular area as per the individual demand of each house.

## 1.3 Organization of the Thesis

This chapter discusses about the background and the environment in which the present work is done.

- In **Chapter 2**, an optimization framework for centralized load scheduling is developed.
- In **Chapter 3**, the algorithm for ladder iterative technique is provided and the method adopted for bid curve generation is discussed.
- In **Chapter 4**, Min-Max fair allocation scheme is discussed and a formulation is developed which is suitable for our framework.
- In **Chapter 5**, a comprehensive case study is carried out considering all the salient features of the framework.
- In **Chapter 6**, conclusions are drawn and future scope of the work is discussed.

# Chapter 2

## Optimal Scheduling

### 2.1 Introduction

The overall framework under which the DSM has been done in this thesis is depicted in Fig. 2.1. A single LSE is considered serving a number of residential consumers. The total load of a consumer can be divided into two parts: base load and schedulable load. The base load is the quantity agreed upon by both the consumer and the LSE so that this quantity will be served under all circumstances at the agreed price. The LSE has to incur financial risk for serving this base load because of volatile wholesale market price. However, in the case of retail side competition, an LSE can get stronger hold in the market by serving more base load quantity by carrying out appropriate risk analysis. There can be multiple economic groups of consumers within the territory served by the LSE. These economic groups can afford energy prices according to their economic statuses. In this thesis, each economic group is represented by an area. The price that the consumers in an area are willing to pay is decided by taking the consensus of consumers within that area. This price is submitted as a bid to the LSE. The schedulable load is that part of consumer's load which can be controlled by LSE and this gives him opportunity to reduce financial risks in the electricity market. The following are the steps carried out in the proposed framework.

- The LSE takes the base load consumption data, appliance data of schedulable loads and price bids of all the areas.
- LSE calculates the revenue for samples of net incremental consumer load by an optimization framework.
- Network loss is added to the net consumer load and the aggregate benefit curve in terms of incremental power drawal from grid is generated.
- Bid curve is derived from the aggregate benefit curve for bidding in the market.
- From the cleared volume of grid power drawal, the net incremental consumer load is calculated through a pre-determined function map, which is, subsequently, optimally distributed over different areas.
- Within an area, allocation of power to houses is done by min-max fair share.



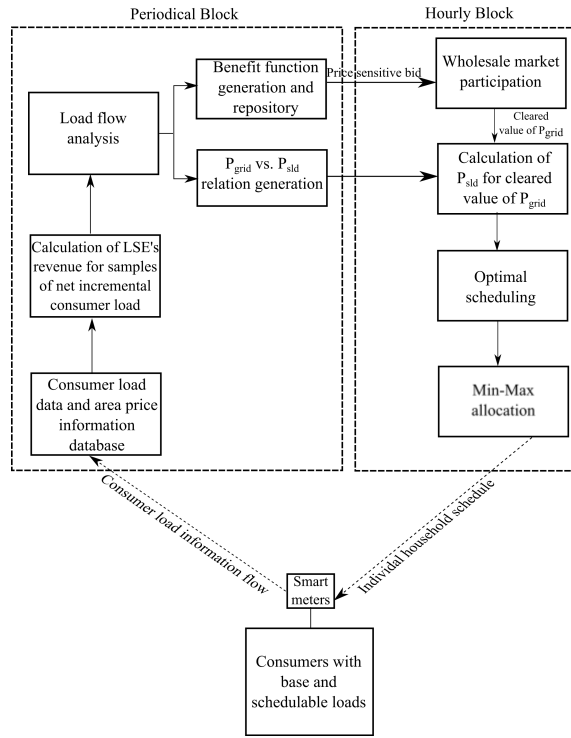


Figure 2.1: The proposed DSM framework.

## 2.2 Problem Formulation

The scheduling problem is framed as an optimization problem. The power in excess of the base load requirement is scheduled to various load groups in a way as to maximize the revenue to the LSE for the net incremental active power scheduled to the consumer loads. The houses connected to a node in the network are grouped into one or more areas. The energy price is fixed for a particular area. The loads to be scheduled in the areas have been categorised into various load groups. Each load group comprises of a set of identical load elements and is characterized by three attributes: the area to which the load group belongs, type of connection and the load element involved. Type of connection refers to the phase to which the load group is connected. Three types of load elements are considered which are described below.

- (i) A load type which can be scheduled for zero or  $P_{max}$  (i.e., fixed load).
- (ii) A load type which can be scheduled for zero or from  $P_{min}$  to  $P_{max}$  (i.e., partially adjustable/dispatchable load).
- (iii) A load type which can be scheduled from zero to  $P_{max}$  (i.e., fully adjustable/dispatchable load).

Each area is assigned a power factor and the same applies to all the houses that belong to that area. The power factor specification is at the discretion of LSE. Any deviation from the assigned power factor is charged according to the real-time price. Considering a unique power factor for an area,

the active power phase balance also indicates reactive power phase balance. Having conceived these factors, the formulation of the optimal load scheduling problem is presented below.

$$\begin{aligned} & \text{maximize} && z \\ & \text{s.t.} && \end{aligned}$$

$$z = z_{actual} + z_{slack} \quad (2.1)$$

$$z_{actual} = \boldsymbol{\pi}^T \left\{ \sum_{k=1}^3 \mathbf{A}_{ld(k)} \mathbf{P}_{sld(k)} \right\} \quad (2.2)$$

$$z_{slack} = -\alpha \mathbf{1}_N^T \left\{ \sum_{p=1}^3 \mathbf{S}_P^{(p)} \right\} \quad (2.3)$$

$$\mathbf{P}_{sld(1)} = \widehat{\mathbf{P}}_{ld(1),max} \mathbf{n}_{ld(1)} \quad (2.4)$$

$$\widehat{\mathbf{P}}_{ld(2),min} \mathbf{n}_{ld(2)} \leq \mathbf{P}_{sld(2)} \leq \widehat{\mathbf{P}}_{ld(2),max} \mathbf{n}_{ld(2)} \quad (2.5)$$

$$\mathbf{0}_{\eta_3} \leq \mathbf{P}_{sld(3)} \leq \widehat{\mathbf{P}}_{ld(3),max} \mathbf{n}_{ld(3)} \quad (2.6)$$

$$\mathbf{0}_{\eta_k} \leq \mathbf{n}_{ld(k)} \leq \mathbf{n}_{ld(k),max} \quad \text{for } k = 1, 2, 3 \quad (2.7)$$

$$\mathbf{P}_{inj}^{(p)} = -\mathbf{A}_{area} \left\{ \mathbf{P}_{bld}^{(p)} + \sum_{k=1}^3 (\mathbf{A}_{ld(k)}^{(p)} \mathbf{P}_{sld(k)}) \right\} + \mathbf{S}_P^{(p)} \quad \text{for } p = a, b, c \quad (2.8)$$

$$\sum_{p=a,b,c} (\mathbf{1}_N^T \mathbf{P}_{inj}^{(p)}) = P_{sld,sp} \quad (2.9)$$

$$\mathbf{P}_{inj}^{(a)} = \mathbf{P}_{inj}^{(b)} \quad (2.10)$$

$$\mathbf{P}_{inj}^{(a)} = \mathbf{P}_{inj}^{(c)}. \quad (2.11)$$

where,

$$\mathbf{A}_{ld(k)} = \sum_{p=1}^3 \mathbf{A}_{ld(k)}^{(p)} \quad \text{for } k = 1, 2, 3 \quad (2.12)$$

$$\widehat{\mathbf{P}}_{ld(1),max} = \text{diag}(P_{ld(1),max,1}; P_{ld(1),max,2}; \dots; P_{ld(1),max,\eta_1}) \quad (2.13)$$

$$\widehat{\mathbf{P}}_{ld(2),min} = \text{diag}(P_{ld(2),min,1}; P_{ld(2),min,2}; \dots; P_{ld(2),min,\eta_2}) \quad (2.14)$$

$$\widehat{\mathbf{P}}_{ld(2),max} = \text{diag}(P_{ld(2),max,1}; P_{ld(2),max,2}; \dots; P_{ld(2),max,\eta_2}) \quad (2.15)$$

$$\widehat{\mathbf{P}}_{ld(3),max} = \text{diag}(P_{ld(3),max,1}; P_{ld(3),max,2}; \dots; P_{ld(3),max,\eta_3}). \quad (2.16)$$

The matrices  $\mathbf{A}_{ld(k)}^{(p)}$  and  $\mathbf{A}_{area}$  are defined as follows:

$$\begin{aligned} A_{ld(k),i,g}^{(p)} &= 1 && \text{if } g\text{th load group of Type } k \text{ is on Phase } p \text{ of Price-area } i \\ &= \frac{1}{3} && \text{if } g\text{th load group of type } k \text{ is on 3 phases of Price-area } i \\ &= 0 && \text{otherwise} \end{aligned} \quad (2.17)$$

$$\begin{aligned}
A_{area,j,i} &= 1 && \text{if Price-area } i \text{ is incident on Bus } j \\
&= 0 && \text{otherwise.}
\end{aligned}
\tag{2.18}$$

The integer variables in the formulation are  $\mathbf{n}_{ld(1)}$  and  $\mathbf{n}_{ld(2)}$ ; and the all other variables are continuous. In Equation (2.1), a slack variable  $z_{slack}$  is added to the  $z_{actual}$  to avoid the infeasibility issue that may happen because of enforcing phase balance. Equations (2.2) and (2.3) show expressions for  $z_{slack}$  and  $z_{actual}$ . In Equation (2.3),  $\alpha$  is the penalty factor for the slack variable  $z_{slack}$ . Equations (2.10) and (2.11) ensure phase balance of nodal active power injections. The incremental loads scheduled for different areas should be equal to the specified value of the net incremental consumer load. This is enforced through Equation (2.9). The other constraints are self-explanatory.

## Chapter 3

# Bid Curve Generation

### 3.1 Introduction

In the previous chapter, we have only considered consumer groups or areas to schedule the loads and have not considered the distribution network through which this power is served to these areas. When a distribution network is kept in place, two factors describe the performance of the system. One is the security of the network in which the line limit plays a role in serving the loads. The other factor is the losses in the lines. Assuming that the distribution network has sufficiently high line limits so that line security does not pose any problem. However, the distribution network loss should be suitably addressed in determining the bid curve (that will be submitted in the wholesale market) of the LSE since the end consumers do not pay for the scheduled network loss as per the proposed framework.

### 3.2 Three Phase Load Flow Analysis

A three phase load flow analysis is carried out to calculate the network losses. Because a distribution network is radial, iterative techniques commonly used in transmission network power-flow studies are not used because of poor convergence characteristics. Instead, an iterative technique designed for a radial system is to be used. After the optimal scheduling is carried out, the active power flow (scheduled) over each phase of an area is known. The reactive power flow details can be obtained from the information of area power factors (that are mandated by LSE). Therefore, the power injections at all the buses are known from the area-bus incidence information. These bus injections are fed as inputs to the three phase load flow program. Ladder iterative technique [20] is employed for performing the three phase load flow in the network.

#### 3.2.1 Ladder iterative technique algorithm

The impedance of a Branch  $l$  in the network is represented by a  $3 \times 3$  matrix as given in Equation (3.1).

$$\mathbf{Z}_l = \begin{bmatrix} Z_{aa,l} & Z_{ab,l} & Z_{ac,l} \\ Z_{ba,l} & Z_{bb,l} & Z_{bc,l} \\ Z_{ca,l} & Z_{cb,l} & Z_{cc,l} \end{bmatrix} \quad (3.1)$$

where  $a, b, c$  are phases of Branch  $l$  in the network;  $Z_{aa,l}, Z_{bb,l}$  and  $Z_{cc,l}$  are self impedances of phase  $a, b$  and  $c$  respectively;  $Z_{ab,l}, Z_{bc,l}, Z_{ca,l}$  etc. are mutual impedances between phases.

If we assume that the network is balanced then all the self impedance values are equal and all the mutual impedance values are equal. Then equation (3.1) can be re-written as follows.

$$\mathbf{Z}_l = \begin{bmatrix} Z_{s,l} & Z_{m,l} & Z_{m,l} \\ Z_{m,l} & Z_{s,l} & Z_{m,l} \\ Z_{m,l} & Z_{m,l} & Z_{s,l} \end{bmatrix} \quad (3.2)$$

where  $Z_{s,l}$  and  $Z_{m,l}$  are self and mutual impedances respectively.

For constant power loads, the node currents at Node  $x$  is given by,

$$\begin{bmatrix} I_a(x) \\ I_b(x) \\ I_c(x) \end{bmatrix} = \begin{bmatrix} SL_a(x)/V_a(x) \\ SL_b(x)/V_b(x) \\ SL_c(x)/V_c(x) \end{bmatrix}^* \quad (3.3)$$

where,  $I_a(x), I_b(x)$  and  $I_c(x)$  are node currents at Node  $x$ ;  $SL_a(x), SL_b(x)$  and  $SL_c(x)$  are complex power demands at the three phases of Node  $x$ ;  $V_a(x), V_b(x), V_c(x)$  are phase voltages at Node  $x$ .

The node voltages are calculated by applying the KVL,

$$\begin{bmatrix} V_a(y) \\ V_b(y) \\ V_c(y) \end{bmatrix} = \begin{bmatrix} V_a(x) \\ V_b(x) \\ V_c(x) \end{bmatrix} - \begin{bmatrix} Z_{s,l} & Z_{m,l} & Z_{m,l} \\ Z_{m,l} & Z_{s,l} & Z_{m,l} \\ Z_{m,l} & Z_{m,l} & Z_{s,l} \end{bmatrix} \begin{bmatrix} I_{al}(l) \\ I_{bl}(l) \\ I_{cl}(l) \end{bmatrix} \quad (3.4)$$

where,  $I_{al}(l), I_{bl}(l), I_{cl}(l)$  are branch currents directed from Node  $x$  to Node  $y$  connected by line  $l$ .

The ladder iterative technique algorithm for the network in Fig. 5.1 would proceed as follows.

1. Assume three-phase voltages at the end nodes (5,7,8,10,13,14, and 15) to be 1.0 pu and angles 0.0, 120, -120 degrees.
2. With the above assumed voltages, the node currents at Nodes 14 and 15 are computed by Equation (3.3).
3. At Node 5, compute the node current and, with this current, apply kirchhoff's voltage law (KVL) given by Equation (3.4) to calculate the voltage at Node 4. This will be referred to as "the most recent voltage at Node 4".
4. Using the most recent value of the voltage at Node 4, the current injection at Node 4 is computed by Equation (3.3).
5. Apply Kirchhoffs current law (KCL) to determine the current flowing from Node 3 toward Node 4.
6. Compute the voltage at Node 3.

7. As Node 3 is a junction node, an end node downstream from Node 3 is selected to start the forward sweep toward Node 3.
8. Select Node 13, compute the node current, and then compute the voltage at Node 12 and Node 11.
9. Compute the node current at Node 3.
10. Apply KCL at Node 3 to compute the current flowing from Node 2 to Node 3.
11. As Node 2 is a junction node, an end node downstream from Node 2 is selected to start the forward sweep toward Node 2.
12. Select Node 8, compute the node current.
13. Go to downstream end Node 7. Compute the node current and then the voltage at Junction Node 6.
14. Compute the node current at Node 6. Apply KCL at Node 6 to compute the current flowing from Node 2 to Node 6.
15. Go to downstream end Node 10. compute the node current, and then compute the voltage at Node 9.
16. Compute the node current at node 9. Apply at Node 9 to compute the current flowing from node 2 to node 9.
17. Calculate the voltage at Node 2 with the current from Node 2 to Node 9 by applying KVL.
18. Compute the node current at Node 2.
19. Apply KCL at Node 2 to compute the current flowing from Node 1 to Node 2.
20. Calculate the voltage at Node 1.
21. Compare the calculated voltage at Node 1 to the specified source voltage.
22. If the difference between the calculated and specified source voltage is not within a specified tolerance, use the specified source voltage and the forward sweep current flowing from Node 1 to Node 2, and compute the new voltage at Node 2.
23. The backward sweep continues, using the new upstream voltages and line segment current from the forward sweep to compute the new downstream voltages.
24. The backward sweep is completed when new voltages at all end nodes have been completed.
25. This completes the first iteration.
26. Repeat the forward sweep, only now using the new end voltages rather than the assumed voltages as was done in the first iteration.
27. Continue the forward and backward sweeps until the calculated voltage at the source is within a specified tolerance of the source voltage.

28. At this point the voltages are known at all nodes, and the currents flowing in all line segments are known.

29. The apparent power,  $S_{grid}$  injected into the network is calculated by,

$$S_{grid} = V_a(1)I_{al}^*(1) + V_b(1)I_{bl}^*(1) + V_c(1)I_{cl}^*(1) \quad (3.5)$$

The total active power drawn at the grid bus is obtained as,  $P_{grid} = \text{Real}\{S_{grid}\}$ .

### 3.3 Benefit Function Construction

The total power drawn from the grid  $P_{grid}$  is equal to the sum of total power to be served to all the areas i.e.,  $(P_{bld} + P_{slid})$  and total network loss. We define  $\Delta P_{grid}$  as,

$$\Delta P_{grid} = P_{grid} - (P_{bld} + P_{bld,loss}) \quad (3.6)$$

where,  $P_{bld,loss}$  is the loss in the network when only the base load is drawn from the grid. In effect,  $\Delta P_{grid}$  indicates the incremental power drawal from the grid to serve the incremental consumer loads. The incremental power drawal from the grid can be calculated for different samples of net incremental consumer load. The complete map between those quantities is obtained by connecting the results for two successive samples with a straight line segment. It is obvious that, with more number of samples, more accurate map can be generated. The map thus generated acts as an interface between the wholesale market and the end consumers. Generating the respective map is a periodical job and a revision is required only after getting some requests from end consumers with regard to modifying the price and load data.

The relationship between LSE's revenue and the net incremental consumer load can be determined in the same way as before based upon sample evaluations. Subsequently, by mapping the net incremental consumer load to incremental grid power drawal, the relationship between LSE's revenue and incremental grid power drawal can be obtained. This defines the benefit function of the LSE in the wholesale market. In order to obtain a closed form of expression, the actual benefit curve is approximated by a concave quadratic function (i.e., of the form  $z_{approx} = a(\Delta P_{grid}) - b(\Delta P_{grid})^2$ ). This is obtained by solving the following optimization problem for  $a$  and  $b$ .

$$\text{maximize} \quad \left\{ \frac{1}{2}a(\Delta P_{grid,N_s})^2 - \frac{1}{3}b(\Delta P_{grid,N_s})^3 \right\}$$

s.t.

$$z_{actual,s} \geq a(\Delta P_{grid,s}) - b(\Delta P_{grid,s})^2, \forall s \quad (3.7)$$

$$a \geq 0 \quad (3.8)$$

$$b \geq 0. \quad (3.9)$$

Here, in the objective, the area between the actual benefit curve (as was obtained by piecewise linear curve fitting on samples) and the quadratic benefit curve is minimized. The number of samples collected to construct the actual benefit curve is indicated by  $N_s$ . Index  $s$  runs for samples. Constraint (3.7) is enforced to ensure that the actual benefit will be no lower than approximated

benefit. However, the particular constraint is applied only on discrete samples. Finally, constraints (3.8) and (3.9) ensure the desired shape of the quadratic benefit function.

The bid curve is the derivative of the benefit function and is given by,

$$\frac{dz_{approx}}{d\Delta P_{grid}} = a - 2b\Delta P_{grid} \quad (3.10)$$



## Chapter 4

# Min-Max Allocation of Consumer Loads

### 4.1 Introduction

In the optimal scheduling problem, individual house is not considered as a separate entity. Instead, all the similar load elements from different houses in a particular area are treated in group. In this chapter, the method by which the power allocated to each area through optimal scheduling is distributed over individual houses in that area is discussed. The load allocation to individual houses can be carried out by means of min-max fairness criterion [22]. The min-max fairness criterion deals with dividing a scarce resource among a set of users, each of whom has an equal right to the resource, but some of whom intrinsically demand fewer resources than others. Intuitively, a fair share allocates a user with a “small” demand what it wants, and evenly distributes unused resources to the “big” users.

### 4.2 Min-Max Formulation

Formally, the min-max fair share allocation can be defined as follows:

- No entity gets a resource share larger than its demand.
- The resource share provided to an entity must be no lower than the resource share provided to any entity of smaller demand.
- The demand of an entity can be fully satisfied only after fully satisfying the demands of entities with smaller requests.
- Entities with partially satisfied demands get equal shares of the resource.

The following formulation is realized for the implementation of min-max fairness allocation.

$$\begin{aligned} & \text{maximize} && \sum_{h=1}^{H_i} w_{h,i} P_{house,h,i} \\ & \text{s.t.} && \end{aligned}$$

$$P_{house,h,i} \geq P_{house,h-1,i} \quad \text{for } h = H_i \text{ to } 2 \quad (4.1)$$

$$P_{house,h,i} = \sum_{k=1}^3 \sum_{g \in G_i} P_{house,h,i,g,k} \quad \forall h \quad (4.2)$$

$$\mathbf{A}_{house,i} \mathbf{P}_{house,i} = \mathbf{P}_i^* \quad (4.3)$$

$$n_{house,h,i,g,k} P_{min,g,k} \leq P_{house,h,i,g,k} \leq n_{house,h,i,g,k} P_{max,g,k} \quad \forall h \text{ and } g \in G_i \quad (4.4)$$

$$0 \leq n_{house,h,i,g,k} \leq n_{house,max,h,i,g,k} \quad \forall h \text{ and } g \in G_i . \quad (4.5)$$

where,

$$\begin{aligned} A_{house,i,p,h} &= 1 && \text{if House } h \text{ is on Phase } p \text{ of Price-area } i \\ &= \frac{1}{3} && \text{if House } h \text{ is on 3 phases of Price-area } i \\ &= 0 && \text{otherwise.} \end{aligned} \quad (4.6)$$

For the above formulation, the houses in an area have to be arranged in the increasing order of their demand requirement. In the objective function,  $w_{h,i}$  is the weight given to a house corresponding to its demand requirement. Inequality (4.1) ensures that no house with a lower demand requirement will get its share more than a house with a higher power requirement. Equation (4.2) breaks up the total power scheduled for a house over different load groups that are associated with the particular house. Equation (4.3) ensures the balance between the house power allocation and the total area power allocation over different phases. Equations (4.4) and (4.5) are simple load and element limit constraints that are self-explanatory.

# Chapter 5

## Case Study

### 5.1 System Description

A consumer conglomerate of 140 houses is considered to be served by an LSE. These houses are supplied by a 15-node three phase radial distribution network shown in Fig. 5.1. Node-1 is the

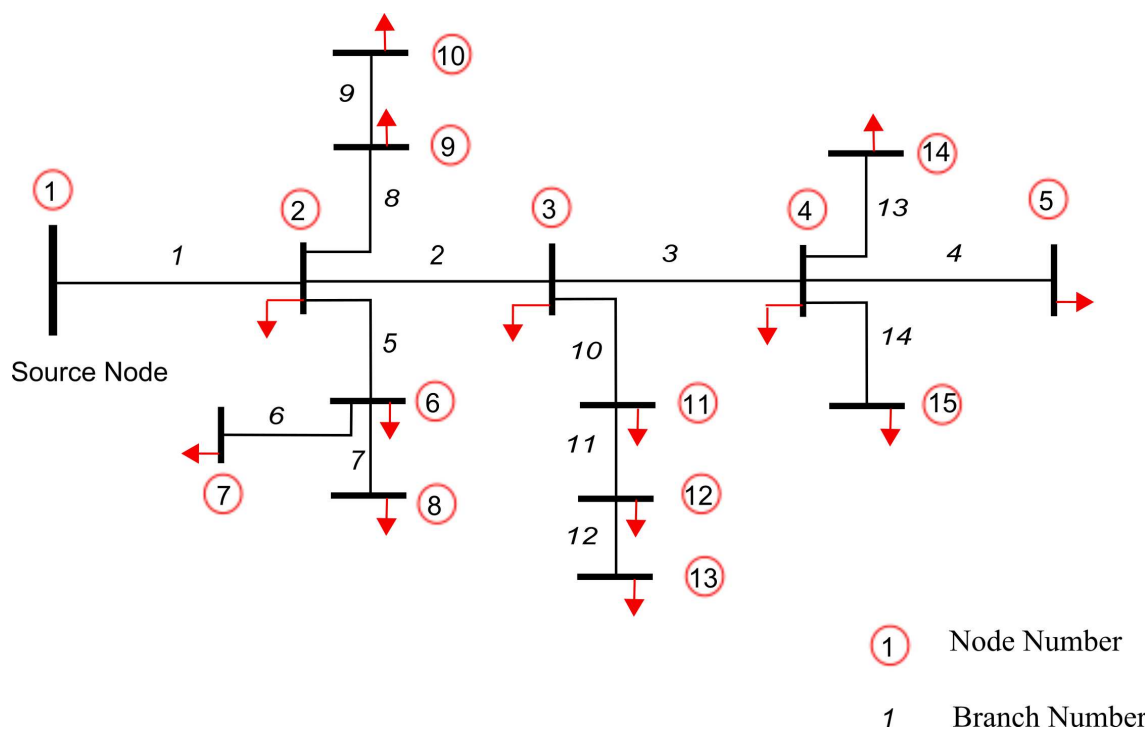


Figure 5.1: 15-node test distribution network.

substation node from where the feeder originates and there is no load connected to this node. The data of line parameters for the particular network is provided in Table A.1. In our study, 14 areas with 10 houses in each area have been considered. For simplicity, the 14 areas are categorised into three area sets such that each area within a particular area set bids the same price. The data of price bids from different areas is given in Table 5.1. Areas 1 to 14 are successively connected from

Buses 2 to 15 respectively. For simplicity of the study, 15 different load elements are considered

Table 5.1: Data Showing the Price Bids from Different Areas

Area set	Areas	Price (Rs./kWh)
I	1,2,5,8,14	8
II	3,4,10,13	6
III	6,7,9,11,12	4.5

including both single phase and 3-phase loads. These 15 load elements with their minimum and maximum power requirements are listed in Table 5.2. The asterisked load elements are the three phase loads in the list. 135 load groups are considered to be present over all the areas. The data for

Table 5.2: Data for Load Elements

Load Element id.	$P_{min}$ (W)	$P_{max}$ (W)	Load Element id.	$P_{min}$ (W)	$P_{max}$ (W)
<i>LE1</i>	90	90	<i>LE9</i>	0	150
<i>LE2</i>	50	150	<i>LE10*</i>	120	120
<i>LE3</i>	0	200	<i>LE11*</i>	30	180
<i>LE4*</i>	150	150	<i>LE12*</i>	0	360
<i>LE5*</i>	60	240	<i>LE13</i>	40	40
<i>LE6*</i>	0	400	<i>LE14</i>	15	90
<i>LE7</i>	60	60	<i>LE15</i>	0	120
<i>LE8</i>	30	120			

load groups is given in Tables B.1 and B.2. The codes used for the phases presented in the Tables B.1 and B.2 are described below.

- 0 Element in the load group is connected to Phase *a*.
- 1 Element in the load group is connected to Phase *b*.
- 2 Element in the load group is connected to Phase *c*.
- 3 Element in the load group is connected to 3 phases.

The base load consumptions in an area per phase is the aggregate of the base load consumptions of the individual houses connected to a particular phase of that area. For the purpose of our study, the power factor at which the load operates (and which is mandated by the LSE) is taken as 0.9 in all the areas. Therefore, the power factor specification of each load element is considered as 0.9. For areas in the Area sets I and II, out of 10 houses that each area is comprised of, 9 houses (three houses per phase) are provided with a single phase supply and one house is provided with a three phase supply. For areas in the Area set III, all the 10 houses are provided with a single phase supply. The data showing the base load consumption in a house, phase to which a house is connected and the schedulable load groups to which a house is associated with is given in Tables C.1 to C.14.

## 5.2 Results

### 5.2.1 Optimal scheduling

The optimal scheduling formulation described in Section 2.2 is solved by GAMS software. The optimization problem takes net consumer load  $P_{slid}$  as the input and gives information about the

corresponding revenue earned  $z_{actual}$  by the LSE. Revenue of LSE for different net consumer load levels are tabulated in Table 5.3. The optimally scheduled powers to various areas for different levels of net consumer load is shown in Table 5.4. The other information that we obtain from the result is the number of load elements of various load groups being scheduled for power supply. The

Table 5.3: Revenue of LSE for Different Net Consumer Load Levels

$P_{sld}$ (kW)	Revenue $z_{actual}$
0	0
78	624
162	1296
246	1968
294	2263.5
342	2551.5
390	2839.2
426	3014.2
462	3176.2
492	3311.2
509.78	3407.4

Table 5.4: Optimally Scheduled Powers to Various Areas

Pslid	Area power in kW													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	17.97	15	0	0	15.03	0	0	15	0	0	0	0	0	15
162	33	32.7	0	0	33	0	0	33	0	0	0	0	0	30.3
246	49.95	49.92	0	0	50.01	0	0	49.95	0	0.03	0	0	0	46.14
294	49.95	49.92	0.51	0	50.01	0	0	49.95	0	15.84	0	0	27.9	49.92
342	49.95	49.92	29.1	18.72	50.01	0	0	49.95	0	15.33	0	0	29.1	49.92
390	49.95	49.92	39.9	29.16	50.01	0	0	49.95	0	31.14	0.15	0	39.9	49.92
426	49.95	49.92	39.9	29.16	50.01	4.8	4.8	49.95	4.8	39.78	7.68	5.43	39.9	49.92
462	49.95	49.92	39.9	29.16	50.01	12	12	49.95	12	39.78	13.35	14.16	39.9	49.92
492	49.95	49.92	39.9	29.16	50.01	20.1	20.1	49.95	13.35	39.78	19.86	20.1	39.9	49.92
509.78	49.95	49.92	39.9	39.96	50.01	20.1	20.1	49.95	20.1	39.78	20.1	20.1	39.9	49.92

functional relationship between LSE's revenue and net incremental consumer load as is obtained by piece-wise linear curve fitting on sample evaluations is plotted in Fig. 5.2.

## 5.2.2 Map generation between net consumer load and the incremental grid power drawal

A three phase load flow program is executed based on the algorithm described in Section 3.2.1. The actual and incremental power drawals (i.e.,  $P_{grid}$  and  $\Delta P_{grid}$ ) from the grid for different levels of net consumer load  $P_{sld}$  are presented in Table 5.5. The mapping between net consumer load and incremental power drawal from the grid is plotted in Fig. 5.3.

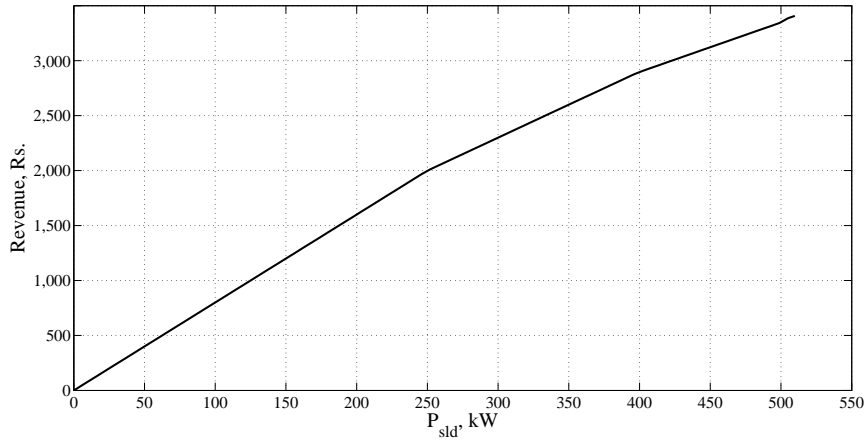


Figure 5.2: Revenue vs. net consumer load.

Table 5.5: Total Active Power Drawn from the Grid vs. Net Consumer Load

$P_{sld}$ (kW)	$P_{grid}$ (kW)	$\Delta P_{grid}$ (kW)
0	699.13	0
78	779.32	80.12
162	865.83	166.69
246	952.44	253.31
294	1002.53	303.40
342	1052.85	353.72
390	1103.14	404.00
426	1140.70	441.56
462	1178.43	479.29
492	1209.86	510.98
509.78	1228.56	529.43

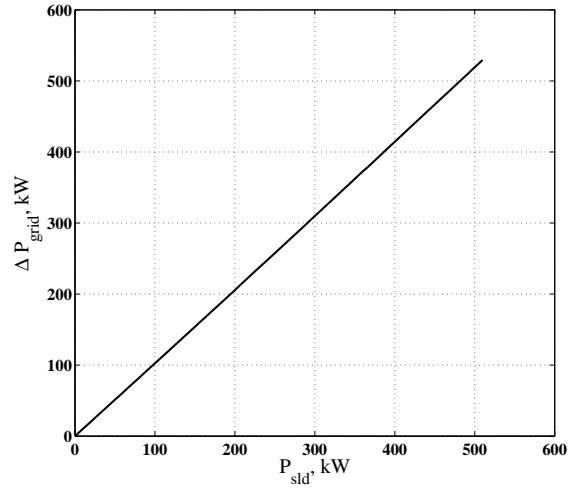


Figure 5.3: Incremental power drawal from the grid vs. net consumer load.

### 5.2.3 Benefit function and bid curve

By co-relating the plots in Fig. 5.2 and Fig. 5.3, the relationship between LSE's revenue and the incremental grid power drawal (which represents the benefit curve of LSE) is obtained as a plot shown in Fig. 5.4.

The approximated benefit curve in the form of concave quadratic function is obtained by solving

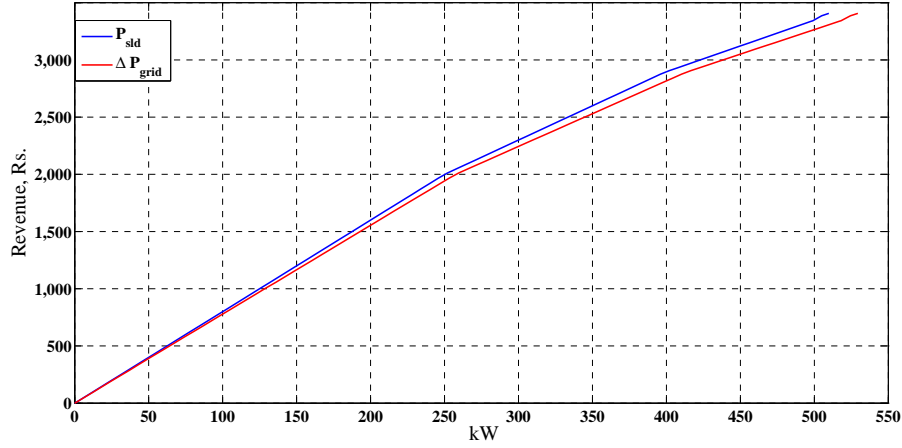


Figure 5.4: Revenue vs. incremental power drawal from the grid.

an optimization problem discussed in Section 3.3 for  $a$  and  $b$ . The values of  $a$  and  $b$  are found to be 7.766811 and 0.002537 respectively. With these values in hand, the approximated benefit curve is shown as a plot in Fig. 5.5.

The bid curve is obtained by Equation (3.10) and is plotted in Fig. 5.6.

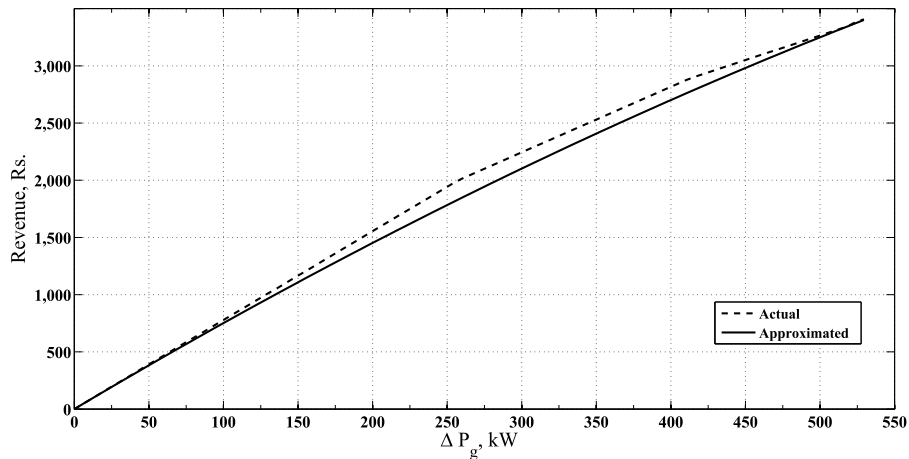


Figure 5.5: Approximated benefit curve.

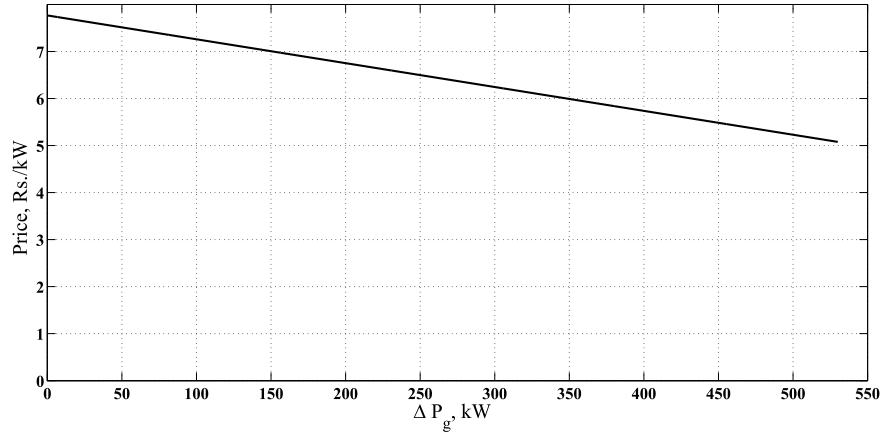


Figure 5.6: Bid curve.

### 5.2.4 Min-Max allocations

For a total schedulable consumer load of 162 kW, the areas that obtain power are Areas-1,2,5,8,14, as seen from Table 5.4. The min-max allocations to houses in these areas are tabulated below (Tables 4.1-4.5). The house id shown in the tables should not be confused with house indexing in the formulation.

Table 5.6: Min-Max Power Allocations in Area-1 Table 5.7: Min-Max Power Allocations in Area-2

House ID	Allocated power (kW)	Maximum power (kW)	House ID	Allocated power (kW)	Maximum power (kW)
0001	4.25	5.75	0011	4.15	6.50
0002	2.50	3.85	0012	2.80	3.88
0003	3.20	5.00	0013	3.15	4.77
0004	2.50	3.85	0014	2.80	3.88
0005	3.20	5.00	0015	3.15	4.77
0006	4.25	5.75	0016	4.15	6.50
0007	3.20	5.00	0017	3.15	4.77
0008	4.25	5.75	0018	4.15	6.50
0009	2.50	3.85	0019	2.80	3.88
0010	3.15	3.90	0020	2.40	4.47



Table 5.8: Min-Max Power Allocations in Area-5 Table 5.9: Min-Max Power Allocations in Area-8

House ID	Allocated power (kW)	Maximum power (kW)	House ID	Allocated power (kW)	Maximum power (kW)
0041	4.30	6.55	0071	4.10	6.35
0042	2.70	3.78	0072	2.50	3.85
0043	3.55	5.17	0073	3.35	5.15
0044	2.70	3.78	0074	2.50	3.85
0045	3.55	5.17	0075	3.35	5.15
0046	4.30	6.55	0076	4.10	6.35
0047	3.55	5.17	0077	3.35	5.15
0048	4.30	6.55	0078	4.10	6.35
0049	2.70	3.78	0079	2.50	3.85
0050	1.35	3.50	0080	3.15	3.90

Table 5.10: Min-Max Power Allocations in Area-14

House ID	Allocated power (kW)	Maximum power (kW)
0131	3.35	6.50
0132	2.80	3.88
0133	3.15	4.77
0134	2.80	3.88
0135	3.15	4.77
0136	3.35	6.50
0137	3.15	4.77
0138	3.35	6.50
0139	2.80	3.88
0140	2.40	4.47

# Chapter 6

## Conclusions

### 6.1 Conclusions

A DSM framework is proposed for the centralized scheduling of consumer loads without any price or demand elasticity forecast. For the framework proposed, the consumer load scheduling is explicitly carried out by the LSE based upon the price contracts. The aggregation of consumer loads by the LSE to participate in the wholesale market is also discussed. Aggregation of residential consumers has facilitated the LSE to represent them as a sizeable demand response resource. Representation of the consumers by an aggregate benefit curve is accomplished by suitably taking the network loss into account through a load flow calculation following the optimal load scheduling. To achieve simplicity, the actual benefit curve obtained is approximated by a quadratic function through curve fitting based upon constrained area minimization. The price sensitive bid which is the resultant of the proposed framework would help the LSE by reducing the financial risk in the wholesale market transactions. The initial round of scheduling is carried out on area basis. Subsequently, the total power scheduled for an area is distributed over individual consumers based upon min-max fairness allocation.

### 6.2 Scope of Future Work

- The framework proposed in this thesis is based on pay-as-bid strategy, in which the consumers pay the price as agreed upon by both LSE and consumer. Other alternative is uniform pricing scheme, in which every consumer pays a common price as cleared by the LSE.
- The above framework can be extended to the case of multiple LSEs serving the consumers over a common territory. The performance of an LSE can be assessed by how effectively the scheduling of loads is done.

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# Appendix A

## Line Data for Test Distribution Network

Table A.1: Line Data for 15-Node Distribution Network

Branch no.	Self impedance		Mutual impedance	
	$\Omega/\text{phase}$		$\Omega/\text{phase}$	
	R	X	R	X
1	4.0593	3.9705	0.5074	0.4963
2	3.5107	3.4339	0.4388	0.4292
3	2.5233	2.4681	0.3154	0.3085
4	4.5704	3.0828	0.5713	0.3853
5	7.6718	5.1747	0.9589	0.6468
6	3.2646	2.202	0.408	0.2752
7	3.7543	2.5323	0.4692	0.3165
8	6.03951	4.0737	0.7549	0.5092
9	5.0601	3.4131	0.6325	0.4266
10	5.3866	3.6333	0.6733	0.4541
11	7.3453	4.9545	0.9181	0.6193
12	6.0395	4.0737	0.7549	0.5092
13	6.6924	4.5141	0.8365	0.5642
14	3.5910	2.4222	0.4488	0.3028

# Appendix B

## Load Group Data

The units of  $P_{min}$  and  $P_{max}$  are watt(W).

Table B.1: Data for Load Groups LG1-LG64

Load Group ID	Area	Phase	Load Element			Load Group ID	Area	Phase	Load Element		
			$P_{min}$	$P_{max}$	Nos.				$P_{min}$	$P_{max}$	Nos.
LG1	1	0	90	90	60	LG33	4	0	0	150	30
LG2	1	0	50	150	40	LG34	4	1	60	60	60
LG3	1	0	0	200	25	LG35	4	1	30	120	40
LG4	1	1	90	90	60	LG36	4	1	0	150	30
LG5	1	1	50	150	40	LG37	4	2	60	60	60
LG6	1	1	0	200	25	LG38	4	2	30	120	40
LG7	1	2	90	90	60	LG39	4	2	0	150	30
LG8	1	2	50	150	40	LG40	4	3	30	180	7
LG9	1	2	0	200	25	LG41	5	0	90	90	60
LG10	1	3	150	150	5	LG42	5	0	50	150	40
LG11	2	0	90	90	60	LG43	5	0	0	200	25
LG12	2	0	50	150	40	LG44	5	1	90	90	60
LG13	2	0	0	200	25	LG45	5	1	50	150	40
LG14	2	1	90	90	60	LG46	5	1	0	200	25
LG15	2	1	50	150	40	LG47	5	2	90	90	60
LG16	2	1	0	200	25	LG48	5	2	50	150	40
LG17	2	2	90	90	60	LG49	5	2	0	200	25
LG18	2	2	50	150	40	LG50	5	3	0	400	2
LG19	2	2	0	200	25	LG51	6	0	40	40	40
LG20	2	3	60	240	3	LG52	6	0	15	90	30
LG21	3	0	60	60	60	LG53	6	0	0	120	20
LG22	3	0	30	120	40	LG54	6	1	40	40	40
LG23	3	0	0	150	30	LG55	6	1	15	90	30
LG24	3	1	60	60	60	LG56	6	1	0	120	20
LG25	3	1	30	120	40	LG57	6	2	40	40	40
LG26	3	1	0	150	30	LG58	6	2	15	90	30
LG27	3	2	60	60	60	LG59	6	2	0	120	20
LG28	3	2	30	120	40	LG60	7	0	40	40	40
LG29	3	2	0	150	30	LG61	7	0	15	90	30
LG30	3	3	120	120	10	LG62	7	0	0	120	20
LG31	4	0	60	60	60	LG63	7	1	40	40	40
LG32	4	0	30	120	40	LG64	7	1	15	90	30

Table B.2: Data for Load Groups LG65-LG135

Load Group ID	Area	Phase	Load Element			Load Group ID	Area	Phase	Load Element		
			$P_{min}$	$P_{max}$	Nos.				$P_{min}$	$P_{max}$	Nos.
LG65	7	1	0	120	20	LG101	11	1	40	40	40
LG66	7	2	40	40	40	LG102	11	1	15	90	30
LG67	7	2	15	90	30	LG103	11	1	0	120	20
LG68	7	2	0	120	20	LG104	11	2	40	40	40 3
LG69	8	0	90	90	60	LG105	11	2	15	90	30
LG70	8	0	50	150	40	LG106	11	2	0	120	20
LG71	8	0	0	200	25	LG107	12	0	40	40	40
LG72	8	1	90	90	60	LG108	12	0	15	90	30
LG73	8	1	50	150	40	LG109	12	0	0	120	20
LG74	8	1	0	200	25	LG110	12	1	40	40	40
LG75	8	2	90	90	60	LG111	12	1	15	90	30
LG76	8	2	50	150	40	LG112	12	1	0	120	20
LG77	8	2	0	200	25	LG113	12	2	40	40	40
LG78	8	3	150	150	5	LG114	12	2	15	90	30
LG79	9	0	40	40	40	LG115	12	2	0	120	20
LG80	9	0	15	90	30	LG116	13	0	60	60	60
LG81	9	0	0	120	20	LG117	13	0	30	120	40
LG82	9	1	40	40	40	LG118	13	0	0	150	30
LG83	9	1	15	90	30	LG119	13	1	60	60	60
LG84	9	1	0	120	20	LG120	13	1	30	120	40
LG85	9	2	40	40	40	LG121	13	1	0	150	30
LG86	9	2	15	90	30	LG122	13	2	60	60	60
LG87	9	2	0	120	20	LG123	13	2	30	120	40
LG88	10	0	60	60	60	LG124	13	2	0	150	30
LG89	10	0	30	120	40	LG125	13	3	120	120	10
LG90	10	0	0	150	30	LG126	14	0	90	90	60
LG91	10	1	60	60	60	LG127	14	0	50	150	40
LG92	10	1	30	120	40	LG128	14	0	0	200	25
LG93	10	1	0	150	30	LG129	14	1	90	90	60
LG94	10	2	60	60	60	LG130	14	1	50	150	40
LG95	10	2	30	120	40	LG131	14	1	0	200	25
LG96	10	2	0	150	30	LG132	14	2	90	90	60
LG97	10	3	0	360	3	LG133	14	2	50	150	40
LG98	11	0	40	40	40	LG134	14	2	0	200	25
LG99	11	0	15	90	30	LG135	14	3	60	240	3
LG100	11	0	0	120	20						

# Appendix C

## House Data

Table C.1: Data for Houses in Area-01

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)	LG1	LG2	LG3	LG10
0001	7500	0	0	3630	0	0	25	15	10	0
0002	7500	0	0	3630	0	0	15	10	5	0
0003	7500	0	0	3630	0	0	20	12	7	0
							LG4	LG5	LG6	LG10
0004	0	7500	0	0	3630	0	15	10	5	0
0005	0	7500	0	0	3630	0	20	12	7	0
0006	0	7500	0	0	3630	0	25	15	10	0
							LG7	LG8	LG9	LG10
0007	0	0	7500	0	0	3630	20	12	7	0
0008	0	0	7500	0	0	3630	25	15	10	0
0009	0	0	7500	0	0	3630	15	10	5	0
0010	7500	7500	7500	3630	3630	3630	0	9	9	5



Table C.2: Data for Houses in Area-02

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG11	LG12	LG13	LG20
0011	7500	0	0	3630	0	0	25	15	10	0
0012	7500	0	0	3630	0	0	12	12	5	0
0013	7500	0	0	3630	0	0	18	13	6	0
							LG14	LG15	LG16	LG20
0014	0	7500	0	0	3630	0	12	12	5	0
0015	0	7500	0	0	3630	0	18	13	6	0
0016	0	7500	0	0	3630	0	25	15	10	0
							LG17	LG18	LG19	LG20
0017	0	0	7500	0	0	3630	18	13	6	0
0018	0	0	7500	0	0	3630	25	15	10	0
0019	0	0	7500	0	0	3630	12	12	5	0
0020	7500	7500	7500	3630	3630	3630	15	0	12	3

Table C.3: Data for Houses in Area-03

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG21	LG22	LG23	LG30
0021	2500	0	0	1210	0	0	30	16	14	0
0022	2500	0	0	1210	0	0	20	12	8	0
0023	2500	0	0	1210	0	0	10	10	5	0
							LG24	LG25	LG26	LG30
0024	0	2500	0	0	1210	0	20	12	8	0
0025	0	2500	0	0	1210	0	10	10	5	0
0026	0	2500	0	0	1210	0	30	16	14	0
							LG27	LG28	LG29	LG30
0027	0	0	2500	0	0	1210	10	10	5	0
0028	0	0	2500	0	0	1210	30	16	14	0
0029	0	0	2500	0	0	1210	20	12	8	0
0030	2500	2500	2500	1210	1210	1210	0	6	9	10

Table C.4: Data for Houses in Area-04

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG31	LG32	LG33	LG40
0031	2500	0	0	1210	0	0	25	18	14	0
0032	2500	0	0	1210	0	0	20	12	8	0
0033	2500	0	0	1210	0	0	10	10	5	0
							LG34	LG35	LG36	LG40
0034	0	2500	0	0	1210	0	20	12	8	0
0035	0	2500	0	0	1210	0	10	10	5	0
0036	0	2500	0	0	1210	0	25	18	14	0
							LG37	LG38	LG39	LG40
0037	0	0	2500	0	0	1210	10	10	5	0
0038	0	0	2500	0	0	1210	25	18	14	0
0039	0	0	2500	0	0	1210	20	12	8	0
0040	2500	2500	2500	1210	1210	1210	15	0	9	7

Table C.5: Data for Houses in Area-05

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG41	LG42	LG43	LG50
0041	7500	0	0	3630	0	0	25	14	11	0
0042	7500	0	0	3630	0	0	12	10	6	0
0043	7500	0	0	3630	0	0	18	13	8	0
							LG44	LG45	LG46	LG50
0044	0	7500	0	0	3630	0	12	10	6	0
0045	0	7500	0	0	3630	0	18	13	8	0
0046	0	7500	0	0	3630	0	25	14	11	0
							LG47	LG48	LG49	LG50
0047	0	0	7500	0	0	3630	18	13	8	0
0048	0	0	7500	0	0	3630	25	14	11	0
0049	0	0	7500	0	0	3630	12	10	6	0
0050	7500	7500	7500	3630	3630	3630	15	9	0	2

Table C.6: Data for houses in Area-06

House ID	$P_{bl d}^{(a)}$	$P_{bl d}^{(b)}$	$P_{bl d}^{(c)}$	$Q_{bl d}^{(a)}$	$Q_{bl d}^{(b)}$	$Q_{bl d}^{(c)}$	Number of load elements in load groups associated		
	(W)	(W)	(W)	(W)	(W)	(W)			
							LG51	LG52	LG53
0051	2666.67	0	0	1290.67	0	0	15	12	9
0052	2666.67	0	0	1290.67	0	0	13	10	6
0053	2666.67	0	0	1290.67	0	0	12	8	5
							LG54	LG55	LG56
0054	0	2666.67	0	0	1290.67	0	12	8	5
0055	0	2666.67	0	0	1290.67	0	13	10	6
0056	0	2666.67	0	0	1290.67	0	15	12	9
							LG57	LG58	LG59
0057	0	0	2000	0	0	968	13	10	8
0058	0	0	2000	0	0	968	6	5	3
0059	0	0	2000	0	0	968	9	6	4
0060	0	0	2000	0	0	968	12	9	5

Table C.7: Data for Houses in Area-07

House ID	$P_{bl d}^{(a)}$	$P_{bl d}^{(b)}$	$P_{bl d}^{(c)}$	$Q_{bl d}^{(a)}$	$Q_{bl d}^{(b)}$	$Q_{bl d}^{(c)}$	Number of load elements in load groups associated		
	(W)	(W)	(W)	(W)	(W)	(W)			
							LG60	LG61	LG62
0061	2666.67	0	0	1290.67	0	0	15	12	9
0062	2666.67	0	0	1290.67	0	0	13	10	6
0063	2666.67	0	0	1290.67	0	0	12	8	5
							LG63	LG64	LG65
0064	0	2666.67	0	0	1290.67	0	12	8	5
0065	0	2666.67	0	0	1290.67	0	13	10	6
0066	0	2666.67	0	0	1290.67	0	15	12	9
							LG66	LG67	LG68
0067	0	0	2000	0	0	968	13	10	8
0068	0	0	2000	0	0	968	6	5	3
0069	0	0	2000	0	0	968	9	6	4
0070	0	0	2000	0	0	968	12	9	5

Table C.8: Data for Houses in Area-08

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG69	LG70	LG71	LG78
0071	7500	0	0	3630	0	0	25	14	10	0
0072	7500	0	0	3630	0	0	15	10	5	0
0073	7500	0	0	3630	0	0	20	13	7	0
							LG72	LG73	LG74	LG78
0074	0	7500	0	0	3630	0	15	10	5	0
0075	0	7500	0	0	3630	0	20	13	7	0
0076	0	7500	0	0	3630	0	25	14	10	0
							LG75	LG76	LG77	LG78
0077	0	0	7500	0	0	3630	20	13	7	0
0078	0	0	7500	0	0	3630	25	14	10	0
0079	0	0	7500	0	0	3630	15	10	5	0
0080	7500	7500	7500	3630	3630	3630	0	9	9	5

Table C.9: Data for Houses in Area-09

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated		
	(W)	(W)	(W)	(W)	(W)	(W)			
							LG79	LG80	LG81
0081	2666.67	0	0	1290.67	0	0	15	12	9
0082	2666.67	0	0	1290.67	0	0	13	10	6
0083	2666.67	0	0	1290.67	0	0	12	8	5
							LG82	LG83	LG84
0084	0	2666.67	0	0	1290.67	0	12	8	5
0085	0	2666.67	0	0	1290.67	0	13	10	6
0086	0	2666.67	0	0	1290.67	0	15	12	9
							LG85	LG86	LG87
0087	0	0	2000	0	0	968	13	10	8
0088	0	0	2000	0	0	968	6	5	3
0089	0	0	2000	0	0	968	9	6	4
0090	0	0	2000	0	0	968	12	9	5

Table C.10: Data for Houses in Area-10

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG88	LG89	LG90	LG97
0091	2500	0	0	1210	0	0	25	15	13	0
0092	2500	0	0	1210	0	0	20	12	10	0
0093	2500	0	0	1210	0	0	10	10	7	0
							LG91	LG92	LG93	LG97
0094	0	2500	0	0	1210	0	20	12	10	0
0095	0	2500	0	0	1210	0	10	10	7	0
0096	0	2500	0	0	1210	0	25	15	13	0
							LG94	LG95	LG96	LG97
0097	0	0	2500	0	0	1210	10	10	7	0
0098	0	0	2500	0	0	1210	25	15	13	0
0099	0	0	2500	0	0	1210	20	12	10	0
0100	2500	2500	2500	1210	1210	1210	15	9	0	3

Table C.11: Data for Houses in Area-11

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated		
	(W)	(W)	(W)	(W)	(W)	(W)			
							LG98	LG99	LG100
0101	2666.67	0	0	1290.67	0	0	15	12	9
0102	2666.67	0	0	1290.67	0	0	13	10	6
0103	2666.67	0	0	1290.67	0	0	12	8	5
							LG101	LG102	LG103
0104	0	2666.67	0	0	1290.67	0	12	8	5
0105	0	2666.67	0	0	1290.67	0	13	10	6
0106	0	2666.67	0	0	1290.67	0	15	12	9
							LG104	LG105	LG106
0107	0	0	2000	0	0	968	13	10	8
0108	0	0	2000	0	0	968	6	5	3
0109	0	0	2000	0	0	968	9	6	4
0110	0	0	2000	0	0	968	12	9	5

Table C.12: Data for Houses in Area-12

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated		
	(W)	(W)	(W)	(W)	(W)	(W)			
							LG107	LG108	LG109
0111	2666.67	0	0	1290.67	0	0	15	12	9
0112	2666.67	0	0	1290.67	0	0	13	10	6
0113	2666.67	0	0	1290.67	0	0	12	8	5
							LG110	LG111	LG112
0114	0	2666.67	0	0	1290.67	0	12	8	5
0115	0	2666.67	0	0	1290.67	0	13	10	6
0116	0	2666.67	0	0	1290.67	0	15	12	9
							LG113	LG114	LG115
0117	0	0	2000	0	0	968	13	10	8
0118	0	0	2000	0	0	968	6	5	3
0119	0	0	2000	0	0	968	9	6	4
0120	0	0	2000	0	0	968	12	9	5

Table C.13: Data for Houses in Area-13

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG116	LG117	LG118	LG125
0121	2500	0	0	1210	0	0	30	16	14	0
0122	2500	0	0	1210	0	0	20	12	8	0
0123	2500	0	0	1210	0	0	10	10	5	0
							LG119	LG120	LG121	LG125
0124	0	2500	0	0	1210	0	20	12	8	0
0125	0	2500	0	0	1210	0	10	10	5	0
0126	0	2500	0	0	1210	0	30	16	14	0
							LG122	LG123	LG124	LG125
0127	0	0	2500	0	0	1210	10	10	5	0
0128	0	0	2500	0	0	1210	30	16	14	0
0129	0	0	2500	0	0	1210	20	12	8	0
0130	2500	2500	2500	1210	1210	1210	0	6	9	10

Table C.14: Data for Houses in Area-14

House ID	$P_{bld}^{(a)}$	$P_{bld}^{(b)}$	$P_{bld}^{(c)}$	$Q_{bld}^{(a)}$	$Q_{bld}^{(b)}$	$Q_{bld}^{(c)}$	Number of load elements in load groups associated			
	(W)	(W)	(W)	(W)	(W)	(W)				
							LG126	LG127	LG128	LG135
0131	7500	0	0	3630	0	0	25	15	10	0
0132	7500	0	0	3630	0	0	12	12	5	0
0133	7500	0	0	3630	0	0	18	13	6	0
							LG129	LG130	LG131	LG135
0134	0	7500	0	0	3630	0	12	12	5	0
0135	0	7500	0	0	3630	0	18	13	6	0
0136	0	7500	0	0	3630	0	25	15	10	0
							LG132	LG133	LG134	LG135
0137	0	0	7500	0	0	3630	18	13	6	0
0138	0	0	7500	0	0	3630	25	15	10	0
0139	0	0	7500	0	0	3630	12	12	5	0
0140	7500	7500	7500	3630	3630	3630	15	0	12	3