Stability Analysis of Machining Processes

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Gurur brahmaa Gurur Vishnu
Gurur devo Maheshwarah
Guru saakshaath Parbrahma
Tasmai shree guruve namaha

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Dedication

This thesis is dedicated to my parents & siblings
for their love, endless support
and encouragement

Abstract

In this work, we do a parametric stability analysis of the turning and milling process. We observe that the governing equation for these processes is a delay differential motion, and therefore use the quasi polynomial and the spectral tau method for DDEs to analyze their stability. In quasi polynomial method, we convert the quasi polynomial into an approximate polynomial expression using Taylor series expansion. The roots of the polynomial expression are then used to determine the stability of the system. Next we discuss the spectral tau method based on the Galerkin series approximation to study the stability of DDEs. Here we first obtain an equivalent PDE representation of the DDE, and then use spectral approximation techniques to obtain a finite dimensional ODE approximation of the DDE. The boundary condition is incorporated using the tau method, where the last row of the system ODE is replaced with the boundary condition. We then use these methods to obtain the stability diagrams for the single and multi degree of freedom turning and milling process and compare them with literature. The numerical examples clearly demonstrate that these methods can be used to determine the stable zones of operation, therefore can be used to demarcate the regions of parametric space for which the machining process will be stable.

Contents

Chapter 1

Introduction

The application of DDEs is ubiquitous in the field of biology, population dynamics, manufacturing, controls, traffic flow modeling, fluid structure interaction. They are widely used to model time delay systems, where the present state is dependent on the previous state. It has been observed that the governing equation of a wide number of manufacturing processes involves DDEs. For example, the well known orthogonal turning process, where the feed force contains time delay terms.

As we know that the dynamic unstable interaction of the tool with workpiece causes chatter in machining process, which results in a bad surface finish. Therefore a major requirement for any machining operation is that, we operate in the stable operational regime. This ensures a smooth surface finish and less wear of the tool. Since the governing equation of motion for these processes is a DDE, therefore we must exploit the existing tools for stability analysis of DDEs to determine the stable zones of operation.

A popular method for stability analysis of DDEs is the quasi polynomial method, where we first assume an exponential solution of the governing DDE. The resulting equation is a quasi polynomial and contains infinite number of roots. Next we expand the exponential term using Taylor series approximation for a finite number of terms to obtain a polynomial expression. The roots of the polynomial expression approximate the original roots of the quasi polynomial. The stability is then determined by analyzing the real parts of the roots. If all the roots have negative real parts, then the system is stable, otherwise it is unstable.

Another widely used method for stability analysis of DDEs is the spectral tau method. Here we use Cauchy transformation to obtain an equivalent PDE representation of the DDE along with the boundary condition. Next we use Galerkin methods to obtain a finite dimensional ODE approximation of the DDE. The boundary condition is incorporated using the tau method, where we replace the last row of the system ODE with the boundary condition. If the real part of all the eigenvalues of the system matrix is less than zero, then the system is stable, otherwise it is unstable.

We also know that the coefficients of a DDE can be constant or periodic. If the coefficients of a DDE are periodic, then the DDE is known as a time periodic DDE. For example, the governing equation for the milling process due to the tooth pass excitation effect is time periodic with time period equal to the delay of the system. Therefore, the ODE approximation of such time periodic DDEs using the spectral method also contains time periodic terms. We then resort to Floquet theory to determine the stability of such time periodic ODEs. Here we integrate the ODEs using identity initial conditions for time equal to the period of the system. The response obtained after integrating the ODEs for each independent initial condition forms the columns of the Floquet transition matrix. If all the eigenvalues of the Floquet transition matrix have magnitude less than or equal to unity, then the system is stable, otherwise it is unstable.

In this work, we use the quasi polynomial and the spectral tau methods to determine the regions of parametric stability for the orthogonal and oblique turning and a single-degree-of-freedom milling model.

1.1 Literature review

The history of machine tool chatter can be traced almost hundred years back. Taylor (1907) [1] described machine tool chatter as the most obscure problem faced during machining processes. Hanna and Tobias (1974) [2] showed that chatter frequencies are related to unstable periodic motions about stationary cutting. Later Shi and Tobias (1984) [3] showed experimentally that unstable periodic motions about stationary cutting is a case of subcritical Hopf bifurcation. Stepan and Kalmar-Nagy (1997) [4] proved the same analytically. Seagalmann et al (2000) [5] investigated turning process where system stiffness was varied periodically. Incorporating the varying stiffness into equation of motion they formed a DDE. They investigated resulting periodic DDE by harmonic balance method. Gouskov et al. used a tool head with two multiple tool rows for chatter suppression. They analyzed mathematical model with two delays and showed influence of technological parameters on stability. Insperger et al. [6] studied stability of two degree of freedom model of turning process. Regenerative delay determined by combination of workpiece rotation and tool vibration is state dependent. An associated linear system corresponding to the state dependent delay equation is derived and stability analysis of this linear system was examined. Gabor Stepan et al. [7] did stability analysis of orthogonal turning process. The cutting force was proportional to feed. An equation of motion was derived using Newton's laws of motion. Equation of motion resulted in the formation of a DDE. The DDE was analyzed to get the stability plots.

1.2 Thesis structure

The entire thesis has been divided in six chapters. The first chapter is the introduction, wherein we discuss the different machining processes and a general class of equations that are used to model them. Next we define the problem that we are going to focus on. In the second chapter, we discuss the quasi-polynomial and the spectral tau methods that can be used to analyse the stability of DDEs. In the subsequent chapters, we present the mathematical model and the stability analysis of the orthogonal turning, oblique turning and the single-degree-of-freedom milling process. Finally conclusions are provided in chapter six.

Chapter 2

Stability Analysis

In this chapter, we discuss the quasi-polynomial and the spectral approximation technique for the stability analysis of DDEs. First we present the class of DDEs in which we are interested in. We then discuss the quasi-polynomial method, in which we expand the solution using the Taylor series approximation and apply polynomial root finding techniques. Next we discuss the spectral tau method based on the Galerkin series approximation to study the stability of DDEs.

2.1 Mathematical Model

Consider the following first order DDE:

$$\dot{x} + ax + bx(t-\tau) = 0, \quad \tau > 0 \tag{2.1}$$

Equation (2.1) is a first order DDE, with delay in $x(t)$. Since the delay (τ) is not present in the highest order derivative, therefore, this class of DDEs is referred to as retarded DDE. We now present the quasi-polynomial and the spectral tau method for the stability analysis of such DDEs.

2.2 Quasi Polynomial Method

In this method, we assume a solution of the form e^{st} and substitute it in (2.1),

$$s + a + be^{-s\tau} = 0 \tag{2.2}$$

Equation (2.2) is a quasi polynomial and has infinite number of roots. Expanding e^{st} using Taylor series approximation as,

$$e^{-s\tau} = 1 + (-s\tau) + \frac{(-s\tau)^2}{2!} + \frac{(-s\tau)^3}{3!} + \text{............} \tag{2.3}$$

We now substitute finite number of terms for $e^{-s\tau}$ in 2.2, and apply polynomial root finding techniques to obtain approximate solution of 2.2. Thus we can see that the roots of the polynomial expression are the approximate solution of the DDE. If all the roots have negative real parts, then the system is stable, otherwise it is unstable.

2.3 Spectral tau method

Consider the following standard transformation:

$$y(s,t) = x(t+s) \tag{2.4}$$

Using the transformation (2.4), the DDE (2.1) can be transformed as:

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial s}, \quad s \in [-\tau,\, 0] \tag{2.5}$$

$$\left.\frac{\partial y}{\partial t}\right|_{0,t} = -ay(0,t) - by(-\tau,t) \tag{2.6}$$

Equation (2.5–2.6) is the equivalent PDE representation of the DDE (2.1). We now assume a series solution of the form as:

$$y(s,t) = \phi(s)^T \eta(t) \tag{2.7}$$

where $\phi(s) = [\phi_1(s),\, \phi_2(s),\, ...\phi_N(s)]^T$ are the basis functions and $\eta(t) = [\eta_1(t),\, \eta_2(t),\, ...\eta_N(t)]^T$ are the time dependent coefficients. We use Shifted Legendre polynomials as the basis functions which are given as:

$$\phi_1(s) = 1 \tag{2.8a}$$

$$\phi_2(s) = 1 + \frac{2s}{\tau} \tag{2.8b}$$

$$\phi_i(s) = \frac{(2i-3)\,\phi_2(s)\phi_{i-1}(s) - (i-2)\,\phi_{i-2}(s)}{i-1}, \quad i = 3, 4, \ldots, N. \tag{2.8c}$$

Now we substitute the series solution (2.7) in (2.5),

$$\phi(s)^T \dot{\eta}(t) = \phi^{'}(s)^T \eta(t) \tag{2.9}$$

Now we pre-multiply both sides of (2.9) and integrate over the domain $s \epsilon [-\tau,\, 0]$ to obtain the following system of ODEs,

$$M\dot{\eta} = K\eta \tag{2.10}$$

where M and K are given as:

$$\mathbf{M} = \int_{-\tau}^{0} \boldsymbol{\phi}(s)\boldsymbol{\phi}(s)\,\mathrm{d}\,s \tag{2.11a}$$

$$\mathbf{K} = \int_{-\tau}^{0} \boldsymbol{\phi}(s)\boldsymbol{\phi}'(s)\,\mathrm{d}\,s. \tag{2.11b}$$

The entries of matrices $\mathbf{M}$ and $\mathbf{K}$ are defined as follows:

$$\mathbf{M}_{cd} = \frac{\tau}{2c-1}\delta_{cd} \qquad c = 1,2,\ldots,N; d = 1,2,\ldots,N \tag{2.12a}$$

$$\mathbf{K}_{cd} = \begin{cases} 2, & \text{if } c < d \text{ and } c+d \text{ is odd} \\ 0, & \text{otherwise} \end{cases} \qquad c = 1,2,\ldots,N; d = 1,2,\ldots,N. \tag{2.12b}$$

Next we substitute the series solution (2.7) into the boundary condition (2.6):

$$\phi(0)^T\dot{\eta}(t) = -a\phi(0)^T\eta(t) - b\phi(-\tau)^T\eta(t) \tag{2.13}$$

We now use the tau method to incorporate the ODE corresponding to the boundary condition (2.13) into the system ODE (2.10). In this method, we replace the last row of matrices M and K with the boundary condition. Thus, we finally arrive at the following system of ODEs:

$$M_{Tau}\dot{\eta} = K_{Tau}\eta \tag{2.14}$$

where M_{Tau} and K_{Tau} are defined as follows:

$$M_{Tau} = \begin{bmatrix} \bar{M} \\ \phi(0)^T \end{bmatrix} \quad \text{and} \quad K_{Tau} = \begin{bmatrix} \bar{K} \\ -a\phi(0)^T - b\phi(-\tau)^T \end{bmatrix} \tag{2.15}$$

Here $\bar{M}$ and $\bar{K}$ are obtained by removing the last row of matrices M and K in (2.12a) and (2.12b) respectively.

$$\dot{\eta} = A\eta \tag{2.16}$$

where $A = M_{Tau}^{-1}K_{Tau}$. For constant coefficient matrix, if the real part of all the eigenvalues of matrix (A) is less than 0, then the system is stable, otherwise it is unstable.

In summary we have discussed the quasi polynomial and the spectral tau method for stability analysis of DDEs. We now use the theory discussed to determine the stability of machining processes governed by these DDEs.

Chapter 3

Orthogonal Turning

In this chapter, we study the mathematical modeling of the orthogonal turning process. First we present the governing equation of motion and then reduce it to a simpler form using standard substitution techniques. Next we study the stability characteristics of the resulting expression using the quasi polynomial and spectral tau methods.

3.1 Mathematical Model

The mechanical model of the one-degree-of-freedom turning process shown in Figure (3.1) is given as:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -k\omega\left(x(t) - x(t-\tau)\right)^{\gamma} + kvt + cv + kl \tag{3.1}$$

where m is the mass of the tool, c is the damping coefficient and ω is the depth of cut. The parameters k and l are the spring stiffness and natural length respectively. The expression $-k\omega\left(x(t) - x(t-\tau)\right)^{\gamma}$ contains time delay terms and represents the feed force and the constant γ is determined experimentally.
We now introduce the following substitution for $x(t)$:

$$x(t) = \xi(t) + vt + C \tag{3.2}$$

On substituting (3.2) in (3.1) we obtain:

$$m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) + kC + kvt + cv = -k_c\omega(v\tau + \xi(t) - \xi(t-\tau))^{\gamma} + kvt + cv + kl \tag{3.3}$$

Next we expand $(v\tau + \xi(t) - \xi(t-\tau))^{\gamma}$ using Taylor series expansion and obtain:

$$m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) + kC = -k_c\omega(v\tau)^{\gamma} - k\omega(v\tau)^{\gamma-1}(\xi(t) - \xi(t-\tau)) + kl \tag{3.4}$$

Assuming the constant C in Eq. (3.4) as $kC = -k_c\omega(v\tau)^{\gamma} + kl$, we obtain the following simplified expression:

$$m\ddot{\xi}(t) + c\dot{\xi}(t) + k\xi(t) = -k\omega(v\tau)^{\gamma-1}(\xi(t) - \xi(t-\tau)) \tag{3.5}$$

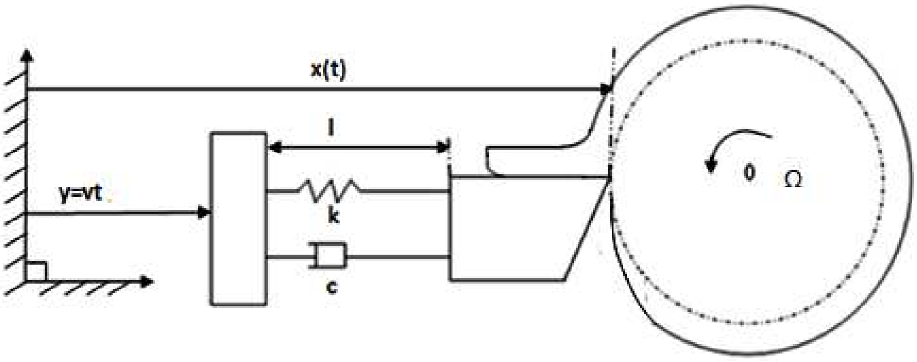


Figure 3.1: Mechanical model of the turning process

We now divide both sides of Eq. (3.5) by m and introduce the following notations $\zeta = \frac{c}{2m\omega_n}$, $\omega_n = \sqrt{\frac{k}{m}}$ to obtain:

$$\ddot{\xi}(t) + 2\zeta\omega_n\dot{\xi}(t) + \omega_n^2\xi(t) = -k\omega(v\tau)^{\gamma-1}(\xi(t) - \xi(t-\tau)) \tag{3.6}$$

Now we substitute $t = \tilde{t}\omega_n$, and by abuse of notation set $t = \tilde{t}$:

$$\ddot{\xi}(t) + 2\zeta\dot{\xi}(t) + \xi(t) = \frac{-k\omega(v\tau)^{\gamma-1}}{\omega_n^2}(\xi(t) - \xi(t-\tau)) \tag{3.7}$$

We now define $p = \frac{k\omega(v\tau)^{\gamma-1}}{\omega_n^2}$ to reduce (3.7) as,

$$\ddot{\xi}(t) + 2\zeta\dot{\xi}(t) + (1+p)\xi(t) - p\xi(t-\tau) = 0 \tag{3.8}$$

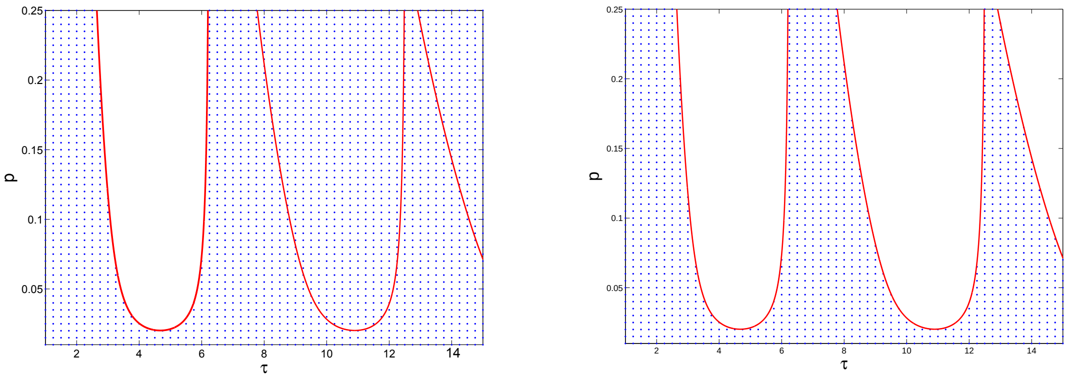
Thus (3.8) is the simplified form of (3.1), the stability analysis of which determines the stability of the system.

3.2 Numerical Studies

In this section, we present the stability diagrams of (3.8) using the quasi polynomial and the spectral tau method. The simulations were performed using Matlab® R2012b on a 2.60-GHz Intel® Xeon® E5-2670 processor.

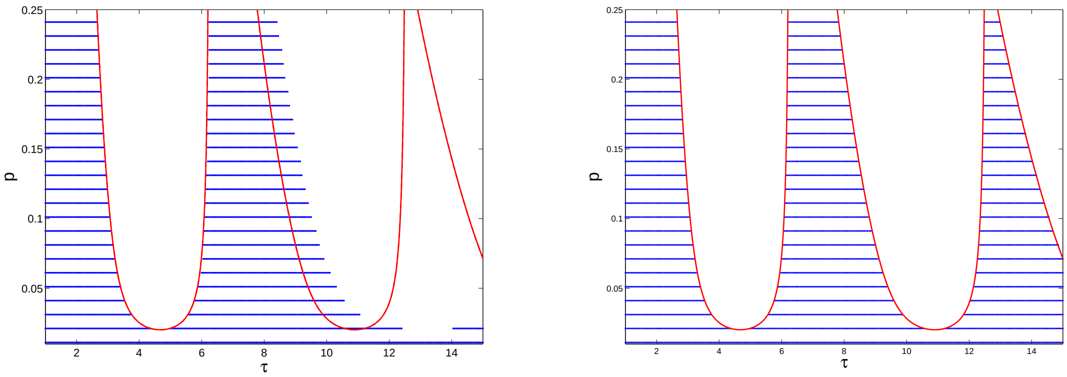
Figure (3.2) shows the stability diagram for the orthogonal turning model (3.8) using the quasi polynomial method (2.2). The red lines indicate the boundary from literature, whereas the blue dots are the stable points obtained using quasi polynomial technique. From Fig. (3.2)(b), we can see that $N = 59$ terms are required in the Taylor series approximation 2.3 for convergence.

Next we plot the stability diagrams of (3.8) using the spectral tau method (2.3) in Figure (3.3). We generate the stability diagrams for N equal to 6 and 12 terms in the Galerkin series approxima-



(a) $N = 30$ (b) $N = 59$

Figure 3.2: Stability diagrams for orthogonal turning model (3.8) using the quasi polynomial method. Dots indicate the point at which the system is stable and the red line is from literature.



(a) $N = 6$ (b) $N = 12$

Figure 3.3: Stability diagrams for orthogonal turning model (3.8) using the spectral tau method. Dots indicate the point at which the system is stable and the red line is from literature.

tion (2.7). From Fig. (3.2)(b), we can see that the stability diagrams are in very good agreement with literature for $N = 12$ terms in the Galerkin series approximation.

Chapter 4

Oblique Turning

In this chapter, we first discuss the governing equation of motion for the oblique turning process. We then apply the spectral tau method to convert the DDEs into a system of ODEs. Finally we present the stability diagrams obtained using this method.

4.1 Mathematical Model

The governing equation of motion for a tool modeled as a 2 degree-of-freedom oscillator in oblique turning is given by:

$$m\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) = p\omega(y(t) - y(t-\tau)^q \tag{4.1}$$

$$m(\ddot{y}(t) - (\ddot{vt})) + c_y(\dot{y}(t) - (\dot{vt})) + k_y(yt - vt) = -u\omega(y(t) - y(t-\tau))^q \tag{4.2}$$

where (4.1) and (4.2) represents the equation of motion along depth of cut (x) and feed (y) direction respectively. Here the coefficients m represents mass of the tool, c_x and c_y are the damping coefficients along x and y direction respectively. Similarly k_x and k_y represents the stiffness along the x and y direction respectively. The parameters p and u indicate the cutting force coefficients along the x and y direction respectively. We now simplify the equations of motion as follows:

4.1.1 Along y direction

$$m(\ddot{y}(t) - (\ddot{vt})) + c_y(\dot{y}(t) - (\dot{vt})) + k_y(yt - vt) = -u\omega(y(t) - y(t-\tau))^q \tag{4.3}$$

We now introduce the transformation $y(t) = \eta(t) + vt + C$ in (4.2) and obtain,

$$m\ddot{\eta}(t) + c_y\eta(t) + k_y\eta(t) - c_y v + k_y C = -u\omega(v\tau + \eta(t) - \eta(t-\tau))^q \tag{4.4}$$

Next we expand $(v\tau + \eta(t) - \eta(t-\tau))^q$ using Taylor series approximation to obtain:

$$m\ddot{\eta}(t) + c_y\eta(t) + k_y\eta(t) - c_y v + k_y C = -u\omega(v\tau)^q - u\omega q(v\tau)^{q-1}(\eta(t) - \eta(t-\tau)) - c_y v + k_y C \tag{4.5}$$

We now choose a constant C such that $-c_y v + k_y C = -u\omega(v\tau)^q$, to finally obtain

$$m\ddot{\eta}(t) + c_y\dot{\eta}(t) + k_y\eta(t) = -u\omega q(v\tau)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.6}$$

4.1.2 Along x direction

$$m\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) = p\omega(y(t) - y(t-\tau)^q$$

Here p is the cutting coefficient along x direction. Now introducing the transformation $x = \xi(t) + C_1$ in (4.2), we have:

$$m\ddot{\xi}(t) + c_x\dot{\xi}(t) + k_x\xi(t) + k_x C1 = p\omega(v\tau + \eta(t) - \eta(t-\tau))^q \tag{4.7}$$

Expanding $(v\tau + \eta(t) - \eta(t-\tau))^q$ using Taylor series approximation, we obtain

$$m\ddot{\xi}(t) + c_x\dot{\xi}(t) + k_x\xi(t) + k_x C_1 = p\omega(v\tau)^q + p\omega q(v\tau)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.8}$$

We now set the constant C_1 such that $k_x C_1 = p\omega(v\tau)^q$ to further simplify (4.8) as follows:

$$m\ddot{\xi}(t) + c_x\dot{\xi}(t) + k_x\xi(t) = p\omega q(v\tau)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.9}$$

Thus, we arrive at the following simplified equations of motion:

$$\textbf{Along X} : m\ddot{\xi}(t) + c_x\dot{\xi}(t) + k_x\xi(t) = p\omega q(v\tau)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.10a}$$

$$\textbf{Along Y} : m\ddot{\eta}(t) + c_y\dot{\eta}(t) + k_y\eta(t) = -u\omega q(v\tau)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.10b}$$

Assuming equal damping coefficients along x and y direction, that is $c_x = c_y$ and introducing the notations $\zeta = \frac{c}{2m\omega_n}$, $k_r = \frac{k_y}{k_x}$ and $\rho = v\tau$ in (4.10a) and (4.10b), we have:

$$\ddot{\xi}(t) + 2\omega_n\zeta\dot{\xi}(t) + \omega_n^2\xi(t) = k_1/k_r(\rho)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.11a}$$

$$\ddot{\eta}(t) + 2\omega_n\zeta\dot{\eta}(t) + \omega_n^2\eta(t) = -k_1(\rho)^{q-1}(\eta(t) - \eta(t-\tau)) \tag{4.11b}$$

Now we set $t = \widetilde{t}\omega_n$, and drop the tilde for simplicity, to finally arrive at the following DDEs:

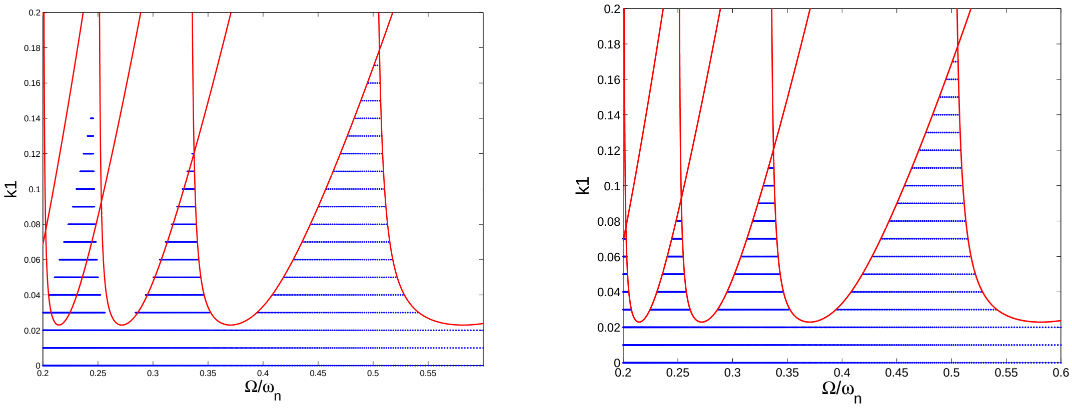
$$\ddot{\xi}(t) + 2\zeta\dot{\xi}(t) + \xi(t) = k_1/k_r(\rho)^{q-1}(\eta(t) - \eta(t-\omega_n\tau)) \tag{4.12a}$$

$$\ddot{\eta}(t) + 2\zeta\dot{\eta}(t) + \eta(t) = -k_1(\rho)^{q-1}(\eta(t) - \eta(t-\omega_n\tau)) \tag{4.12b}$$

where $k_1 = \frac{p\omega q}{\omega_n^2}$.

4.2 Numerical Studies

In this section, we plot the stability diagrams of the oblique turning model (4.12) using the spectral tau method (2.3). We can see that Eq. (4.12) is a system of coupled second order DDEs. Therefore, we first convert the DDEs into a system of first order DDEs and then use the spectral tau method to determine the stability of the system.



(a) $N = 14$ (b) $N = 21$

Figure 4.1: Stability diagrams for the oblique turning model (4.12) using the spectral tau method. Dots indicate the point at which the system is stable and the red line is from literature.

In Figure (4.1), we show the stability diagram of (4.12) using the parameter values $\zeta = 0.01$, $\tau = 5$, and $k = 2$. The red line is the stability boundary from literature, whereas the blue dots are the stable points obtained using the spectral tau technique. We can see that $N = 21$ terms are sufficient in the Galerkin series approximation (2.7) to achieve convergence. Thus, using this method we can successfully demarcate the stable and unstable operating regimes for the oblique turning process.

Chapter 5

Single-degree-of-freedom Milling

In this chapter, we first discuss the mathematical model for the single-degree-of-freedom milling process. We then convert the DDEs into a set of ODEs using the spectral tau method. Finally we present the stability and bifurcation diagrams using this method.

5.1 Mathematical Model

The mathematical model for the single-degree-of-freedom milling process is given as:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = -\frac{wh(t)}{m_t}(x(t) - x(t-\tau)) \tag{5.1}$$

where ζ is the damping coefficient, ω_n is the angular natural frequency, w is the depth of cut, and m_t is the modal mass of the tool. The function $h(t)$ is given by,

$$h(t) = \sum g(\phi_j(t))\sin(\phi_j(t))(K_t\cos(\phi_j(t)) + K_n\sin(\phi_j(t))) \tag{5.2}$$

Here K_t and K_n are the tangential and normal linearized cutting force coefficients, and N_t is the number of teeth in the tool. The angular position $(\phi_j(t))$ of the tool is calculated as:

$$\phi_j(t) = (2\pi\Omega/60)t + j2\pi/N_t \tag{5.3}$$

where Ω is the spindle speed in rpm. The function $g(\phi_j(t))$ is a screen function and is defined as:

$$g(\phi_j(t)) = \begin{cases} 1 & \text{if } \phi_{st}(t) < \phi_j(t) < \phi_{ex}(t) \\ 0 & \text{otherwise} \end{cases}$$

Therefore, when the tooth j is in cut $g(\phi_j(t)) = 1$ and when out of cut $g(\phi_j(t)) = 0$. The variables $\phi_{st}(t)$ and $\phi_{ex}(t)$ denotes the start and exit angles of the tooth respectively. When up-milling, $\phi_{st} = \pi$, $\phi_{ex} = \arccos(1 - 2a/D)$. When down-milling, $\phi_{st} = \arccos(2a/D - 1)$ and $\phi_{ex} = \pi$ and the ratio a/D is the radial depth of cut ratio.

The function $h(t)$ because of the tooth pass excitation effect is time periodic, with time period equal to the time delay of the system. The time delay (τ) is given as,

$$\tau = T = 60/(N_t\Omega) \tag{5.4}$$

We now employ the spectral approximation technique (2.3) to convert the time-periodic DDE (5.1) into the following ODE form:

$$\dot{\eta} = A(t)\eta \tag{5.5}$$

where $A(t)$ is periodic with period T, that is $A(t+T) = A(t)$. The stability of such time-periodic ODEs (5.5) can be determined through Floquet theory. Let Φ denote the Floquet transition matrix, which is obtained by integrating the following matrix differential equation,

$$\dot{\Phi} = A(t)\Phi \tag{5.6}$$

We now integrate (5.6) with initial conditions $\Phi(0)$ =I for time period T, to obtain the Floquet transition matrix. The system is stable, if the magnitude of all the eigenvalues of Φ is less than or equal to 1, and is unstable if the magnitude is greater than 1.

5.2 Stability Diagrams

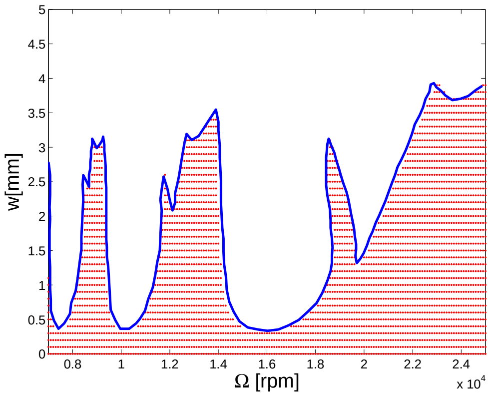
In this section, we present the stability and bifurcation diagrams of the single-degree-of-freedom milling model (5.1). The time response of the ODEs was obtained using $ode15s$ solver in Matlab with default absolute and relative integration tolerances of $1e^{-3}$ and $1e^{-6}$ respectively.

In Fig. (5.1), we show the stability diagrams of (5.1) for four different values of radial depth of cut ratio (a/D). The parameter values used are $\zeta = 0.011$, $\omega_n = 5793$ rad/s, $m_t = 0.03993$ kg, $K_t = 6 \times 10^8$ N/m^2 and $K_n = 2 \times 10^8$ N/m^2. The red dots indicate the stable points, where the magnitude of the dominant eigenvalue of the Floquet transition matrix (Φ) is less than 1 and the blue solid line is the stability boundary reported by Insperger and Stépán ([8]). The convergence analysis reveals that $N = 25$ terms are sufficient in the galerkin series approximation (2.7) to accurately capture the stability diagrams. From Fig. (5.1) we can see that the stability diagrams are in very good agreement with literature.

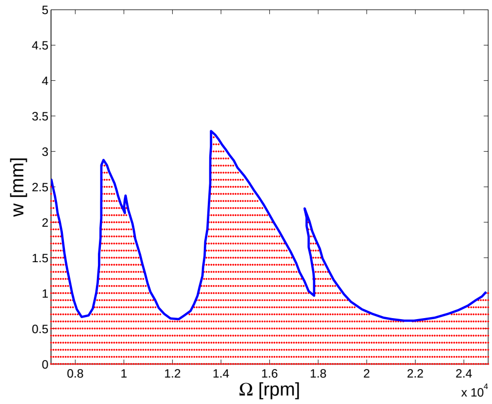
Therefore, we see that the spectral tau method gives satisfactory results for milling and can be used to determine the regions of parametric space for which the system is stable. If we operate in the stable region, where the magnitude of the dominant eigenvalues of the Floquet transition matrix (Φ) is less than 1, the system is stable and it will result in a smooth surface finish.

5.3 Bifurcation diagrams

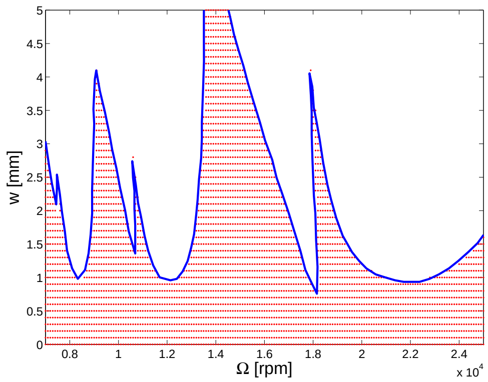
We now discuss the bifurcation analysis which can be carried out using the eigenvalues of the Floquet transition matrix (Φ). Recalling the fact that in stable regions, the magnitude of the dominant eigenvalues of Φ is no greater than 1, whereas in unstable regions it is greater than 1. However at the stability boundary, the magnitude of the most dominant eigenvalue of Φ is exactly equal to 1. We denote these dominant eigenvalues at the boundary as λ_{bound}. The location where λ_{bound} lies on the unit circle in the complex plane gives rise to different types of bifurcations as follows:



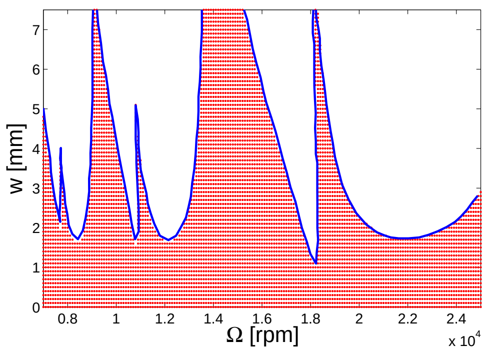
(a) $a/D = 1$



(b) $a/D = 0.5$



(c) $a/D = 0.1$



(d) $a/D = 0.05$

Figure 5.1: Stability diagrams for the single-degree-of-freedom milling model (5.1) using the spectral tau method. Dots indicate the points at which the system is stable and the solid blue line is from literature [8].

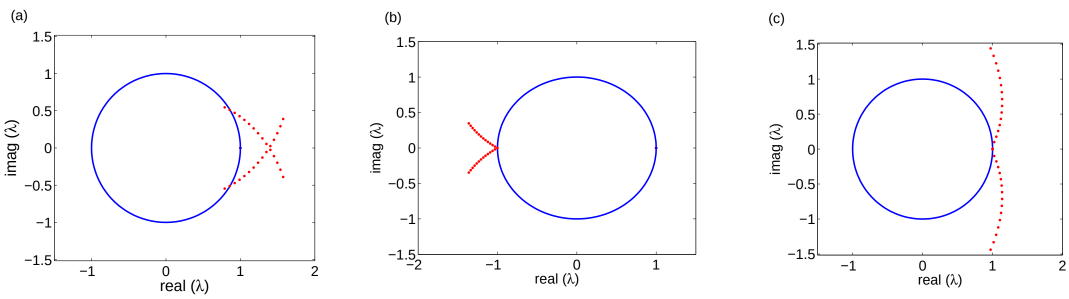
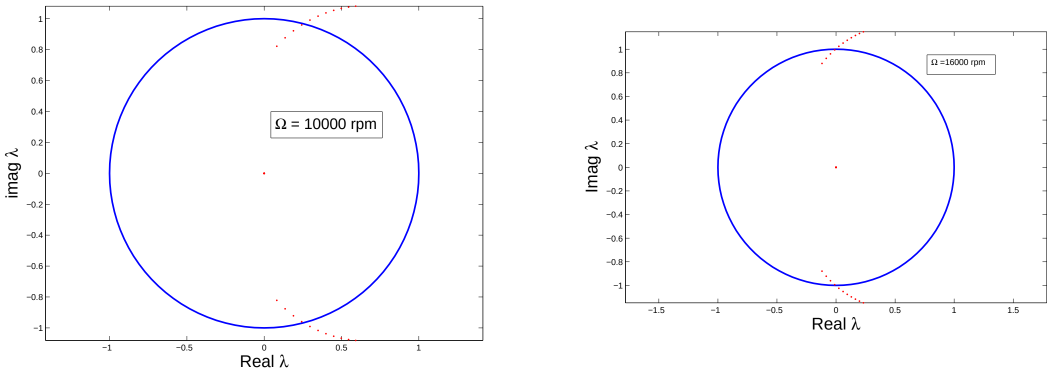


Figure 5.2: Possible bifurcation diagrams for time periodic systems: (a) Secondary Hopf (b) Period-doubling (c) Period one



(a) $\Omega = 10000$ rpm

(b) $\Omega = 16000$ rpm

Figure 5.3: Bifurcation diagrams for the single-degree-of-freedom milling model (5.1).

- As shown in Fig. (5.2)(a), when the eigenvalues escape the unit circle as a complex pair, the resulting bifurcation is known as secondary hopf bifurcation
- As shown in Fig. (5.2)(b), the eigenvalues escape the unit circle through -1, which gives rise to period doubling bifurcation
- As shown in Fig. (5.2)(c), the eigenvalues escape the unit circle through 1, resulting in period doubling bifurcation

The bifurcation analysis therefore gives an idea about the path through which the system loses its stability. We now plot the bifurcation diagrams for two different values of Ω in Eq. (5.1). In fig. (5.2)(a) and fig. (5.2)(b), we can see that the eigenvalues at the boundary have non-zero imaginary parts. This situation is known as secondary hopf bifurcation.

Thus, we see that the spectral tau can also be used to generate the bifurcation diagrams for the single-degree-of-freedom milling process. The bifurcation analysis not only gives an idea about the path through which the system loses its stability, but also helps to calculate the arising chatter frequencies in case of unstable cutting process [9].

Chapter 6

Conclusions

In this work, we have studied the mathematical models of turning and milling process. We observed that the governing equation of motion for these processes is a delay differential equation and therefore exploited the existing tools for DDEs to carry out the stability analysis. First we study the stability of a single-degree-of-freedom orthogonal turning model using the quasi polynomial method, where we expand the exponential terms using the Taylor series expansion. We also study the stability analysis of this system using the spectral tau method based on the galerkin projections of the DDE. Here we first convert the DDE into an equivalent system of PDE and then use galerkin series approximation to obtain a finite dimensional ODE approximation of the DDE. The boundary condition is coupled using the tau method, in which we replace the last row of the system ODE with the boundary condition.

Next we study the stability analysis of a two-degree-of-freedom oblique turning model using the spectral tau method. We observed that the governing equation for this process is a coupled second order DDE. The convergence study reveals that the spectral tau method requires very few terms to accurately capture the stable zones of operation.

Finally, we generate the stability diagrams of a single-degree-of-freedom milling model using the spectral tau method. The governing equation for this process is a time-periodic DDE, and therefore the ODE approximation of the DDE also contains time-periodic terms. We then resort to Floquet theory and construct the Floquet transition matrix by integrating the system ODEs for independent initial conditions. We observed that again the spectral method works really well and accurately captures the stability and bifurcation diagrams. These diagrams are in very good agreement when compared with literature.

In summary, we have generated the stability diagrams for different machining processes. These stability diagrams help us to determine the regions of parametric space for which the system is stable. When the process is modeled using these stable set of parameters, it results in smooth surface finish, less wear and higher tool life.

Bibliography

[1] F. W. Taylor. The art of cutting metals. *Scientific American* 63, (1907) 25,942–25,944.

[2] N. Hanna and S. Tobias. A theory of nonlinear regenerative chatter. *Journal of Manufacturing Science and Engineering* 96, (1974) 247–255.

[3] H. Shi and S. Tobias. Theory of finite amplitude machine tool instability. *International Journal of Machine Tool Design and Research* 24, (1984) 45–69.

[4] G. Stépán and T. Kalmár-Nagy. Nonlinear regenerative machine tool vibrations. In Proceedings of the 1997 ASME Design Engineering Technical Conferences, Sacramento, California. 1997 .

[5] D. J. Segalman and E. A. Butcher. Suppression of regenerative chatter via impedance modulation. *Journal of Vibration and Control* 6, (2000) 243–256.

[6] T. Insperger, G. Stépán, and J. Turi. State-dependent delay in regenerative turning processes. *Nonlinear Dynamics* 47, (2007) 275–283.

[7] Insperger, T., Stépán, G. Stability chart for the delayed Mathieu equation. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences* 458, (2002) 1989–1998.

[8] T. Insperger and G. Stépán. Updated semi-discretization method for periodic delay-differential equations with discrete delay. *International Journal for Numerical Methods in Engineering* 61, (2004) 117–141.

[9] T. Insperger and G. Stepan. Stability analysis of turning with periodic spindle speed modulation via semidiscretization. *Journal of vibration and control* 10, (2004) 1835–1855.