Reliability and Delay Analysis of Slotted Anycast Multi-hop Wireless Networks Targeting Dense Traffic IoT Applications

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Abstract—**Studies on IEEE 802.15.4 MAC in the current literature for anycast multi-hop networks do not capture a node's behaviour accurately. Due to the inaccurate modeling of state-wise behaviour of a node, the optimization of network parameters has not been efficient so far. In this work, we include the state-wise behaviour of a relay node into a 3D Markov model to more accurately investigate the protocol performance. Performance analysis of the proposed analytical model is evaluated for different variants of active state length, packet length and wake up rates considering reliability and delay as key performance metrics. Performance analysis shows that the model captures the behaviour of relay nodes most accurately.**

Keywords-**Anycast clustered multi-hop network, 3D Markov chain, Analytical model of IEEE 802.15.4 MAC.**

I. INTRODUCTION

Analytical models for IEEE 802.15.4 MAC proposed in the literature [1], [2] & [3] do not accurately capture the state-wise behaviour of a relay node with generic routing strategy for large dense networks. In [1], a two-dimensional Markov model for IEEE 802.15.4 MAC with anycast routing is proposed and in [2], a three-dimensional Markov model for IEEE 802.15.4 multi-hop scenario with reduced sensing and non-homogenous traffic is analyzed. The analytical study proposed in [3] incorporates joint sleep and contention control guaranteeing throughput and SINR requirements for extending network life time. The effect of adaptive MAC parameters on single hop and multi-hop wireless sensor networks are well studied using three-dimensional Markov models in [4] and [5] respectively. This work primarily investigates the accuracy of a three-dimensional Markov chain model for slotted IEEE 802.15.4 CSMA/CA MAC with slot-wise state modelling for Sleep, Idle-Listen and Active-Tx states of a node. Our analysis concludes that node wake up rates and active state time periods have a significant impact on the performance of the network. The rest of this letter is organized as follows. Section

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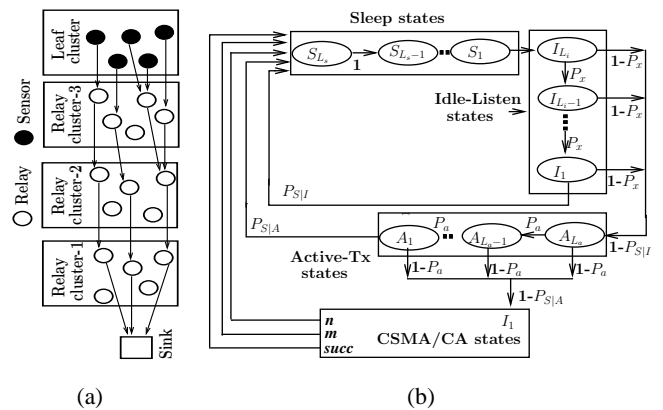

Fig. 1: a) Network scenario b) Relay node state model

2 discusses the system model based on a generic 3D Markov chain and Section 3 provides the mathematical formulation of the proposed model. The performance analysis from the emulations and the analytical model is discussed in Section 4 and finally, Section 5 concludes the letter.

II. SYSTEM MODEL

We consider a generic Cyber Physical Systems (CPS) wireless network scenario with four clusters as shown in Fig. 1(a). Sensor data collected at the leaf nodes have to be routed to a network sink via intermittent random relay nodes known as anycast routing. The functional model of relay nodes is captured accurately in four different states namely, Sleep (S_i), Idle-Listen (I_i), Active-Tx (A_i) and CSMA/CA as shown in Fig. 1(b). In Sleep state relay nodes sleep until assigned a wake up time. In Idle-Listen state, relay nodes broadcast a beacon to the predecessor cluster (cluster-3 is the predecessor to cluster-2) and wait for a packet from it. After successfully receiving a packet in Idle-Listen state relay nodes jump to Active-Tx state and wait for a beacon from their next cluster (cluster-2 is the next cluster to cluster-3). Time intervals between beacons observed by a relay in Active-Tx are Poisson distributed which determine the waiting time in Active-Tx state described later in the delay model of this

letter. The relay nodes that are successful in receiving a beacon within a maximum of L_a slots of the Active-Tx state follow CSMA/CA flow depicted using a 3D Markov chain shown in Fig. 2(a) with backoff stages (m), backoff counter (k) and collision retries (n) as the three dimensions.

Our primary investigation focuses on the effect of CSMA/CA retries in multi-hop scenarios with accurate state-wise behaviour of relay nodes. In this letter busy channel probabilities α and β in Clear Channel Assessment (CCA1 and CCA2) states and channel sensing probability τ along with collision probability P_c shown in Fig. 2(a) are derived considering the effect of Sleep, Idle-Listen and Active-Tx state probabilities along with the CSMA/CA model. With basic understanding of busy probabilities in [4], [5] and [6], one can drive the mathematical model in the following section.

III. MATHEMATICAL MODEL

We formulate the proposed model in two stages by deriving the transition probabilities for all states in Fig. 1(b) in the first stage and then follows the formulation of CSMA/CA model in second stage. In the rest of our discussion μ_w indicates average wake up rate per node in a cluster consisting of N nodes, L_a and L_i are length of active and idle slots respectively.

In Eq. (1), P_x is the transition probability of a relay node to move into the next Idle-Listen slot from the current one, when there is no packet arrivals at a given slot with an average of λ Poisson arrivals and P_a indicates the transition probability of a relay node to move into the next Active-Tx slot from the current, when there is no beacon arrival at a given slot with an average of λ_a Poisson arrivals shown in Eq. (4). Using Eq. (1), the probability of a node transitioning to Sleep state from the last slot of Idle-Listen ($P_{S|I}$) and Active-Tx ($P_{S|A}$) states can be obtained as shown in Eq. (2) and (3) respectively. Finally $P_{S|CSMA}$ in Eq. (5) indicates the transition probability of a node from CSMA/CA state to the first slot of Sleep state which should always equal to one.

$$P_x = \exp(-\lambda) \quad , \quad P_a = \exp(-\lambda_a) \quad (1)$$

$$P_{S|I} = P_{S_{L_s}|I_1} = P_x^{L_i} \quad (2)$$

$$P_{S|A} = P_{S_{L_s}|A_1} = P_a^{L_a} \quad (3)$$

$$\lambda_a = \mu_w * N \quad (4)$$

$$P_{S|CSMA} = P_{S_{L_s}|CSMA_m} + P_{S_{L_s}|CSMA_n} \quad (5)$$
$$+ P_{S_{L_s}|CSMA_{succ}} = 1$$

$$P_{S_0} = P_{S_0}P_x^{L_i} + P_{S_0}(1-P_x^{L_i})P_a^{L_a} + P_{S|CSMA}b_{0,0,0} \quad (6)$$

In Eq. (5), $P_{S_{L_s}|CSMA_m}$ and $P_{S_{L_s}|CSMA_n}$ are the probabilities to enter into the first slot of Sleep state when the received packet is discarded in CSMA/CA after exceeding maximum m and n respectively. $P_{S_{L_s}|CSMA_{succ}}$ is the probability of the node to enter the first slot of Sleep state after the node successfully forwards the packet. Eq. (6) can be simplified to arrive at probability P_{S_0} of a node to stay in the first sleep slot at any given time slot in terms of $b_{0,0,0}$, where $b_{0,0,0}$ is the probability of a node to stay in the first CCA1 slot of the CSMA/CA model.

$$P_{CSMA} = \sum_{i=0}^{m} \sum_{k=0}^{W_i-1} \sum_{j=0}^{n} b_{\text{i,k,j}} + \sum_{j=0}^{n} \sum_{i=0}^{m} b_{\text{i,-1,j}}$$
$$+ \sum_{j=0}^{n} \left(\sum_{k=0}^{L_s-1} b_{\text{-1,k,j}} + \sum_{k=0}^{L_c-1} b_{\text{-2,k,j}} \right) = Z_1 * b_{0,0,0} \quad (7)$$

$$P_S + P_I + P_A + P_{CSMA} = 1 \quad (8)$$

$$P_{S_0}\left[L_s + \sum_{i=1}^{L_i} P_x^{i-1} + (1 - P_x^{L_i}) \sum_{i=1}^{L_a} P_a^{i-1} \right] \quad (9)$$
$$+ Z_1 b_{0,0,0} = 1$$

The probability of a node to reside in CSMA/CA state (P_{CSMA}) at a randomly given time slot is the sum of backoff, CCA2, success and failure state probabilities respectively given in Eq. (7). Z_1 indicates the proportionality factor after expressing all probabilities in Eq. (7) in terms of $b_{0,0,0}$. In Eq. (8) P_S, P_I, P_A and P_{CSMA} indicate the probabilities for a node to reside in Sleep, Idle-Listen, Active-Tx and CSMA/CA states respectively in any given random time slot. All probabilities P_S, P_I, P_A and P_{CSMA} of Fig. 1(b) can be written in terms of P_{S_0} and $b_{0,0,0}$ as shown in Eq. 9. Now using normalization Eq. (8), $b_{0,0,0}$ can be derived as in Eq. (10). After getting the relation for $b_{0,0,0}$, we derive α, β and τ expressions utilizing $b_{0,0,0}$ in a similar methodology shown in [6]. Numerical methods are used to solve highly nonlinear α, β, τ and $b_{0,0,0}$ to arrive at a closed form solution. Both node and cluster probabilities (α, β, τ and P_c) are in close agreement with less than 5% deviation due to equal number of nodes in each cluster and random selection of relay nodes. Reliability and delay of the proposed model are derived and analyzed in the following section.

A. Reliability model

The reliability of a relay node can be determined by deriving the failure probabilities. Failure can occur due to exceeding m, n and active timeout. Considering these cases, the reliability of node k (R_k) is given in Eq. (11).

$$b_{0,0,0} = \frac{(1-P_x)(1-P_a)(1-P_x^{L_i})(1-P_a^{L_a})}{(1-P_a)*\left[L_s(1-P_x)+L_i(1-P_x^{L_i})\right]+(1-P_x^{L_i})(1-P_a^{L_a})(1-P_x)*[L_a+Z_1(1-P_a)]} \quad (10)$$
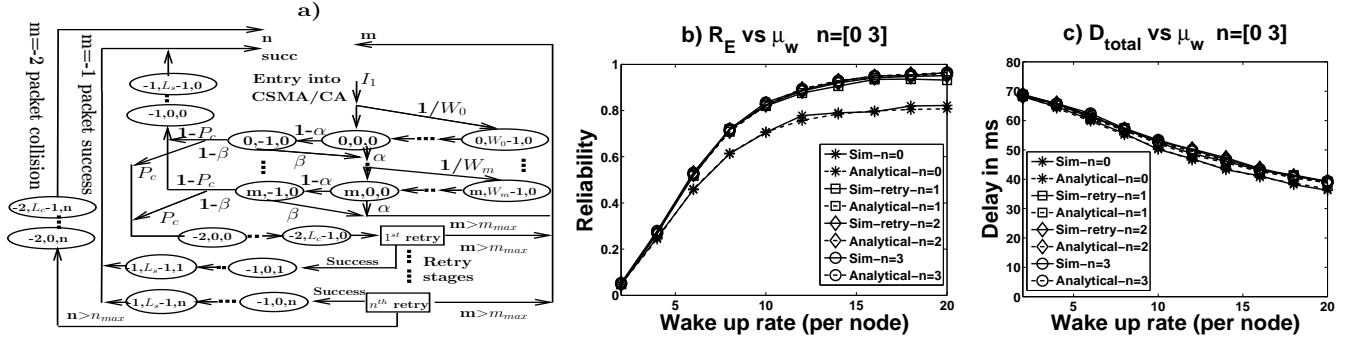

Fig. 2: 2a) Three-dimensional Markov model of IEEE 802.15.4 CSMA/CA; Fig. 2b and 2c: Simulation parameters for 2a) and 2b) are $m_0 = 3$, m = 4, $\lambda = 0.01$, n=[0 3], μ_w=[2 20], $L_a = 100$, $L_p = 10$, L_i=100, $L_s = 100$.

R_E in Eq. (12) depicts the end-to-end reliability calculated over h independent links/clusters/hops. $\widetilde{y}$ indicates the probability of a node transitioning to next retry after successfully sensing the channel from any of the m stages shown in Fig. 2(a).

$$R_k = 1 - \left(x^{m+1}(1+\widetilde{y}) - \widetilde{y}^{n+1}\right)\left(1-P_a^{L_a}\right) - P_a^{L_a} \quad (11)$$

$$R_E = \prod_{k=1}^{h} R_k \quad (12)$$

B. Delay Model

Total delay incurred by an individual node for forwarding a packet successfully is contributed by CSMA/CA delay and active state delay. Delay due to CSMA/CA is given in Eq. (13).

$$D_{\text{csma}} = T_s + D_{\text{avg}} + (T_s + D_{\text{avg}})\left[\frac{y}{1-y} - \frac{((n+1)*(y^{(n+1)}))}{1-y^{n+1}}\right] \quad (13)$$

$$D_{\text{avg}} = 2S_b\left[1 + 0.25\{\frac{1-b_l}{1-b_l^{m+1}}\left[2W_0\frac{1-2b_l^{m+1}}{1-2b_l} - \frac{3(m+1)b_l^{m+1}}{1-b_l}\right] + \frac{3b_l}{1-b_l} - (W_0+1)\}\right] \quad (14)$$

$$P_{(delay=i)|success} = \frac{P_a^{i-1}(1-P_a)}{(1-P_a^{L_a})} \quad (15)$$

$$D_{active} = \sum_{i=1}^{L_a}(i)\frac{P_a^{i-1}(1-P_a)}{(1-P_a^{L_a})} \quad (16)$$

$$D_{\text{total}} = (D_{\text{csma}} + D_{\text{active}} * S_b) * h \quad (17)$$

In Eq. (13), D_{avg} indicates the backoff delay and is derived in Eq. (14) by taking expectation over m stages of CSMA/CA, where D_{csma} of a link can be obtained from computing average probability of success after j retries. T_s and S_b are packet transmission time and unit backoff time, b_l is max$\{\alpha,(1-\alpha)\beta\}$, W_0 indicates minimum backoff window. L_p and L_c are length of time slots required for successful packet transmission and collision respectively and T_c indicates packet collision time. In Eq. (16), the average number of active slots (D_{active}) that a node waits before a beacon arrives is obtained. Finally total delay which is the sum of delays incurred by CSMA/CA and Active states computed over h independent links is given by Eq. (17).

IV. ANALYTICAL RESULTS

The proposed anycast clustered multi-hop analytical model's accuracy is validated by emulating a scenario similar to that shown in Fig. 1(a) which has 4 clusters with 10 nodes each. The proposed emulation model has the following assumptions: Congestion due to ACK and interference from other 2.45 GHz users is negligible. Each relay node switches among 3 different channels for Tx, Rx and beacon modes to reduce interference between nearby nodes of different clusters.

We first analyze the effect of CSMA/CA retries (n) on R_E and D_{total}. Fig. 2(b) and Fig. 2(c) plots R_E and D_{total} versus μ_w respectively for 4 different values of n. R_E and D_{total} are observed to be increasing and decreasing respectively with increase in μ_w as the average waiting time to receive a beacon in active state decreases and failures in active state due to active timeout are reduced. R_E is observed to increase by 15% and D_{total} is increased
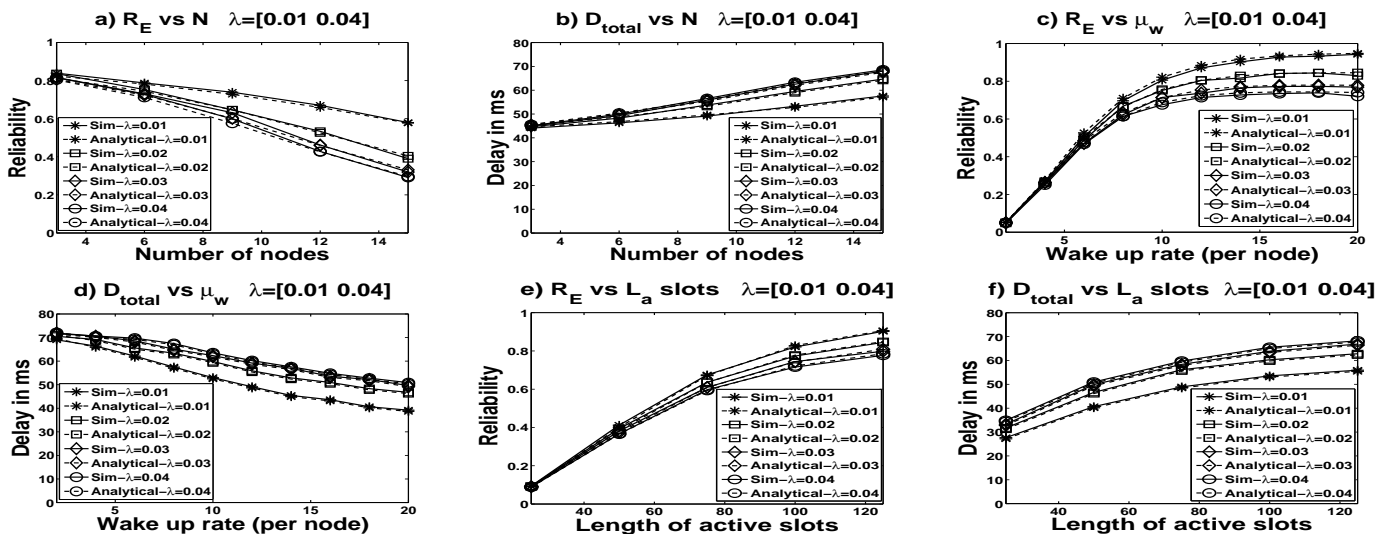
Fig. 3: Parameters for Fig. 3a and 3b: $m_0 = 3$, m = 4, $\mu_w = 10$, n = 1, $L_a = 100$, N=[3 15], λ =[0.01 0.04]; Fig. 3c and 3d: $m_0 = 3$, m = 4, n=1, $L_a = 100$, $\mu_w = [2\ 20]$, $\lambda = [0.01\ 0.04]$; Fig. 3e and 3f: $m_0 = 3$, m = 4, n=1, $L_a = [25\ 125]$, $\lambda = [0.01\ 0.04]$, $\mu_w = 10$; parameters fixed for all 6 scenarios: $L_p = 10$, $L_c = 10$, L_s=100 and L_i=100

by 6 slots with single retry after collision compared to the "no-retries" scenario. Improvement in R_E and D_{total} with higher retries (n=2 & 3) compared to n=1 is merely visible, as the probability to have successive collisions for a node is minimal. Degradation in R_E and D_{total} is observed when analysis is performed by incrementing number of nodes (N) for 4 different λ as shown in Fig. 3(a) and 3(b). Degradation in R_E is valid since increase in channel congestion values (α and β) results in more packet drops and collisions due to exceeding maximum m and n. More delay (D_{total}) with increase in λ is primarily due to degradation in D_{csma}.

Fig. 3(c) and Fig. 3(d) plots R_E and D_{total} versus μ_w for four different λ. From the figures one can infer the importance of λ in the performance of the network. Increase in μ_w reduces average waiting time in Active-Tx state and the chances for packet being dropped because of active timeout are less. The increase in λ results in channel congestion, leading to more packet failures due to active timeout and more delay due to backoff stages.

Fig. 3(e) and 3(f) plots R_E and D_{total} versus L_a for four different λ. R_E was enhanced and D_{total} was growing higher with increase in L_a as the chances of beacon reception before active timeout increases significantly.

V. CONCLUSION

In this letter, a slotted anycast model for clustered multi-hop networks with the state-wise behaviour injected into 3D Markov chain is developed and analyzed.

Reliability and delay performance metrics are analyzed with variation in parameters such as CSMA/CA retries, number of nodes, wake up rate and active time for different packet arrival values, and are validated using both analytical and emulation results with less than 0.5% error. The proposed integrated model and the analysis can greatly aid in driving the future research in modelling of dense traffic wireless multi-hop sensor networks. Optimization of model parameters is a focus of future research.

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