Sparseland Model for Speckle Suppression of B-mode Ultrasound Images

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***Abstract*—Speckle is a multiplicative noise which is inherent in medical ultrasound images. Speckles contributes high variance between neighboring pixels reducing the visual quality of an image. Suppression of speckle noise significantly improves the diagnostic content present in the image. In this paper, we propose how sparseland model can be used for speckle suppression. The performance of the model is evaluated based on variance to mean ratio of a patch in the filtered image. The algorithm is tested on both software generated images and real time ultrasound images. The proposed algorithm has performed similar to past adaptive speckle suppression filters and seems promising in improving diagnostic content.**

***Index Terms*—Dictionary learning, K-SVD, multiplicative noise, speckle, sparse representations**

I. INTRODUCTION

Medical ultrasound scanning is the widely used non-invasive modality to acquire real time images of the organs including kidney, liver, cardiac, fetus monitoring etc. The signal to noise ratio of ultrasound images is poor compared to magnetic resonance imaging and computed tomography reducing the diagnostic precision of the machine. Ultrasound images are inherently affected by a multiplicative noise called speckle. The speckle appears as small worms or snake like structures in the image revealing no significant information about the tissue structure and so it is considered as noise.

Speckle is uncorrelated with respect to spatial, temporal and frequency of operation of ultrasound probe. The uncorrelated property of speckle is used to suppress the noise at RF acquisition stage using different compounding techniques. In frequency compounding, the same imaging sector is scanned with multiple frequencies and the corresponding RF signals are averaged. Phase compounding involves averaging the RF data acquired by scanning the same position with multiple probe positions [1]. Compounding techniques are employed in the ultrasound machines at a cost of increased computations. Speckle suppression also done after complete formation of an ultrasound image by estimating the random behavior of speckles in the image.

Mathematically speckle can be modeled as sum of large number of complex phasors which results in constructive and destructive interference at the receiver side [2]. The constructive interference leads to bright spots and destructive interference leads to dark spots similar to dense *salt and pepper* like noise in the image. The intensity of envelop detected RF data *J(n, m)* affected by speckle noise is given as

$$J(n,m) = (P(n,m) * I(n,m))\ N_{\times}(n,m)$$

where the multiplicative noise $N_{\times}(n,m)$ is sample wise independent of past, future samples and uncorrelated to the image pixel value *I(n, m)*, *P(n, m)* is the point spread function (PSF) of the ultrasound imaging system, (n,m) represents the spatial position of the tissue in the scan plane. The log transformation, which is used to compress the dynamic range of envelope detected data in ultrasound imaging system modify the multiplicative model into an additive model.

$$J(n,m) = (P(n,m) * I(n,m)) + N_{+}(n,m)$$

$N_{+}(n,m)$ is the additive noise term. The behavior of $I(n,m)$ in fully formed image is modeled as summation of complex phasors in a random walk model which results bright pixels due to constructive interference and dark pixels due to destructive interference [2].

$$I(n,m) = \sum_{1}^{p} a_p(n,m)\ e^{j\varphi_p(n,m)}$$

p is a positive integer which is generally considered very large, a_p and φ_p are amplitudes and phases of scattering echoes from tissues.

Smoothing the image is one common solution seen in literature to address speckle suppression. These techniques are mainly differed based on how to smooth and diverse criteria employed to determine the degree of smoothness. The operation of the filter on a particular pixel depends upon local statistics of the pixel surrounding around it. These filters are biased to the size of the window used for finding local statistics and smoothing.

Sparse and redundant representations over learned dictionaries looks promising for image deblurring [3] and denoising applications [4]. In this paper, we show how sparse and redundant representation over learned dictionaries of speckle affected ultrasound image effectively leads to speckle suppression. The algorithm is tested individually on coherent, diffused speckle pattern images and real time ultrasound B-mode (Liver and Kidney) images. Coherent speckles in the ultrasound image appears as dense *salt and pepper* like noise and diffusion speckle pattern appears as *small worm* like

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structures. The speckle suppression is performed on images, which is generated from Matlab using K-wave toolbox [5].

The performance of the algorithm is evaluated based on how efficiently it can smoothen the image. The smoothness of image is computed by variance to mean ratio over some fixed patches. The performance is compared with well defined speckle reduction filters like Frost [6], Lee [7] and Speckle reduction anisotropic diffusion (SRAD) [8] filters. Speckle suppression by sparse and redundant representation over trained dictionaries significantly improved visual content of an image similar to Frost, Lee and SRAD filters.

The rest of the paper is organized in the following way. Section II deals with construction of sparseland model for speckle suppression. Results of sparseland model with respect to other models is discussed in section III and section IV concludes the paper.

II. Representation of an image over sparse and redundant over-complete Dictionary.

A. Problem formulation

Sparse modeling is used in many image processing applications like denoising, inpainting, mosaicing etc. Speckle suppression in ultrasound images is mathematically formulated in the following way.

$$Minf(\underline{X}) = \frac{1}{2} \parallel \underline{X} - \underline{Y} \parallel_2^2 + G(\underline{X}) \tag{1}$$

$\underline{Y}$: Given measurements.
$\underline{X}$: Unknown to be recovered.
$\underline{Y}$ is the speckle image and we want to recover clean image $\underline{X}$. We do not want to recover the image that is not to far from the noisy image and that is the penalization that we have here i.e.,$\parallel \underline{X} - \underline{Y} \parallel_2^2$. $\parallel \underline{X} - \underline{Y} \parallel_2^2$ gives the mean square error between the speckle image and restored image. In (1), $\parallel \underline{X} - \underline{Y} \parallel_2^2$ also seen as variance of speckle noise. The image that minimizes (1) is noisy image itself and so we have not done much. The second term in (1) indicates the prior information regarding the image that has to be recovered. This is simply a Bayesian point of view, adapting the Maximum-A-Posterior (MAP) estimation and the basic idea is computing the $\underline{X}$ that minimizes $f(\underline{X})$. (1) is seen as prior and likelihood estimation with probabilistic interpretation. In this scenario we choose a prior to be

$$G(\underline{X}) = \lambda \parallel \alpha \parallel_0 \tag{2}$$

where $\parallel \alpha \parallel_0$ gives sparsity of the signal representing number of non-zero coefficients.

B. Sparseland modeling of an image.

To construct sparse modeling of an image, we will begin by constructing the sparse model for overlapping fixed image patches. An image patch of size $\sqrt{m} \times \sqrt{m}$ pixels is arranged lexicographically as single column $x \in \Re^m$. To define a sparseland model for this column vector, we need to construct a dictionary of size $D \in \Re^{m \times k}$. If $k > m$ then the dictionary is said to be overcomplete and it is redundant. We simplify the model assuming the matrix D is fixed and it is known to us. Every image patch x in the image is represented sparsely over the dictionary by solving

$$\hat{\alpha} = arg \min_{\alpha} \parallel \alpha \parallel_0 \quad s.t \quad D\alpha \approx x \tag{3}$$

s.t stands for such that.
we will get $\parallel \alpha \parallel_0 \ll m$. Mathematically speaking, the signal is represented by a linear combination of few column vectors of redundant dictionary matrix D. The constraint $D\alpha \approx x$ is equivalent to $\parallel D\alpha - X \parallel_2^2 \leq \epsilon$, ϵ is the magnitude of error allowed. We need to define the parameter Q such that $\parallel \alpha \parallel_0^0 \leq Q \ll m$. It states that it uses atmost Q columns of D for representing the image patch.

We have (ϵ, Q, D) with us and need to find the sparse representation $\hat{\alpha}$

$$\hat{\alpha} = arg \min_{\alpha} \parallel \alpha \parallel_0 \quad subject \quad to \quad \parallel D\alpha - Y \parallel_2^2 < T \tag{4}$$

T is the threshold and the recovered image is given by $D\hat{\alpha} = \hat{X}$. The optimization of (4) is an NP hard problem and cannot be solved as it is. (4) can be solved using relaxation and greedy algorithms by modifying the equation as

$$\hat{\alpha} = arg \min_{\alpha} \parallel D\alpha - Y \parallel_2^2 \quad + \quad \mu \parallel \alpha \parallel_1 \tag{5}$$

Orthogonal matching pursuit (OMP) optimization algorithm is employed for obtaining the $\hat{\alpha}$ due to its efficiency and simplicity. .

C. Despeckling of image from local patches.

Despeckling an ultrasound image Y of size $\sqrt{M} \times \sqrt{M}$ where $M \gg m$ is done by constructing a larger dictionary. The larger dictionary is obtained by just scaling the dictionary of image patch containing the basis of curvelet or contourlet transforms [9]. This is not possible if we use small and fixed dictionary $D \in \Re^{m \times k}$. We can solve this problem by another way by tiling the results of all patches in the image forming the complete despeckled image. Blocking artifacts is seen on the resulted image due to tiling of patches and is overcomed by constructing the dictionary for overlapping patches and averaging the results of the patches accordingly [10], [11].

Considering every patch in the image belonging to sparseland model (ϵ, Q, D), MAP estimator can be rewritten as

$$\begin{aligned} \{\alpha_{ij}, \bar{X}\} = arg \min_{\alpha_{ij}, X} \lambda \parallel \underline{X} - \underline{Y} \parallel_2^2 \\ + \sum_{ij} \mu_{ij} \parallel \alpha_{ij} \|_0 + \sum_{ij} \parallel D\alpha_{ij} - B_{ij}\underline{X} \parallel_2^2 \end{aligned} \tag{6}$$

The first term in the equation represents the error between the recovered image and noise image. B_{ij} is binary image used to extract the patch of an image of size $\sqrt{m} \times \sqrt{m}$. (6) has two unknowns $\alpha_{ij}, \underline{X}$ and is solved by fixing $\underline{X} = \underline{Y}$ and seeking for $\hat{\alpha_{ij}}$. The optimum value of $\hat{\alpha_{ij}}$ for each image patch is obtained by solving

$$\hat{\alpha_{ij}} = arg \min_{\alpha} \mu_{ij} \parallel \alpha \parallel_0 + \parallel D\alpha - x_{ij} \parallel_2^2 \tag{7}$$

OMP algorithm is employed to obtain $\hat{\alpha_{ij}}$ for each image patch by picking one column at a time from a dictionary and stopping it when $\| D\alpha - x_{ij} \|_2^2 < T$. This is operated for every image patch of size $\sqrt{m} \times \sqrt{m}$ one at a time on sliding window model basis. (6) is in quadratic form and its closed form solution is given by

$$\bar{X} = (\lambda I + \sum_{ij} B_{ij}^T B_{ij})^{-1} (\lambda Y + \sum_{ij} B_{ij}^T D\hat{\alpha}_{ij}) \quad (8)$$

(8) is seen as averaging the result of sparseland model of shifted overlapping patches of a speckle image. We formulated all the above equations guessing the dictionary matrix D is given to us. Various dictionaries are proposed in the literature and we choose to have discrete cosine transform as dictionary due its uncorrelated basis structure which tends to have high sparsity. We have to update the parameters D and α_{ij} based on the image patches iteratively unless the required condition is met. D and α_{ij} can be updated using patches from a set of clean images or from the corrupted image itself. D is the common dictionary used to represent all the patches in the image. In this paper, we updated D and α_{ij} using the patches from the speckled image. The final generalized problem is formulated as

$$\begin{aligned} \bar{X} = arg \min_{\{\alpha_{ij}\}_{ij}, X, D} \lambda \| \underline{X} - \underline{Y} \|_2^2 \\ + \sum_{ij} \| D\underline{\alpha_{ij}} - B_{ij} X \|_2^2 \quad s.t \quad \| \alpha_{ij} \|_0 < Q \end{aligned} \quad (9)$$

(i, j) in (9) corresponds to spatial location of image patch in the image. Representing the image with few number of columns naturally reduces the noise (noise is represented in lower dimension where it cannot well) and averaging the patches leads to smoothness of the image. The algorithm for finding sparseland model for speckle suppression is shown in Algorithm.1.

The sparseland model algorithm for speckle suppression requires to initialize few parameters, they are listed below.

1) **Initial dictionary (I)** :Initial dictionary for K-SVD training- overcomplete DCT.
2) **Overlapping stepsize** Δ: Interval between neighboring blocks.
3) **Sigma**: used to determine the target error for sparse-coding each block.
4) **Training block T**: Number of training blocks extracted for training.
5) **Dict D** :Dictionary size.
6) **Block size** $\sqrt{m} \times \sqrt{m}$: size of image patch to process.
7) **Iteration number I**: number of K-SVD training iterations to perform.
8) λ:Specifies the relative weight attributed to the noisy input signal in determining the output.

Algorithm 1 Algorithm for finding Sparse representation of an image.

1) In (9) we have to fix three terms X,D and α_{ij}. To find this, we are going to fix two terms and find the third term. Let us fix $\underline{X}$=$\underline{Y}$ and overcomplete dictionary D.
2) Repeat J times
   - **Sparse matrix update** using OMP algorithm, Compute sparse vector α_{ij} for each patch $B_{ij}X$
   $$\forall_{ij} \min_{\underline{\alpha}_{ij}} \| \alpha \|_0 \ s.t \sum_{ij} \| B_{ij}X - D\underline{\alpha} \|_2^2 \leq \epsilon$$
   - **Dictionary update stage**: update each atom a=1,2,...k in D by
   -Find the set of blocks in image that uses this atom $v_a = \{(i, j) | \alpha_{ij}(a) \neq 0\}$
   -Find the corresponding error for each index $(i, j) \in w_a$
   $$e_{ij}^a = B_{ij} X_{ij} - \sum_{n \neq a} d_n \alpha_{ij}(n) \quad (10)$$
   -Matrix E is formed with columns $\{e_{ij}^a\}_{(ij) \in w_a}$
   -E_a is factorized as $U \nabla V^T$ using SVD algorithm. Fix the updated dictionary column d_l be the first column of U. Coefficients of $\{\alpha_{ij}^a\}_{(ij) \in w_a}$ is updated by the entries of V multiplied by $\nabla(1, 1)$. Iteration of 2) is called K-SVD.
3) Now D and α_{ij} known, compute $\underline{X}$ by
   $$\bar{X} = (\lambda I + \sum_{ij} B_{ij}^T B_{ij})^{-1} (\lambda Y + \sum_{ij} B_{ij}^T D\hat{\alpha}_{ij})$$
   which is a simple averaging of shifted patches.

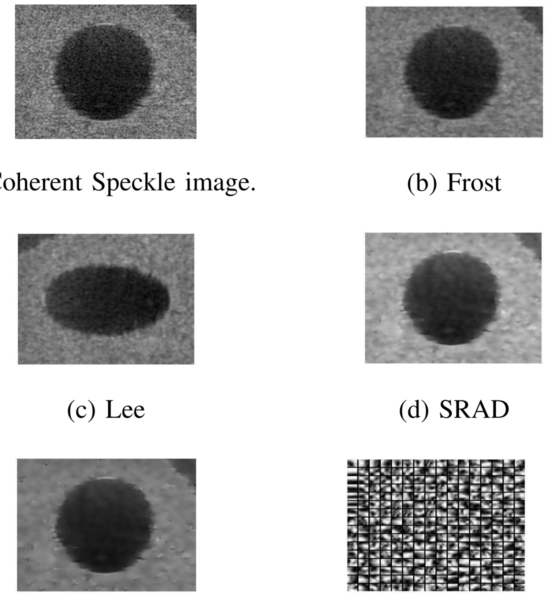
TABLE I
K-WAVE PARAMETERS USED TO SIMULATE THE ULTRASOUND IMAGE SHOWN IN FIG. 2(A).

Transducer width	14.1593 mm (64 grid points)
Number of elements	64
Number of active elements	64
Element width	221.2389 um (1 grid points)
Sound speed	1540 m/s
Focus distance	30 mm
Elevation focus distance	30 mm

The order of complexity for each pixel in sparseland model is of the order $O(mkQJ)$, where m is patch size, k is number of columns in dictionary, Q is the sparsity of each column in coefficient matrix and J is number of stages used for updating the dictionary.

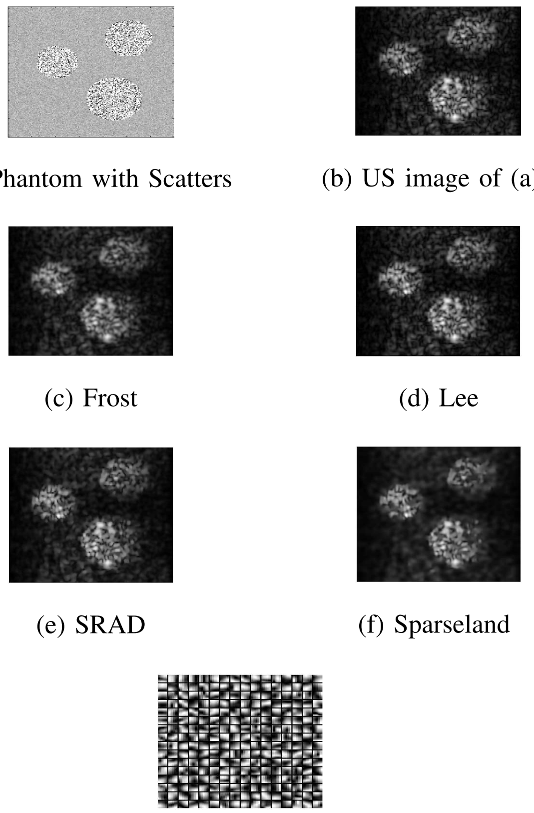
III. RESULTS

The algorithm is applied on four types of ultrasound images. In first case a phantom image with coherent speckle is considered. The Frost, Lee, SRAD and Sparseland filtered images



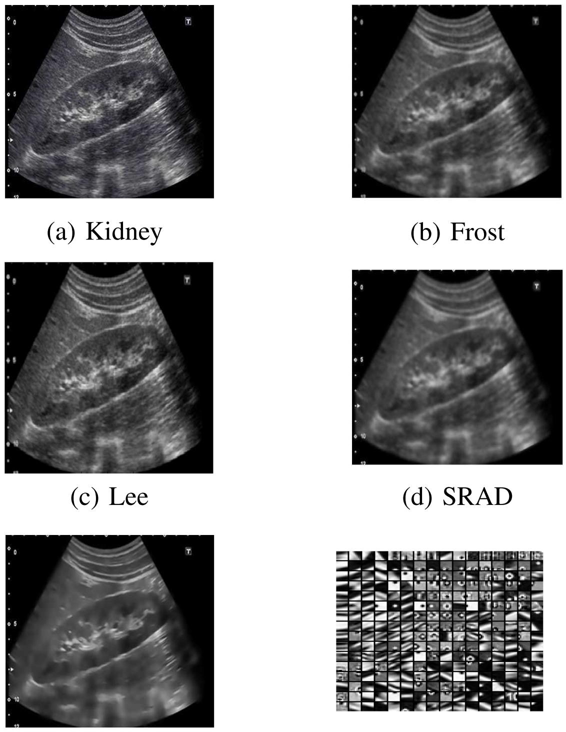
(a) Coherent Speckle image. (b) Frost (c) Lee (d) SRAD (e) Sparseland (e) Trained dictionary

Fig. 1. (b), (c), (d). Speckle suppressed images of various filters on (a). Coherent speckled image. (f) Trained dictionary of sparseland model.



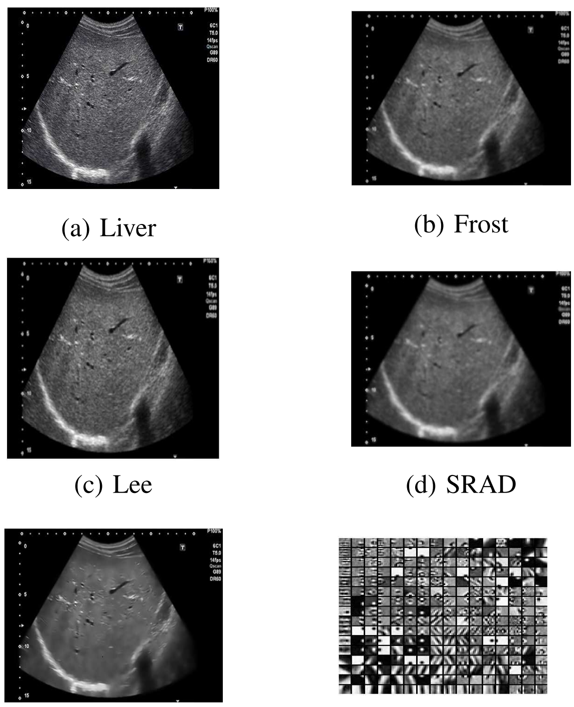
(a) Phantom with Scatters (b) US image of (a). (c) Frost (d) Lee (e) SRAD (f) Sparseland (e) Trained dictionary

Fig. 2. (a) Scattering phantom. (b) Ultrasound image (US) of (a). (c), (d), (e), (f) Filtered images of various filters on (b). (g) Trained dictionary of sparseland model.



(a) Kidney (b) Frost (c) Lee (d) SRAD (e) Sparseland (e) Trained dictionary

Fig. 3. (b), (c), (d), (e) Filtered images of various filters on (a). Kidney image. (f) Trained dictionary of sparseland model.



(a) Liver (b) Frost (c) Lee (d) SRAD (e) Sparseland (e) Trained dictionary

Fig. 4. (b), (c), (d), (e) Filtered images of various filters on (a) Liver image. (f) Trained dictionary of sparseland model.

along with trained dictionary is shown in Fig. 1. A fully formed speckle ultrasound phantom image is generated using K-wave tool box in MATLAB is shown in Fig. 2. The K-wave parameter specifications used for generating the ultrasound image for the template Fig. 2(a) is shown in Table I. The small snake like structures in the image Fig. 2(b) corresponds to speckle patterns. The result of various filters on diffused speckle is shown in Fig. 2(c)-Fig. 2(f). Trained dictionaries of sparseland model for corresponding images are represented in the form of image for better visualization. Fig. 3 and Fig. 4 shows the results of various filters on kidney images and liver images respectively. The B-mode ultrasound kidney and liver images are collected from Toshiba Capasee SSA-220A US scanner from Nitya Diagnostic Centre, Hyderabad.

The algorithm is conducted by varying the parameters listed in section II. The optimal suppression based on smoothing is obtained for the values σ=20, I=20, λ=25, T=40000, dictionary size D-8 x 256, Block size- 8 x 8. The algorithm is applied by fixing the same parameters for all the images i.e., coherent speckled, diffused speckle, kidney and liver images.

The variance to mean ratio of the filtered coherent speckle phantom image at different patches of size 20x20 is shown in Table. II. The window size for computing the local statistics of the filter Lee and Frost is 5 x 5. The SRAD filter is fixed for 30 iterations. The high variance to mean ratio of coherent speckle signifies the patch has edge and low variance to mean ratio signifies the patch is from flat region. By fixing the number of iterations to 20, the performance (variance to mean ratio) of the sparseland model with respect to sigma σ for coherent speckle image is shown in Fig. 5. The graph follows approximately the same trend for rest of the images, so σ is fixed to 20. Fixing the σ, the performance of the sparseland model with respect to number of iterations for updating the dictionary for the coherent image is shown in Fig. 6. The variance to mean ratio saturates after 20 iterations and there is no change in the performance afterwards, so it is taken as bench mark value.

The pixel variations of 98^{th} column of coherent speckle of various filtered images is shown in Fig. 7. All the filters reduce the variance to mean ratio, steep rise and fall of pixel intensities signifies the present of edges in the image. All the filters mimic the edges as all lines coincide with each other at that location, which is the anticipated result of speckle suppression filters. The stats signifies speckle suppression by sparseland model performs similar to Frost, Lee and SRAD filters. The algorithm is implemented in MATLAB 7.9 version on core i5 processor with 2.8 Ghz clock speed. Sparseland model took 10.2 seconds for execution for an image of size 256×256.

IV. Conclusion

The sparseland modeling representation of speckled images has the natural capability of removing outliers present in the image. Averaging the results of sparseland model on overlapping patches of the image lead to smoothing. The

TABLE II
VARIANCE/MEAN OF PIXELS WITH WINDOW SIZE 20X20 OF IMAGE SHOWN IN FIG.1 AT DIFFERENT LOCATIONS.

Phantom	Frost	Lee	SRAD	Sparseland
4.08	1.26	0.49	0.08	0.04
4.71	2.34	3.19	2.53	1.84
4.98	2.03	2.73	2.24	1.493
11.32	7.54	9.16	9.58	7.68
16.63	13.58	14.73	16.66	11.99

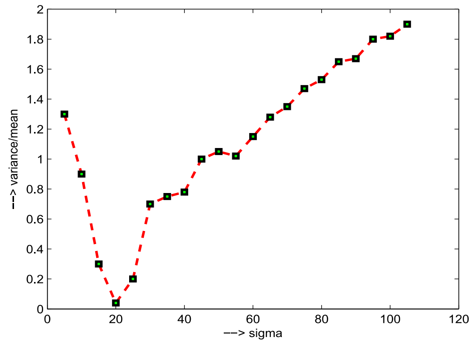


Fig. 5. Performance of Sparseland model with respect to sigma.

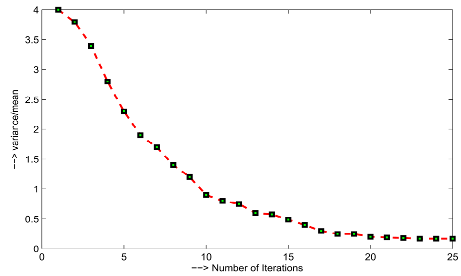


Fig. 6. Performance of Sparseland model with respect to number of iterations.

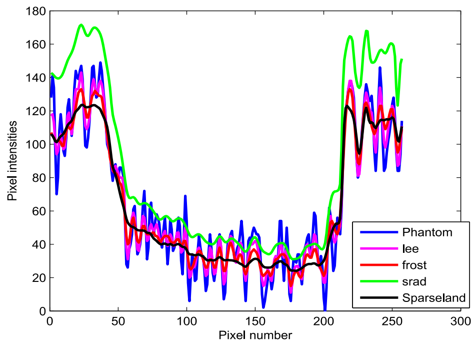


Fig. 7. Pixel variations of filtered images.

suppression of speckles by sparseland method performed similar to other adaptive speckle suppression filters. The speckle suppression using sparseland model significantly improved the visual content of the image, sonographers can infer more information from the despeckled ultrasound images to do accurate diagnosis.

We need to study the effectiveness of sparse modeling using different initialization of dictionaries and training of dictionaries on clean images for speckle suppression of ultrasound images.

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