Maximum Likelihood Detection for Decode and Forward Cooperation with Interference

Y. Aditya, G.V.S.S. Praneeth Varma and G.V.V. Sharma
Department of Electrical Engineering
Indian Institute of Technology
Hyderabad, India
Email: {ee11b039, ee14resch11007, gadepall}@iith.ac.in

***Abstract*—In this paper, we obtain the maximum likelihood (ML) decision for a decode and forward (DF) cooperative system in Nakagami-m fading in the presence of co-channel interference at the relay as well as the destination. Through simulation results, we first show that conventional ML designed for interference free systems fails to combat the deleterious effect of interference. An optimum ML decision for combating interference is then derived for integer m. This receiver is shown to be superior to conventional ML through bit error rate (BER) performance simulations. Further, our results also indicate that optimum ML preserves relay diversity in the presence of interference.**

***Index Terms*—Cooperative diversity, co-channel interference, decode and forward, ML**

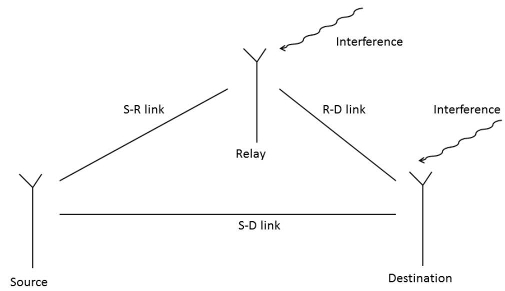

Fig. 1. Cooperative diversity system with a relay and an interferer.

I. Introduction

The performance of a cooperative system in interference limited networks has attracted attention recently. In [1], [2], expressions for the outage probability in an interference limited amplify and forward (AF) cooperative system are derived. [3], [4] have investigated outage probability for DF systems employing relay selection. The BER is also derived for the system considered in [4]. In [5], [6], an interference limited Nakagami-m fading channel is considered for DF cooperation. The outage probability was then evaluated for this system. [7], [8] extended the ML decision rule for higher order modulation schemes and investigated their performance. However, they did not account for the effect of interference on the performance of the cooperative system.

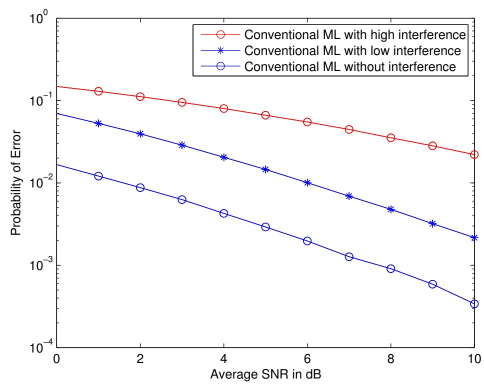

Fig. 2. Conventional ML performance deteriorates with increasing interference.

A. Motivation

In Fig. 2, the performance of conventional ML [9] in the presence of varying levels of interference is shown for Nakagami-m fading. Clearly, the figure indicates that conventional ML is susceptible to interference. While this is expected, this motivates the quest for an optimum ML scheme that performs better in the presence of interference.

B. Approach

This problem is addressed in this paper for Nakagami-m fading for integer m. First, an optimum ML detector is derived for interference mitigation. The decision rule is derived using the Maximum Likelihood criterion for cooperative systems introduced in [9]. Through simulations, it is then shown that the performance of optimum ML is better than conventional ML. The effect of interference on relay diversity is also investigated for the optimum ML receiver.

$$p_{y_{rs}}(y_{rs}|x_s,h_{rs}) = \begin{cases} \sum_{x_i^{(1)}} P(x_i^{(1)}) \frac{2(m_{ri})^{m_{ri}}}{\sqrt{\pi}\Gamma(m_{ri})} \frac{\exp\left\{-\frac{m_{ri}Y_{rs}^2}{\left(m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}\right)}\right\}}{\left(m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}\right)^{m_{ri}}} \sum_{r=0}^{2m_{ri}-1} \binom{2m_{ri}-1}{r} N_0^{\left(\frac{r}{2}\right)} \left(\frac{E_i\left\{x_i^{(1)}\right\}^2 Y_{rs}^2\Omega_{ri}}{m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}}\right)^{\left(\frac{2m_{ri}-r-1}{2}\right)} & \\ \times\left[\frac{1}{2}\Gamma(\frac{r+1}{2}) + \frac{(-1)^r}{2}\left\{\Gamma(\frac{r+1}{2}) - \Gamma\left(\frac{r+1}{2}, \frac{E_i\left\{x_i^{(1)}\right\}^2 Y_{rs}^2\Omega_{ri}}{N_0\left(m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}\right)}\right)\right\}\right] & x_i^{(1)}Y_{rs} \geq 0 \\ \sum_{x_i^{(1)}} P(x_i^{(1)}) \frac{2(m_{ri})^{m_{ri}}}{\sqrt{\pi}\Gamma(m_{ri})} \frac{\exp\left\{-\frac{m_{ri}Y_{rs}^2}{\left(m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}\right)}\right\}}{\left(m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}\right)^{m_{ri}}} \sum_{r=0}^{2m_{ri}-1} \binom{2m_{ri}-1}{r} N_0^{\left(\frac{r}{2}\right)} \left(\frac{E_i\left\{x_i^{(1)}\right\}^2 Y_{rs}^2\Omega_{ri}}{m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}}\right)^{\left(\frac{2m_{ri}-r-1}{2}\right)} & \\ \times\frac{1}{2}\Gamma\left(\frac{r+1}{2}, \frac{E_i\left\{x_i^{(1)}\right\}^2 Y_{rs}^2\Omega_{ri}}{N_0\left(m_{ri}N_0+E_i\left\{x_i^{(1)}\right\}^2\Omega_{ri}\right)}\right) & x_i^{(1)}Y_{rs} < 0 \end{cases} \quad (5)$$

II. System Model

The cooperative system in Figure 1 is considered, with system equations

$$\begin{aligned} y_{d,s} &= \sqrt{\xi_s} h_{d,s} x_s + z_{d,s} \\ y_{r,s} &= \sqrt{\xi_s} h_{r,s} x_s + \sqrt{\xi_i} h_{r,i} x_i^{(1)} + z_{r,s} \\ y_{d,r} &= \sqrt{\xi_r} h_{d,r} x_r + \sqrt{\xi_i} h_{d,i} x_i^{(2)} + z_{d,r} \end{aligned} \quad (1)$$

and variables described in Table I, without loss of generality. We assume that full channel state information (CSI) is available for the signal fades, while only partial CSI in the form of the second order statistics is available for the interference. We have one interferer each, in the $s-r$ and $r-d$ links.

h	Nakagami fading coefficient
m, Ω	Nakagami fading figures
ξ	Transmit power at a node
y	Received symbol at a node
x	Transmitted symbol at a node
s, r, d	Source relay and destination subscripts
i	Interferer subscript
l	Relay location
z	Additive white Gaussian noise (AWGN)
N_0	Two sided noise power spectral density

TABLE I
NOTATION USED THROUGHOUT THE PAPER

Definition II.1. *The probability density function (PDF) of a Nakagami distributed random variable $X \sim Nakagami(m,\Omega)$ is given by*

$$p_X(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} e^{-\frac{mx^2}{\Omega}}, \quad x,\Omega > 0, m > 0.5 \quad (2)$$

In (1), $h_{k,j} \sim Nakagami(m_{k.j}, \Omega_{kj}), z_{k,j} \sim \mathcal{N}(0, N_0/2)$, where $k,j \in \{s,r,d,i\}$.

III. Decision Rules

Lemma III.1. *For $X \sim Nakagami(m,\Omega), a > 0$,*

$$E_X\left[e^{2bX-aX^2}\right] = \begin{cases} \frac{2m^m}{\Gamma(m)} \frac{e^{\frac{b^2}{\frac{m}{\Omega}+a}}}{(m+a\Omega)^m} \\ \times \sum_{r=0}^{2m-1} \binom{2m-1}{r} \\ \times \left(\sqrt{\frac{b^2}{\frac{m}{\Omega}+a}}\right)^{2m-r-1} & b < 0 \\ \times \left(\frac{1}{2}\Gamma(\frac{r+1}{2}, \frac{b^2}{\frac{m}{\Omega}+a})\right) \\ \frac{2m^m}{\Gamma(m)} \frac{e^{\frac{b^2}{\frac{m}{\Omega}+a}}}{(m+a\Omega)^m} \\ \times \sum_{r=0}^{2m-1} \binom{2m-1}{r} \\ \times \left(\sqrt{\frac{b^2}{\frac{m}{\Omega}+a}}\right)^{2m-r-1} & b \geq 0 \\ \times \left[\frac{1}{2}\Gamma(\frac{r+1}{2}) \right. \\ + \frac{(-1)^r}{2}\left\{\Gamma(\frac{r+1}{2}) \right. \\ \left.\left. - \Gamma(\frac{r+1}{2}, \frac{b^2}{\frac{m}{\Omega}+a})\right\}\right] \end{cases} \quad (3)$$

where E_X is the expectation with respect to X and $\Gamma(\cdot), \Gamma(\cdot,\cdot)$ are the Gamma and incomplete Gamma functions [10].

Proof. See Appendix A.

Theorem III.1. *The ML decision made at the relay is*

$$x_r = \max_{x_s} p_{y_{rs}}(y_{rs}|x_s,h_{rs}) \quad (4)$$

where $p_{y_{rs}}(y_{rs}|x_s,h_{rs})$ is given in (5) *and* $Y_{rs} = y_{rs} - \sqrt{\xi}_s h_{rs} x_s$.

Proof. See Appendix B.

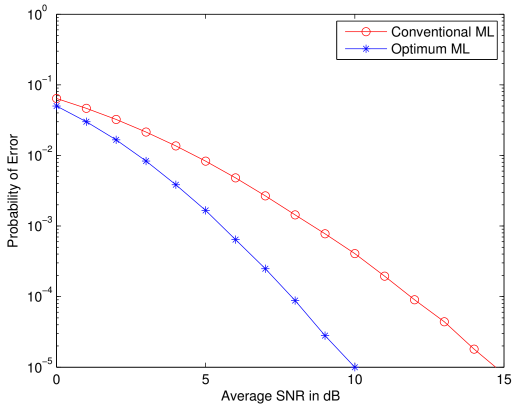

Fig. 3. Optimum ML outperforms conventional ML. $m_{k,l}$ = 2, k,l ∈ $\{s, r, d\}$, $m_{j,i}$ = 1 $l_{r,s}$ = 0.8, $l_{j,i}$ = 0.7, $j \in \{r, d\}$

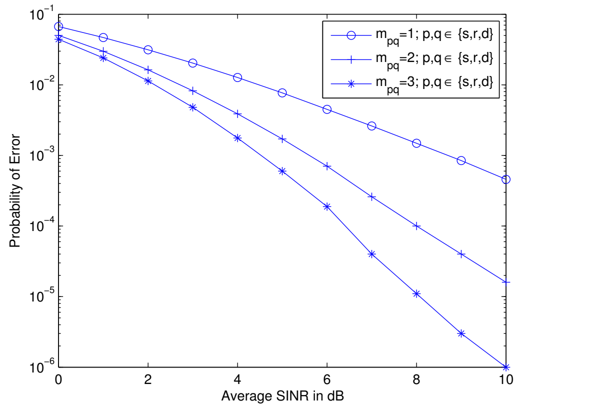

Fig. 5. Diversity performance of optimum ML in the presence of interference. $m_{k,l}$, k,l ∈ $\{s, r, d\}$ values of 1,2 and 3 and $m_{p,i}$ = 1, p ∈ $\{d, r\}$ respectively for $l_{r,s}$ = 0.8, $l_{j,i}$ = 0.7, $j \in \{r, d\}$

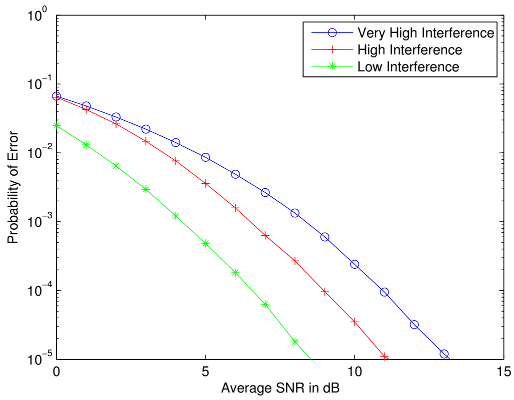

Fig. 4. Performance of optimum ML for varying interference levels. $m_{k,l}$ = 2, $m_{j,i}$ = 1 k,l ∈ $\{s, r, d, i\}$ and $l_{j,i}$ = 0.4, 0.6, 1 respectively. $j \in \{r, d\}$

Theorem III.2. *ML decision at the destination is given by*

$$\begin{aligned} \hat{x}_s &= \max_{x_s} \; p_{y_{ds}, y_{dr}}(y_{ds}, y_{dr} | x_s, h_{ds}, h_{dr}) \\ &= \max_{x_s} \sum_{x_i^{(2)}} p_{y_{ds}}(y_{ds} | x_s, h_{ds}) \\ &\qquad \times E_{h_{di}} \left[p_{y_{dr}}(y_{dr} | x_s, h_{dr}, x_i^{(2)}, h_{di}) \right] P\left(x_i^{(2)}\right) \end{aligned}$$

where

$$p(y_{ds} | x_s, h_{ds}) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y_{ds} - \sqrt{E_s} h_{ds} x_s)^2}{N_0}}, \qquad (6)$$

$E_{h_{di}^{(1)}} \left[p_{y_{ds}}(y_{ds} | x_s, h_{ds}, x_i^{(2)}, h_{di} \right]$ *is obtained by substituting* $b = \frac{\sqrt{\xi_i}\left(x_i^{(2)} Y_{dr}\right)}{N_0}$ *and* $a = \frac{\xi_i \left(x_i^{(2)2}\right)}{N_0}$ *in Lemma III.1, with* $Y_{dr} = y_{dr} - \sqrt{\xi_r} h_{dr} x_r$.

Proof. See Appendix C.

IV. Results and Discussion

The simulation setup is similar to the one in [11]. Conventional ML refers to the decision rule in [9] while optimum ML refers to the decision rules derived in this paper. For simplicity, the transmit powers of the interferers is kept constant. Also, the location of the interference node relative to the relay is equal to location of the interference node relative to the destination. The cooperating links are assumed to experience similar fading. Similar assumption is made for the interfering links as well, though their fading figures are taken to be less than those of the cooperating links. All these assumptions have been made for generating simulation results, but the decision rule is valid for different fading parameters and node locations.

Fig. 3 shows that the BER performance of optimum ML outperforms conventional ML, justifying the effort involved in designing a new receiver. The performance of the optimum receiver for varying interference levels is shown in Fig. 4. In Fig. 5, the BER is plotted for increasing fading figures on the cooperative links. The fading figures of the interfering links are kept constant. The results indicate that relay diversity is preserved by optimum ML in the presence of interference. This observation, however, needs to be substantiated theoretically.

V. Conclusion

In this paper, we have shown that for a DF cooperative system, the performance of conventional ML deteriorates in the presence of interference. To mitigate the effect of interference, an optimal ML decision rule was obtained for Nakagami-m fading. Through simulation results, it was shown that optimum ML outperforms conventional ML. Also, results indicate that relay diversity is preserved by optimal ML despite interference. While the usefulness of optimum ML decision has been established, simpler suboptimal receivers with similar performance need to be designed for practical systems.

APPENDIX A

Substituting from (2),

$$E_X\left[e^{2bX-aX^2}\right] = \frac{2m^m}{\Gamma(m)\Omega^m} \times \int_0^\infty e^{2bx-ax^2} x^{2m-1} e^{-\frac{m}{\Omega}x^2} dx \quad (7)$$

Letting

$$k = \frac{b}{\sqrt{\frac{m}{\Omega}+a}}, t = \frac{bx}{k} - k,$$

and completing the squares with appropriate substitution, (7) can be expressed as

$$\frac{2m^m}{\Gamma(m)\Omega^m}\frac{e^{k^2}}{(\frac{m}{\Omega}+a)^m}\int_{-k}^{\infty}(t+k)^{2m-1}e^{-t^2}dt$$
$$= \frac{2m^m}{\Gamma(m)}\frac{e^{k^2}}{(m+a\Omega)^m}\sum_{r=0}^{2m-1}\binom{2m-1}{r}k^{2m-r-1} \times \int_{-k}^{\infty} t^r e^{-t^2} dt$$

Let

$$\mathcal{I} = \int_{-k}^{\infty} t^r e^{-t^2} dt = \int_{-k}^{k} t^r e^{-t^2} dt + \int_{k}^{\infty} t^r e^{-t^2} dt$$

For $k > 0, r$ even,

$$I = 2\int_0^k t^r e^{-t^2} dt + \int_k^\infty t^r e^{-t^2} dt = \int_0^\infty t^{\frac{r-1}{2}} e^{-t}\, dt - \frac{1}{2}\int_{k^2}^\infty t^{\frac{r-1}{2}} e^{-t}\, dt = \Gamma\left(\frac{r+1}{2}\right) - \frac{1}{2}\Gamma\left(\frac{r+1}{2}, k^2\right) \quad (8)$$

For $k > 0, r$ odd,

$$\mathcal{I} = \int_k^\infty t^r e^{-t^2} dt = \frac{1}{2}\Gamma\left(\frac{r+1}{2}, k^2\right) \quad (9)$$

Combining the expressions for r even and odd in (8) and (9),

$$\mathcal{I} = \frac{1}{2}\Gamma\left(\frac{r+1}{2}\right) + \frac{(-1)^r}{2}\left[\Gamma\left(\frac{r+1}{2}\right) - \Gamma\left(\frac{r+1}{2}, k^2\right)\right] \quad (10)$$

For $k < 0$,

$$\int_{-k}^\infty t^r e^{-t^2} dt = \int_{k^2}^\infty \sqrt{t}^r e^{-t}\frac{dt}{2\sqrt{t}} = \frac{1}{2}\Gamma\left(\frac{r+1}{2}, k^2\right) \quad (11)$$

From (10) and (11), we obtain (3).

APPENDIX B

From (1),

$$p_{y_{rs}|x_s,h_{rs},h_{ri}} = \frac{1}{\sqrt{\pi N_0}}e^{-\frac{(y_{rs}-\sqrt{\xi_s}h_{rs}x_s-\sqrt{\xi_i}h_{ri}x_i^{(1)})^2}{N_0}} = \frac{1}{\sqrt{\pi N_0}}e^{-\frac{Y_{rs}^2}{N_0}}e^{\frac{2\sqrt{\xi_i}x_i^{(1)}Y_{rs}h_{ri}-\xi_i\left(x_i^{(1)}\right)^2h_{ri}^2}{N_0}}$$

Unconditioning over h_{ri} and $x_i^{(1)}$,

$$p_{y_{rs}}(y_{rs}|x_s,h_{rs}) = E_{x_i^{(1)},h_{ri}}\left[p_{y_{rs}}(y_{rs}|x_s,h_{rs},x_i^{(1)})\right] = \sum_{x_i^{(1)}} p_{y_{rs}}(y_{rs}|x_s,h_{rs},x_i^{(1)})P(x_i^{(1)}) \quad (12)$$

where

$$p_{y_{rs}}(y_{rs}|x_s,h_{rs},x_i^{(1)}) = \frac{1}{\sqrt{\pi N_0}}e^{-\frac{Y_{rs}^2}{N_0}}E_{h_{ri}}\left[e^{\frac{2\left\{\sqrt{\xi_i}x_i^{(1)}Y_{rs}\right\}h_{ri}-\left\{\xi_i\left(x_i^{(1)}\right)^2\right\}h_{ri}^2}{N_0}}\right] \quad (13)$$

Using Lemma III.1 in (13), (12) can be expressed as (5).

APPENDIX C

The decision for x_s at the destination is obtained by maximizing

$$p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr}) = E_{x_i^{(2)}}\left[p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr},x_i^{(2)})\right] = \sum_{x_i^{(2)}} p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr},x_i^{(2)})P(x_i^{(2)}) \quad (14)$$

In (14),

$$p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr},x_i^{(2)}) = E_{h_{di}}\left[p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr},x_i^{(2)},h_{di})\right]$$

Assuming all fading coefficients to be mutually independent, we have

$$p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr},x_i^{(2)},h_{di}) = p(y_{ds}|x_s,h_{ds})p(y_{dr}|x_s,h_{dr},x_i^{(2)},h_{di}) \quad (15)$$

Unconditioning with respect to h_{di}, (15) can be expressed as

$$p(y_{ds},y_{dr}|x_s,h_{ds},h_{dr},x_i^{(2)}) = p(y_{ds}|x_s,h_{ds})E_{h_{di}}\left[p(y_{dr}|x_s,h_{dr},x_i^{(2)},h_{di})\right]$$

where

$$p(y_{ds}|x_s,h_{ds}) = \frac{1}{\sqrt{\pi N_0}}e^{-\frac{(y_{ds}-\sqrt{\xi_s}h_{ds}x_s)^2}{N_0}}$$

Since,

$$
\begin{aligned}
&p(y_{dr}|x_s h_{dr}, x_i^{(2)}, h_{di}) \\
&\quad = \sum_{x_r} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y_{dr}-\sqrt{\xi_r}h_{dr}x_r-\sqrt{\xi_i}h_{d,i}x_i^{(2)})^2}{N_0}} P(x_r|x_s) \\
&\quad = \frac{1}{\sqrt{\pi N_0}} \sum_{x_r} e^{-\frac{Y_{dr}^2}{N_0}} \\
&\qquad \times e^{\frac{2\sqrt{\xi_i}h_{di}\left(x_i^{(2)}Y_{dr}\right)-\xi_i\left(x_i^{(2)2}\right)h_{di}^2}{N_0}} P(x_r|x_s),
\end{aligned}
$$

where $Y_{dr} = y_{dr} - \sqrt{\xi_r}h_{dr}x_r$, $E_{h_{di}}\left[p(y_{dr}|x_s, h_{dr}, x_i^{(2)}, h_{di})\right]$ is evaluated from Lemma III.1 by substituting $b = \frac{\sqrt{\xi_i}\left(x_i^{(2)}Y_{dr}\right)}{N_0}$ and $a = \frac{\xi_i\left(x_i^{(2)2}\right)}{N_0}$

REFERENCES

[1] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Max-min relay selection for legacy amplify-and-forward systems with interference," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3016–3027, June 2009.

[2] H. Suraweera, H. Garg, and A. Nallanathan, "Performance analysis of two hop amplify-and-forward systems with interference at the relay," *IEEE Commun. Lett.*, vol. 14, no. 8, pp. 692–694, August 2010.

[3] J.-B. Kim and D. Kim, "Exact and closed-form outage probability of opportunistic decode-and-forward relaying with unequal-power interferers," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3601–3606, December 2010.

[4] S.-I. Kim, S. Kim, J. Kim, and J. Heo, "Opportunistic decode-and-forward relaying with interferences at relays," *Wireless Personal Communications*, vol. 68, no. 2, pp. 247–264, 2013.

[5] H. Yu, I.-H. Lee, and G. Stuber, "Outage probability of decode-and-forward cooperative relaying systems with co-channel interference," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 266–274, January 2012.

[6] D. Benevides da Costa, H. Ding, and J. Ge, "Interference-limited relaying transmissions in dual-hop cooperative networks over nakagami-m fading," *IEEE Commun. Lett.*, vol. 15, no. 5, pp. 503–505, May 2011.

[7] M. Bhatnagar and A. Hjorungnes, "Ml decoder for decode-and-forward based cooperative communication system," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4080–4090, December 2011.

[8] M. Ju and I.-M. Kim, "Ml performance analysis of the decode-and-forward protocol in cooperative diversity networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3855–3867, July 2009.

[9] D. Chen and J. N. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, p. 17851794, July 2006.

[10] G. E. Andrews, R. Askey, and R. Roy, *Special Functions*, 1st ed. Cambridge University Press, 1999.

[11] G. V. V. Sharma, U. B. Desai, and S. N. Merchant, "Conditionally Gaussian Distributions and their Application in the Performance of Maximum Likelihood Decode and Forward Cooperative Systems," *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, p. 2732, january 2011.