Neutrinoless double Beta-decay in See-saw Mechanism

Supriya Senapati

A Thesis Submitted to
Indian Institute of Technology Hyderabad
In Partial Fulfillment of the Requirements for
The Degree of Master of Science

Department of Physics

April 2015

Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.

Supriya Senapati

(Signature)

Supriya Senapati

(Supriya Senapati)

PH13M1015

(Roll No.)

Approval Sheet

This thesis entitled " **Neutrinoless Double Beta Decay in See-saw Mechanism**" by Supriya Senapati is approved for the degree of Master of Science from IIT Hyderabad.

Dr. Prem Pal
Coordinator/Examiner

Dr. Narendra Sahu
Advisor/Examiner

06·05·15
Dr. J.Suryanarayana
Examiner

Dr. J.Mohanty
Examiner

Vandana 05/05/15
Dr. Vandana Sharma
Examiner

Dr. S.Hundi
Examiner

Acknowledgements

I would like to thank my guide **Dr. Narendra Sahu** for his guidance and constant supervision as well as for providing necessary information regarding the project and also for his support in completing the project .

I would also like to thank Chayan Majumdar for supporting me as a project partner and for many useful discussion, Paswa Nath for helping me in programming , Animesh Adhikari for helping me in LaTeX, Arunima Bhattacharya for supporting me.

Abstract

In the standard model the total lepton number is conserved.Thus the neutrinoless double beta decay ,in which lepton number violated by 2 units is a probe of Physics beyond the Standard Model.Here we first discuss about the Standard Model and then we discuss about the Seesaw mechanism to generate the small Majorana mass of the neutrinos , in which the lepton number is violated by 2 units. After a brief summary we discuss then about the neutrino masses and mixing i.e.Cabbibo-Kobayashi-Maskawa (CKM) matrix and PMNS matrix, normal and inverted hierarchy , the absolute scale of neutrino masses ,effective Majorana mass.After brief discussion we plot the graphs for normal and inverted hierarchy and find out the range of the normal and inverted spectrum and we discuss a little bit about the Majorana phases and we plot the graphs between two Majorana phases.In the next portion we discuss about the theory of neutrinoless double beta decay.In this case at first we calculate the matrix element, then the decay rate of the neutrinoless double beta decay and finally we calculate the half life-time of this process. Then we plot the half life-time vs lightest mass graphs for normal and inverted hierarchy and make some conclusion.

Contents

Chapter 1

The Standard Model and Neutrino Masses

1.1 Introduction

We present here the basics of the Standard Model of the electroweak interaction and we make some remarks on the perspectives for the discovery of the Higgs boson, the most important challenge of the Standard Model.The electroweak interaction is described by gauge theory based on the $SU(2)_L \otimes U(1)_Y$ group which is spontaneously broken through the Higgs mechanism.The model proposed by Glashow, Salam and Weinberg in the middle sixties, has been extensively tested during the last 30 years. The discovery of neutral weak interactions and the production of intermediate vector bosons ($W^{\pm}$ and Z^0) with the expected properties increased our confidence in the model.The matter fields - leptons and quarks are organized in families, with the left-handed fermions belonging to weak isodoublets while the right-handed components transform as weak isosinglets. The vector bosons, $W^{\pm}$, Z^0 and γ, that mediate the interactions are introduced via minimal coupling to the matter fields. An essential ingredient of the model is the scalar potential that is added to the Lagrangian to generate the vectorboson (and fermion) masses in a gauge invariant way, via the Higgs mechanism. In 1957, Schwinger suggested a model based on the group $O(3)$ with a triplet gauge fields (V^+, V^-, V^0). The charged gauge bosons were associated to weak bosons and the neutral was identified with the photon. This model was proposed before the structure V - A of the weak currents have been established. Glashow, in 1961 suggested the gauge group $SU(2) \otimes U(1)$, where $U(1)$ was associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge by the **Gell-MannNishijima formula** $Q = (T_3 + Y/2)$.The theory now requires four gauge bosons: a triplet (W^1,W^2,W^3) associated to the generators of SU(2) and a neutral field (B) related to U(1).The charged weak bosons are the linear combination of W^1 and W^2, while the photon and a neutral weak boson Z^0 are both given by a mixture of W^3 and B. The **Glashow-Weinberg-Salam** model is known as the Standard Model of Electroweak Interactions.

1.2 Right and Left Handed Fermions

The Dirac spinors at high energies (i.e. for $E \gg m$), $u(p,s)$ and $v(p,s) \equiv C\overline{u}^T(p,s) = i\gamma_2 u^*(p,s)$ are eigenstates of the γ_5 matrix.
The helicity projectors are,

$$L \equiv \frac{1}{2}(1-\gamma_5)$$
$$R \equiv \frac{1}{2}(1+\gamma_5)$$

which satisfy the properties of projection operators given below,

$$L+R=1$$
$$LR+RL=0$$
$$L^2=L$$
$$R^2=R$$

We have the conjugate spinors ,

$$\overline{\psi_L} = \overline{\psi}R$$
$$\overline{\psi_R} = \overline{\psi}L$$

The fermion mass term contains both the right and lefthanded fermion components,

$$\overline{\psi}\psi = \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L$$

But on the other hand , the vector current, does not mix those components, i.e.

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi_R}\gamma^\mu\psi_R + \overline{\psi_L}\gamma^\mu\psi_L$$

So,we can write the $(V-A)$ fermionic weak current as,

$$\overline{\psi_L}\gamma^\mu\psi_L = \frac{1}{2}\overline{\psi}\gamma^\mu(1-\gamma_5)\psi$$

From here we can see that only lefthanded fermions play a role in weak interactions.

1.3 Choosing the Gauge Group

We have to find the gauge group which would be able to unify the electromagnetic and weak interactions.

Consider the charged weak current for leptons. Since electrontype and muontype lepton numbers are separately conserved, they must form separate representations of the gauge group.

Consider any lepton flavor ℓ ($\ell = e, \mu, \tau$) and the final Lagrangian will be given by a sum over all these flavors.

The weak current, for a lepton ℓ, is given by,

$$J_\mu^+ = 2\overline{\ell_L}\gamma_\mu\nu_L$$

Then we introduce the lefthanded isospin doublet ($T = \frac{1}{2}$),

$$L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \tag{1.1}$$

Since, there is no righthanded component for the neutrino, the righthanded part of the charged lepton is accommodated in a weak isospin singlet ($T = 0$)

$$R = \ell_R$$

The charged weak current is now given by,

$$J_{\mu^i} = \overline{L}\gamma_\mu\frac{\tau^i}{2}L$$

where τ^i are the Pauli matrices.

Explicitly we can write,

$$\begin{aligned} J_\mu^1 &= \tfrac{1}{2}(\overline{\ell_L}\gamma_\mu\nu_L + \overline{\nu}_L\gamma_\mu\ell_L) \\ J_\mu^2 &= \tfrac{i}{2}(\overline{\ell_L}\gamma_\mu\nu_L - \overline{\nu}_L\gamma_\mu\ell_L) \\ J_\mu^3 &= \tfrac{1}{2}(\overline{\nu_L}\gamma_\mu\nu_L - \overline{l_L}\gamma_\mu\ell_L) \end{aligned}$$

Therefore, the weak charged current that couples with intermediate vector boson W_μ^- can be written in terms of J_1 and J_2 as,

$$J_\mu^+ = 2(J_\mu^1 - iJ_\mu^2)$$

To accommodate the neutral current J_3, we can define the hypercharge current by,

$$J_\mu^Y = -(\overline{L}\gamma_\mu L + 2\overline{R}\gamma_\mu R)$$

The electromagnetic current can be written as,

$$J_\mu^{em} = J_\mu^3 + \tfrac{1}{2}J_\mu^3$$

now, the Gell-Mann-Nishijima relation between Q and T_3 ,

$$Q = T_3 + \frac{Y}{2}$$

So. we have the candidate for the gauge group given by,

$$SU(2)_L \otimes U(1)$$

Next we have to introduce the gauge fields corresponding to each generator,

$$SU(2)_L \rightarrow W^1_\mu, W^2_\mu, W^3_\mu$$
$$U(1)_Y \rightarrow B_\mu$$

The strength tensors for the gauge fields is defined as,

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

The free Lagrangian for the gauge fields is given by,

$$L_{gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

The free Lagrangian for the leptons is,

$$L_{leptons} = \bar{\ell} i \eth \ell + \bar{\nu} i \eth \nu$$

Now we introduce the fermiongauge boson coupling via covariant derivative, i.e.

$$L : \partial_\mu + i\frac{g}{2}\tau^i W^i_\mu + i\frac{g^{'}}{2} Y B_\mu$$
$$R : \partial_\mu + i\frac{g^{'}}{2} Y B_\mu$$

where g and $g^{'}$ are the coupling constant associated to the $SU(2)_L$ and $U(1)_Y$ groups respectively.

The charged leptons can be written as,

$$L^{\pm}_{leptons} = -\frac{g}{2\sqrt{2}} (\bar{\nu}\gamma^\mu(1-\gamma_5)\ell W^+_\mu + \bar{\ell}\gamma^\mu(1-\gamma_5)\nu W^-_\mu)$$

where

$$W^{\pm}_\mu = \frac{1}{2}(W^1_\mu \mp W^2_\mu)$$

For low energy phenomenology we obtain the relation,

$$\frac{g}{2\sqrt{2}} = \sqrt{\frac{M_w^2 G_F}{\sqrt{2}}}$$

Now consider a neutral piece of Leptons containing both left and right fermion components,

$$L^0_{leptons} = -gJ_3^\mu W_\mu^3 - \frac{g'}{2} J_Y^\mu B_\mu$$

The charges respect the Gell-MannNishijima relation and currents satisfy the relation given by,

$$J_{em} = J_3 + \frac{J_Y}{2}$$

For obtaining the right combination of fields which couples to the electromagnetic current, we make a rotation of the neutral fields and define the new fields A and Z by,

$$\begin{pmatrix} A_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} cos\theta_W & sin\theta_W \\ -sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \tag{1.2}$$

where θ_W is called Weinberg angle.
And we have the relation,

$$sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$
$$cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

In terms of the new fields, the neutral leptons can be written as,

$$L^0_{leptons} = -gsin\theta_W(\bar{\ell}\gamma^\mu \ell)A_\mu - \frac{g}{2cos\theta_W}\Sigma\overline{\psi_i}\gamma^\mu(g_V^i - g_A^i\gamma_5)\psi_i Z_\mu$$

The electromagnetic charge,

$$e = gsin\theta_W = g' cos\theta_W$$

And we have

$$g_V^i = T_3^i - 2Q_i sin\theta_W^2$$
$$g_A^i = T_3^i$$

Up to now we have in the theory:

4 massless gauge fields : $W_\mu^\pm$,Z_μ and A_μ
2 massless fermions : ν and ℓ

Now we have to add scalar fields to break the symmetry spontaneously and use the Higgs mechanism to give mass to the three vector bosons, but the photon will remain massless.

1.4 The Higgs Mechanism and The W and Z mass

For applying the Higgs mechanism to give mass to $W^\pm$ and Z^0, we introduce the scalar doublet,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1.3}$$

We introduce the Lagrangian given below,

$$L_{scalar} = \partial_\mu \phi \dagger \partial^\mu \phi - V \phi \dagger \phi$$

where the potential,

$$V(\phi \dagger \phi) = \mu^2 \phi \dagger \phi + \lambda(\phi \dagger \phi)^2$$

For gauge invariance under the $SU(2)_L \otimes U(1)_Y$, we should introduce the covariant derivative,

$$\partial_\mu \to D_\mu = \partial_\mu + i\frac{g}{2}\tau^i W_\mu^i + i\frac{g^{'}}{2} Y B_\mu$$

We can choose the vacuum expectation value of the Higgs field as,

$$< \phi >_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{1.4}$$

where,

$$v = \sqrt{\frac{-\mu^2}{\lambda}}$$

To maintain the electric charged conserved, we must break the original symmetry group as,

$$SU(2)_L \otimes U(1)_Y \to U(1)_{em}$$

In this case the corresponding gauge boson, the photon will remain massless.Let the vacuum invariant under $U(1)_{em}$. This invariance requires that,

$$e^{i\alpha Q} < \phi >_0 = < \phi >_0$$

The electric charge of the vacuum is zero,

$$Q < \phi >_0 = 0$$

The gauge bosons, corresponding to the broken generators T_1 , T_2 , $(2T_3 - Q)$ should acquire mass.

Now parametrize the Higgs doublet,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\omega^+ \\ v + H - iZ^0 \end{pmatrix} \tag{1.5}$$

where $\omega^\pm$ and Z^0 are the goldstone bosons.

Now, if we make a $SU(2)_L$ gauge transformation then the field becomes ,

$$\phi \to \phi' = \frac{v + H}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.6}$$

The quadratic terms in the vector fields are,

$$\frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{g^2 v^2}{8(cos\theta_W)^2} Z_\mu Z^\mu$$

When it is compared with the usual mass terms for a charged and neutral vector bosons , then we get ,

$$M^2_W W^+_\mu W^{-\mu} + \tfrac{1}{2} M^2_Z Z_\mu Z^\mu$$

and also we can have ,

$$M_W = \frac{gv}{2}$$
$$M_Z = \frac{gv}{2cos\theta_W}$$

For lowenergy phenomenology , we obtain the vacuum expectation value,

$$v = 246Gev$$

The term containing the scalar field H given by,

$$\tfrac{-1}{2}(-2\mu^2)H^2 + \tfrac{1}{4}\mu^2 v^2(\tfrac{4}{v^2}H^3 + \frac{H^4}{v^4} - 1)$$

The Higgs boson mass term is given by,

$$M_H = \sqrt{-2\mu^2}$$

In spite of predicting the existence of the Higgs boson, the Standard Model does not give a hint on the value of its mass since μ^2 is a priori unknown.

1.5 Introducing the Quarks

For introducing the strong interacting particles in the Standard Model we shall first examine what happens with the hadronic neutral current when the Cabibbo angle is taken into account.

$$J^H_\mu(0) = \overline{u}\gamma_\mu(1-\gamma_5)u + cos\theta_c^2\overline{d}\gamma_\mu(1-\gamma_5)d + sin\theta_c^2\overline{s}\gamma_\mu(1-\gamma_5)s + cos\theta_c sin\theta_c(\overline{d}\gamma_\mu(1-\gamma_5)s + \overline{s}\gamma_\mu(1-\gamma_5)d)$$

The Lagrangian for the quarks given by,

$$L_{quarks} = \overline{L_u}i\eth L_u + \overline{L_c}i\eth L_c + \overline{R_u}i\eth R_u + \cdots + \overline{R_c}i\eth R_c$$

The charged weak couplings quarkgauge bosons is,

$$L^{\pm}_{quarks} = \frac{g}{2\sqrt{2}}(\overline{u}\gamma^\mu(1-\gamma_5)d' + \overline{c}\gamma^\mu(1-\gamma_5)s')W^+_\mu + h.c.$$

The neutral current interaction of the quarks becomes,

$$L^0_{quarks} = -\frac{g}{2cos\theta_W}\Sigma\overline{\psi_q}\gamma^\mu(g^q_V - g^q_A\gamma_5)\psi_q Z_\mu$$

1.5.1 The Quark Masses

For generating the mass for both the up ($U_i = u, c, t$) and down ($D_i = d, s, b$) quarks, we need a $Y = -1$ Higgs doublet.The conjugate doublet Higgs can be defined as,

$$\overbrace{\phi} = i\sigma_2\phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \quad (1.7)$$

The Yukawa Lagrangian for three generations of quarks can be written as,

$$L^{q}_{quarks} = -\Sigma G^{U}_{ij}\overline{R_{U_i}}(\overbrace{\phi\dagger} L_j) + G^{D}_{ij}\overline{R_{D_i}}(\phi \dagger L_j)$$

The weak eigenstates ($q^{'}$) are the linear superposition of the mass eigenstates (q) given by the unitary transformations:

$$\begin{pmatrix} u^{'} \\ c^{'} \\ t^{'} \end{pmatrix} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix} \tag{1.8}$$

$$\begin{pmatrix} d^{'} \\ s^{'} \\ b^{'} \end{pmatrix} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.9}$$

where $U_{L,R}$ and $D_{L,R}$ are unitary matrices.These matrices diagonalize the mass matrices.

The quark mixing, by convention, is restricted to the down quarks,with $T^{q}_{3} = -\frac{1}{2}$,

$$\begin{pmatrix} d^{'} \\ s^{'} \\ b^{'} \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.10}$$

V is the Cabibbo-Kobayashi-Maskawa matrix.It can be parametrized as,

$$V = R_1(\theta_{23})R_2(\theta_{13}, \delta_{13})R_3(\theta_{12})$$

where $R_i(\theta_{ij})$ are rotation matrices around the axis i, the angle θ_{ij} describes the mixing of the generations j and k and δ_{13} is the phase.

The Cabibbo-Kobayashi-Maskawa matrix can be written as,

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{1.11}$$

where

$$S_{ij}(c_{ij}) = sin(cos)\theta_{ij}$$

Taking into account the limit $\theta_{23} = \theta_{13} \to 0$ we have $\theta_{12} \to \theta_c$ and we obtain

$$V = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ o & o & 1 \end{pmatrix} \tag{1.12}$$

1.6 The Standard Model Lagrangian

The standard (Weinberg-Salam) model lagrangian is given by ,

$$L = L_1 + L_2 + L_3 + L_4$$

where,

$$L_1 = -\tfrac{1}{4} W_{\mu\nu} W^{\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$L_2 = \overline{L}\gamma^\mu (i\partial_\mu - \frac{g}{2}\tau W_\mu - \frac{g^{'}}{2} Y B_\mu) L + \overline{R}\gamma^\mu (i\partial_\mu - \tfrac{g^{'}}{2} Y B_\mu) R$$
$$L_3 = (i\partial_\mu - \frac{g}{2}\tau W_\mu - \frac{g^{'}}{2} Y B_\mu)^2 - V(\phi)$$
$$L_4 = -G_1 \overline{L}\phi R + G_2 \overline{L}\phi_c R + h.c.$$

L_1 is the lagrangian for the $W^\pm$, Z,γ kinetic energy and self interaction term .
L_2 is for the leptons and quarks kinetic energy and their interaction with $W^\pm$,Z ,γ .
L_3 is for higgs masses and coupling.
L_4 is for lepton and quark masses and coupling to Higgs.

1.7 Beyond the Standard Model

Physics beyond the Standard Model refers to the theoretical evidence that needs to explain the deficiencies of the Standard Model, like the origin of mass, the strongCP problem, neutrino oscillations, matter-antimatter asymmetry etc.

1.7.1 Problem with the Standard Model

There are fundamental physical phenomena in nature that the Standard Model does not properly explain:

1.**Neutrino masses:** According to the standard model, **neutrinos are massless particles**. But, neutrino oscillation experiments have shown that neutrinos do have mass. Mass terms for the neutrinos can be added to the standard model by hand, but these lead to new theoretical problems.

2.**Matter-antimatter asymmetry:** The Universe is made out of mostly matter. However, the standard model predicts that matter and antimatter should have been created in equal amounts if the initial conditions of the Universe did not involve disproportionate matter relative to antimatter. Yet, no mechanism sufficient to explain this asymmetry exists in the Standard Model.

3.**Hierarchy problem :** The standard model introduces particle masses through a process known as spontaneous symmetry breaking caused by the Higgs field. Within the standard model, the mass of the Higgs gets some very large quantum corrections due to the presence of virtual particles (mostly virtual top quarks). These corrections are much larger than the actual mass of the Higgs. This means that the bare mass parameter of the Higgs in the standard model must be fine tuned in such a way that almost completely cancels the quantum corrections. This level of fine-tuning is deemed unnatural by many theorists.There are also issues of Quantum triviality, which suggests that it may not be possible to create a consistent quantum field theory involving elementary scalar particles.

4.**Strong CP problem :** Theoretically it can be argued that the standard model should contain a term that breaks CP symmetry relating matter to antimatter in the strong interaction sector. Experimentally, however, no such violation has been found, implying that the coefficient of this term is very close to zero. This fine tuning is also considered unnatural.

1.7.2 Neutrinos

In the standard model, **neutrinos have exactly zero mass**. **This is a consequence of the standard model containing only left-handed neutrinos**. With no suitable right-handed partner, it is impossible to add a renormalizable mass term to the standard model. Measurements however indicated that neutrinos spontaneously change flavour, which implies that neutrinos have a mass. These measurements only give the relative masses of the different flavours.

One approach to add masses to the neutrinos, the so-called **seesaw mechanism**, is to add right-handed neutrinos and have these couple to left-handed neutrinos with a **Dirac mass term**. **The right-handed neutrinos have to be sterile**, meaning that they do not participate in any of the standard model interactions. Because they have no charges, the right-handed neutrinos can act as their own anti-particles, and have a **Majorana mass term**. Like the other Dirac masses in the standard model, the neutrino Dirac mass is expected to be generated through the Higgs mechanism, and is therefore unpredictable. On the other hand, the Majorana mass for the right-handed neutrinos does not arise from the Higgs mechanism, and is therefore expected to be tied to **some energy scale of new physics beyond the standard model, for example the Planck scale**. Therefore, any process involving right-handed neutrinos will be suppressed at low energies. The correction due to these suppressed processes effectively gives the left-handed neutrinos a mass that is inversely proportional to the right-handed Majorana mass, a mechanism known as the **see-saw**. The presence of heavy right-handed neutrinos thereby explains both the small mass of the left-handed neutrinos and the absence of the right-handed neutrinos in observations.

The mass terms mix neutrinos of different generations. This mixing is parameterized by the PMNS matrix, which is the neutrino analogue of the CKM quark mixing matrix. Unlike the quark mixing, which is almost minimal, the mixing of the neutrinos appears to be almost maximal.The mixing matrix could also contain several complex phases that break CP invariance, although there has been no experimental probe of these. These phases could potentially create a surplus of leptons over anti-leptons in the early universe, a process known as **leptogenesis**.

Dirac Masses

The Dirac mass terms in the Standard Model are obtained from the Yukawa couplings when the Higgs field gets a vev, i.e. after a spontaneous symmetry breaking.
The Dirac mass term is given by,

$$M\overline{\psi}\psi = M(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

It is invariant under $U(1)_{em}$ symmetry. The lepton no. is conserved here.The Dirac mass term destroys a particle and creates a new one, or destroys/creates a particle-antiparticle pair, or destroys an anti-particle and creates a new one. This conserves the charge.

Majorana Masses

The Majorana mass for the right-handed neutrinos does not arise from the Higgs mechanism, and is therefore expected to be tied to **some energy scale of new physics beyond the standard model, for example the Planck scale**.
The Majorana mass term is given by ,

$$M(\overline{(\psi_L)^c}\psi_L + \overline{(\psi_R)^c}\psi_R)$$

It is not invariant under $U(1)$ symmetry. The lepton no. is not conserved here.The charge conservation would be violated by this term.

1.8 Constraint from Planck Data

Since light massive neutrinos constitute hot dark matter, cosmological data give information on the sum of neutrino masses. The analysis of cosmological data in the framework of the standard Cold Dark Matter model with a cosmological constant disfavors neutrino masses larger than some fraction of eV, because free streaming neutrinos suppress small-scale clustering. The value of the upper bound on the sum of neutrino masses depends on model assumptions and on the considered data set.

Assuming a spatially flat universe, from the recent result of the Planck experiment the upper bound is found to be $\Sigma m_k = 0.23eV$.

Chapter 2

Mechanism for generating Neutrino Masses

2.1 Introduction

According to the standard model, neutrinos are massless particles. However, neutrino oscillation experiments have shown that neutrinos do have mass.For generating the neutrino masses we use some mechanism called SEESAW mechanism.The seesaw mechanism is a generic model used to understand the relative sizes of observed neutrino masses, of the order of eV, compared to those of quarks and charged leptons, which are millions of times heavier.This mechanism serves to explain why the neutrino masses are so small.This model produces a light neutrino, for each of the three known neutrino flavours, and a corresponding very heavy neutrino for each flavour, which has yet to be observed.

Here we discuss about the basic principle of See-saw mechanism and the three type of See-saw mechanism.

2.2 Dimension 5 Operator

In the SM, the lowest-dimension operator that violates lepton or baryon number is dimension 5 operator. So, we can write,

$$L_{eff} = L_{SM} + \frac{L_{d=5}}{\Lambda} + \frac{L_{d=6}}{\Lambda^2} + \cdots$$

The dimension 5 operator can be written as,

$$O_5 = \frac{LLHH}{\Lambda} = \frac{(L^T CL)(H^T CH)}{\Lambda}$$

Here lepton no is violated by 2 units.

2.3 Seesaw Mechanism

The principle of the seesaw mechanism can be understood by looking at the neutrino mass matrix.

One has to assume that besides the usual left handed (LH) neutrinos ν_L, there are right handed (RH) neutrinos ν_R, which are not strictly forbidden by the SM.

Therefore one can construct a Dirac mass term for neutrinos ,

$$L_D = M_D(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

and the Majorana mass term for neutrinos ,

$$L_M = \frac{1}{2}M_L(\overline{\psi_L}\psi_R^c + \overline{\psi_R^c}\psi_L) + \frac{1}{2}M_R(\overline{\psi_R}\psi_L^c + \overline{\psi_L^c}\psi_R)$$

Now one can introduce a mass matrix M, so that

$$L_{mass} = \frac{1}{2}(\overline{\nu_L}\overline{\nu_L^c})M \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} \tag{2.1}$$

where,

$$M = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix} \tag{2.2}$$

The positive mass eigenstates for this matrix are ,

$$M_{1,2} = \frac{1}{2}(M_L + M_R \pm \sqrt{(M_L - M_R)^2 + 4M_D^2})$$

In the seesaw case the RH neutrino fields $\nu_R = N_R$ are assumed to be fields with a heavy mass, whereas M_D is of the electroweak scale. Therefore $M_D << M_R$. Since ν_L possesses non-zero isospin and hypercharge, the LH Majorana term is forbidden by the SM gauge symmetries, hence $M_L = 0$. This means in a fundamental theory that respects the SM symmetries one obtains the mass eigenstates,

$$M_1 = -\frac{M_D^2}{M_R}$$
$$M_2 = M_R$$

As a consequence, one has a neutrino at a mass scale $\Lambda = M_R$ of new physics and a very light neutrino, the mass of which is suppressed by $\frac{M_D}{\Lambda}$. To explain the low experimental upper limit for the neutrino mass, the new mass scale has to be close to the GUT scale.

2.3.1 Rise of the Masses

Dirac mass M_D: around electroweak scale ($100GeV$)
Majorana mass M_R: expected in the order of GUT scale ($10^{15}GeV$)

$$M_1 = 0.01eV = M_{\nu_L} \text{ (extremely light neutrino)}$$
$$M_2 = 10^{15}GeV = M_{\nu_R} \text{ (very heavy neutrino)}$$

With a fixed M_D,we can obtain a heavy neutrino M_1, a light neutrino M_2 that is why this mechanism is called Seesaw mechanism.

2.4 Scale of Seesaw Mechanism

For Seesaw mechanism we have mass of neutrino,

$$m_\nu \rightarrow \frac{1}{\Lambda}$$

where Λ is called the scale of Seesaw mechanism and it has mass dimension 1 .

2.5 The Three Type of Seesaw Mechanism

Exchange of 3 different types of new particles can generate neutrino masses.

Right-handed neutrinos → type I see-saw

Scalar $SU(2)_L$ triplets → type II see-saw

Fermion triplets → type III see-saw

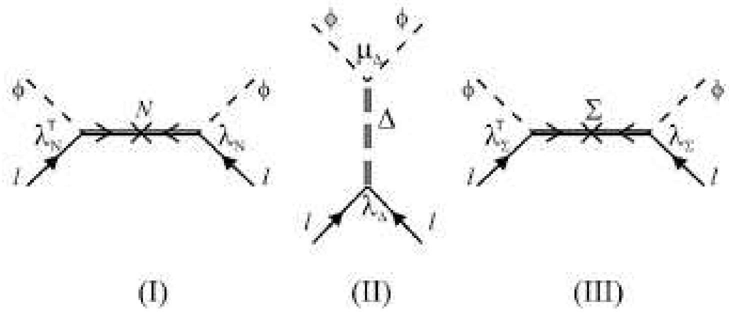

Figure 2.1: Three types of see-saw mechanism

Type I : The first type of the seesaw mechanism couples the lepton and the Higgs fields via the exchange of a heavy virtual fermion N_R, which is a singlet under all SM gauge groups.

Type II : By replacing the fermion $SU(2)$ singlet by a scalar $SU(2)$ triplet Δ, one obtains the second type of the seesaw mechanism.

Type III : The third type is nearly the same as the first one, except for the replacement of the fermion $SU(2)$ singlet by a fermion $SU(2)$ triplet.

In the following we will discuss the structure of these three realizations of the seesaw mechanism.

2.5.1 Type I Seesaw Mechanism(Fermion Singlet) :

In type I the left-handed lepton fields couple to the right handed heavy fermion singlet fields. They also have to couple to the Higgs field, which gives the neutrinos their mass after electroweak symmetry breaking.

The diagrammatic representation is given here,

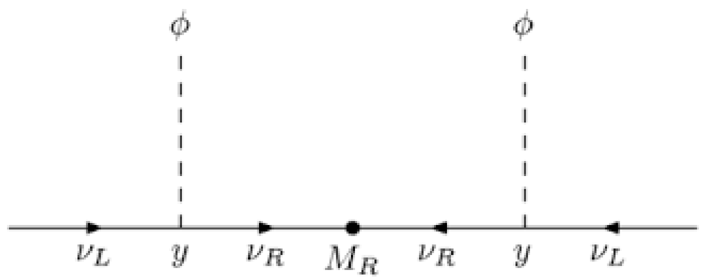

Figure 2.2: Type I see-saw mechanism

The lagrangian is then given by,

$$\Delta L = y\overline{l_L}\sigma_2\phi^*\nu_R + \frac{M_R}{2}\nu_R^T c\nu_R + h.c.$$

From here we can get a mass matrix ,

$$M = \begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix} \tag{2.3}$$

If $M_R << M_D$, neutrinos would be predominantly Dirac particles.
For$M_R \cong M_D$, we have a messy combination of Majorana and Dirac,

whereas for $M_D << M_R$ we would have a predominantly Majorana case.
In this case the approximate eigenstates are N with mass $M_N = M_D$ and ν with a tiny mass ,

$$M_\nu = -M_D^T \frac{1}{M_N} M_D$$

This is called type I seeesaw formula. With heavy ν_R, neutrino mass must be of this type.

2.5.2 Type II Seesaw Mechanism (Scalar Triplet) :

Instead of ν_R a $Y = 2$ triplet Δ we add here.
where,

$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix} \tag{2.4}$$

The diagrammatic representation is given here,

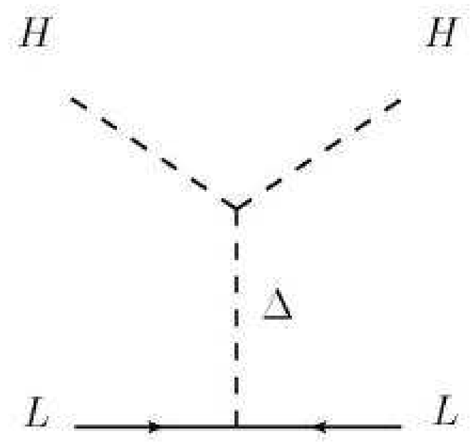

Figure 2.3: Type II see-saw mechanism

The lagrangian is give by ,

$$L = L_{SM} + V(\Delta, H)$$

where,

$$V(\Delta, H) =$$
$$M_\Delta^2 \Delta \dagger \Delta + \lambda_\Delta(\Delta \dagger \Delta)^2 + f\overline{L^c}\Delta L + \mu\overline{H^c}\Delta H + M_H^2 H \dagger H + \lambda_H(H \dagger H)^2 + \lambda(\Delta, H)(\Delta \dagger \Delta)(H \dagger H)$$

We have to minimize the potential $V(\Delta, H)$ at $< H >= v$ and $< \Delta >= u$

So we have now,

$$V = M_\Delta^2 u^2 + \lambda_\Delta u^4 + \mu u v^2 + M_H^2 v^2 + \lambda_H v^4 + \lambda(\Delta, H) u^2 v^2$$

Now by minimizing we get ,

$$u =< \Delta > \simeq -\frac{\mu v^2}{M_\Delta^2}$$

where one expects μ of order M_Δ. If $M_\Delta >> v$, neutrinos are naturally light. So,for large scales of new physics, neutrino mass must come from $d = 5$ operator.
From this type of seesaw mechanism we can get the mass of the neutrino is,

$$M_\nu = \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Also we can obtain,

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \tag{2.5}$$

2.5.3 Type III Seesaw Mechanism (Fermion Triplet) :

The type III seesaw mechanism is obtained by replacing the fermion singlet of type I with a triplet with $Y = 0$ Σ
where,

$$\Sigma = \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \tag{2.6}$$

The diagrammatic representation is given here,

The lagrangian is given by ,

$$L = Y_\Sigma \overline{L^c} i\tau_2 \Sigma H + h.c.$$

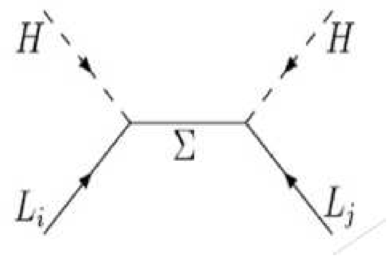

Figure 2.4: Type III see-saw mechanism

where,

$$\Sigma = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} \end{pmatrix} \tag{2.7}$$

where,

$$\Sigma^{\pm} = \frac{\Sigma^1 - i\Sigma^2}{\sqrt{2}}$$
$$\Sigma^0 = \Sigma^3$$

In exactly the same manner as before in Type I, one gets a Type III seesaw for $M_\Sigma >> v$

$$M_\nu = -Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

2.6 Consequences

If neutrinos are Majorana particles and if the see-saw machanism is the correct theory then the smallness of the neutrino mass has an explanation and the transition of neutrino into corresponding anti-neutrino is possible and therefore the lepton number is violated by $\Delta L = 2$.
For testing if neutrino is a Majorana particle and whether see-saw mechanism could be the right theory we have to perform the double beta decay experiment and neutrinoless duble beta decay experiment which is a probe of Physics beyond the Standard Model.

Chapter 3

Neutrinoless Double Beta-Decay

3.1 Introduction

Neutrinoless double beta decay is a postulated very slow radioactive process in which two neutrons inside a nucleus transform into two protons emitting two electrons.The discovery of this process would demonstrate that neutrinos are majorana particles and that total lepton number is not violated in nature two findings with far-reaching implications in particle physics and cosmology.First,the existence of Majorana neutrinos implies a new energy scale at a level inversely proportional to the observed neutrino masses. Such a scale, besides providing a simple explanation for the striking lightness of neutrino masses, is probably connected to several open questions in particle physics, like the origin of mass or the flavour problem. Second, Majorana neutrinos violate the conservation of lepton number, and this, together with CP violation, could be responsible, through the mechanism known as leptogenesis, for the observed cosmological asymmetry between matter and antimatter.

Here we discuss about the normal and inverted hierarchy ,effective Majorana mass and the theory of neutrinoless double beta decay.

3.2 Normal and Inverted Hierarchy

In the framework of three neutrino mixing with the convention the solar squared mass difference

$$\Delta m_s^2 = \Delta m_{12}^2$$

and mixing angle

$$\theta_s = \theta_{12}$$

and taking into account that

$$\Delta m_s^2 << \Delta m_A^2$$

where Δm^2_A is the atmospheric squared mass difference

we have

$$\Delta m^2_A = \frac{1}{2}|\Delta m^2_{13} + \Delta m^2_{23}|$$

and mixing angle $\theta_A = \theta_{23}$

The absolute value in the definition of Δm^2_A is necessary, because there are the two possible spectra for the neutrino masses.

The normal mass spectrum (NH) :

$$m_1 < m_2 < m_3 \text{ with } \Delta m^2_{12} << \Delta m^2_{23}$$

The inverted mass spectra (IH) :

$$m_3 < m_1 < m_2 \text{ with } \Delta m^2_{12} << |\Delta m^2_{13}|$$

The two spectra differ by the sign of Δm^2_{13} and Δm^2_{23} , which is positive in the normal spectrum and negative in the inverted spectrum.

3.3 Absolute Scale of Neutrino Masses

The determination of the absolute scale of neutrino masses is an open problem which cannot be resolved by neutrino oscillations, that depend only on the differences of the squares of the neutrino masses. However, the measurement in neutrino oscillation experiments of the neutrino squared-mass differences allows us to constraint the allowed patterns of neutrino masses. A convenient way to see the allowed patterns of neutrino masses is to plot the values of the masses as functions of the unknown lightest mass m_{min}, , where we used the squared-mass differences given by 3σ range with

The normal mass spectrum (NH) :

$$m_{min} = m_1$$
$$m_2 = \sqrt{m^2_{min} + \Delta m^2_s}$$
$$m_3 = \sqrt{m^2_{min} + \Delta m^2_A + \frac{\Delta m^2_s}{2}}$$

The inverted mass spectra (IH) :

$$m_{min} = m_3$$
$$m_1 = \sqrt{m_{min}^2 + \Delta m_A^2 - \frac{\Delta m_s^2}{2}}$$
$$m_2 = \sqrt{m_{min}^2 + \Delta m_A^2 + \frac{\Delta m_s^2}{2}}$$

After plotting I got this graphs given below,

For normal hierarchy,

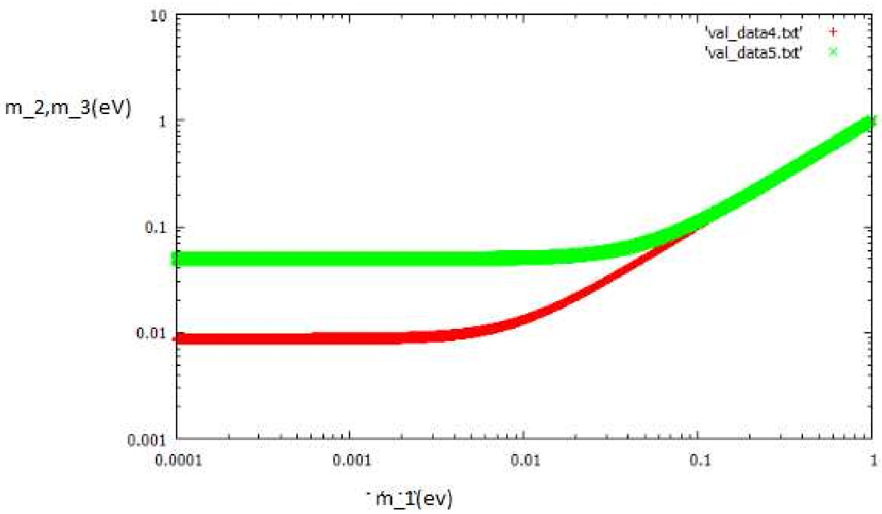

Figure 3.1: Normal Mass Spectrum

A normal hierarchy

In this case

$$m_1 << m_2 \simeq \sqrt{\Delta m_s^2} \simeq 9 \times 10^{-3} eV$$
$$m_3 \simeq \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2} eV$$

For inverted hierarchy ,

A inverted hierarchy

In this case

$$m_3 << m_1 \lesssim m_2 \simeq \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2} eV$$

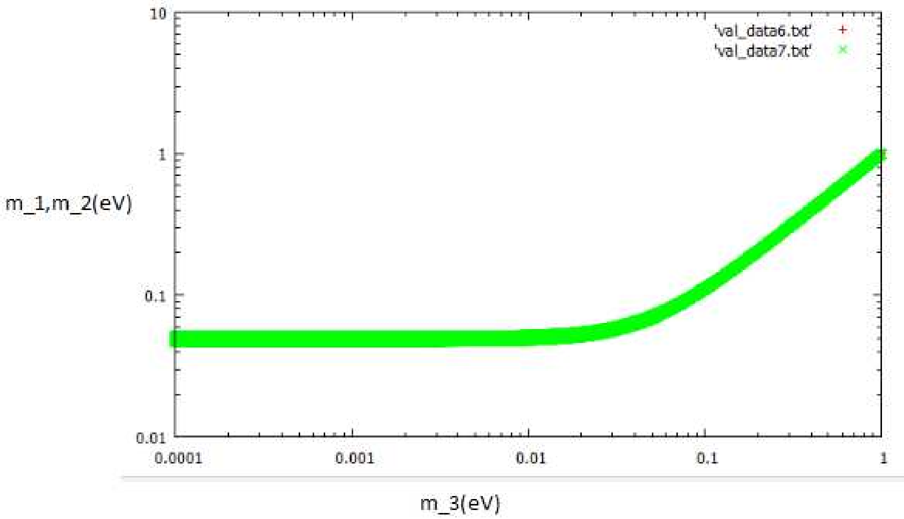

Figure 3.2: Inverted Mass Spectrum

3.4 Beta Decay

In a nuclear beta decay , a neutrino inside a nucleus decays into a proton , an electron and a $\widehat{\nu_e}$, inflicting the following process at the nuclear level,

$$(A, Z) \rightarrow (A, Z + 1) + e^- + \widehat{\nu_e}$$

The electron and the anti-neutrino escape from the nucleus.Although the $\widehat{\nu_e}$ is hard to detect , measurements on the escaping electron (called a beta particle) can provide information about the $\widehat{\nu_e}$.

3.5 Double Beta Decay

$$n + n \rightarrow p + p + e^- + e^- + \widehat{\nu_e} + \widehat{\nu_e}$$

On the nucleus with Z proton, this would inflict a transformation like ,

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- + \widehat{\nu_e} + \widehat{\nu_e}$$

This process is called double beta decay since two β rays (or electrons) emerge in the final states. Usually this process is denoted by $\beta\beta_{2\nu}$ since it is accompanished by two anti-neutrinos. The amplitude of the process has a strength G_F^2 an therefore the process occurs very rarely.

For double beta decay to occur naturally, the arrangement of nuclei of different neighbouring Z values must be such that single beta decay ,

$$(A, Z) \to (A, Z+1) + e^- + \widehat{\nu_e}$$

is energetically forbidden.

Thus double β decay is a very rare process indeed. Infact , only recently , nearly a hundred years after Becquerel first observed β decay , was double beta decay process observed by Elliot , Hahn and Moe for the Selenium nucleus ,

$$Se^{82} \to Kr^{82} + 2e^- + 2\widehat{\nu_e}$$

with half lifetime $1.1 \times 10^{20} years$.

The process conserves the lepton numbers.Therefore , they provide a confirmation of the standard model of weak interaction.

3.6 $0\nu\beta\beta$ Decay

Neutrinoless double-beta decay is a decay mode of an atomic nucleus in which two neutrons convert to two protons and two electrons. This process has not been observed, and it is not known whether it exists. If this decay occurs, the neutrino is its own antiparticle, or a Majorana particle. The observation of neutrinoless double-beta decay would determine whether the neutrino is a Majorana particle and provide information on the absolute scale of neutrino mass. If the neutrino is a Majorana particle, neutrinos could provide a mechanism for the matter-anti-matter imbalance of our universe. The search for $0\nu\beta\beta$ provides the physics community with the opportunity to build on our successes in understanding the neutrino and further our understanding of the universe.

The $0\nu\beta\beta$ process is the process where two electrons are emitted in a nuclear transmution without being accompanished by two neutrinos. Such process violates lepton numbers by two units .

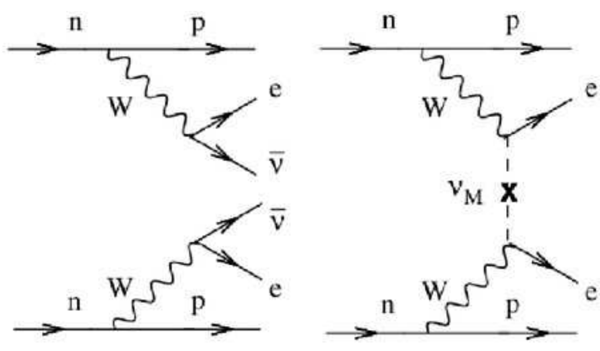

Figure 3.3: $\beta\beta$ and $0\nu\beta\beta$ processes

The neutrinos in the above diagram are virtual particles. With only two electrons in the final state, the electrons total kinetic energy would be approximately the binding energy difference of the initial and final nuclei. To a very good approximation, the electrons are emitted back-to-back.

3.7 Effective Majorana Mass

The existing atmospheric , solar and long-baseline reactor and accelerator neutrino oscillation data are perfectly described by the three neutrino mixing paradigm with

$$\nu_{lL} = \Sigma U_{li} \nu_{il} (l = e, \mu, \tau)$$

Standard parameterization of the CKM matrix uses three Euler angle $(\theta_{12}, \theta_{23}, \theta_{13})$ and one CP violation phase (δ_{13}). θ_{12} is the Cabbibo angle.

And the U_{PMNS} is given by,

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} diag(e^{i\lambda_1}, e^{i\lambda_2}, 1) \tag{3.1}$$

where $S_{ij}(c_{ij}) = sin(cos)\theta_{ij}$

Now using the standard parameterization equation (1) of the three neutrino mixing matrix, the **effective majorana mass** can be written as ,

$$|m_{\beta\beta}| = \Sigma U_{ei}^2 m_i$$
$$|m_{\beta\beta}| = |c_{13}^2 c_{12}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{2i\alpha_2} m_2 + s_{13}^2 m_3|$$
$$\text{where } \alpha_i = (\lambda_i + \delta)$$

Since the values of mixing angle and squared mass differences are known from oscillation data,the value of $|m_{\beta\beta}|$ can be plotted as a function of the lightest neutrino mass $m_{min} = m_1$ in the normal spectrum and $m_{min} = m_3$ in the inverted spectrum.

The plot for normal spectrum is given below ,

In this graph we set the lightest mass m_1 in the x-axis and set the range of the axis 10^{-4} to 1 and the effective Majorana mass $|m_{\beta\beta}|$ in the y-axis and also set the y range from 10^{-4} to 1 and we plot the graph.

The plot for inverted spectrum is given below ,

In this graph we set the lightest mass m_3 in the x-axis and set the range of the axis 10^{-4} to 1 and the effective Majorana mass $|m_{\beta\beta}|$ in the y-axis and also set the y range from 10^{-4} to 1 and we

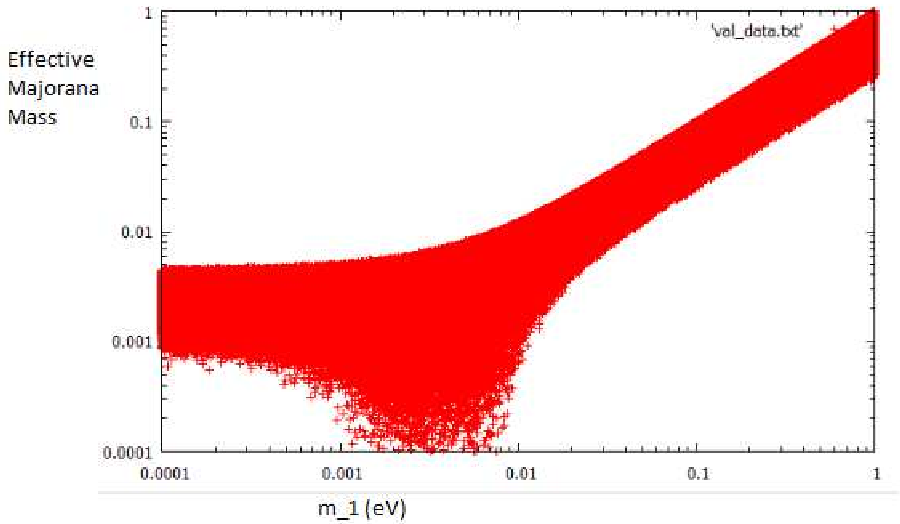

Figure 3.4: Normal Hierarchy Mass spectrum

plot the graph.

The lower bound of $|m_{\beta\beta}|$ for normal hierarchy is approx $(2-7)\times 10^{-3}eV$

The lower bound of $|m_{\beta\beta}|$ for inverted hierarchy is approx $2\times 10^{-2}eV$

The lower bound in case of inverted hierarchy provides a strong encouragement for the experimental searches of $\beta\beta_{0\nu}$ decay in the near future, with the aim of measuring $\beta\beta_{0\nu}$ decay if the neutrino masses have an inverted spectrum or excluding the inverted spectrum if no signal is found.

However if $\beta\beta_{0\nu}$ decay is discovered in these experiment , the problem of the determination of the type of neutrino mass spectrum will remain unsolved.

Also from figures we have that the case of an inverted hierarchy can be established only if it is known independently that $m_{min} \lesssim 10^{-2}eV$. otherwise , the neutrino mass spectrum can be either normal or inverted , with nearly quasi-degenerate states.

The allowed regions of normal and inverted hierarchy has a large overlaps. therefore if $|m_{\beta\beta}|$ is found to be larger than about $2\times 10^{-2}eV$, it may be difficult to distinguish the normal and inverted spectra with absolute neutrino mass experiment .

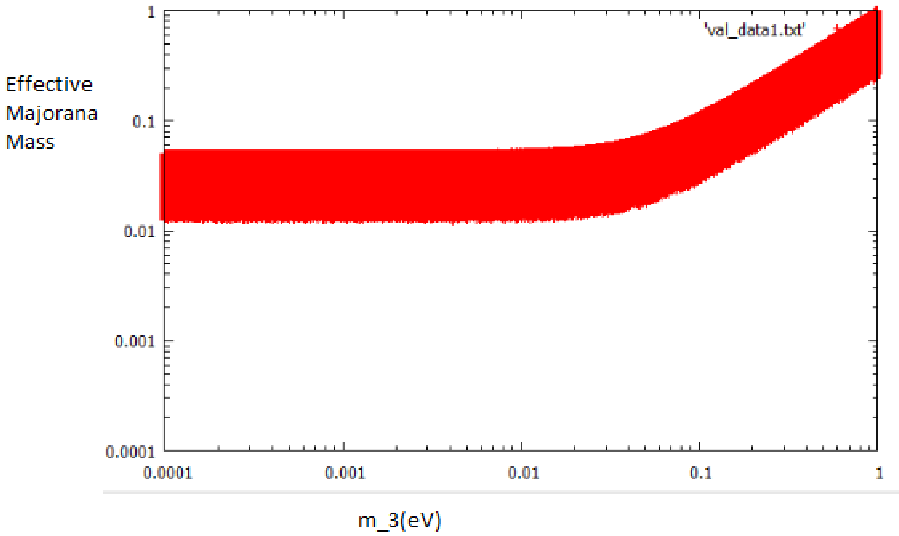

Figure 3.5: Inverted Hierarchy Mass spectrum

3.7.1 Normal hierarchy of Neutrino Masses

In this case the contribution to the $|m_{\beta\beta}|$ of the first term in the equation ,

$$|m_{\beta\beta}| = |c_{13}^2 c_{12}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{2i\alpha_2} m_2 + s_{13}^2 m_3|$$

where

$$\alpha_i = (\lambda_i + \delta)$$

can be neglected.

so, we have ,

$$|m_{\beta\beta}| \simeq |cos^2_{\theta_{13}} sin^2_{\theta_{12}} e^{2i\alpha_2} \sqrt{\Delta m_s^2} + sin^2_{\theta_{13}} \sqrt{\Delta m_A^2}$$

The first term in the righthand side is small becase of the smallness of the solar squared mass difference Δm_s^2 and in the second term the contribution of the large atmospheric mass difference Δm_s^2 is suppressed by the small factor $sin^2_{\theta_{13}}$.

using the best fit values of the parameters we have ,

$$cos^2_{\theta_{13}} sin^2_{\theta_{12}} \sqrt{\Delta m_s^2} \simeq 3 \times 10^{-3} eV$$
$$sin^2_{\theta_{13}} \sqrt{\Delta m_A^2} \simeq 1 \times 10^{-3} eV$$

Thus the absolute values of the two terms are of the same order of amplitude.

Taking into account the 3σ range of the mixing parameters we obtain the upper bound ,

$$|m_{\beta\beta}| \leqslant 4 \times 10^{-3} eV$$

Since the value of the upper bound is significantly smaller than the sensitivity of future planned experiment on the search of $\beta\beta_{0\nu}$ decay , it will very difficult to explore the NH with future $\beta\beta_{0\nu}$ decay experiment.

On the other hand if there are light sterile neutrinos at the eV scale , their additional contribution to $|m_{\beta\beta}|$ cannot be canceled by that of the standard three light neutrinos with a normal hierarchy. in this case the future planned $\beta\beta_{0\nu}$ decay experiment can find a signal.

3.7.2 Inverted Hierarchy for Neutrino Masses

In this case , the contribution of the small m_3 , which is suppressed by the small $sin\theta_{13}^2$ coeffecient , can be neglected leading to ,

$$\sqrt{\Delta m_A^2} \simeq \sqrt{\Delta m_A^2} \sqrt{1 - sin\theta_{12}^2 sin\alpha^2}$$

where the Majorana phase difference $\alpha = \alpha_1 - \alpha_2$ is the only unknown parameter.

Hence , in this case $|m_{\beta\beta}|$ is bounded in the interval ,

$$\sqrt{\Delta m_A^2} cos\theta_{12}^2 \lesssim \sqrt{\Delta m_A^2} \lesssim \sqrt{\Delta m_A^2}$$

Taking into account the 3σranges we obtain the interval,

$$2 \times 10^{-2} eV \lesssim |m_{\beta\beta}| \lesssim 5 \times 10^{-2} eV$$

Now if we considered the phase factor α_1 and α_2 then from the equation of $|m_{\beta\beta}|$ we can see that the allowed region is for ,

$$\text{When } \alpha_1 = 0 \text{ , then } \alpha_2 = \frac{\pi}{2}$$
$$\text{When } \alpha_2 = 0 \text{ , then } \alpha_1 = \frac{\pi}{2}$$
$$\text{and so on } \cdots$$

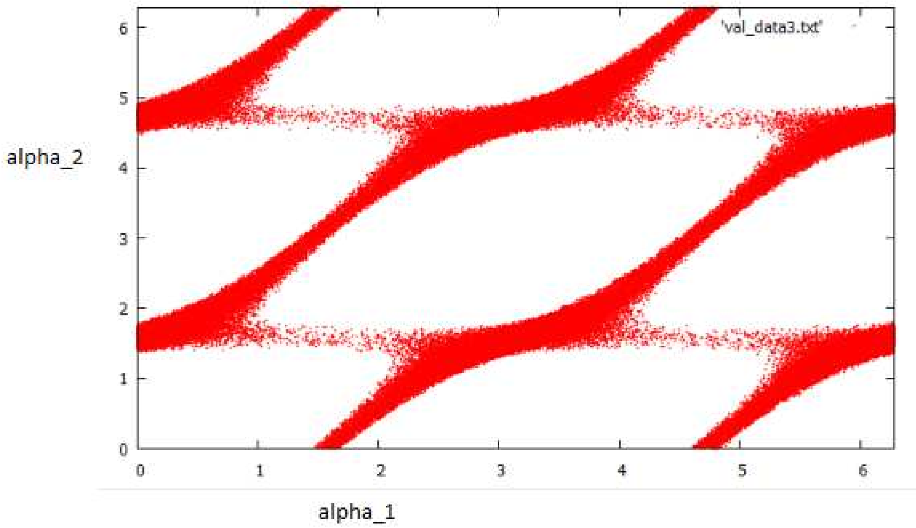

Figure 3.6: $\alpha_1 - \alpha_2$ graph

Then I plot $\alpha_1 - \alpha_2$ graph and I obtain ,

From the graph we can see that the graph is matching with the equation.The Majorana phase α_1 set in the x-axis and the range is 0 to 2π . And another majorana phase α_2 set in the y-axis and the range is 0 to 2π and we plot the graph.

3.8 Theory of $\beta\beta_{0\nu}$ Decay

In this section we present the basic elements of the phenomenological theory of neutrinoless double beta decay of even-even nuclei.

In the following section we assume that,
The interaction lagrangian is the charged current lagrangian of the Standard Model,

$$L_I(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{cc}(x) W^\alpha(x) + h.c.$$

Here,

$$j_\alpha^{cc}(x) = 2\Sigma\overline{\nu_{lL}}(x)\gamma_\alpha l_L(x) + j_\alpha(x)$$

where, $j_\alpha(x)$ is the hadronic charged current.

The massive neutrino fields

$$\nu_i(x) = \nu_{iL}(x) + c\overline{\nu_{iL}}(x)$$

satisfy the majorana condition ,

$$\nu_i(x) = C\overline{\nu_i^T}(x)$$

The effective Hamiltonian of beta decay is given by ,

$$H_I(x) = \frac{G_F cos\theta_c}{\sqrt{2}} 2\overline{e_L}(x)\gamma_\alpha \nu_{eL}(x) j^\alpha(x) + h.c.$$

where G_F is the Fermi constant with

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

θ_c is the cabbibo angle.

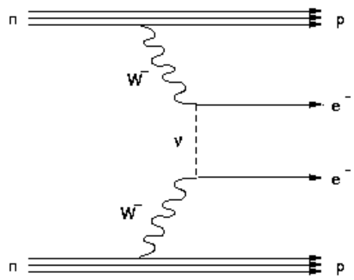

Figure 3.7: $\beta\beta_{0\nu}$ decay

The matrix element is given by ,

$$< f|S^2|i >= 4\frac{(-i)^2}{2!}\frac{G_F cos\theta_c}{\sqrt{2}}^2 N_{p_1}N_{p_2}\int d^4x_1 d^4x_2 \overline{u_L}(p_1)e^{ip_1x_1}\gamma_\alpha < 0|T(\nu_{eL}(x1)\nu_{eL}^T(x2))|0 >$$
$$\gamma_\beta^T \overline{u_L^T}(p_2)e^{ip_2x_2} < N_f|T(J^\alpha(x_1)J^\beta(x_1))|N_i > -(p_1 \leftrightarrows p_2)$$

Here p_1 and p_2 are the electron momentua , $J^\alpha(x)$ is the weak charged current in Heisenberg representation , N_i and N_f are the initial and final nuclei with respective four momenta p_i and p_f and N_p is the standard normalization factor.

Here the neutrino propagator is proportional to m_i .

In this case only the left handed neutrino fields enter into the hamiltonian of weak interaction . In case of massless neutrinos in accordance with the theorem on the equivalance of the theories with massless Majorana and Dirac neutrinos , the matrix element of neutrinoless double beta decay is equal to zero.

Also we have to consider some approximations ,

1.Smallness neutrino masses can be safely neglected in the expression for the neutrino energy q_i^0 . Thus

$$|\overrightarrow{q}^2| >> m_i^2$$

so,

$$q_i^0 = |\overrightarrow{q}|$$

2. Long-Wave approximation : Two electrons are predominantly in the S-state.

3.Closer approximation : We can replace the energies of the intermediate states E_n with the average energy $\overline{E}$.

4.The impulse approximation .

Now we have the matrix element ,

$$< f|S^2|i >= -im_{\beta\beta}\frac{G_F cos\theta_c}{\sqrt{2}}^2 \frac{1}{(2\Pi)^3\sqrt{p_1^0 p_2^0}}\frac{1}{R}[\overline{u}(p_1)(1+\gamma_5 c\overline{u}^T(p_2)M^{0\nu}\delta(p_1^0+p_2^0+M_f-M_i)$$

where R is the radius of the nucleus and $M^{0\nu}$ is the nuclear matrix element.

The probability of $0\nu\beta\beta$ decay ,

$$\Sigma|\overline{u^{r_1}}(p_1)(1+\gamma_5)c\overline{u^{r_2}}(p_2)^2|^2 = 8p_1.p_2$$

Now the decay rate of the $\beta\beta_{0\nu}$ decay is given by,

$$d\tau^{0\nu} = |m_{\beta\beta}|^2|M^{0\nu}|^2\frac{4(G_F cos\theta_c)^4}{(2\Pi)^5 R^2}(E_1E_2 - p_1p_2cos\theta)F(E_1, Z+2)F(E_2, Z+2)|\overrightarrow{p_1}||\overrightarrow{p_2}|sin\theta d\theta dE_1$$

The function F(E, Z) describes final state electromagnetic interaction of the electron and the nucleus. For a point-like nucleus it is given by the Fermi function ,

$$F(E,Z) \simeq \frac{2\Pi\eta}{1-e^{-2\Pi\eta}}$$

where,

$$\eta = Z\alpha\frac{m_e}{p}$$

The inverse half life-time for the $\beta\beta_{0\nu}$ decay is,

$$|T^{0\nu}_{\frac{1}{2}}|^{-1} = \frac{\tau^{0\nu}}{ln2} = |m_{\beta\beta}|^2|M^{0\nu}|^2G^{0\nu}(Q,Z)$$

where $G^{0\nu}(Q,Z)$ is the phase space factor that depends on the transition Q value and on the nuclear charge Z and $M^{0\nu}$ is the nuclear matrix element for this process.

If the light majorana neutrino exchange is the dominant mechanism for $\beta\beta_{0\nu}$ decay then the decay is directly connected to neutrino oscillation phenomenology and it also provides direct information about the absolute neutrino mass scale, as cosmology and β decay experiment do. The relation between $m_{\beta\beta}$ and the actual neutrino masses m_i is affected by the uncertainities in the measured oscillation parameters , the unknown neutrino mass ordering (normal or inverted) and the unknown phases in the neutrino mixing matrix (both Dirac and Majorana) .

Then I took the value of the parameters in the half life-time expression of very recent experiment GERDA experiment i.e.

$$Ge^{76}_{82} = Se^{76}_{34}$$

and I plot the graph between inverse half life-time and the lighest mass for normal and inverted hierarchy. From the graph we make some conclusion.

Both the graphs are given in the next page. In case of NH the width of the lifetime is smaller than the width of the lifetime in case of IH. For both cases we set the lightest neutrino mass in the x-axis and set the range 10^{-4} to 1 . In the y-axis we set the inverse of the half lifetime in y^{-1} and set the range from 10^{-25} to $10^{-30}$and we plot the graph.

In case of NH the width of the lifetime is smaller than the width of the lifetime in case of IH. For both cases we set the lightest neutrino mass in the x-axis and set the range 10^{-4} to 1 . In the y-axis we set the inverse of the half lifetime in y^{-1} and set the range from 10^{-25} to $10^{-30}$and we plot the graph.

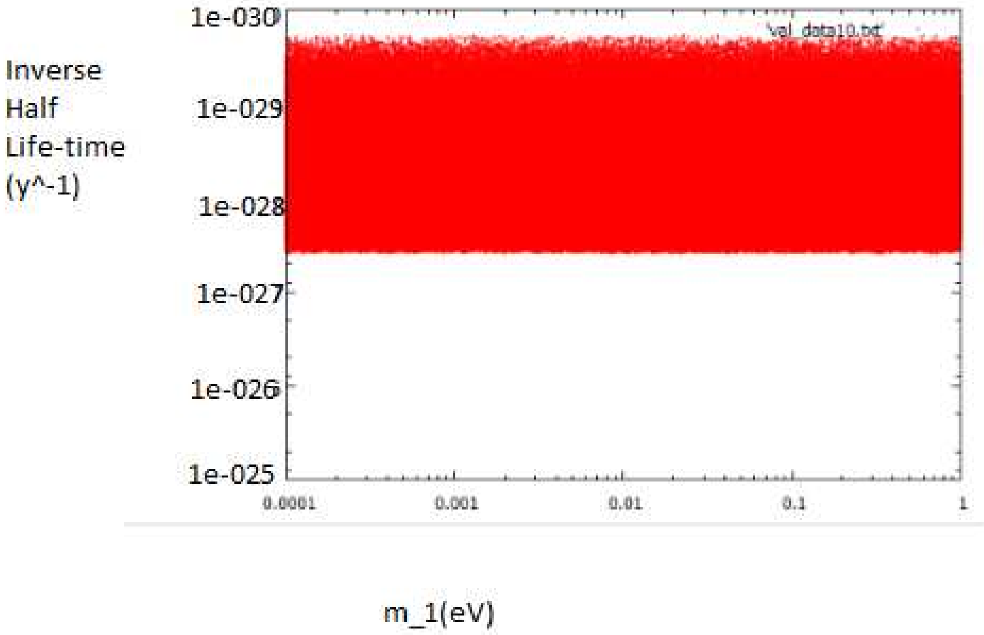

Figure 3.8: Inverse Half Life-time vs m_1 graph for NH

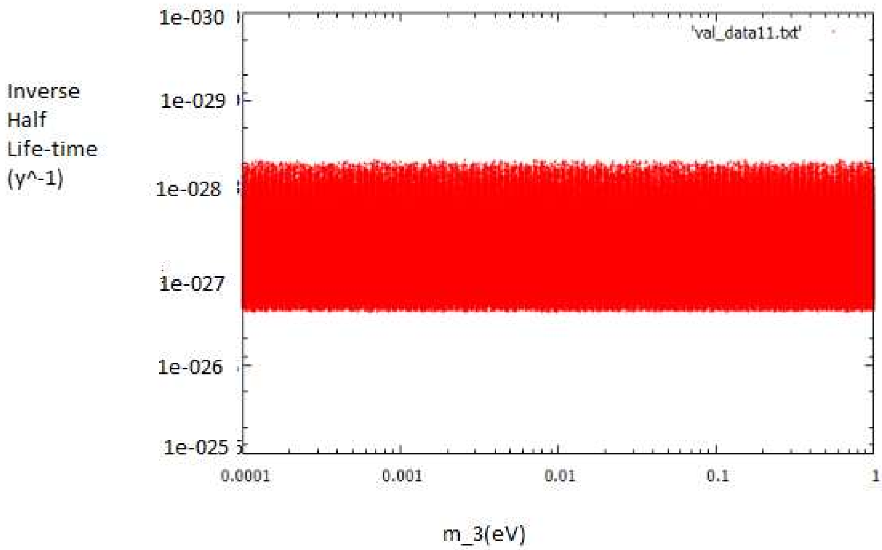

Figure 3.9: Inverse Half Life-time vs m_3 graph for IH

Chapter 4

CONCLUSION

Neutrinoless double beta decay is the most promising process which could allow near future experiment to reveal the Majorana nature of massive neutrino which is expected from the Physics beyond the Standard model .
The experiment on the study of neutrino oscillation enter now a new stage of high-precision measurement of the neutrino oscillation parameters . The last unknown parameter of the three neutrino mixing matrix in case of Dirac neutrinos was the CP violating phase δ . If the massive neutrinos are Majorana particles , there are two additional Majorana phases that are observable in process which violate the total lepton no. as neutrinoless double beta decay. The effective Majorana mass $m_{\beta\beta}$ in $\beta\beta_{0\nu}$ decay depends on the Majorana phases. We take account the 3σ range and we obtained the graph given in chapter 3 .

Upto now no $\beta\beta_{0\nu}$ dacay has been observed , which is currently strongly disfavoured by the direct lower limit on lifetime (Ge^{76}_{32}) of the GERDA expt.The next generation of $\beta\beta_{o\nu}$ experiment is aimed at the exploration of values of $m_{\beta\beta}$ below 0.1 eV , with the purpose of reaching the inverted hierarchy interval between about 0.02 and 0.05 eV .For $m_{\beta\beta} \geqslant 0.2eV$ then inverted hierarchy is excluded ,only the possible hierarchy is normal.For the standard seesaw mechanism the total lepton number is violated at the GeV scale .So, Majorana neutrino exchange is the only mechanism of the neutrinoless double beta decay. We also plot the graph between half life-time and the lightest mass of the neutrino in case of normal and inverted hierarchy as given in chapter 3. From the graph we can see that the width of the inverse NH half life-time is much wider than the width of the inverse IH half life-time . If we put a cosmological bound to 0.2ev then we can get both the normal and inverted hierarchy region.If the neutrino is a Majorana particle then there is a explanation for the smallness of neutrino masses.

Chapter 5

References

1. Relativistic Quantum Mechanics by W. Greiner .

2.A first Book of Quantum Field Theory by Amitabha Lahiri and Palash B. Pal

3.Fundamentals of Neutrino Physics and Astrophysics by Carlo Giunti and Chung W. Kim

4.Massive neutrinos in Physics and Astrophysics by Rabindra N. Mohapatra and Palash B. Pal

5.The standard Model by Dr. T. Teubuer

6.Standard Model : An Introduction by S. F. novaes

7.Neutrinoless Double Beta Decay : A Probe of Physics Beyond the Standard Model by S. M. Bilenky and C. Giunti

8.Phenomenology of Neutrinoless Double Beta Decay by J.J. Gomez-Candenas and J. Martin-Albo

9.Neutrinoless Double Beta Decay and Seesaw Mechanism by Somoil M. Bilenky , Amand Faessler , Walter Potzel , Fedor Simkovi C

10.Neutrinoless Double Beta Decay in Seesaw Model by Mattias Blenbow , Enrique Fernandez-Msrtinez , Jaobo Lopez-Pavon and Javier Menendez