Passive Dynamic Quadruped Robot Bounding with Front and Back Legs Coupled

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This thesis entitled "Passive Dynamic Quadruped Robot Bounding with Front and Back Leg-Coupling" by Ch Sivanand is approved for the degree of Master of Technology from IIT Hyderabad.

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Dedicated to

MY FAMILYAND FRIENDS.

Abstract

Numerical model of quadruped robot traversing with bounding gait is used for analysis of its stability and control. Understanding the properties of fixed points of quadruped model, a control law is proposed. With the help of controlled rotation using motors at the hip joint of the front and back leg this control is implemented. The touchdown and lift off angles are absolute angles which are defined with respect to vertical line. Stability and control of passive dynamic quadruped robot model is tested successfully at some unstable fixed points with this control.

Contents

Chapter 1

Introduction

The legged robots are preferred as they can provide a lot of mobility in locomotion and flexibility in design. The robot design has been inspired from the way animals adapt in rough terrain. One of the reasons for animal adaptability is their possession of elasticity in muscles and tendons which can be replicated in robots by providing compliant elements like springs and dampers in torso and legs instead of rigid links. Usage of compliant elements also offers other benefits like power reduction, stability and autonomous behavior.

1.1 Motivation and Objective

The main motivation is to make a robot that moves more on its natural dynamics with simple controller or in other words make it more autonomous. As stated by Raibert and Hodgins in [1] that “we believe the mechanical system has mind of its own, governed by the physical structure and laws of physics. Rather than issuing commands, the nervous system can only make suggestions, which are reconciled with the physics of system and task”.

Therefore the objective is to formulate similar robot model with minimal human intervention and also to understand various aspects of passive dynamics and its control.

1.2 Literature Survey

The basics of quadrupedal locomotion is understood in book [2] .De Santos et al. gave introduction about quadruped robots and described it in two parts, the first one dealing with the theoretical aspects and second on the general design and control techniques

- The stability properties of sagittal plane model of bounding gait in quadruped robot Scout II is studied in [3]. Also the self stabilization technique that can be generated passively using proper initial conditions is discussed by authors.
- Controller design to stabilize the periodic bounding gait based on touchdown angle control policy and energy controller is discussed in [4].
- Control strategy using PD control algorithm to control the forward speed is discussed in [5] using robot model in sagittal plane with bounding gait. The robot here has legs with both hip joint and knee joint and the link at knee joint has compliant elements.

- In [6] the touchdown angle impact on quadruped robot in various energy levels and parameter configuration is studied. It also discussed the influence of touchdown angle on cycle bifurcation and chaos phenomenon.

1.3 Thesis Overview

The modeling aspect and assumptions made while designing of numerical model of quadruped and various stances of bounding gait is discussed in Chapter 2. The representation of robot using equation of motion in different stances and the necessary conditions for transition from one stance to other is also discussed. The numerical modeling of robot and solving the equations of various stances of bounding gait in Matlab with a brief note on non holonomic constraints and why these constraints can be neglected here is described.

In Chapter 3 the study of model so developed by using properties of fixed points is done. Based on the analysis of properties of fixed points, a control strategy and model of robot is developed.

In Chapters 5, discussion of results with and without proposed model is done and in chapter 6, conclusion of the work done on quadruped robot model with possible future work is given.

Chapter 2

Modeling

2.1 Physical Quadruped Model

Robot model has a rigid torso with compliant legs. The legs have springs in between as shown in Figure 2.1. It has motor as actuator at each of the hip revolute joints, so each leg has one degree of freedom. The physical parameters like mass, inertia etc are taken from Scout II [3]. They are given in Table 2.1

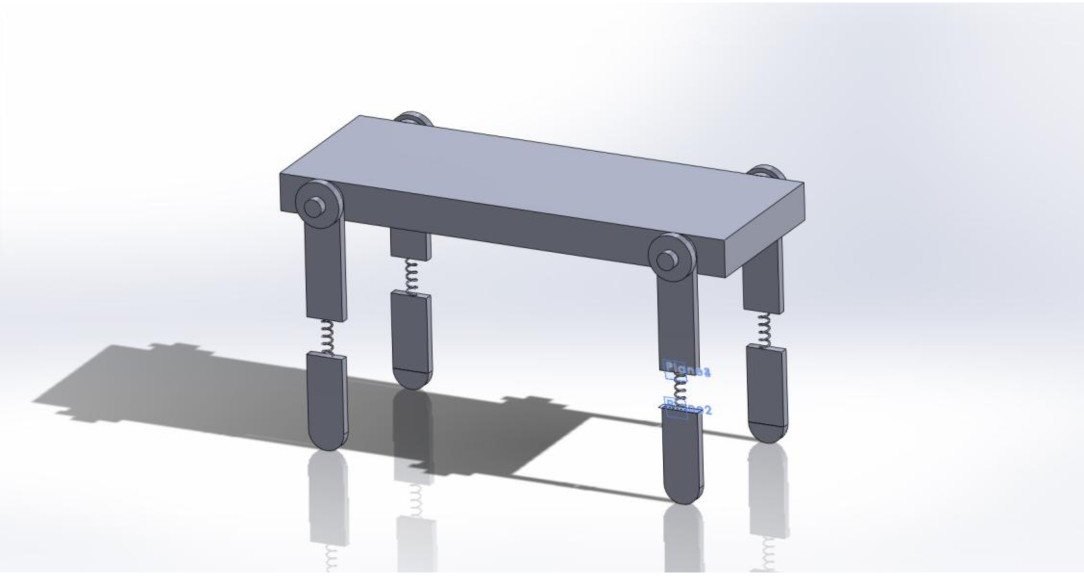

Figure 2.1: Schematic Representation of Quadruped Robot using SOLIDWORKS® Software

Table 2.1 Physical parameters

Parameter	**Value**
Body Mass	20.865 kg
Body Inertia	1.3 kg m^3
Spring Constant k	3520 $^{N/m}$
Hip Separation 2L	.0552 m
Leg rest length	.323m

2.1.1 Assumptions

- For simplifying the model legs are assumed to be mass-less. This can be accomplished in the actual physical model by taking torso to leg mass ratio quite high.
- A revolute joint is assumed between ground and leg whenever there is contact with the ground at the point of contact.
- Due to symmetry in design, the motion can take in sagittal plane with virtual leg concept [7] as shown in Figure 2.2.
- The conditions taken while using virtual leg concept are
    1. Both the torque delivered at the hip and axial force exerted by spring of the actual physical legs is assumed to be half of that delivered to the virtual leg. The reason is obvious as we are replacing a pair of legs with one virtual leg .
    2. Forward position of leg with respect to hip and leg striking the ground has to be same for both the physical leg and the virtual leg.

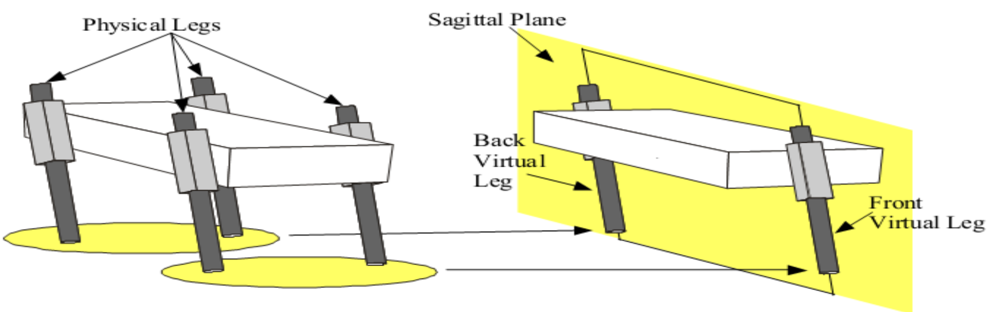

Figure 2.2: Virtual Leg Concept

Diagram courtesy: Ioannis Poulakakis , 2002[8]

2.2 Bounding Gait

Typical leg movements used by animals for locomotion is called gaits. Most gaits are symmetric and periodic in nature. Some of the gait mechanisms are trotting, pacing, galloping, bounding, pronking etc.

Bounding has the following phases in its motion (refer 2.3)

1. Flight phase
2. Back leg stance
3. Front leg stance
4. Double leg stance

Bounding gait can happen in two ways while on the periodic cycle

1. With double stance (1-2-4-3)
2. Without double stance(1-2-1-3)

This is based on what happens after the back leg touches the ground in flight phase, the robot can either go on to air again or its front leg touches the ground. When the former happens second one happens else the first one. This is shown in Figure 2.3.

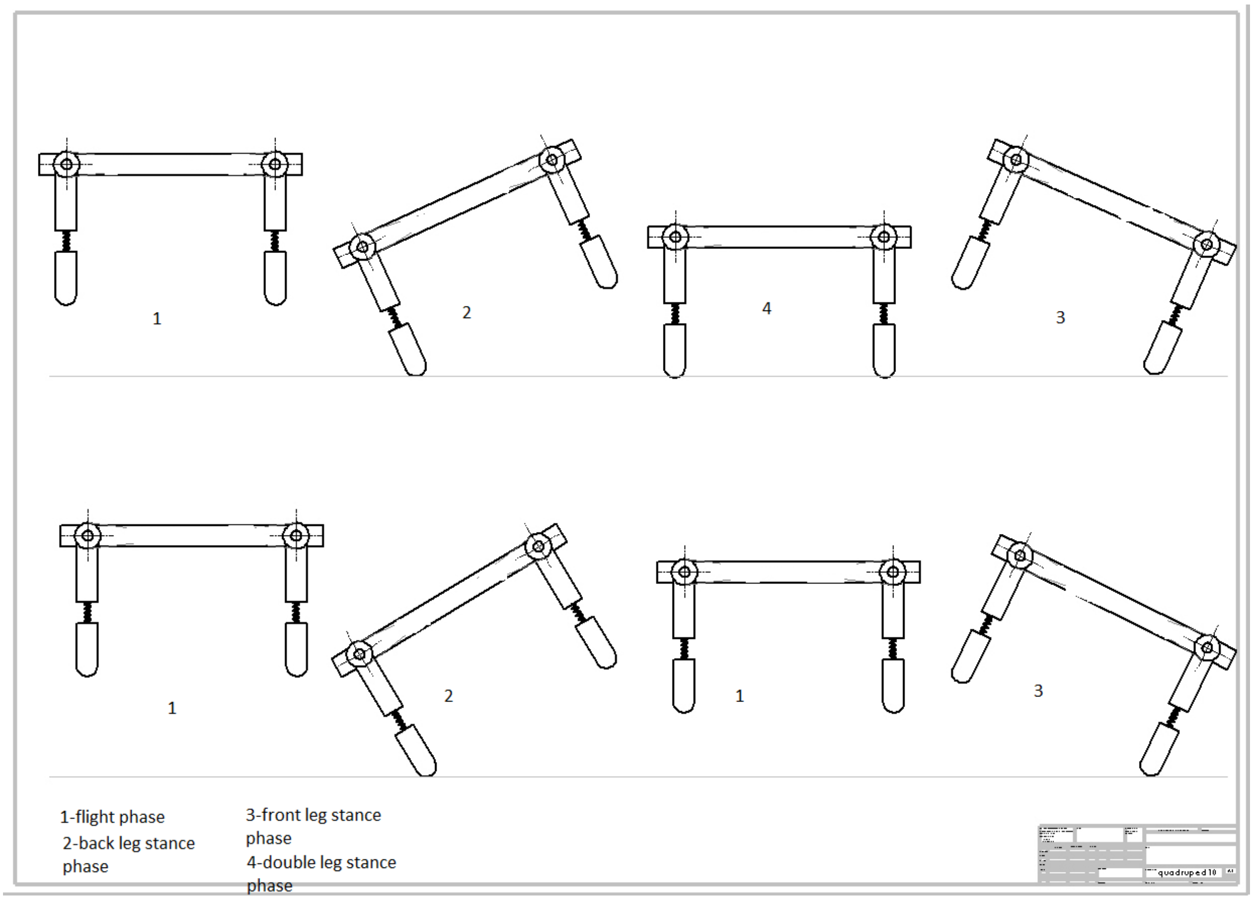

Figure 2.3: Bounding gait with and without double stance

2.2.1 Non Holonomic Constraints

Constraints in the form of velocities or accelerations, which cannot be integrated back to the position configurations, are non holonomic constraints. These arise in two kinds of situations as in [9]

1. Bodies in contact with each other which roll without slipping
2. Where there is conservation of angular momentum in a multibody system

During the flight phase the robot is in non holonomic constraint configuration. But this can be neglected as happens for very less period of time and also leg mass is considered negligible.

2.3 Equations of Motion

The equations of motion are written in Newtonian methodology. Equations are written for different stances of bounding gait and the conditions necessary for transition from one stance to another is given

$$\boldsymbol{M}\ddot{x} + \boldsymbol{F}(\boldsymbol{X}) = 0$$

Where $\boldsymbol{M}$ is mass matrix and $\boldsymbol{F}$ is force vector

With (x, y) the variables of center of mass (c.o.m) and θ is pitch angle of torso of quadruped robot in Cartesian space

$$\boldsymbol{X} = [x \ y \ \theta]$$

Mass matrix $\boldsymbol{M}$ is of the form

$$\boldsymbol{M} = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix}$$

m is mass and I is inertia

And force vector $\boldsymbol{F}$

$$\boldsymbol{F} = [f_x \ f_y \ M]$$

f_x, f_y are forces in x and y directions respectively and M is moment

Robot can be assumed to be starting from a specific point in bounding gait which are given as initial conditions, here let's assume it as from the apex points in flight phase. The notations used in equations are

k_b, k_f - spring constant of back and front legs respectively.

l_b, l_f - Length of back and front legs respectively.

$\Delta l_b, \Delta l_f$ - Change in length of back and front legs respectively due to spring present.

l_i - Actual uncompressed length of back and front legs.

L – Half length of torso.

γ_b^{td} - Angle of back leg with respect to vertical during back leg touchdown.

γ_f^{td} - Angle of front leg with respect to vertical during front leg touchdown.

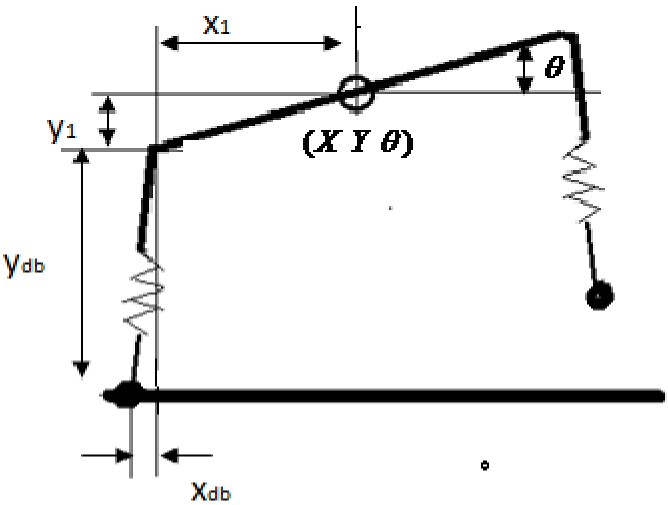

Figure 2.4: Reference diagram for equations

2.3.1 Flight Phase

During flight stance, since there is no contact with ground, except the gravitational force in downward y- direction other forces and the moment are zero.

$$F = [0 \; -mg \; 0]$$

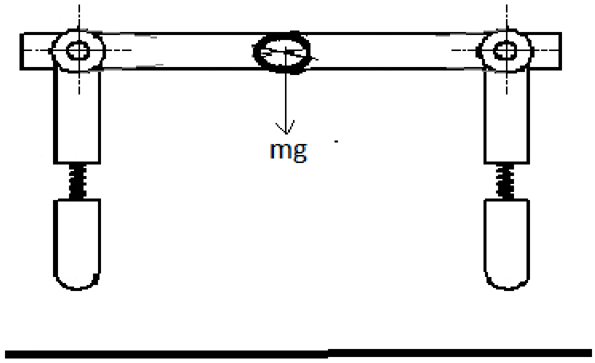

Figure 2.5: Flight Phase

2.3.2 Back Leg Stance

In back leg stance, since back leg is in contact with ground, there is reaction force as well as moment. On solving the equilibrium equations of free-body diagram, the external reaction forces at the joint are

$$R_{xb} = (k_b\,\Delta l_b\,\frac{x_{db}}{l_{db}})$$

$$R_{yb} = (k_b\,\Delta l_b\,\frac{-y_{db}}{l_{db}})$$

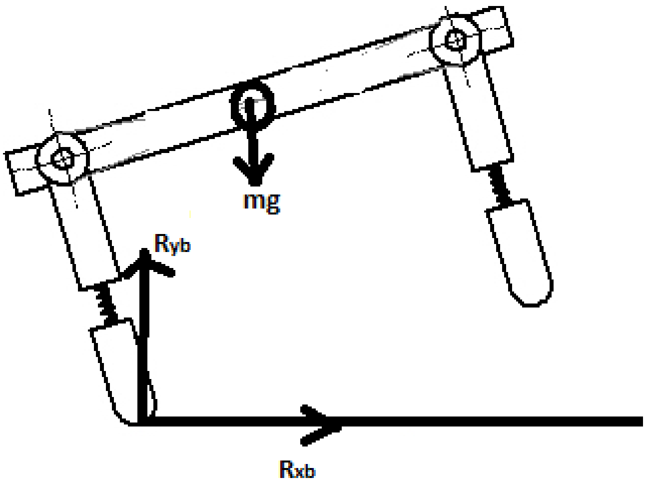

Figure 2.6: Backleg stance Phase

And the Force and Moment

$$F_x = R_{xb}$$

$$F_y = R_{yb} - mg$$

$$M = R_{xb} y_1 - R_{yb} x_1$$

2.3.3 Front Leg Stance

In front leg stance, front leg is in contact with ground, so there are reaction forces and moment

$$R_{xf} = k_f \Delta l_f \frac{x_{df}}{l_f}$$

$$R_{yf} = k_f \Delta l_f \frac{y_{df}}{l_f}$$

And the Force and Moment

$$F_x = R_{xf}$$

$$F_y = R_{yf} - mg$$

$$M = (R_{xf}\, y_1) + (R_{yf}\, x_1)$$

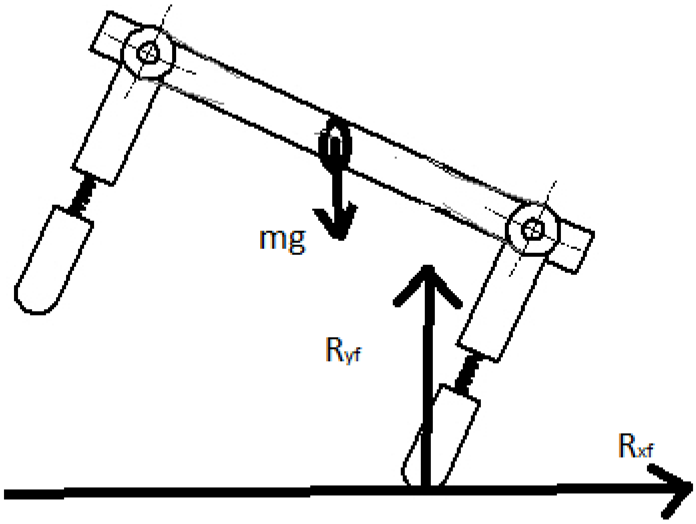

Figure 2.7: Front leg stance

2.3.3 Double Leg Stance

In this stance both the legs are in contact with ground therefore there is reaction from both the joints between leg and ground

$$R_{xb} = (k_b\, \Delta l_b \frac{x_{db}}{l_{db}}) \qquad R_{xf} = k_f \Delta l_f \frac{x_{df}}{l_f}$$

$$R_{yb} = (k_b\, \Delta l_b \frac{-y_{db}}{l_{db}}) \qquad R_{yf} = k_f \Delta l_f \frac{y_{df}}{l_f}$$

And the Force and Moment

$$F_x = R_{xb} + R_{xf}$$

$$F_y = R_{yb} + R_{yf} - mg$$

$$M = (R_{xb} y_1) - (R_{yb} x_1) + (R_{xf} y_1) + (R_{yf} x_1)$$

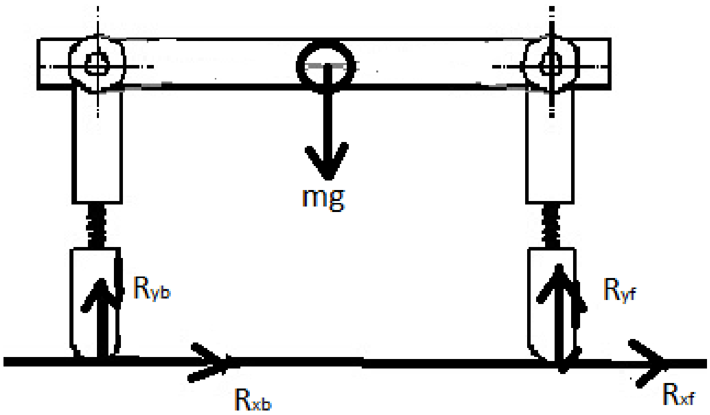

Figure 2.8: Double leg stance phase

2.3.4 Transition Equations for Changing from one stance to another

While modeling the bounding gait numerically, as the robot goes from one phase to another, the end of one phase and start of another is detected mathematically by satisfying certain equations. These equations are implemented in Matlab through inbuilt event detection.

The end of flight phase and start of back leg phase is back leg touchdown and the corresponding satifying equation is

$$Y - L\sin\theta - l_i \cos(\gamma_b^{td}) \leq 0$$

And the end of back leg phase and start of front leg phase or double stance is back leg lift-off which is

$$l_b^{lo} = l_i$$

Then the front leg touchdown is

$$Y + L\sin\theta - l_i \cos(\gamma_f^{td}) \leq 0$$

And front leg lift off is

$$l_f^{lo} = l_i$$

These equations of motion of bounding gait are integrated numerically using inbuilt Matlab function ode45 and transition between phases using event detection. So given the proper initial conditions the robot transcends from one phase to another to complete a cycle of bounding gait.

Chapter 3

Stability Analysis and Control

The stability depends on the properties of fixed points, so proper understanding of fixed points and its behavior is necessary. So, in this chapter first definition of fixed points and then given the initial conditions how fixed points are found is discussed. The method for finding fixed points when some of the state variables are constant is also discussed. Finally, the properties of fixed points, their stability and control of quadruped model are discussed.

3.1 Fixed points

Fixed points are the points in the function's domain which maps into themselves by function, in other words the value of function will remain equal to its initial values every time. In this context value of state variables of robot will remain same after every cycle with respect to particular point of reference in cycle [3].

$$G : X \rightarrow X$$

There exits

$$x \in X \;\; such\, that\, G(x) = x$$

Here G is the whole set of bounding gait functions. And X can have any set of state variables at one particular point in cycle, but for convenience we take variables at apex point. The state variables generally have $(x\, y\, \theta\, \dot{x}\, \dot{y}\, \dot{\theta})$ as variables but we can eliminate x and $\dot{y}$. As x updates after every cycle since robot has to move forward and $\dot{y}$ is zero at apex points.

Therefore in this case the necessary condition for fixed points is

$$[y\, \theta\, \dot{x}\, \dot{\theta}\,]_{n+1} = G([y\, \theta\, \dot{x}\, \dot{\theta}]_n, (\gamma_b^{td}, \gamma_f^{td})_n) \quad \text{…….. Eq.3.1}$$

Where n is the nth cycle of bounding gait

3.2 Finding Fixed points

For finding fixed points the necessary condition to be satisfied is the equation

$$X - G(X, (\gamma_b^{td}, \gamma_f^{td})) = 0 \qquad \text{……… Eq.3.2}$$

Where X is $[y\,\theta\,\dot{x}\,\dot{\theta}]$

Solving above equation using Newton-Raphson Method using an initial guess will give the converged values after a set of iterations. Here the convergence error is taken as 10^ (-7). For the particular cycle apex points the equation is

$$X^{j+1} = X^j + (I - \nabla G(X^j))^{-1}(G(X^j) - X^j)$$

Where j is iteration number

$$\nabla G = [\frac{dG}{dy}\ \frac{dG}{d\theta}\ \frac{dG}{d\dot{x}}\ \frac{dG}{d\dot{\theta}}] \quad \text{……. Eq.3.2}$$

And

$$\frac{dG}{dx_i} = \frac{G(x_1,..x_i + dx,.., x_4) - G(x_1,..x_i - dx,.., x_4)}{2dx}$$

One of the fixed points found out using above method with initial guess of $[y\,\theta\,\dot{x}\,\dot{\theta}]$ as (.35 0 4 200deg/s) and touchdown angles of backleg and forward leg (35°, 34°) is (.3299 0 4.0998 289.029deg/s).

If our interest lies in finding fixed points at particular state variables which means the state variables $[y\,\theta\,\dot{x}\,\dot{\theta}]$ are kept constant and the touchdown angles $(\gamma_b^{td}, \gamma_f^{td})$ are to be found out [9] . This can be done by taking initial values in Newton-Raphson method at desired state variables with proper guess of front leg and back leg touchdown angles and the function G in eq.3.2 can be changed to

$$G(y, \theta, \dot{x}, \dot{\theta}, \gamma_b^{td}, \gamma_f^{td}) = \begin{Bmatrix} (y_i - y)^2 + (\theta_i - \theta)^2 \\ (\dot{x}_i - \dot{x})^2 + (\dot{\theta}_i - \dot{\theta})^2 \end{Bmatrix}$$

Where $(y_i, \theta_i, \dot{x}_i, \dot{\theta}_i)$ are initial state variables

In this method, we should know at least one fixed point at any state variables previously for proper guessing of initial touchdown angles at desired state variables. Through repeated iterations we can get close to desired state variables for proper initial guess in the Newton-Raphson method.

3.3 Properties of Fixed points

The fixed points show the following properties [3]

1. There is symmetric behavior of the state variable in particular bounding cycle.
2. The touchdown angle of back leg is found to be equal to negative of lift-off angle of front leg and touchdown angle for front leg is equal to negative of lift-off angle of back leg.

$$\gamma_b^{td} = -\gamma_f^{lo}$$
$$\gamma_f^{td} = -\gamma_b^{lo}$$

3. Angle at apex point is always equal to zero

$$\theta_{at\,apex\,pt.} = 0$$

3.4 Stability of Fixed points

Stability can be tested in two ways

1. Run the robot model for number of cycles (here 1000 cycles) and find the norm of error between initial variables and final variables, if norm is very less, then the initial values (fixed points) are stable fixed points.
2. Find all the Eigen values of matrix ∇G (Eq.3.2). If all are less than one then stable fixed point else unstable.

3.5 Control

3.5.1 Control Approach

Using the properties of fixed points, the control policy is formed. The touchdown angle of back leg is so adjusted to equal to negative of lift-off angle of forward leg of previous cycle and the touchdown angle for forward leg is made equal to negative of lift-off angle of back leg of previous cycle.

$$\gamma_b^{td} = -\gamma_f^{lo}$$
$$\gamma_f^{td} = -\gamma_b^{lo}$$

3.5.2 Control Model

The robot model incorporates the control approach using the controller which adjusts the leg angles using the motors at the hip joints. The angles to be rotated by legs are based on the calculated values of touchdown angles found out from lift off angles using control policy.

Chapter 4

Results and Discussion

The stability is tested at various situations for the passive dynamic quadruped robot with bounding gait. They are

1. Running quadruped model with Stable fixed points as initial conditions.
2. Running with perturbation at same energy level and constant touchdown angle.
3. Running with unstable fixed points
4. Running with unstable fixed points and the proposed model.

4.1 With Stable Fixed Points As Initial Conditions

A sample stable fixed point of $(x\, y\, \theta\, \dot{x}\, \dot{y}\, \dot{\theta})$ as (0 .35 0 4 0 161.2434deg/s) and $(\gamma_{td}^{b}, \gamma_{td}^{f})$ as (33.2724, 31.2228) is taken as initial conditions for gait cycle , found out using method in section 3.2. The error with respect to initial fixed point for 200 cycles is checked out as shown in Figure: 5.1. The error is of 1e^-3 range.

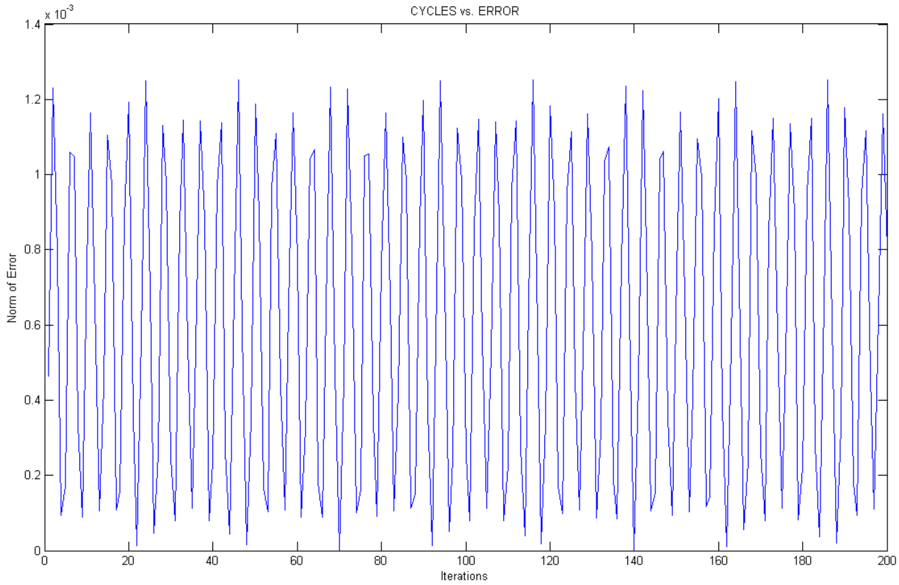

Figure 5.1 Cycles Vs Error (for stable fixed points)

4.2 With Perturbation At Same Energy Level And Constant Touchdown Angle

A perturbation of 1e^-3 is given to $(y, \dot{\theta})$ and $\dot{x}$ is found out by substituting in the same energy level as that fixed point as (0 .35 0 4 0 161.2434deg/s). This perturbed values are given keeping constant touchdown angles of (33.2724, 31.2228) for every cycle. The norm observer to be stabilized to particular value as shown in Figure 5.2 .

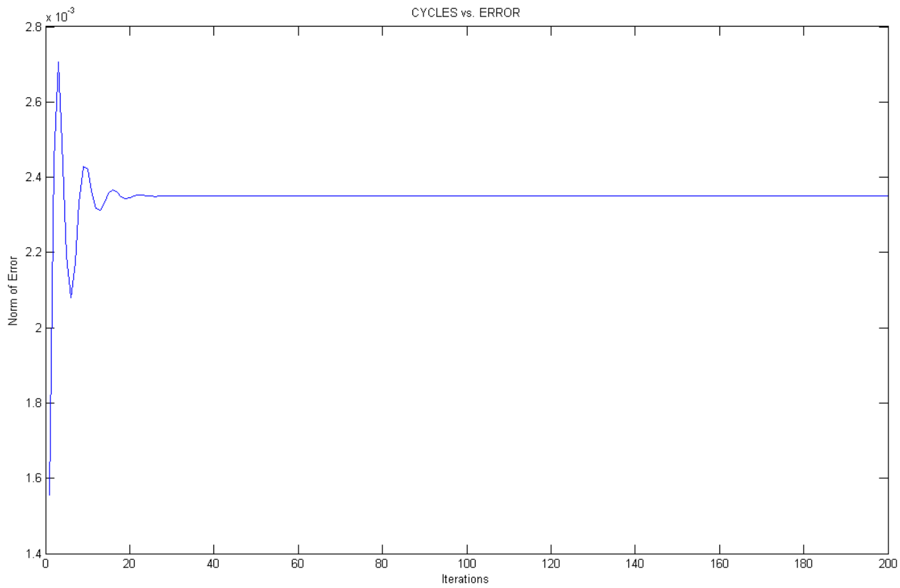

Figure 5.2: Cycles Vs Error (for perturbed values)

4.3 Unstable Fixed Points and the Control Law

Initial values are taken as (0 .35 0 2 0 279.7967deg/s) with touchdown angles as(21.0567,20.0211) which are unstable fixed points, the norm is observed to be quite high and the robot failed after 31 cycles**.**

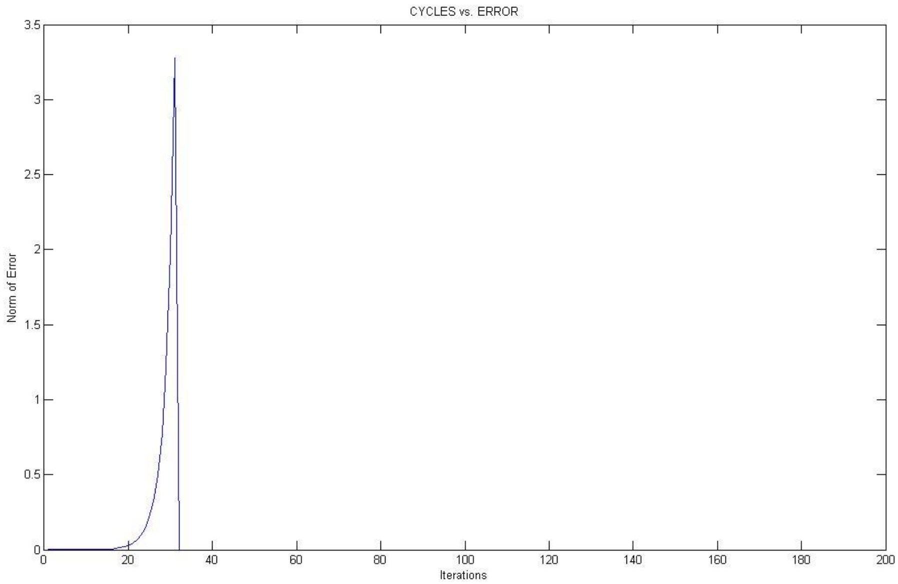

Figure 5.3: Cycles Vs Error (for unstable fixed points and no control)

4.4 Proposed model

Using one of the properties of fixed points where touchdown angle of front leg is equal to negative of lift off angle of back leg and vice versa (refer section 3.3) a controller is developed.

4.5 With Unstable Fixed Points and the proposed model

Taking the initial conditions as unstable fixed points (0 .35 0 2 0 279.7967deg/s) with touchdown angles as (21.0567, 20.0211) and the proposed model . Two hundred cycles are run and the norm is found out to be of order 1e^-6.

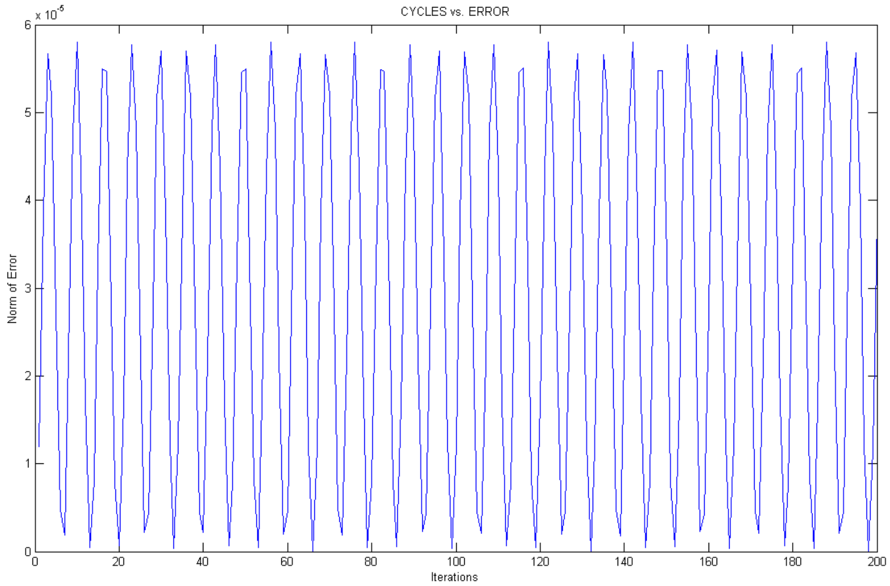

Figure 5.4: Cycles Vs Error (for unstable fixed points with control)

Chapter 5

Conclusion and future work

5.1 Conclusion

Even with unstable fixed points as initial conditions the quadruped model is able to maintain stability by the property of fixed points where the touchdown angle of back leg is to equal to negative of lift-off angle of forward leg of previous cycle and the touchdown angle for forward leg is made equal to negative of lift-off angle of back leg of previous cycle is used. The control design and actuation is reduced by using simple controller and passive dynamics of compliant elements in robot.

5.2 Future work

- Although tested for some unstable fixed points a generalization is not made. In future attempts should be made for generalized control of unstable fixed points.
- Build the physical model of robot and test the above results obtained.
- Body fixed touchdown angle can be used while modeling in future which has an added advantage of requirement of no additional control.

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