

# **In search for the best wavelet for denoising low SNR RF Signal for FMCW Radar Altimeter**

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A Dissertation Submitted to  
Indian Institute of Technology Hyderabad  
In Partial Fulfillment of the Requirements for  
The Degree of Master of Technology



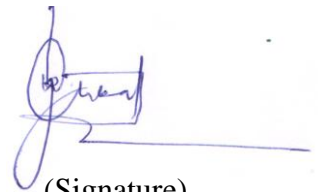
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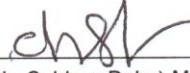
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## Approval Sheet

This Thesis entitled IN Search of Best wavelet for Demodulating ~~5th~~  
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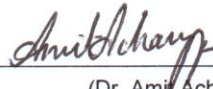
LOW SWR RF  
Signal For  
Radar Altim-  
eter.



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## **Dedicated to**

God, My Family members, True Friends and Gurus

## Abstract

This research project / thesis proposes the best wavelet for Denoising under low Signal to Noise Ratio (SNR) conditions and Discrete Wavelet Transform Architecture based design for RF signal denoising, targeting the real time applications like FMCW (Frequency Modulated Continuous Wave) Radar Altimeter used in Anti-Radiation Missiles, Smart Bombs, Fighter aircrafts, Helicopters etc., and other Defense and RF carrier based applications like Cellular communication. DWT (Discrete Wavelet Transform) & IDWT(Inverse DWT) Architecture Models designed in MATLAB for the wavelets under study like dmey, coif1, sym2, & debouches db1, db2, db3, db4, db6. The reconstructed signal results after denoising are compared in various aspects. The results show that db3 is the best wavelet for denoising application point of view. Finally the db3 based architecture design implemented in VHDL(*VHSIC Hardware Description Language*) and the simulation results compared, synthesis has been done using Xilinx ISE Design suite targeting an FPGA.

This project involves study and implementation of De-noising algorithms using Discrete Wavelet Transform.

In a system the signal to noise ratio (SNR) is important for reliable information retrieval. Analysis of signals with poor SNR may lead to wrong interpretation of results. Conventional techniques like filtering in time domain and frequency domain has its own limitations in estimating and characterizing noise.

Wavelet transforms is a very useful tool in the analysis of non-stationary signals. Wavelet transform has been used in signal processing fields such as de noising or data compression. This method consists of decomposing the data recursively into a sum of details and approximations at different levels of resolution. The details represent the high frequency components while the approximations represent the low frequency components of the signal.

The unwanted frequencies are eliminated by zeroing the output of the filter corresponding to those frequencies, and where signal corresponding to altitude information has no effect. The decomposed wavelet coefficients smaller than given amplitude is suppressed by means of thresholding and finally the data is transformed into the original domain using inverse wavelet transform.

To achieve low complexity, low complexity multiplier is implemented and combinational circuit for repeated use in VHDL Program apart from selecting db3 as best choice (having less number of filter coefficients) and only 80 samples only selected for denoising and found sufficient to recover the wanted difference signal representing altitude information.

By DWT based Denoising using db3 improved SNR appx. 5db more compared to conventional methods and Correlation, Regression,  $R^2$  statistics gave very good results compared to filter methods even with 80 samples.

The difference signal which is useful for altitude information is a Non-Stationary Signal. Hence as DWT is very much suitable for such signals and time information is very important (where conventional methods like FT fails to provide time where the frequency of interest found) for strategic missile applications like Anti-Radiation Missiles, and smart bombs, which are used to destroy enemy radars, communication networks and infrastructure.

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# Chapter 1

## Introduction

### 1.1 Motivation

Wavelet Transform (WT) has wide range of emerging applications including Communication systems, Digital Signal Processing, Bio-Medical Engineering and many other diversified fields. With a curiosity to know DWT based denoising for defense applications it is found that to denoise the low SNR signal with DWT architecture using best wavelet almost no literature is present on denoising with best wavelet for FMCW Radar altimeter application. If at all altitude detection is done, it is by conventional filtering methods and followed by FT. But it can be easily understood that for advanced applications like strategic missiles applications like anti-radiation missiles, smart bombs and other similar applications, where the missile trajectory needs altitude information (which is non-stationary) accurately along with time under low SNR situations where baseband difference signal output from mixer becomes difficult to deal with because of challenges that exists in removing unwanted signals or Denoising.

With a aim to use DWT based denoising for such applications and to compare how well it can perform compared to conventional filtering method this project is taken up.

### 1.2 Literature Survey

While doing literature on applications of DWT for denoising, it is found that the important applications of RF signal denoising like altimeter for defense applications like Aircrafts (Civil & Military), Missiles, still the conventional approach only followed, i.e filtering and Fourier Transform, whereas better methods for low SNR cases involving non-stationary signal like DWT can be used with best suitable applications. Following are important points understood from literature survey.

#### 1.2.1 Outcome of Literature survey:

There are two main types of Radar altimeters

1. FMCW (Frequency Modulated Continuous Wave) Radar Altimeter
2. Pulse Radar altimeter

The differences between CW, FMCW, Pulsed Radar altimeters are as explained below.

**CW Radars :**

They have disadvantage that they cannot measure distance, because they lack the timing mark necessary to allow the system, to time accurately the transmit and receive cycle and convert the measured round-trip-time into range

**FMCW Radars:**

In order to correct for time marking problem, phase or frequency shifting methods can be used. In the frequency shifting method, a signal that constantly changes in frequency around a fixed reference is used to detect stationary objects and to measure the range. In such a Frequency-Modulated Continuous Wave radars (FMCW), the frequency is generally changed in a linear fashion, so that there is an up-and-down or a saw tooth-like alternation in frequency. If the frequency is continually changed with time, the frequency of the echo signal will differ from that transmitted and the difference  $\Delta f$  will be proportional to round trip time  $\Delta t$  and so the range  $R$  of the target too. When a reflection is received, the frequencies can be examined, and by comparing the received echo with the actual step of transmitted frequency, we can do a range calculation similar to using pulses:

<b>FMCW</b>	<b>Pulse</b>
a) Used for lower altitudes b) Low power requirement c) Less complex d) Better resolution e) Applications: All the aircrafts (military & civil) , Strategic missiles like Anti-Radiation Missiles , Smart bombs.	a) Used for long ranges b) High power requirement c) More complex d) Less resolution e) Applications: Satellite applications, e.g : to measure the topography of ocean surface.

So, FMCW Radar Altimeter is targeted for Denoising in view of Defense applications.

### **1.2.2 Major Challenges / Problems in FMCW Received signals**

- a. Because of existence of *phase noise, thermal noise and frequency jitter lead to a non-static frequency spectrum*
- b. Baseband signal is dominated by the leaky ramp frequency which strongly interferes with received signals.
- c. *The ramp signal is mirrored into the baseband spectrum and limits the traditional signal processing performance*
- d. *Due to the steep down-ramp edges that are present in the baseband signal spectrum will have strong peaks at the overtone frequencies (2<sup>nd</sup> Harmonic) of the ramp frequency*
- e. Received signal strength is several magnitudes smaller

### **1.2.3 More about FMCW Radar**

- a. Signal timing is critical to accuracy. (one microsecond error results in a distance error of almost 500ft.)
- b. Position accuracy is directly related to the accuracy of the timing device used, so is important.
- c. FMCW is considered a much more accurate and therefore safer technology.
- d. Inaccuracies may occur over mediums with less than perfect reflectivity qualities (deep snow, ice) or over rapidly changing terrain.
- e. Radio altimeter technology is employed in military applications, most commonly in low-flying craft to avoid radar detection
- f. Most radio altimeter units operate between 4.2 and 4.4GHz in frequency, but only use 150 megahertz within that range.
- g. Despite the proliferation of Global Positioning Satellite ([GPS](#)) technology, almost all *civil aircraft still carry and use at least one radio altimeter due to legislative restrictions on the use of GPS.*

#### **1.2.4 Characteristic features of an FMCW radar:**

The distance measurement is done by comparing the actual frequency of the received signal to a given reference (direct transmitted signal)

The duration of the transmitted signal is much larger than the time required for measuring the installed maximum range of the radar

By suitable choice of frequency deviation per time unit the radar resolution can be varied

With the length of the time of the frequency shift, the maximum range can be varied

A [ferrite circulator](#) shall make the separation of transmit and receive path, when using a single antenna

But using of separate transmitting and receiving antennas is much cheaper in today's commonly used patch antennas in strip-line technology

### **1.3 Contribution of Thesis**

#### **1.3.1 Why DWT based Denoising?**

FT gives the spectral content of the signal, but it gives no information regarding where in time those spectral components appear. Therefore, FT is not a suitable technique for non-stationary signal

STFT gives a fixed resolution at all times

In WT the width of the window is changed as the transform is computed for every single spectral component, which is probably the most significant characteristic of the wavelet transform.

The difference signal which is useful for altitude information is a Non-Stationary Signal. Hence as DWT is very much suitable for such signals and time information is very important for strategic missile applications like Anti-Radiation Missiles, and smart bombs, which are used to destroy enemy radars, communication networks and infrastructure.

DWT has inherent Denoising & Data compression advantages

#### **1.3.2 The main aim of the project is to cover the following points:**

- a. To understand which signal is used for altimeters.
- b. To study & overcome Challenges by signal denoising
- c. Finding of best wavelet for DWT based denoising

- d. Existing method is not Robust to noise-to overcome that
- e. To denoise the low SNR signal with DWT architecture using best wavelet (No literature is present on denoising with best wavelet for altimeter application )
- f. To compare existing conventional filtering method

### 1.3.3 Project Stages and activities performed:

The main work of the project is divided in to two stages consisting of important activities as listed below. The required outcome is achieved by the contributions made in various stages.

#### Stage:1 (activities for finding best wavelet)

- **Finding out the best** wavelet among db1, db2, db3, db4, db6, dmay , sym2, coif1
- Analysis and Synthesis **filter bank architecture development** for above wavelets by MATLAB Programming
- **Suppressing the unwanted signals** from mixed signal in decomposition
- Reconstructing the signal
- **Comparison of reconstructed signals** for all of above wavelets to choose the best

**OUTCOME:** db3 is found as the best choice

#### Stage:2 (activities for Denoising using best wavelet)

- Modeling of low SNR RF Signal and denoising using db3 based DWT architecture and by thresholding
- Comparing o/p **SNR and other parameters** to that of conventional 3 stage HPF
- Study with 1280 samples & 80 samples using MATLAB Models and VHDL implementation and synthesis, comparison of VHDL output with that of MATLAB.
- Implementing of the model ( 80 samples) in VHDL and simulation
- Comparing the VHDL o/p result with that of MATLAB o/p, Synthesis

**OUTCOME:** db3 based DWT found superior in terms of SNR and other Parameters

### 1.3.4 The major programming tasks performed by using MATLAB, and VHDL

#### MATLAB Modelling

- a. I/P Signal modeling
- b. Analysis & Synthesis Filter banks modeling
- c. Denoising
- d. Simulations

#### VHDL Modelling targeting an FPGA Using Xilinx

- Architecture development for Analysis and Synthesis filter banks
- Denoising
- Simulations
- Synthesis

## 1.4 Thesis Organization

**Chapter 1:** Is the introduction describing the motivation behind the work, literature survey, objectives and contributions of the present work.

**Chapter 2:** In this introductions to Wavelet theory is covered and it describes of basics DWT and IDWT

**Chapter 3:** In this basic principles of operation of FMCW Radar altimeter is covered.

**Chapter 4:** Denoising using DWT & IDWT architectures with various wavelets is discussed to identify best wavelet by comparing with conventional filter and discussing the results

**Chapter 5:** Describes Experimental Setup for FMCW received signal modelling & simulations and other signal specifications used.

**Chapter 6:** In this is it is discussed about Denoising of FMCW Radar altimeter RF signal with 3 level DWT Architecture using best wavelet i.e db3 , comparison with conventional 3 stage HPF method, results and Conclusions.

# Chapter 2

## Introduction to Wavelet Theory

### 2.1 What are wavelets?

A wavelet is a waveform of effectively limited duration that has an average value of zero. Sinusoids do not have limited duration -- they extend from minus to plus infinity. And where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric.



Sine wave



Wavelet (10db)

### 2.2 What is Wavelet Analysis?

For many signals, **FOURIER ANALYSIS** is extremely useful because the signal's frequency content is of great importance. So why do we need other techniques, like wavelet analysis?

Fourier analysis has a serious *drawback*. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell **when** a particular event took place. If the signal properties do not change much over time -- that is, if it is what is called a **stationary** signal -- this drawback isn't very important. However, most interesting signals contain numerous non stationary or transitory characteristics: drift, trends, abrupt changes, and



beginnings and ends of events. These characteristics are often the most important part of the signal, and Fourier analysis is not suited to detecting them.



Everywhere around us are signals that can be analyzed. For example, there are seismic tremors, human speech, engine vibrations, medical images, financial data, music, and many other types of signals. **Wavelet analysis** is a new and promising set of tools and techniques for analyzing these signals.

## 2.3 Why Wavelet Transform?

### Firstly, what is a transform and why is it needed?

Mathematical transformations are applied to signals to obtain further information from that signal that is not readily available in the raw signal. Let us consider a time-domain signal as a **raw** signal, and a signal that has been "transformed" by any of the available mathematical transformations as a **processed** signal. There are a number of transformations that can be applied, among which the Fourier transforms are probably by far the most popular. Most of the signals in practice are **TIME-DOMAIN** signals in their raw format. That is, whatever that signal is measuring, is a function of time. In other words, when we plot the signal one of the axes is time (independent variable), and the other (dependent variable) is usually the amplitude. When we plot time-domain signals, we obtain a **time-amplitude representation** of the signal. This representation is not always the best representation of the signal for most signal processing related applications. In many cases, the most distinguished information is hidden in the frequency content of the signal. The **frequency spectrum** of a signal is basically the frequency components (spectral components)

of that signal. The frequency spectrum of a signal shows what frequencies exist in the signal.

Intuitively, frequency is the rate of change of a particular parameter. If the parameter changes rapidly, then it is said to possess a HIGH frequency and if it changes slowly it is said to possess a LOW frequency. If this parameter does not change at all, then we say it has zero frequency, or no frequency. Frequency is measured in cycles/second, or with a more common name, in "Hertz".

Let us compare the following figures with different frequencies.

The first one is a sine wave at 3 Hz, the second one at 10 Hz, and the third one at 50 Hz.

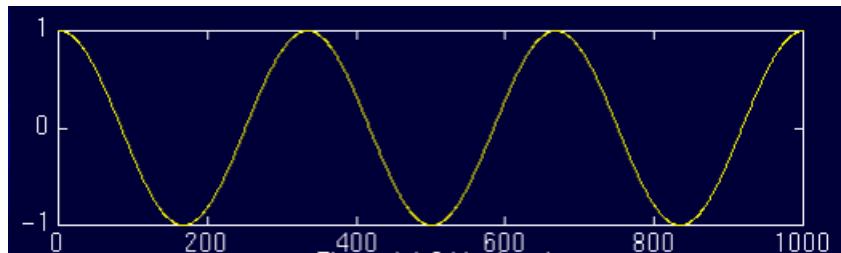


Figure 2.1

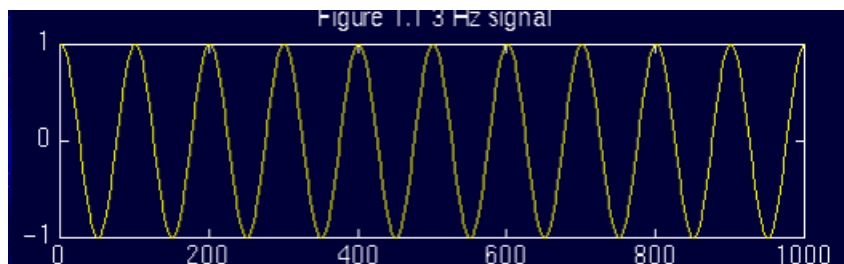


Figure 2.2

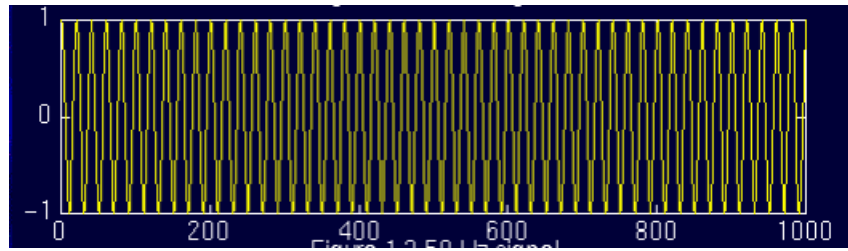


Figure 2.3

So how do we measure frequency, or how do we find the frequency content of a signal? The answer is **FOURIER TRANSFORM (FT)**. If the FT of a signal in time domain is taken, the frequency-amplitude representation of that signal is obtained. In other words, we now have a plot with one axis being the frequency and the other being the amplitude. This plot tells us how much of each frequency exists in our signal.

The following figure shows the FT of the 50 Hz signal:

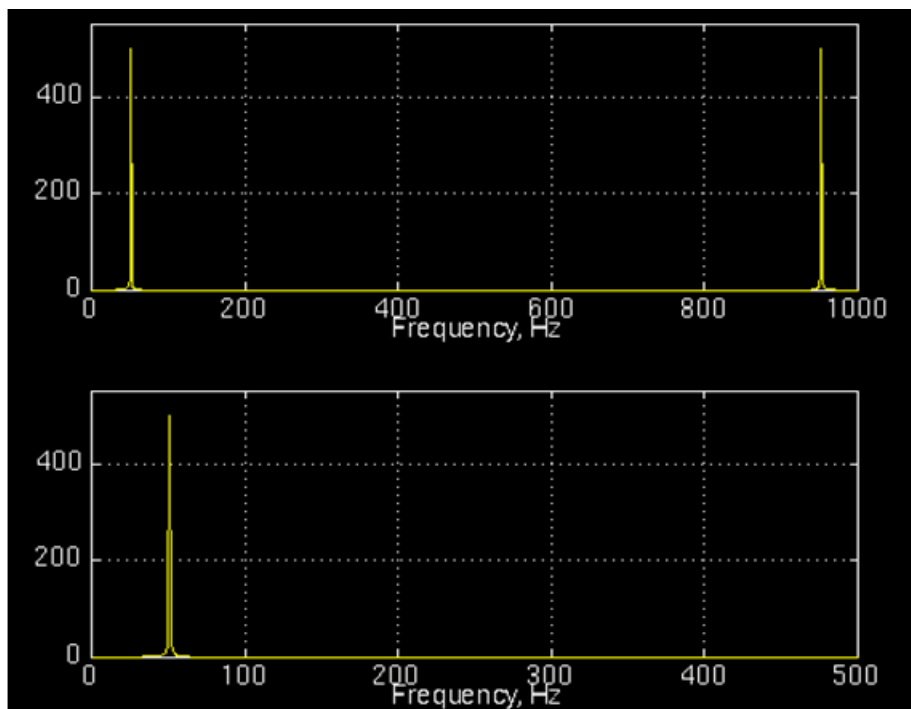


Figure 2.4

**Note that two plots are given in Figure 2.4.** The bottom one plots only the first half of the top one. Due to reasons that are not crucial to know at this time, the

frequency spectrum of a real valued signal is always symmetric. The top plot illustrates this point. However, since the symmetric part is exactly a mirror image of the first part, it provides no additional information, and therefore, this symmetric second part is usually not shown.

### **Why do we need the frequency information?**

Often times, the information that cannot be readily seen in the time-domain can be seen in the frequency domain. Consider an ECG signal (Electro Cardiograph, graphical recording of heart's electrical activity). The typical shape of a healthy ECG signal is well known to cardiologists. Any significant deviation from that shape is usually considered to be a symptom of a pathological condition. This pathological condition, however, may not always be quite obvious in the original time-domain signal. Cardiologists usually use the time-domain ECG signals which are recorded on strip-charts to analyze ECG signals. Recently, the new computerized ECG recorders/analyzers also utilize the frequency information to decide whether a pathological condition exists. A pathological condition can sometimes be diagnosed more easily when the frequency content of the signal is analyzed. This, of course, is only one simple example why frequency content might be useful. Today Fourier transforms are used in many different areas including all branches of engineering. There are many other transforms that are used quite often by engineers and mathematicians. Hilbert transform, short-time Fourier transform, Wigner distributions, the Radon Transform, and of course our **featured transformation**, the **Wavelet Transform**, constitute only a small portion of a huge list of transforms that are available at engineer's and mathematician's disposal. Every transformation technique has its own area of application, with advantages and disadvantages, and the wavelet transform (WT) is no exception.

For a better understanding of the need of the **WT** let's look at the FT more closely. FT (as well as WT) is a reversible transform, that is, it allows going back and forwarding between the raw and processed (transformed) signals. However, only either of them is available at any given time. That is, no frequency information is

available in the time-domain signal, and no time information is available in the Fourier transformed signal. The natural question that comes to mind is that is it necessary to have both the time and the frequency information at the same time?

The answer depends on the particular application and the nature of the signal in hand. Recall that the FT gives the frequency information of the signal, which means that it tells us how much of each frequency exists in the signal, but it does not tell us **when in time** these frequency components exist. This information is not required when the signal is so-called **stationary**.

Signals whose frequency content does not change in time are called **stationary signals**. In other words, the frequency content of stationary signals does not change in time. In this case, one does not need to know **at what times frequency components exist**, since **all frequency components exist at all times!!!** .

**Example:**

$x(t) = \cos(2\pi \cdot 10 \cdot t) + \cos(2\pi \cdot 25 \cdot t) + \cos(2\pi \cdot 50 \cdot t) + \cos(2\pi \cdot 100 \cdot t)$  is a stationary signal, because it has frequencies of 10, 25, 50, and 100 Hz at any given time instant.

This signal is plotted below:

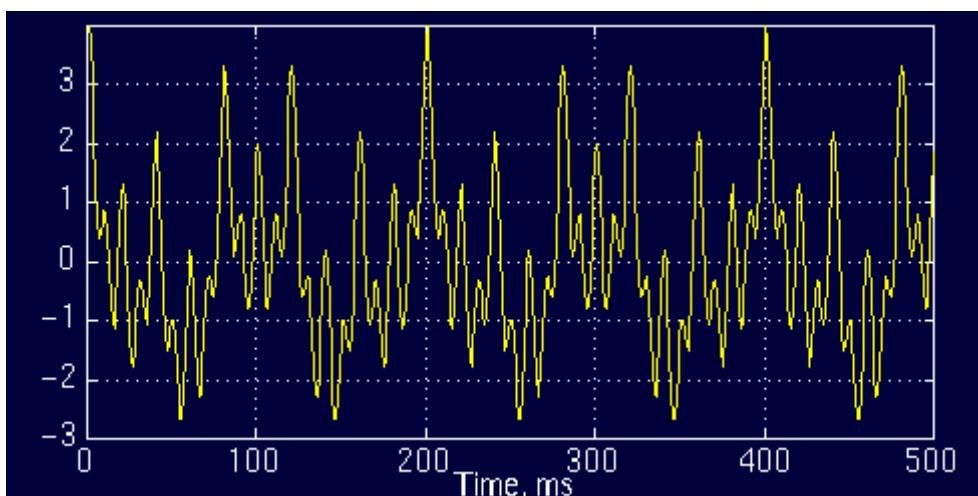


Figure 2.5

And the following is its FT:

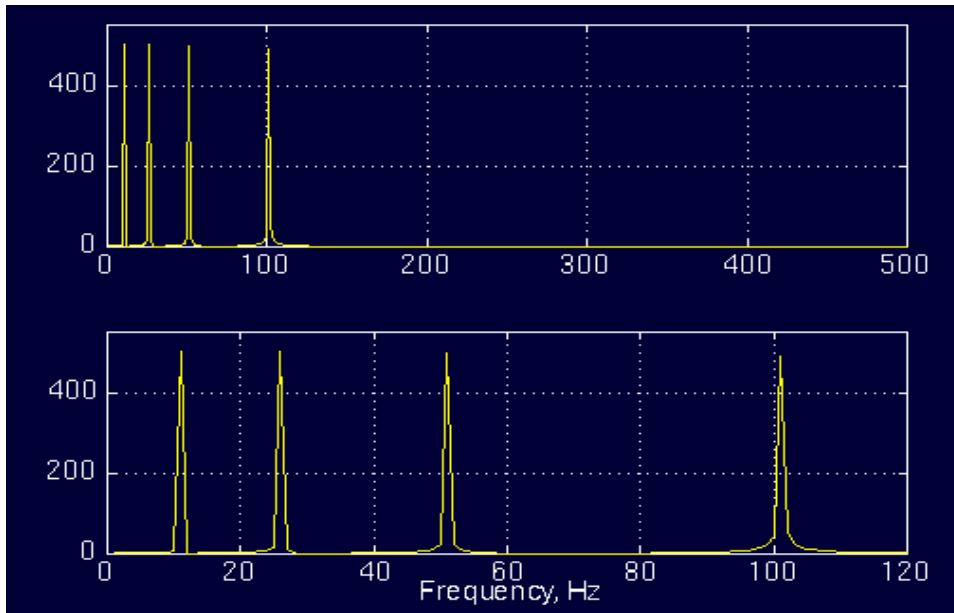


Figure 2.6

The top plot in Figure 2.5 is the (half of the symmetric) frequency spectrum of the signal in Figure 2.4. The bottom plot is the zoomed version of the top plot, showing only the range of frequencies that are of interest to us. Note the four spectral components corresponding to the frequencies 10, 25, 50 and 100 Hz.

Contrary to the signal in Figure 2.4, the following signal is not stationary. Figure 2.7 plots a signal whose frequency constantly changes in time. This signal is known as the "chirp" signal. This is a non-stationary signal.

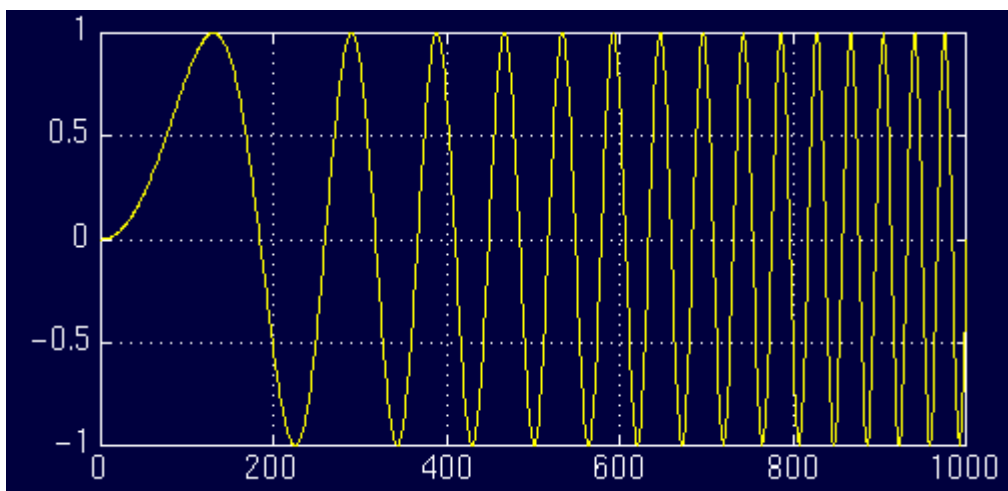


Figure 2.7

**Example:**

Figure 2.8 plots a signal with four different frequency components at four different time intervals, hence a non-stationary signal. The interval 0 to 300 ms has a 100 Hz sinusoid, the interval 300 to 600 ms has a 50 Hz sinusoid, the interval 600 to 800 ms has a 25 Hz sinusoid, and finally the interval 800 to 1000 ms has a 10 Hz sinusoid.

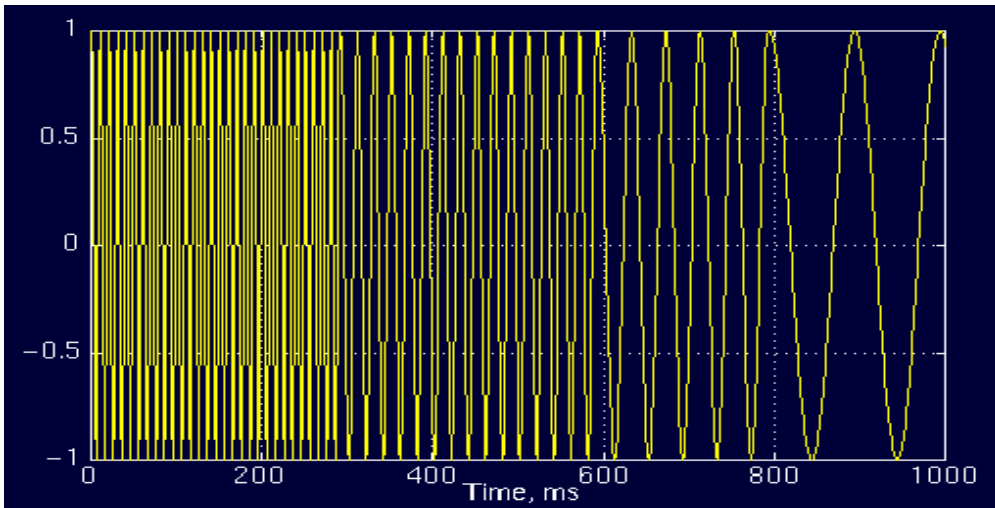


Figure 2.8

And the following is its FT:

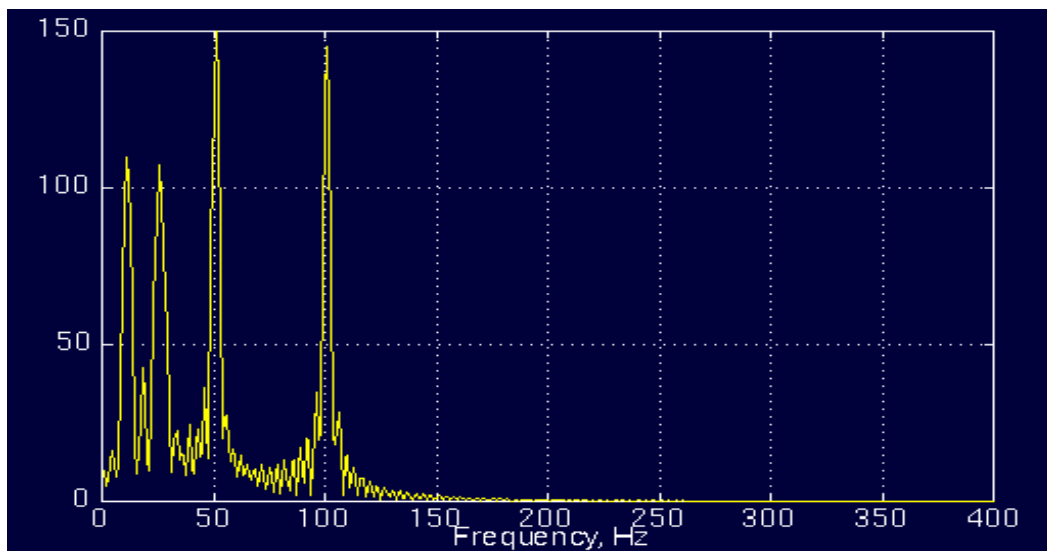


Figure 2.9

The ripples are due to sudden changes from one frequency component to another, which have no significance. Note that the amplitudes of higher frequency

components are higher than those of the lower frequency ones. This is due to fact that higher frequencies last longer (300 ms each) than the lower frequency components (200 ms each).

For the first signal, plotted in Figure 2.5, consider the following question:

**At what times (or time intervals), do the frequency components occur?**

**Answer:** At all times! Remember that in stationary signals, all frequency components that exist in the signal exist throughout the entire duration of the signal. There is 10 Hz at all times, there is 50 Hz at all times, and there is 100 Hz at all times.

Now, consider the same question for the non-stationary signal in Figure 2.7 or in Figure 2.8.

**At what times the frequency components occur?**

For the signal in Figure 2.8, we know that in the first interval we have the highest frequency component, and in the last interval we have the lowest frequency component. For the signal in Figure 2.7, the frequency components change continuously. Therefore, for these signals the frequency components **do not** appear at all times!

Now compare the Figures 2.6 and 2.9. The similarity between these two spectrums should be apparent. Both of them show four spectral components at exactly the same frequencies, i.e., at 10, 25, 50, and 100 Hz. Other than the ripples, and the difference in amplitude (which can always be normalized), the two spectrums are almost identical, although the corresponding time-domain signals are not even close to each other. Both of the signals involve the same frequency components, but the first one has these frequencies at all times, the second one has these frequencies at different intervals. So, how come the spectrums of two entirely different signals look very much alike? Recall that the FT gives the spectral content of the signal, but it gives no information regarding **where in time those spectral components appear**.



Therefore, FT is not a suitable technique for non-stationary signal, with one exception:

FT can be used for non-stationary signals, if we are only interested in what spectral components exist in the signal, but not interested where these occur. However, if this information is needed, i.e., if we want to know, what spectral component occur at what time (interval) , then Fourier transform is not the right transform to use. For practical purposes it is difficult to make the separation, since there are a lot of practical stationary signals, as well as non-stationary ones. Almost all biological signals, for example, are non-stationary. Some of the most famous ones are ECG (electrical activity of the heart, electrocardiograph), EEG (electrical activity of the brain, electroencephalograph), and EMG (electrical activity of the muscles).

Once again please note that, the FT gives what frequency components (spectral components) exist in the signal. When the time localization of the spectral components is needed, a transform giving the **TIME-FREQUENCY REPRESENTATION** of the signal is needed.

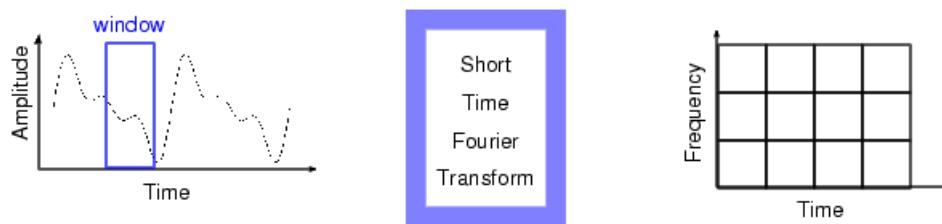
**ULTIMATE SOLUTION:    WAVELET TRANSFORM**

## **2.4 WAVELET TRANSFORM**

It provides the time-frequency representation. Wavelet transform is capable of providing the time and frequency information simultaneously, hence giving a time-frequency representation of the signal. For example, in EEGs, the latency of an event-related potential is of particular interest (Event-related potential is the response of the brain to a specific stimulus like flash-light, the latency of this response is the amount of time elapsed between the onset of the stimulus and the response).The knowledge of **Short Time Fourier Transform (STFT)** is essential to study WT since the WT was developed as an alternative to the STFT.

## 2.4.1 SHORT TIME FOURIER TRANSFORM:

In an effort to correct the deficiency caused by Fourier analysis, Dennis Gabor (1946) adapted the Fourier transform to analyze only a small section of the signal at a time -- a technique called *windowing* the signal. Gabor's adaptation, called the *Short-Time Fourier Transform* (STFT), maps a signal into a two-dimensional function of time and frequency.

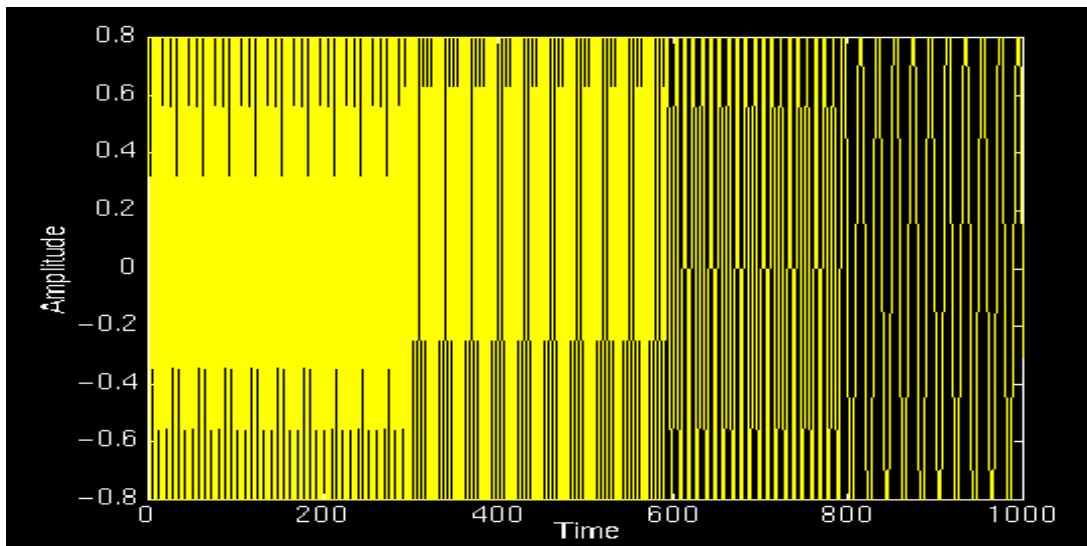


The STFT represents a sort of compromise between the time- and frequency-based views of a signal. It provides some information about both when and at what frequencies a signal event occurs. However, you can only obtain this information with limited precision, and that precision is determined by the size of the window. While the STFT compromise between time and frequency information can be useful, the drawback is that once you choose a particular size for the time window, that window is the same for all frequencies. Many signals require a more flexible approach -- one where we can vary the window size to determine more accurately either time or frequency.

There is only a minor difference between STFT and FT. In STFT, the signal is divided into small enough segments, where these segments (portions) of the signal can be assumed to be stationary. For this purpose, a window function " $w$ " is chosen. The width of this window must be equal to the segment of the signal. This window function is first located to the very beginning of the signal. That is, the window function is located at  $t=0$ . Let's suppose that the width of the window is " $T$ " sec. At this time instant ( $t=0$ ), the window function will overlap with the first  $T/2$  seconds (Assume that all time units are in seconds). The window function and the signal are then multiplied. By doing this, only the first  $T/2$  seconds of the signal is being chosen, with the appropriate weighting of the window (if the window is a rectangle,

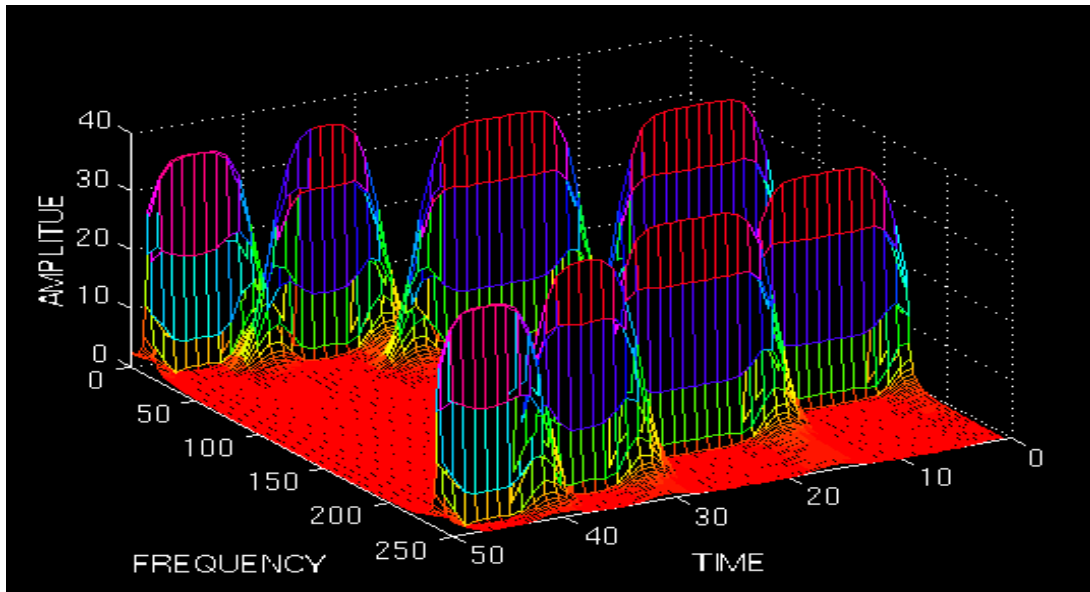
with amplitude "1", then the product will be equal to the signal). Then this product is assumed to be just another signal, whose FT is to be taken. In other words, FT of this product is taken, just as taking the FT of any signal. The result of this transformation is the FT of the first  $T/2$  seconds of the signal. If this portion of the signal is stationary, as it is assumed, then there will be no problem and the obtained result will be a true frequency representation of the first  $T/2$  seconds of the signal.

The next step would be shifting this window (for some  $t_1$  seconds) to a new location, multiplying with the signal, and taking the FT of the product. This procedure is followed; until the end of the signal is reached by shifting the window with " $t_1$ " seconds intervals. Consider the following non stationary signal



Figure

In this signal, there are four frequency components at different times. The interval 0 to 250 ms is a simple sinusoid of 300 Hz, and the other 250 ms intervals are sinusoids of 200 Hz, 100 Hz, and 50 Hz, respectively. Apparently, this is a non-stationary signal. Now, let's look at its STFT:



Figure

Note that the graph is symmetric with respect to midline of the frequency axis. Remember that, although it was not shown, FT of a real signal is always symmetric, since STFT is nothing but a windowed version of the FT, it should come as no surprise that STFT is also symmetric in frequency. The symmetric part is said to be associated with negative frequencies, an odd concept which is difficult to comprehend, fortunately, it is not important; it suffices to know that STFT and FT are symmetric.

Note that there are four peaks corresponding to four different frequency components. Also note that, unlike FT, **these four peaks are located at different time intervals along the time axis.** Remember that the original signal had four spectral components located at different times.

Now we have a true time-frequency representation of the signal. We not only know what frequency components are present in the signal, but we also know where they are located in time.

The problem with STFT is the fact whose roots go back to what is known as the **Heisenberg Uncertainty Principle**. This principle originally applied to the momentum and location of moving particles, can be applied to time-frequency

information of a signal. Simply, this principle states that one cannot know the exact time-frequency representation of a signal, i.e., one cannot know what spectral components exist at what instances of times. What one can know is the **time intervals** in which certain **band of frequencies** exist, which is a **resolution** problem.

The problem with the STFT has something to do with the **width** of the window function that is used. To be technically correct, this width of the window function is known as **the support** of the window. If the window function is narrow, then it is known as **compactly supported**.

Narrow windows give good time resolution, but poor frequency resolution. Wide windows give good frequency resolution, but poor time resolution; furthermore, wide windows may violate the condition of stationary. The problem, of course, is a result of choosing a window function, once and for all, and uses that window in the entire analysis. The answer, of course, is application dependent: If the frequency components are well separated from each other in the original signal, then we may sacrifice some frequency resolution and go for good time resolution, since the spectral components are already well separated from each other. However, if this is not the case, then a good window function could be more difficult than finding a good stock to invest in.

By now, it is quite evident how the wavelet transforms have come into play.

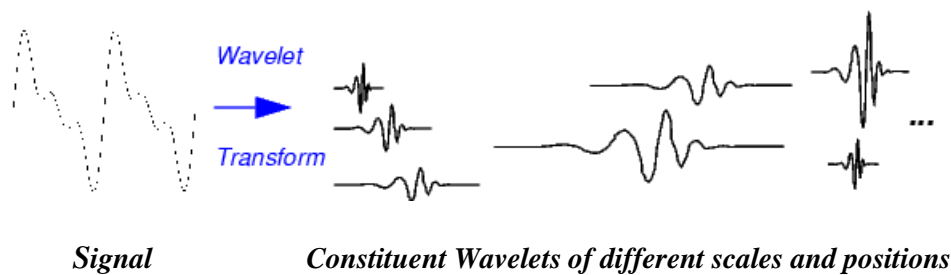
**NUMBER OF DIMENSIONS:** Wavelet analysis can be applied to two-dimensional data (*images*) and, in principle, to higher dimensional data.

## 2.5 CONTINUOUS WAVELET TRANSFORM

The continuous wavelet transform was developed as an alternative approach to the short time Fourier transforms to overcome the resolution problem. The ***continuous wavelet transform*** (CWT) is defined as the sum over all time of the signal multiplied by scaled, shifted versions of the wavelet function. The CWT equation is as follows:

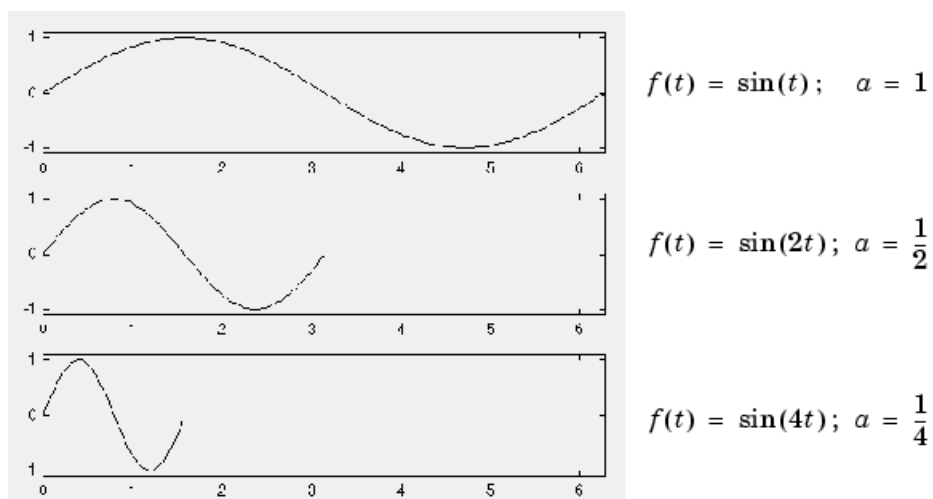
$$C(\text{scale}, \text{position}) = \int_{-\infty}^{\infty} f(t)\psi(\text{scale}, \text{position}, t)dt$$

The results of the CWT are many *wavelet coefficients*  $C$ , which are a function of scale and position. Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelets of the original signal.

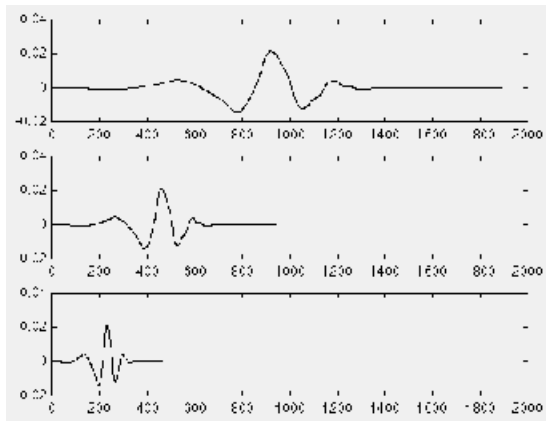


### 2.5.1 SCALING:

Scaling a wavelet simply means stretching (or compressing) it. To go beyond colloquial descriptions such as "stretching," we introduce the *scale factor*, often denoted by the letter 'a'. If we're talking about sinusoids, for example, the effect of the scale factor is very easy to see.



The scale factor works exactly the same with wavelets. The smaller the scale factor, the more "compressed" the wavelet.



$$f(t) = \psi(t) ; a = 1$$

$$f(t) = \psi(2t) ; a = \frac{1}{2}$$

$$f(t) = \psi(4t) ; a = \frac{1}{4}$$

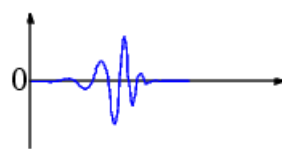
It is clear from the diagram that, for a sinusoid, the scale factor is related (inversely) to the radian frequency. Similarly, with wavelet analysis, the scale is related to the frequency of the signal.

### The main differences between the STFT and the CWT:

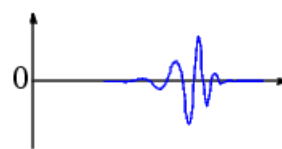
1. The Fourier transforms of the windowed signals are not taken, and therefore single peak will be seen corresponding to a sinusoid, i.e., negative frequencies are not computed.
2. The width of the window is changed as the transform is computed for every single spectral component, which is probably the most significant characteristic of the wavelet transform.

### 2.5.2 SHIFTING:

Shifting a wavelet simply means delaying (or hastening) its onset.



Wavelet function  
 $\psi(t)$



Shifted wavelet function  
 $\psi(t-k)$

## 2.6 DISCRETE WAVELET TRANSFORM

Calculating wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data. What if we choose only a subset of scales and positions at which to make our calculations?

It turns out, rather remarkably, that if we choose scales and positions based on powers of two -- so-called *dyadic* scales and positions -- then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the *discrete wavelet transform* (DWT).

An efficient way to implement this scheme using filters was developed in 1988 by Mallat. The Mallat algorithm is in fact a classical scheme known in the signal processing community as a *two-channel sub band coder*. This very practical filtering algorithm yields a *fast wavelet transform* -- a box into which a signal passes, and out of which wavelet coefficients quickly emerge. Let's examine this in more depth.

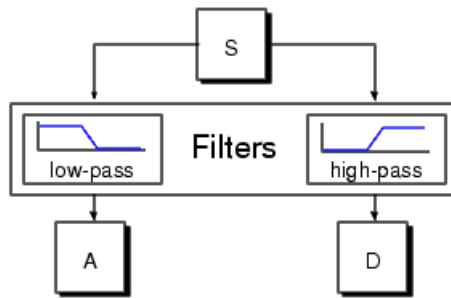
### 2.6.1 ONE STAGE FILTERING – APPROXIMATIONS AND DETAILS

For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavor or nuance. Consider the human voice. If you remove the high-frequency components, the voice sounds different, but you can still tell what's being said. However, if you remove enough of the low-frequency components, you hear gibberish.

In wavelet analysis, we often speak of *approximations and details*. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components.

The filtering process, at its most basic level, looks like this.

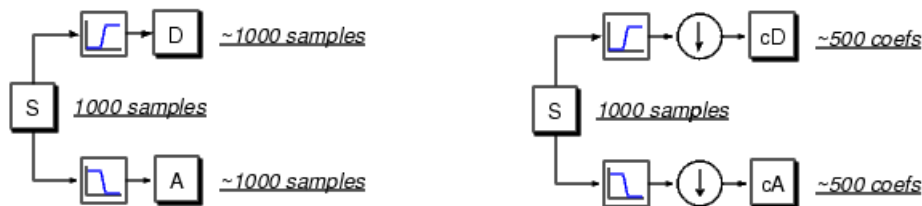




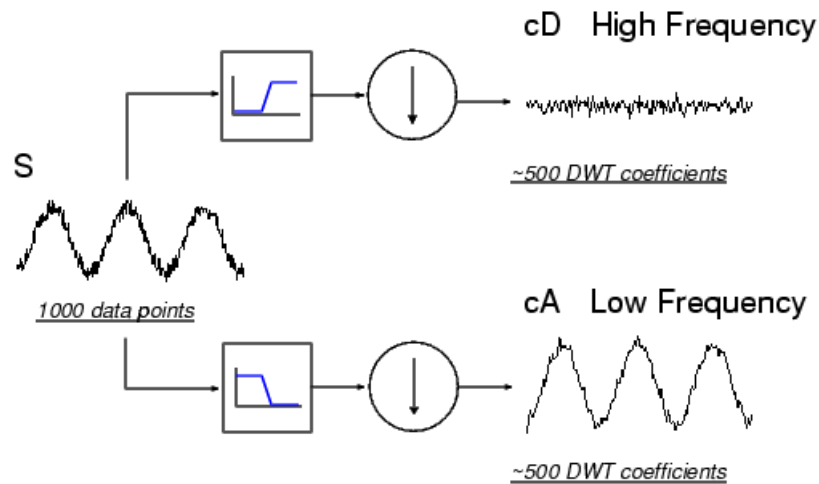
The original signal,  $s$ , passes through two complementary filters and emerges as two signals.

Unfortunately, if we actually perform this operation on a real digital signal, we wind up with twice as much data as we started with. Suppose, for instance, that the original signal  $S$  consists of 1000 samples of data. Then the resulting signals will each have 1000 samples, for a total of 2000.

These signals  $A$  and  $D$  are interesting, but we get 2000 values instead of the 1000 we had. There exists a more subtle way to perform the decomposition using wavelets. By looking carefully at the computation, we may keep only one point out of two in each of the two 2000-length samples to get the complete information. This is the notion of down sampling. We produce two sequences called  $cA$  and  $cD$ .



The process on the right, which includes down sampling, produces DWT coefficients. To gain a better appreciation of this process, let's perform a one-stage discrete wavelet transform of a signal. Our signal will be a pure sinusoid with high-frequency noise added to it. Here is our schematic diagram with real signals inserted into it.



The MATLAB<sup>®</sup> code needed to generate *s*, *cD*, and *cA* is

```
s = sin (20.*linspace (0, pi, 1000)) + 0.5.*rand (1, 1000);
[cA, cD] = dwt(s,'db2');
```

Where *db2* is the name of the wavelet we want to use for the analysis.

Note that the detail coefficients *cD* are small and consist mainly of a high-frequency noise, while the approximation coefficients *cA* contain much less noise than does the original signal.

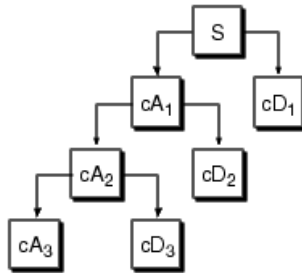
```
[length (cA) length (cD)]
```

```
Answer = 501 501.
```

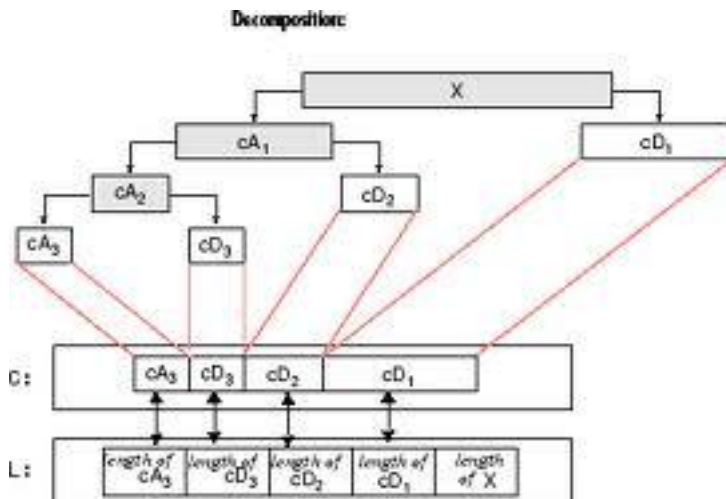
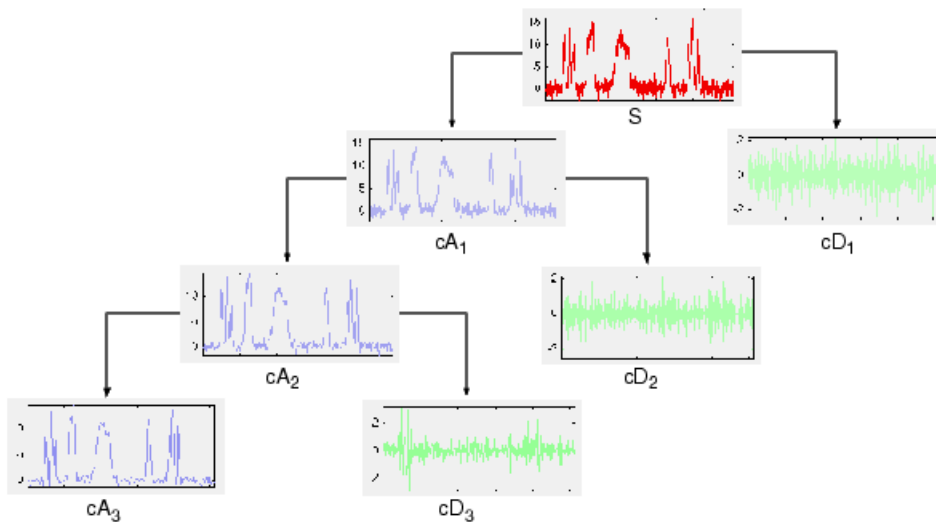
You may observe that the actual lengths of the detail and approximation coefficient vectors are slightly *more* than half the length of the original signal. This has to do with the filtering process, which is implemented by convolving the signal with a filter. The convolution "smears" the signal, introducing several extra samples into the result.

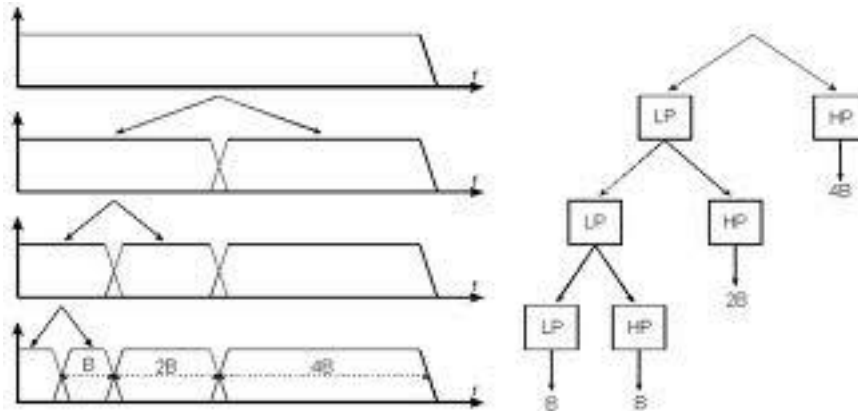
## 2.6.2 MULTILEVEL DECOMPOSITION

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower resolution components. This is called the *wavelet decomposition tree*.



Looking at a signal's wavelet decomposition tree can yield valuable information.

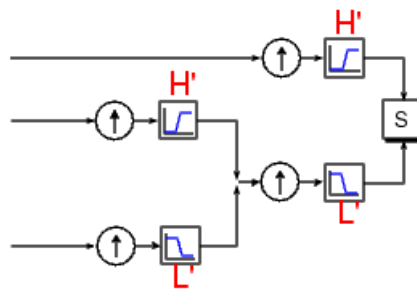




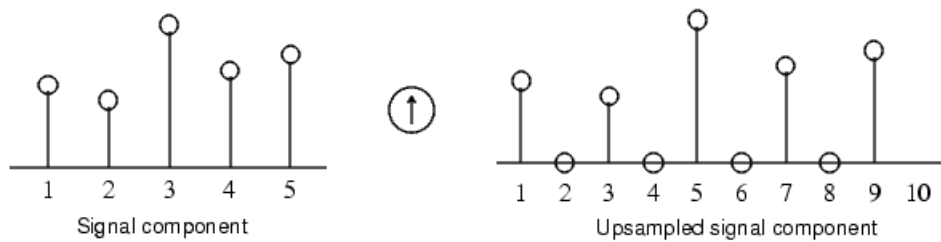
**Fig: Frequency spectrum by decomposing**

### 2.6.3 WAVELET RECONSTRUCTION

We've learned how the discrete wavelet transform can be used to analyze, or decompose, signals and images. This process is called *decomposition* or *analysis*. The other half of the story is how those components can be assembled back into the original signal without loss of information. This process is called *reconstruction*, or *synthesis*. The mathematical manipulations that effect synthesis is called the inverse discrete wavelet transform (IDWT).



Where wavelet analysis involves filtering and down sampling, the wavelet reconstruction process consists of up sampling and filtering. Up sampling is the process of lengthening a signal component by inserting zeros between samples.

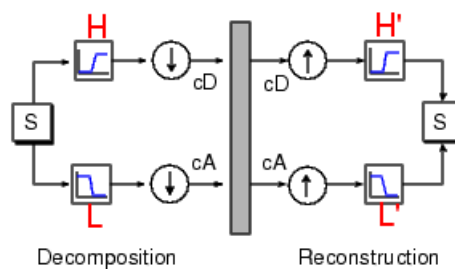


### 2.6.4 RECONSTRUCTION FILTERS

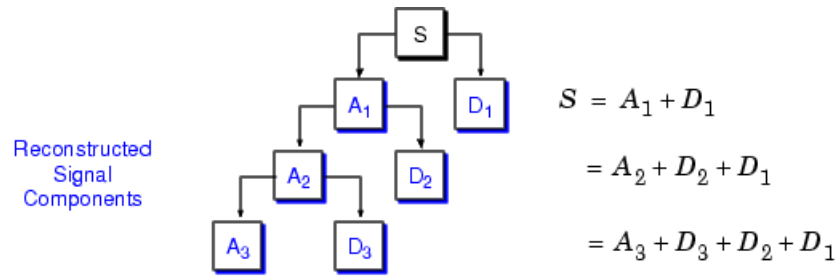
The down sampling of the signal components performed during the decomposition phase introduces a distortion called aliasing. It turns out that by carefully choosing filters for the decomposition and reconstruction phases that are closely related (but not identical), we can "cancel out" the effects of aliasing. The low- and high-pass decomposition filters ( $L$  and  $H$ ), together with their associated reconstruction filters ( $L'$  and  $H'$ ), form a system of what is called *quadrature mirror filters*

### 2.6.5 RECONSTRUCTING APPROXIMATIONS AND DETAILS

We have seen that it is possible to reconstruct our original signal from the coefficients of the approximations and details. It is also possible to reconstruct the approximations and details themselves from their coefficient vectors.

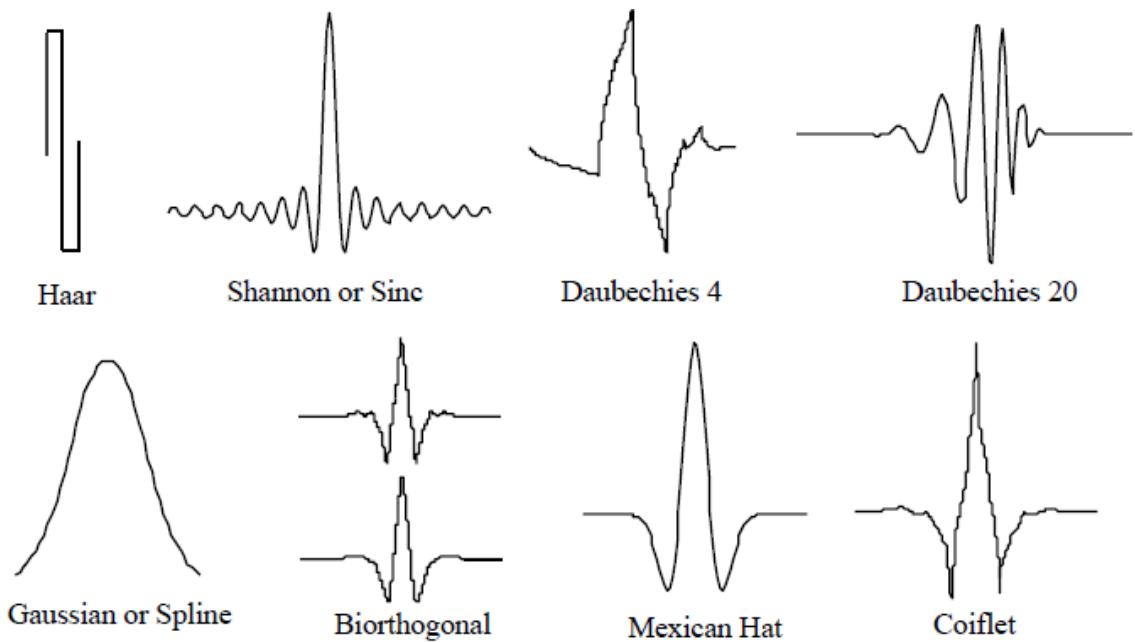


Extending this technique to the components of a multilevel analysis, we find that similar relationships hold for all the reconstructed signal constituents. That is, there are several ways to reassemble the original signal:

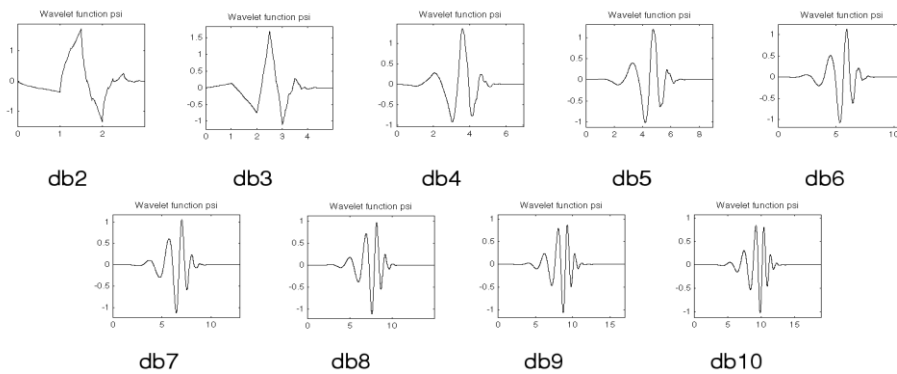


Where S is the signal, A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>... are the Approximations and D<sub>1</sub>, D<sub>2</sub>, D<sub>3</sub>... are the Details.

## 2.7 DIFFERENT TYPES OF WAVELETS



### 2.7.1 Types of Daubechies wavelets



# Chapter 3

## 3.1 Basics of FMCW Radar Altimeter

Radar Altimeter is also called as RF Altimeter or Radio Altimeter.

**Radar Altimeter:** It is an instrument for determining **elevation / altitude / range** for the Applications like Aeroplanes, Helicopters, Missiles, Satellites etc.. The types of Radar altimeter are Pulse modulated, Frequency Modulated Continuous Wave (FMCW) Radar altimeter.

### 3.1.1 FMCW Radar Altimeter:

- It is considered as a much more accurate and therefore safer technology for applications other than in satellites & FMCW radar in general is cheaper and it uses Continuous transmitting energy.
- Sweeping of carrier frequency is done between the two frequencies and with the **length of the time of the frequency shift, the maximum range can be varied**
- It uses the **time taken for a radio signal to reflect** from the surface back to the aircraft to measure the distance above ground
- The distance measurement is done by **comparing the actual frequency of the received signal** to a given reference (direct transmitted signal)

### 3.1.2 Applications of FMCW Radar altimeter:

- Is used to **measure height above ground level** during landing in commercial and military aircraft, altitude of missiles during level flights.
- It is also a component of **terrain avoidance warning** systems, warning the pilot if the aircraft is flying too low, or if there is rising terrain ahead

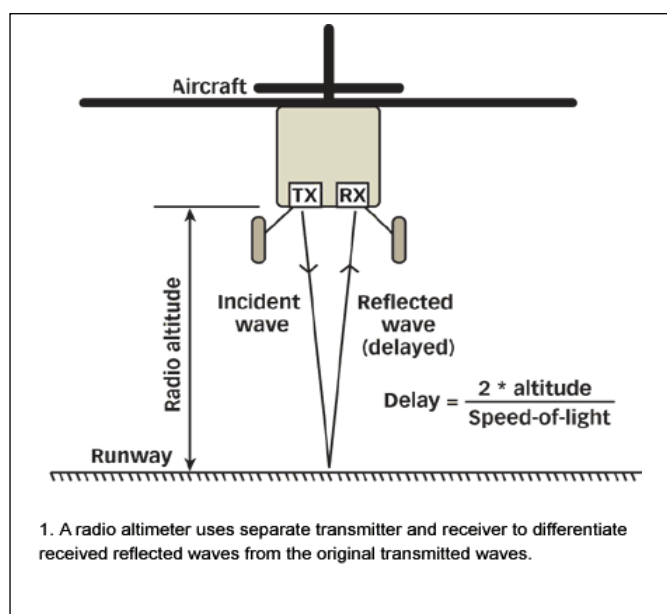
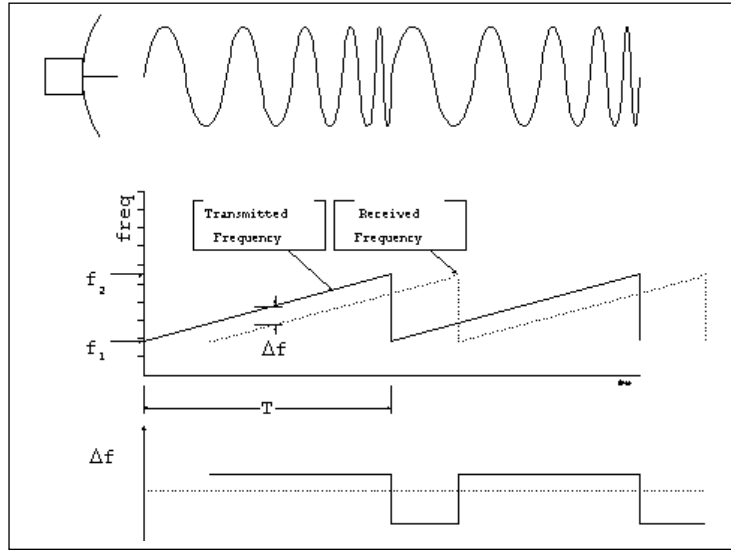


Fig3.1: Showing Radar Altimeter application



**Fig.3.2:** Tx, Rx signals used to get altitude information.

### 3.1.3 Range formulae

$$R = \frac{c_0 \cdot |\Delta t|}{2} = \frac{c_0 \cdot |\Delta f|}{2 \cdot (df/dt)}$$

Where:

$c_0$  = speed of light =  $3 \cdot 10^8$  m/s

$\Delta t$  = measured time-difference [s]

R = distance (altimeter to terrain) [m]

$df/dt$  = transmitters frequency shift per unit time

$\Delta f$  = measured frequency-difference [Hz]

The reflected signal is a noisy one and hence denoising is very important process to be done before estimation of altitude / range is done.

In general Digital signal processing is done for denoising and extracting received signal. For this, the method proposed in this paper i.e DWT Based denoising with db3 wavelet (even at low SNR cases) also can be used for noisy received signal in FMCW Radar Application. The Tx, Rx signals can be represented as

**Transmitted signal (with saw tooth FM modulation):**

$$s_t(t) = \cos(2\pi f_c t + 2\pi \int_0^t f_t d\tau)$$

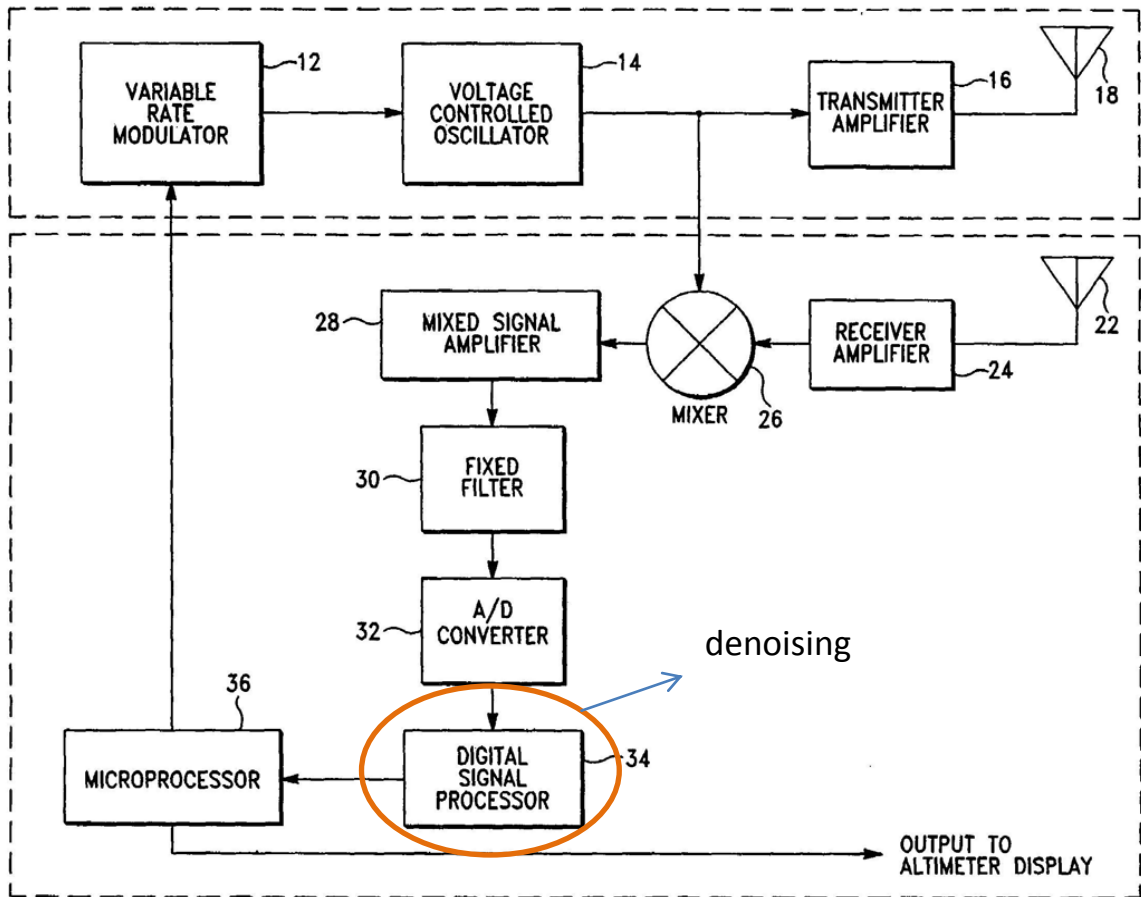
**Received Signal (delayed):**

$$S_r(t) = S_t(t-t_d)$$



Mixer process :  $S_t(t) * S_r(t) = \text{Cos}(f_t+f_r) - \text{Cos}(f_t-f_r)$   
 $\Delta f = f_r - f_t = \text{Base band frequency}$

**Fig.3.3: Functional block diagram having DSP Block**



# Chapter 4

## 4.1 Denoising using DWT & IDWT architectures with Various wavelets

A three level of Decomposition and Reconstruction is shown in below figures 4.1 & 4.2.

If we have a sequence of discrete values  $s(n) = \text{carrier} + \text{noise}$ , sampled from a continuous function  $s(t)$  at intervals of  $t_s$ . A discrete wavelet transform step decomposes the sequence  $S(n)$  into two sequences  $a^1$  and  $d^1$  by means of a low-pass filter  $h = (h_n)=h_0$  and a high-pass filter  $g = (g_n)=h_1$  followed by a down sampling of order 2. The sequence  $a^1$  is called approximation and contains low-frequency information of  $s(n)$ . The sequence  $d^1$  is said to be the details and contains high-frequency information.  $h$  and  $g$  are called Decomposition filters.

### 4.2 MATLAB MODEL FOR DENOISING

The filtering process can be modeled / written as

$$a^1 = [s(n) \circledast h] \downarrow 2, \text{ followed by left shift by 2}$$

$$d^1 = [s(n) \circledast g] \downarrow 2, \text{ followed by left shift by 2}$$

$\circledast$  indicates circular convolution, and  $\downarrow$  indicates down sampling.

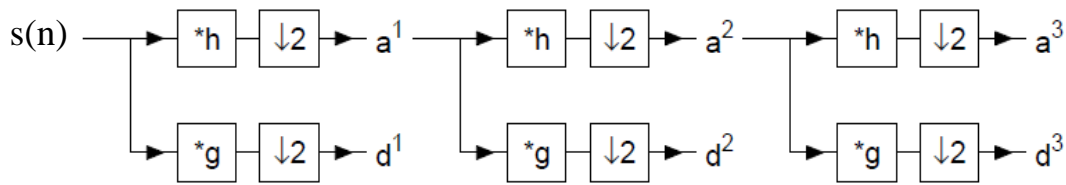
In a numerical implementation, a sequence of finite length is convoluted with filters  $h$  and  $g$ , which are also of finite length (one talks of FIR filters). For the convolution to be well defined, the signal has to be extended (padded) at both ends e.g. periodically or by zeros.

The down sampling operator  $\downarrow p$ , maps a sequence  $(x_n)$  to the sequence  $(y_n) = (x_{pn})$ , i.e. only every  $p$ -th sample is kept.  $S(n)$  can be reconstructed from  $a^1$  and  $d^1$  by means of the reconstruction filters  $g_0, g_1$  and up sampling or say

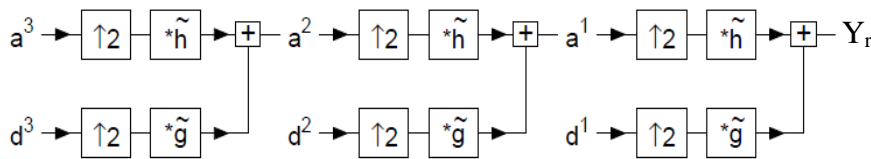
$$\tilde{h} = g_0 \text{ and } \tilde{g} = g_1, \text{ i.e.}$$

$$\text{Reconstructed output} = y_r = (a^1 \uparrow 2 \circledast \tilde{h}) + (d^1 \uparrow 2 \circledast \tilde{g})$$

The up sampling operator  $\uparrow$  inserts zeros into a sequence.



**Figure 4.1:** 3 levels of a DWT decomposition filter bank algorithm. The sequence  $s(n)$  is split into details  $d^1$ ,  $d^2$  and  $d^3$  and an approximation  $a^3=0$  ( $a^3$  will be made as 0 to eliminate unwanted low frequency that lie in  $a^3$  approximation)



**Figure 4.2:** 3 levels of a DWT reconstruction filter bank algorithm. The sequence  $Y_r$  is reconstructed from details  $d^1$ ,  $d^2$  and  $d^3$  and an approximation  $a^3$  (was made=0).

Unwanted signal frequency of known range (in this case 10Hz signal) is removed by making the corresponding filter coefficients ( in this case  $a^3$ ) as zero and we reconstruct the signal.

The decomposition & reconstruction filter coefficients for various wavelets under study like dmey, coif1, sym2, & debouches db1, db2, db3, db4, db6 can be generated in MATLAB and used along with signal as inputs , i.e. for e.g. for db3 wavelet we use following to generate decomposition low and high pass filters coefficients  $h_0, h_1$  and reconstruction low and high pass filter coefficients  $g_0, g_1$ .

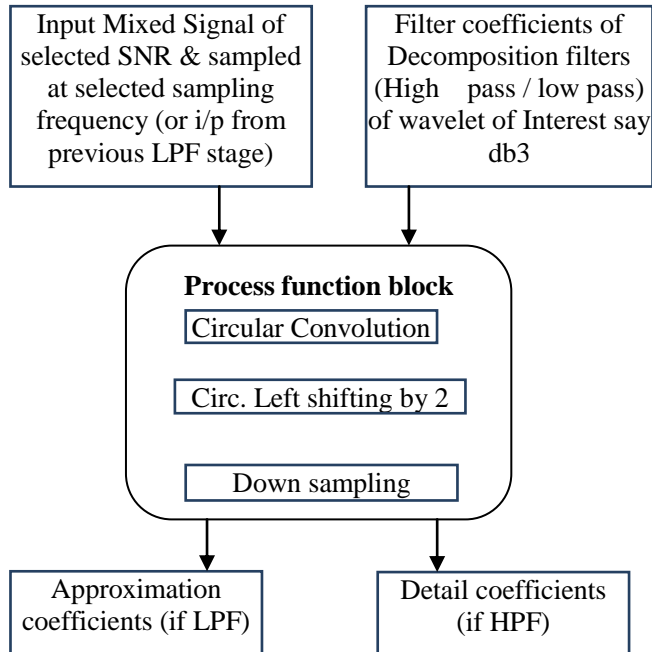
```
dwtmode('per');
wname='db3';
[h0,h1,g0,g1] = wfilters('db3');
```

Input mixed signal that is to be denoised is generated as  $s(n)=160\text{Hz}$  sine wave (carrier ) + 10Hz sine wave (as noise). Signal frequency in this case is chosen more than noise frequency, as this is quite common in microwave frequency applications such as FMCW Radar, cellular communications etc..

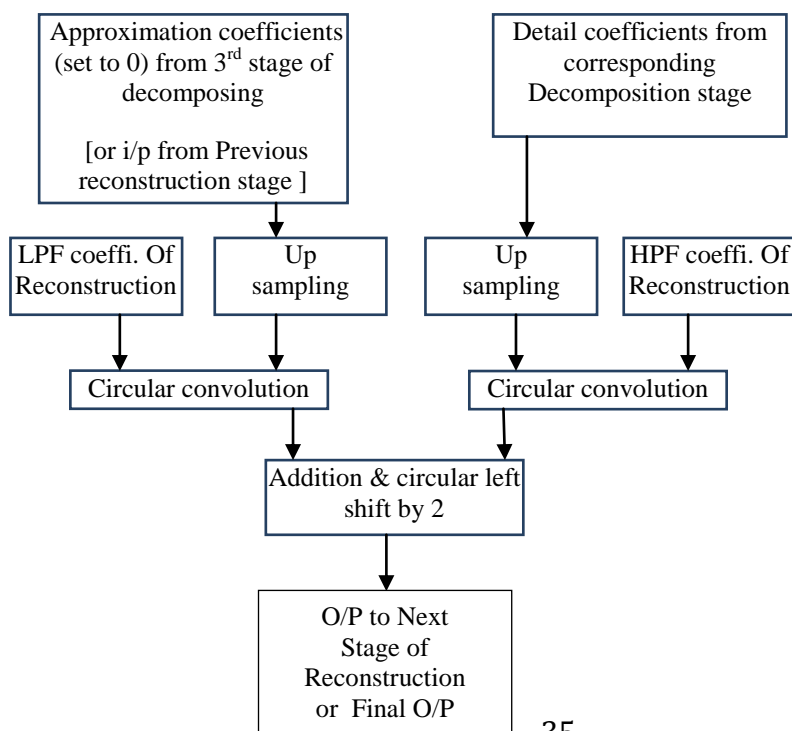
As the carriers of communication systems are of basic sine of cosine wave, and noise if it is of similar in nature and of more amplitude, it will be difficult case of signal separation and any realistic signal can be represented as a combination of sine/cosine waves. Most of the noisy signals seen practically are confined to known range of frequency, hence the basic input mixed signal chosen is two sine waves of 160Hz (carrier or message) and 10Hz (Noise) are selected keeping a view of microwave communications, in which the noise will be of lower side of the frequency range of operation.

Denoising was done by making approximation coefficients to zero to remove unwanted frequency components. In this case it is output of third stage analysis filter bank. Now, a very important activity is modeling of analysis and synthesis filter banks of wavelet chosen, which is very essential for Hardware architecture design in VHDL and prior to which MATLAB model is proved.

#### **4.2.1 Decomposition filter function Model**



#### **4.2.2 Reconstruction filter functional Model**



### 4.3 MATLAB CODE (of main functionality), SNR=0.1 (-20dB) with sampling frequency fs=1280Hz & db3 wavelet

```
fs=1280;
if rem(fs,2)==0; fs=fs-1;end;
ts =1/fs;
t=0:ts:1;

c=60*(sin(2*pi*160*t)+1);
ns=600*(sin(2*pi*10*t)+1);
s=(c+ns);

% Decomposition and reconstruction filters

dwtmode('per');
wname='db3';

[h0,h1,g0,g1] = wfilters('db3');

c_out=convwoconv(s,h0);
x0_lpfc=c_out';
x0_lpfc=circshift(x0_lpfc,-2);
%Circular left shift

a0 = x0_lpfc(2:2:length(x0_lpfc));
% downsampling

function c_out=convwoconv(x,h)

[rx cx]=size(x);
[rhch]=size(h);
if rx>cx x=x';end;
if rh>ch h=h';end;
m=length(x);
n=length(h);
X=[x,zeros(1,n)];
H=[h,zeros(1,m)];
for i=1:n+m-1
Y(i)=0;
for j=1:m
if(i-j+1>0)
Y(i)=Y(i)+X(j)*H(i-j+1);
else
end
end
end
y=Y/128;
y=floor(y);
lx=length(x);
ly=length(y);
y1=y(lx+1:end);
ly1=length(y1);
y1=[y1 zeros(1,ly-ly1)];
y(lx+1:end)=0;
c_out=y1+y;
c_out=c_out(1:lx);
```

### Reconstruction algorithm

```
y3r = zeros(1,2*length(w2));
y3r(1:2:2*length(w2)) = w2(1:length(w2));
% Up sampling
y3r_inv=y3r';

%to calculate cconv(y3r_inv,g1)

c_out=convwoconv(y3r_inv,g1);
yr3c=c_out';

alr = xr3c + yr3c;
alr=circshift(alr,-2);
```

### Plotting of reconstructed signal

```
a=alr;
N1=2^nextpow2(length(a))*2;
az=abs(fft(a,N1));
f1=(0:N1/2-1)*((fs)/N1);
figure;
plot(f1,az(1:N1/2));
title('WL=db3,fs=1280,reconstructed signal s_(rec)');

figure;
[v_azidx_az]=max(az);
stem(ceil(f1(idx_az)),ceil(v_az),'b-o');
```

Denosing and Reconstruction is experimented with **dmey**, **coif1**, **sym2**, & **debouches db1, db2, db3, db4, db6** wavelets at various sampling frequencies (more than nyquist rate) i.e. 512, 1120, 1280Hz. SNR of the input signal varied from 0.1 to 0.0333 (i.e. noise amplitude is 10 times to 30 times that of carrier). All these results of denoised signal are tabulated in **Table-I** & denoised result need to contain only 160Hz component. Based on these results the individual wavelet performance is listed out as follows.

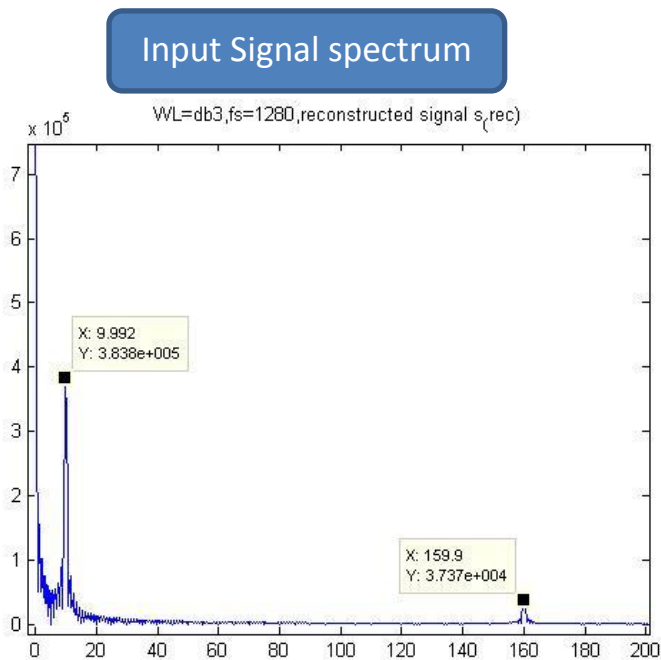
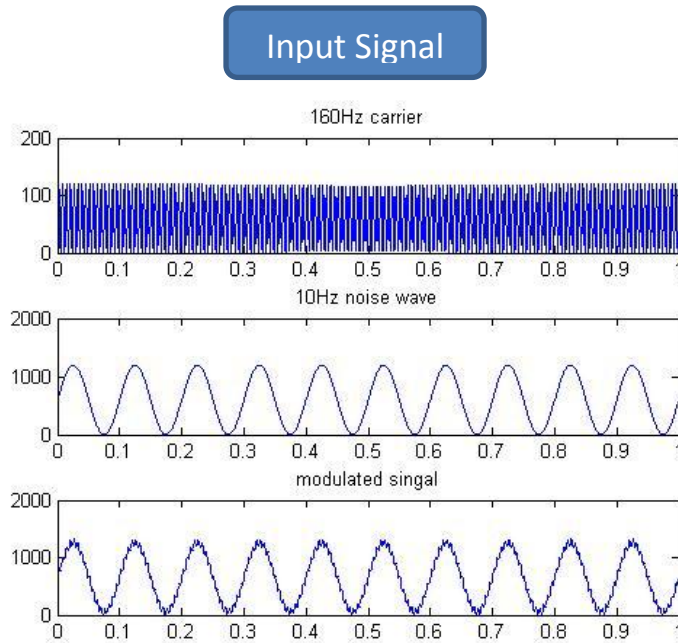
### Best Results:

Wname =db3, fs=1280 (No. of filter coefficients: 6)  
Wname =db4, fs=1280 (No. of filter coefficients: 8)  
Wname =db6, fs=1280 (No. of filter coefficients: 12)  
Wname =dmey, fs=1280 (No. of filter coefficients: 102)

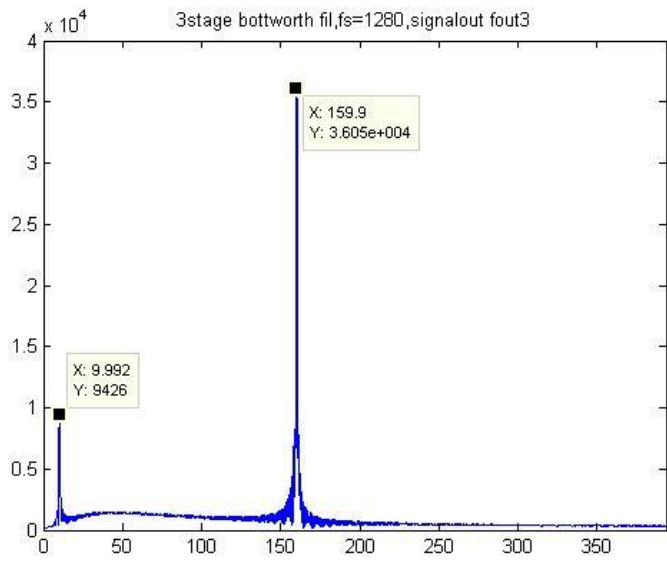
Note: db3 is the best because it offers excellent result & has few no. of filter coefficients.

Keeping in view of Hardware complexity, the wavelet that has less no of filter coefficients is finalized i.e. db3 is the best choice among all.

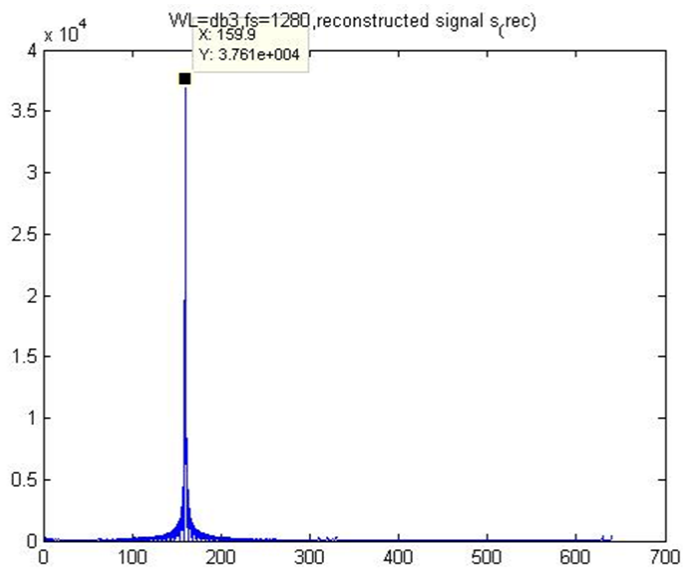
#### 4.4 Signal Model & its frequency spectrum for best wavelet finding



### HPF O/P Signal

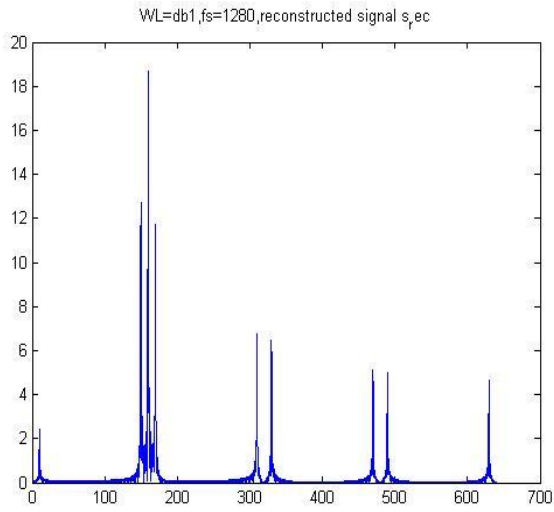


### DWT O/P Signal

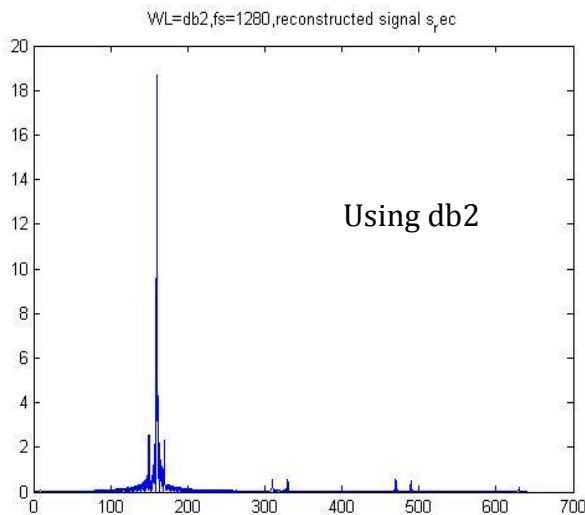




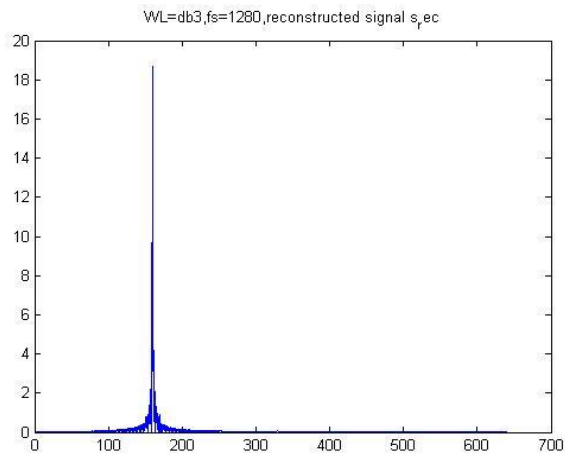
### 4.5 DWT Architecture denoising results using various wavelets (SNR=0.1, -20db) with 0.03 signal and 0.3 noise level



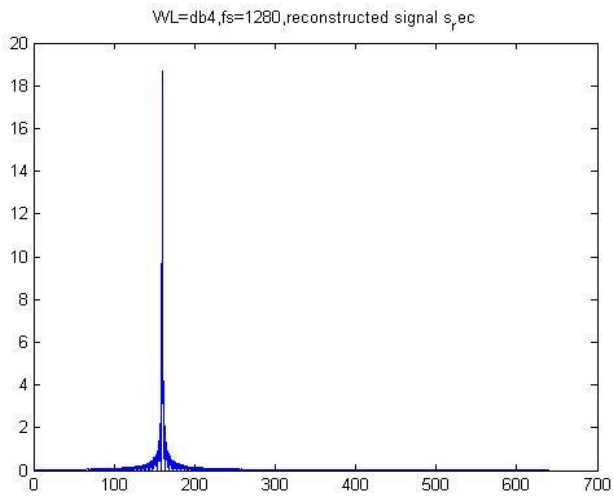
Reconstructed signal Using db1



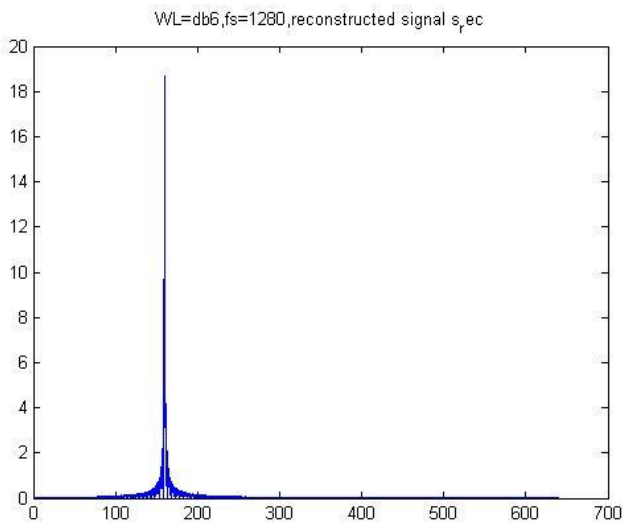
Reconstructed signal Using db2



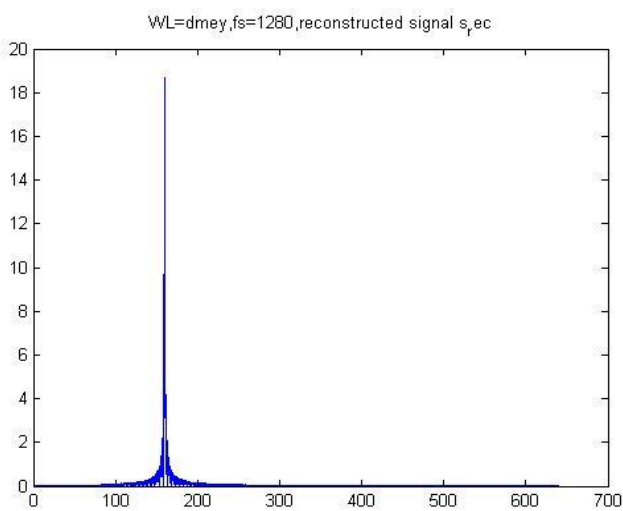
Reconstructed signal Using db3



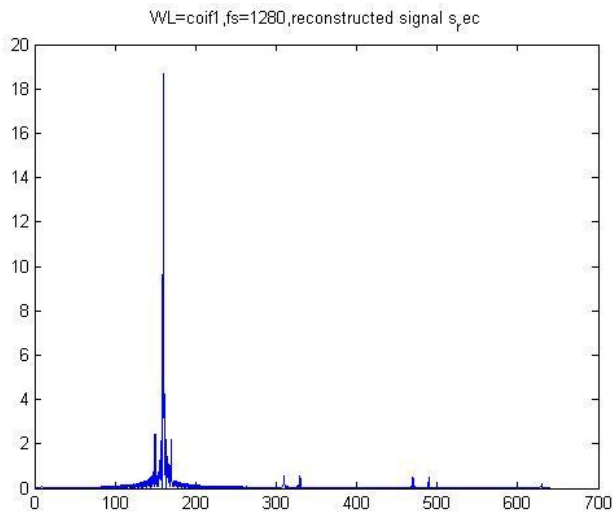
Reconstructed signal Using db4



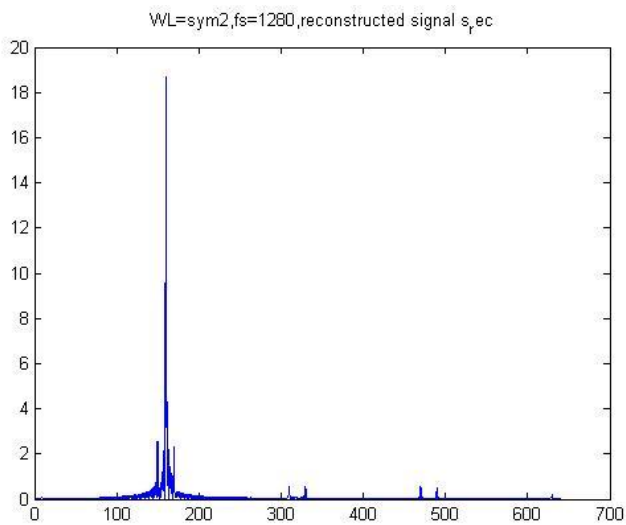
Reconstructed signal Using db6



Reconstructed signal Using dmey

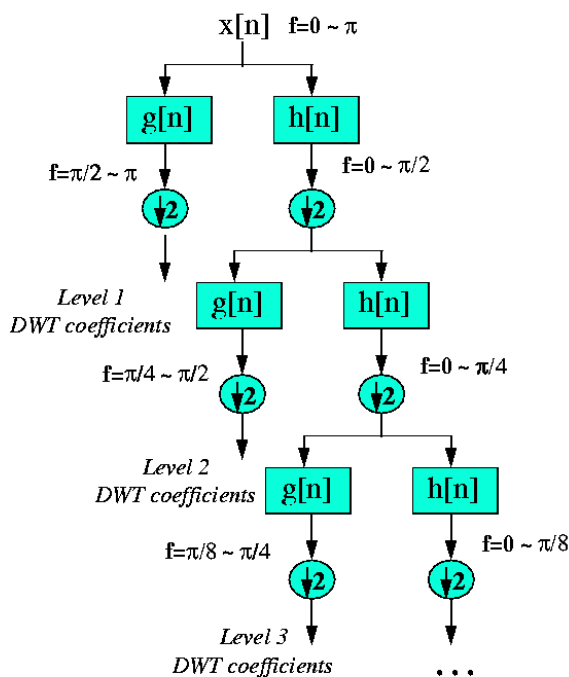


Reconstructed signal Using coif1



Reconstructed signal Using sym2

#### 4.6 DWT performance parameters comparison of various wavelets for denoising



If a signal under study is  $x(t)$  and its sampled version having maximum frequency component  $\pi$  radians is  $x[n]$ . then DWT can be obtained by filtering and down sampling as shown below called decomposition, where  $h[n]$ ,  $g[n]$  are low pass and high pass wavelet filters

Mixed signal SNR selected is 0.1 or -20dB

Note that due to successive subsampling by 2, the signal length must be a power of 2, or at least a multiple of power of 2, in order that the DWT scheme to be efficient.

The input X,Y data set (1280 Samples each) used to calculate Correlation, Regression, R-Square values are:

Mixed signal => 160KHz (i.e X) pure sine wave (which is desired signal after denoising the mixed signal ) + noise ( 5 KHz ramp and its 2<sup>nd</sup> harmonic component). Amplitudes are selected such that SNR is -20dB.

Y=> denoised & reconstructed signal data using DWT or HP Filtering

The parameters results obtained using MATLAB are tabulated in Table-I

The input data set (1280 Samples) used to calculate correlation, regression, R-Square values are

X=> 160Hz pure sine wave (which is desired signal after denoising the mixed signal ) and

Y=> denoised & reconstructed signal data

#### **4.6.1 The correlation, regression, R-Square parameters arrived at by using following MATLAB Functions**

```
tmps=zeros(length(X),1);  
  
for i=1:length(X)  
    tmps(i,1)= (X(i,1)-mean(X))*(Y(i,1)-mean(Y));  
end  
  
CORR = 100*((mean(tmps))/(std(X)*std(Y)));  
  
REGR = regress(X,Y);  
  
RSQR = 100*(1-(norm(X-Y)/(norm(X-mean(X)))));
```

**Table-I: Performance parameter comparison**

Wavelet	Correlation	Regression	R-square
Db1 (nfc=2)	66.4354552	1.000195248	25.30444455
Db2 (nfc=4)	98.4701111	1.001659334	83.01484430
<b>Db3 (nfc=6)</b>	<b>99.8557156</b>	<b>0.999193320</b>	<b>96.36072248</b>
Db4 (nfc=8)	99.9047657	0.999267696	98.14808313
Db6 (nfc=12)	99.9046037	0.999186848	98.13901349
Coif1 (nfc=6)	98.6023453	1.001307221	83.80169801
Dmey (nfc=102)	99.9041269	0.999170852	98.11348185
Sym2 (nfc=4)	98.4701111	1.001659334	83.01484430

**Note:** nfc stands for number of filter coefficients of wavelet filter

B. Using 3 stage High Pass Butterworth filter to remove unwanted signal frequencies.

X=> 160kHz pure sine wave and

Y=> 3 stage HP filter output for same mixed signal input

**Table-II:**

Filter	Correlation	Regression	R-square
1 <sup>st</sup> Order fc=160kHz	80.7955393	0.790323	37.3726827

**SNR i/p:** -20db min

**SNR O/P Results :** a. With DWT denoising: 11db,  
b. With HPF denoising: 5db

From the above results it is confirmed that for low complexity architecture having less number of filter coefficients db3 wavelet is the best. So, this is selected for denoising of FMCW Radar altimeter signal.

# Chapter 5

## 5.1 Experimental Setup and Signal specifications i.r.o FMCW Radar signal for denoising

FMCW Radar Signal Modeling and Denoising Method using DWT with finalized best wavelet db3 in MATLAB

I/p noisy signal to DWT based Denoising architecture consists of following for MATLAB db3 wavelet based denoising Model.

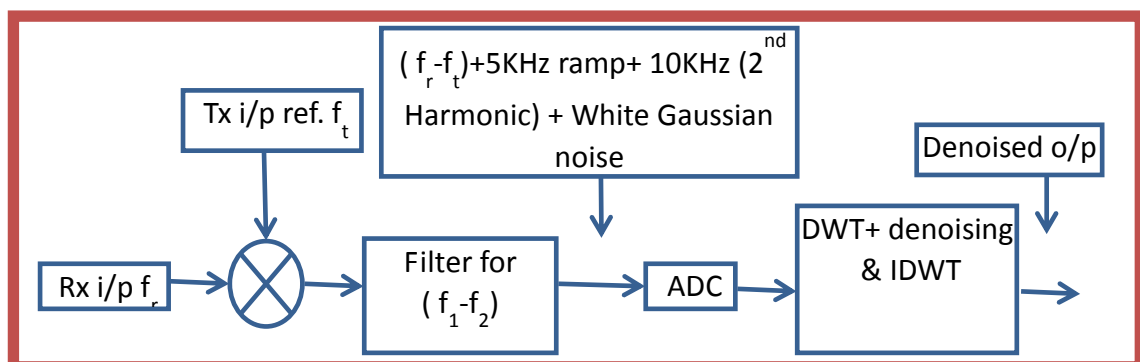
**Difference signal** is  $(f_r - f_t)$  i.e Rx, Tx frequency difference (for 4Km altitude is 160KHz)

**Noise:** Ramp signal 5KHz + 10KHz ( $2^{\text{nd}}$  Harmonic) both with amplitude 10 times to that of difference signal and White Gaussian Noise.

Sampling frequency for MATLAB to be 6 to 20 times in general so it is selected as 1280KHz and number of samples selected for case1 is 1280, case2 is 80 and SNR, Correlation, Regression, R-Square parameters are compared with filtered o/p of 3 stage Butterworth HPF with  $f_c=20\text{KHz}$ .

Denoising is done by hard thresholding on detail coefficients to remove WGN specifically and setting level 3 approximation coefficients to zero to remove 5KHz and 10KHz components along with white noise.

I/P SNR : -26dB (for 1280 samples case)  
-29.86dB (for 80 samples case)



Experimental Setup for Received signal for denoising of FMCW Rxd signal

## 5.2 Specifications

- Transmit frequency would be 4GHz, **Wavelet** : db3 for DWT
- A **baseband (distance) frequency of 160KHZ (for distance of 4 KM)** is used
- Linear Ramp of 5KHz => **a period tramp= 0.2 msec** ,
- $df/dt$  => 1.2MHz for 0.2 msec of ramp period (in general)
- **Two types of Noise:**
  - a) **Random Noise (Optional) : WGN (using randn function of MATLAB)**
  - b) Strong Ramp and its Harmonic leakage interference : 5KHz, 10KHz (its 2<sup>nd</sup> harmonic)
- **Two methods of Denoising studied**
  - High Pass Filter : 1<sup>st</sup> order High pass Filter,  $f_c = 20$  KHz
  - DWT Filtering and thresholding of random noise
- Two cases of study : 1280 samples & 80 samples
- Performance parameters : SNR, Correlation, Regression, R-Square

**I/P SNR : Amplitudes of difference signal and Ramp and its harmonics selected such That SNR of Mixed signal for two cases is**

- a. **-26dB for 1280 samples case**
- b. **-29.86dB for 80 samples case**

# Chapter 6

## 6.1 FMCW Radar Signal Modeling and Denoising Method using DWT with db3 in MATLAB and Comparison with 3 Stage Butterworth HPF

I/p noisy signal to DWT based Denoising architecture consists of following for MATLAB db3 wavelet based denoising Model.

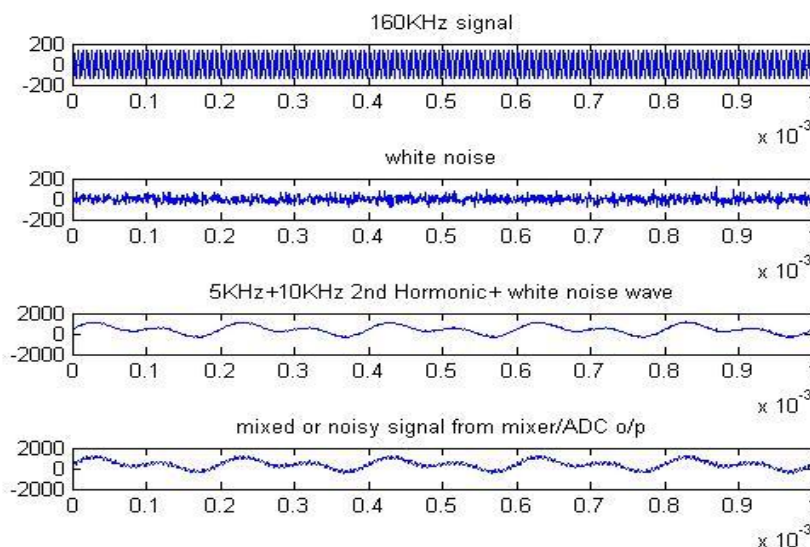
- a. **Difference signal** is  $(f_r - f_t)$  i.e Rx, Tx frequency difference (for 4Km altitude is 160KHz)
- b. **Noise:** Ramp signal 5KHz + 10KHz (2<sup>nd</sup> Harmonic) both with amplitude 10 times to that of difference signal and White Gaussian Noise.

Sampling frequency for MATLAB to be 6 to 20 times in general so it is selected as 1280KHz and number of samples selected for case1 is 1280, case2 is 80 and SNR, Correlation, Regression, R-Square parameters are compared with filtered o/p of 3 stage Butterworth HPF with  $f_c=20$ KHz.

Denoising is done by hard thresholding on detail coefficients to remove WGN specifically and setting level 3 approximation coefficients to zero to remove 5KHz and 10KHz components along with white noise.

I/P SNR : -26dB (for 1280 samples case)  
-29.86dB (for 80 samples case)

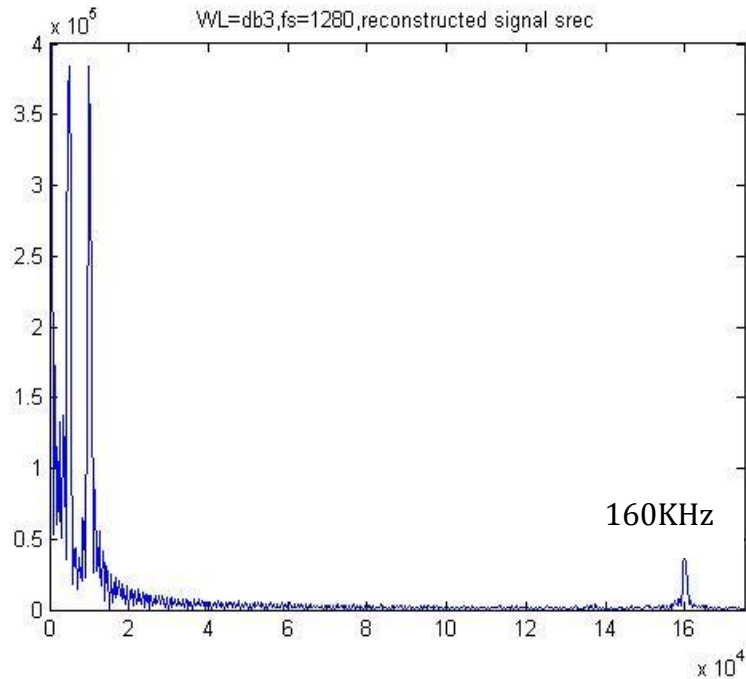
### 6.1.1 Input Signal Model for FMCW Altimeter



Pic: Screen shot of mixed signal to be denoised: Signal:160KHz carrier, Noise:WGN+5KHz+10KHz , and noisy / mixed signal.



6.1.2. Input mixed / Noisy signal frequency spectrum having 5KHz,10KHz, WGN(randn), difference signal of160KHz, 1280 samples, with fs=1280KHz.



## MATLAB Modeling for FMCW Altimeter Denoising

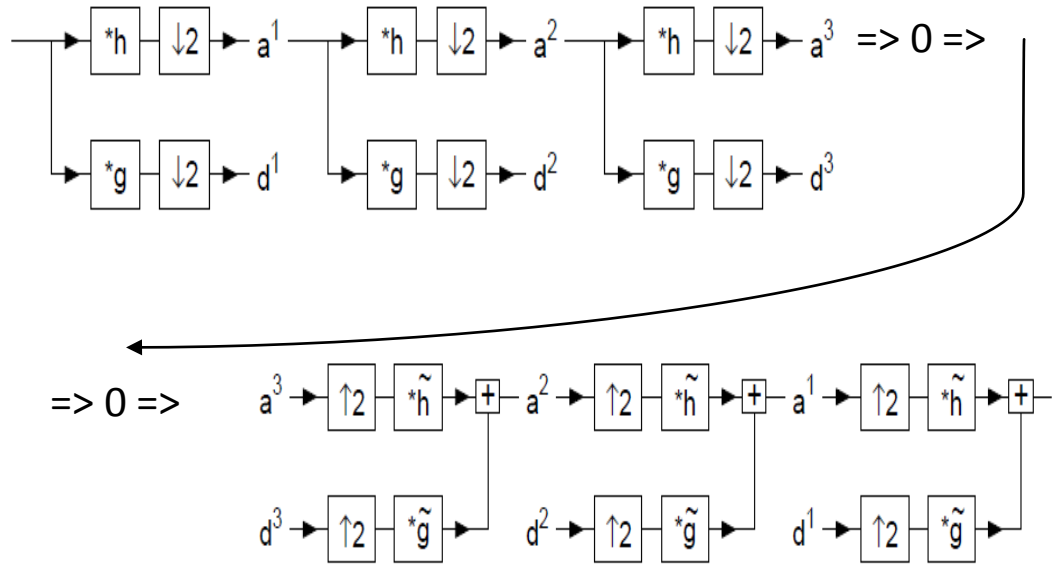
```

1 % Enter fs preferably such that it is 2^n or (2^n) - 1, n is integer
2 %
3 %
4 %
5 clc; clear all; close all;
6 fs=1280e3;
7 if rem(fs,2)==0; fs=fs-1;end;
8
9 ts=1/fs;
10 t=0:ts:1279*ts;
11
12 cs= 60*(sin(2*pi*160e3*t));
13 cs1=cs';
14 ns=600*(sin(2*pi*10e3*t)+ sin(2*pi*5e3*t)+1);
15 wn=10*randn(1,length(cs));
16 n1=wn+ns ;
17 a=cs+n1 ;
18
19 [c,1] = wavedec(n1,3,'db3');
20 [thrv,nkeep] = wdcbm(c,1,3);
  
```

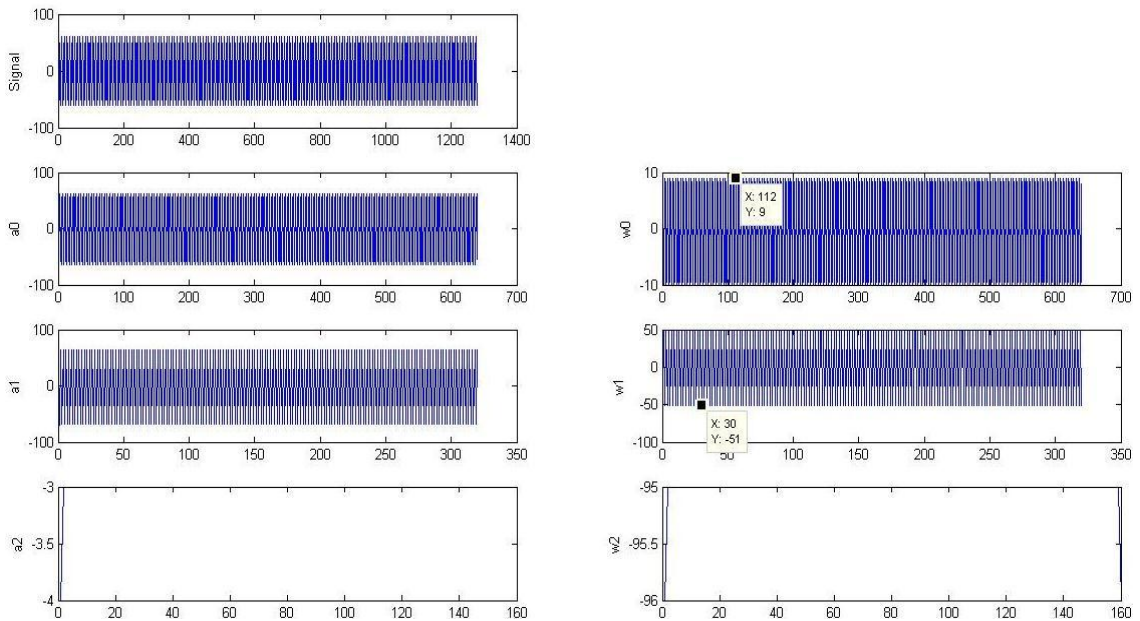
## 6.2 Denoising

Denoising is done by hard thresholding on detail coefficients to remove WGN specifically and setting level 3 approximation coefficients to zero to remove 5KHz and 10KHz components along with white noise.

### 6.2.1. By setting approximation coef. $A_3=0$ : Removal of 5KHz & 10KHz

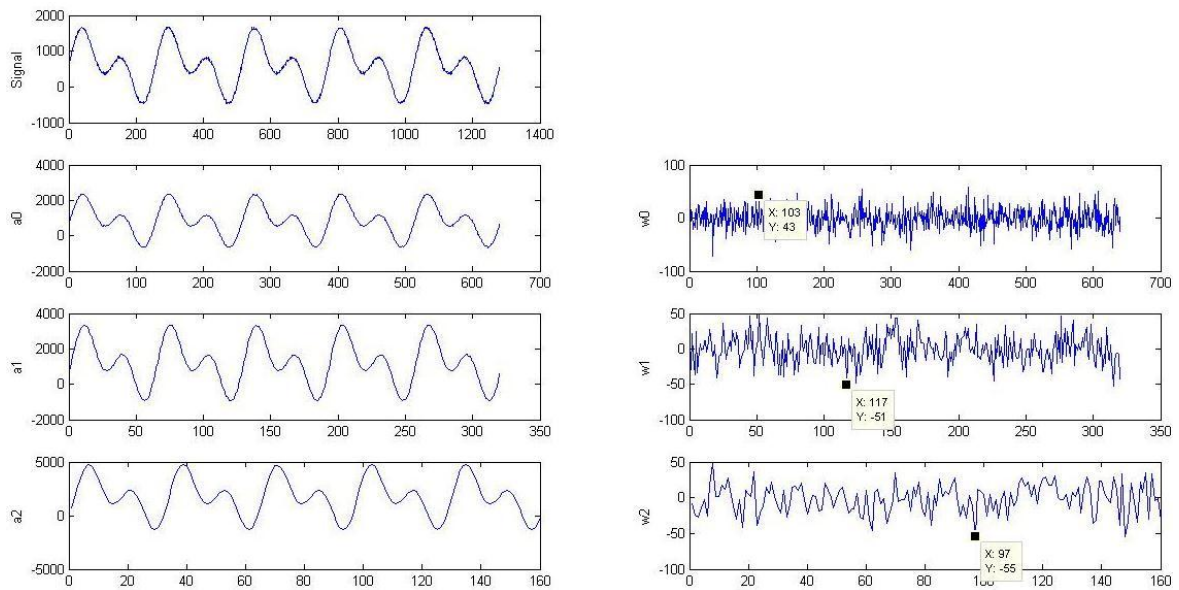


### 6.2.2 By setting detailed coef. $d_1, d_2, d_3$ thresholding such that the threshold is less than signal amplitude (when signal is applied without noise)



**Pic: Showing amplitudes of signal alone in  $w_0, w_1, w_2$  detailed coefficients in MATLAB**

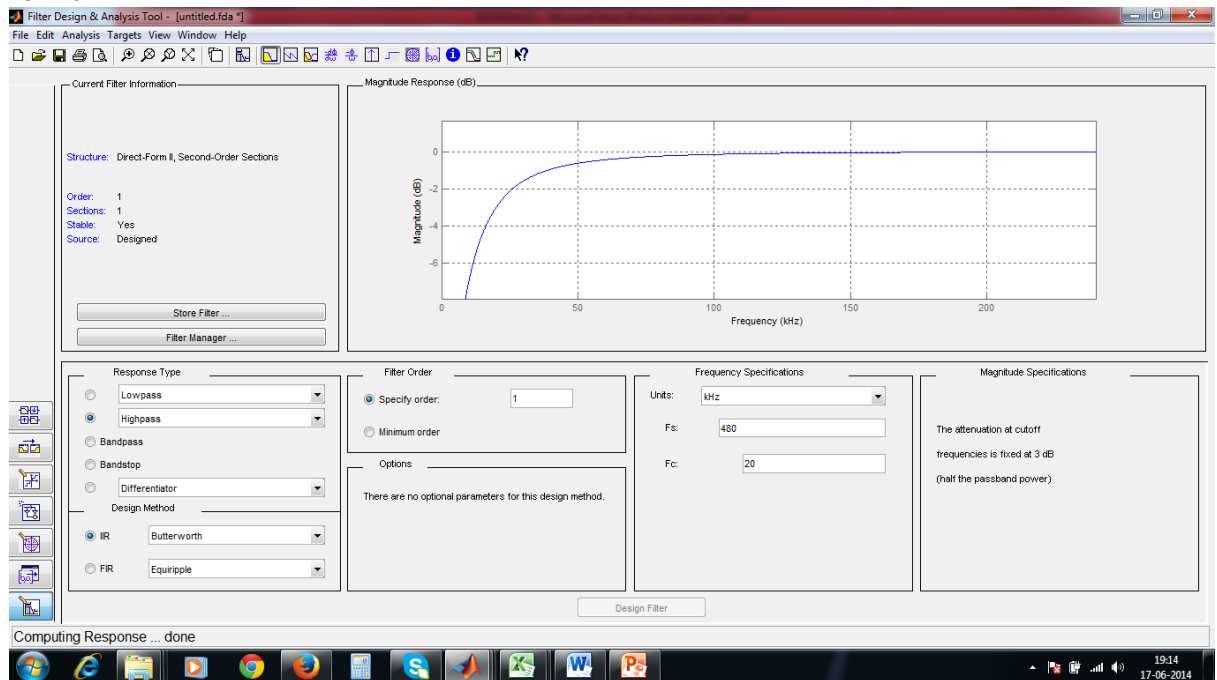
6.2.3 By setting detailed coef.  $d_1, d^2, d^3$  thresholding such that the threshold is less than signal amplitude (below is noise signal alone is shown to decide threshold)



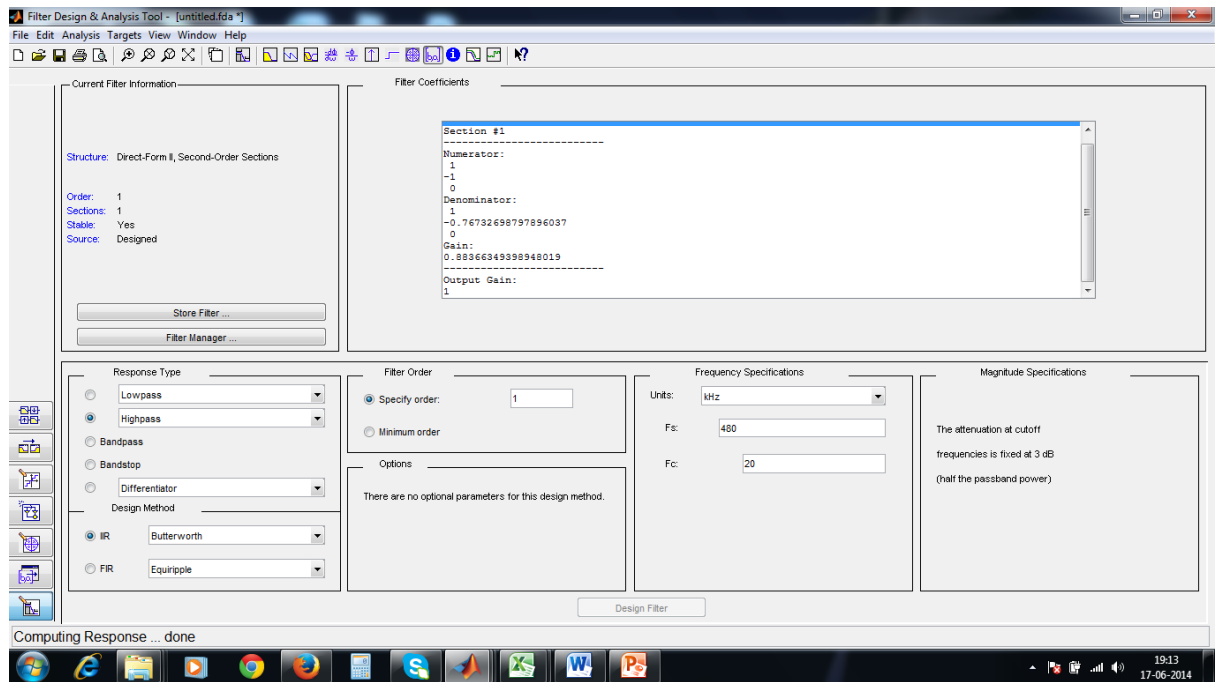
**Pic: Showing amplitudes of noise alone in w0, w1, w2 detailed coefficients in MATLAB to decide hard threshold level.**

```
% Hard thresholding function in MATLAB
for i =1:length(w0)
    if abs(w0(i)) <= 7
        w0(i) =0;
    end;
end;
```

**6.2.4 Butterworth High Pass filter design using MATLAB FDA Tool, with  $f_c=20\text{KHz}$**



## 6.2.5 HPF Coefficients generated by FDA Tool



**Filter function implementation in MATLAB:** here  $s$  is input mixed signal,  $y_{22}$  is filter output.

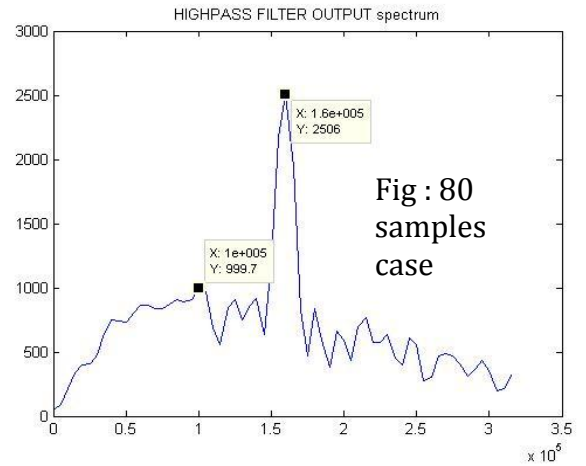
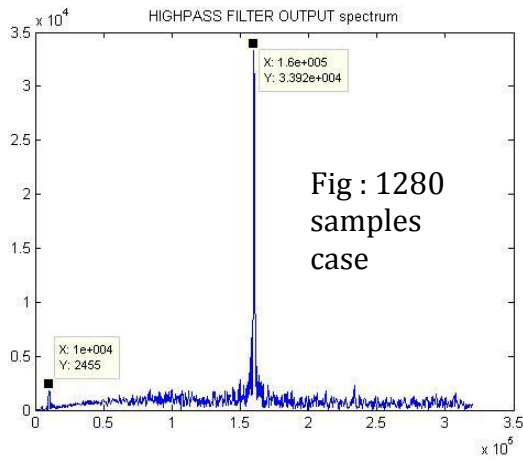
```
y22=filter(0.88366349398948019*[1 -1 0],[1 -0.767326987978960370],s);
```

## 6.3 Consolidated Results with 1280 & 80 sample cases after Noise reduction / denoising using db3 based DWT and HPF (3 stage )

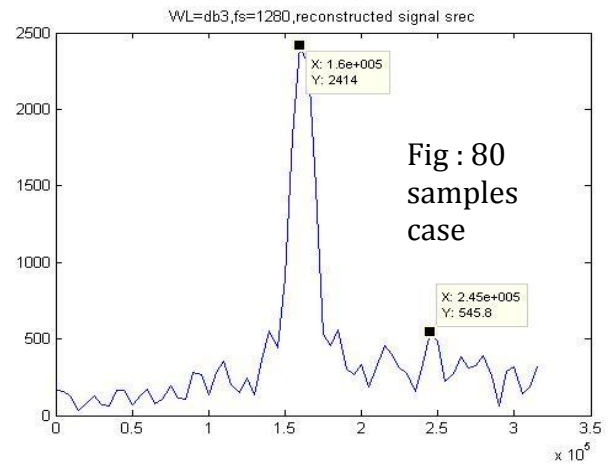
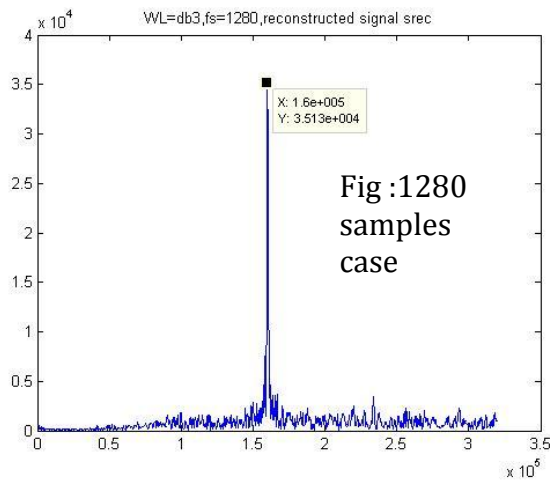
### 6.3.1 Results Table-III: Denoised o/p signal assessed parameter results comparison

	Db3 DWT(1280 samples)	HPF o/p (1280)	Db3 DWT (80 samples)	HPF o/p (80)
<b>Denoised signal SNR in dB</b>	8.89	4.1	6.23	-4
<b>Correlation</b>	86	50.74	81.42	36.21
<b>Regression</b>	0.92	0.52	0.89	0.63
<b>R^2</b>	48.39	1.96	42.3	4.39

### 6.3.2 HPF output for 1280 or 80 samples of mixed signal as input



### 6.3.3 DWT (Using db3 architecture) output for 1280 or 80 samples of mixed signal as input



Pic: Comparison of Denoised Outputs of MATLAB Models for 3 stage HPF with  $f_c=20K$ , and using DWT architecture with db3 wavelet

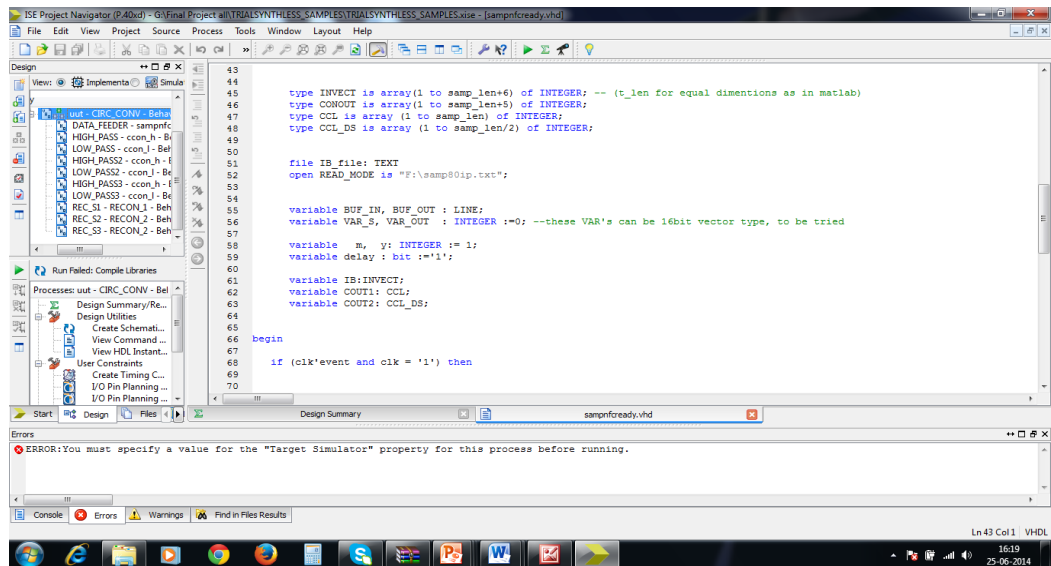
From the above results it is confirmed that for even at low input SNR the DWT method using db3 wavelet gives very good results by improving SNR of the signal. For low complexity architecture with less number of samples the performance is still optimum. So VHDL architecture can be implemented with 80 samples and can be used for denoising of FMCW Radar altimeter signal.

## 6.4 Hardware Architecture Design using VHDL and Simulations:

**A. VHDL Coding:** Xilinx ISE Design suite 14.3 tool is used for FPGA as target device, VHDL structural modules are developed to implement the Hardware architecture of db3 Wavelet consisting of 3 stage Analysis and Synthesis banks by designing high pass and low pass filters (decomposition & Reconstruction) and component instantiation is done for 3 stages. A module for file reading and feeding the data as i/p to decomposition and also for writing into a txt file is designed. To reduce hardware complexity four points are exercised in the code they are, a) 'for' loops are implemented by counter method, b) multiplier is implemented using shifter and add/subtraction in combinational logic subroutine, c) taking only 80 input samples , d) db3 has less no. of filter coefficients. Further reduction is possible by resource sharing like memories and registers.

### 6.4.1 VHDL Scree shots:

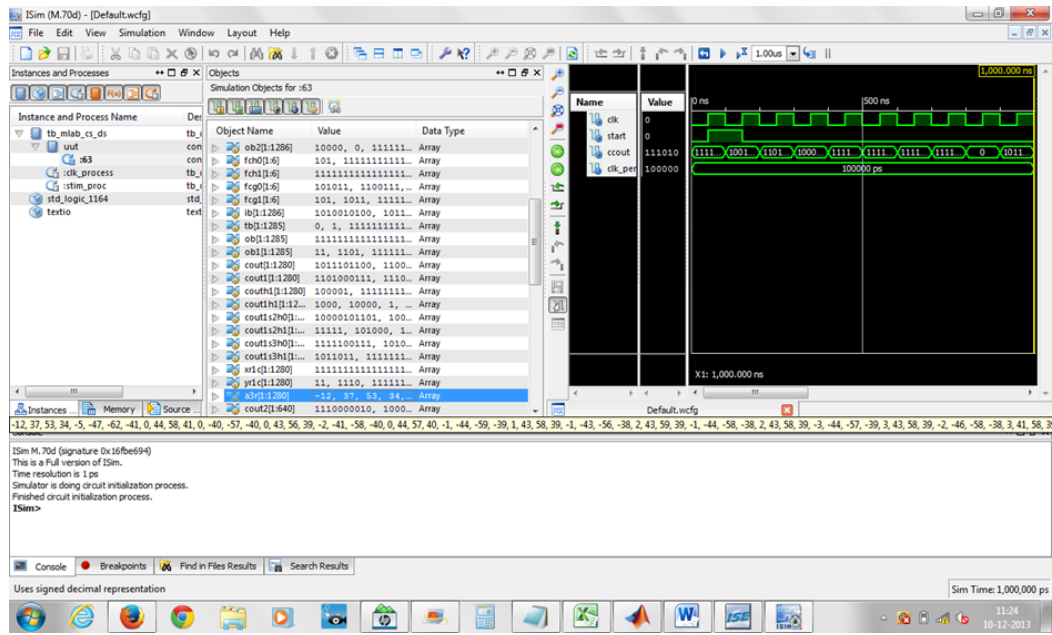
VHDL input signal reading from txt file : Screen Shot



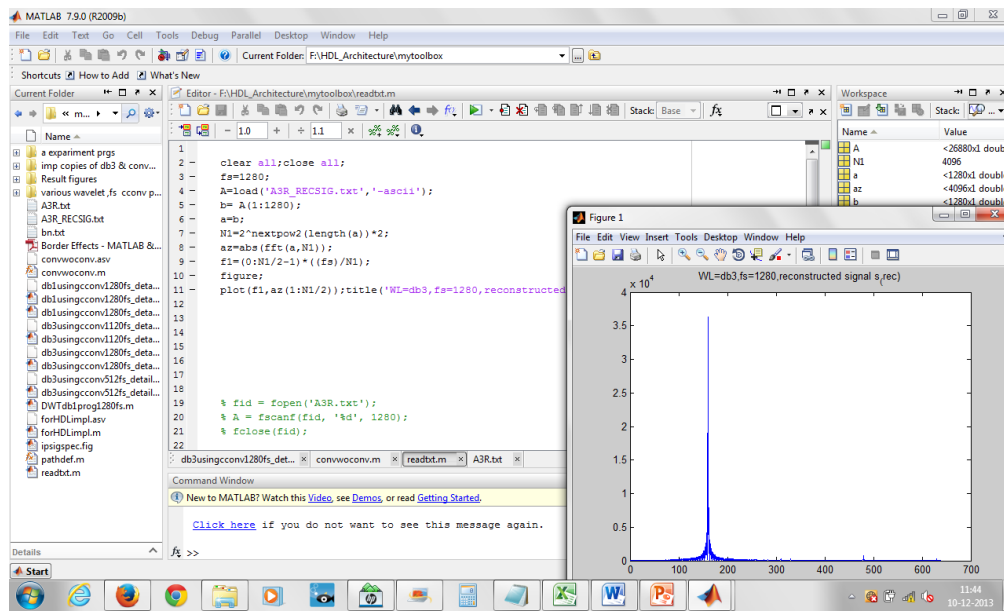
```
43
44
45     type INVECT is array(1 to samp_len+6) of INTEGER; -- (t_len for equal dimentions as in matlab)
46     type CONOUT is array(1 to samp_len+5) of INTEGER;
47     type CCL is array (1 to samp_len) of INTEGER;
48     type CCL_DS is array (1 to samp_len/2) of INTEGER;
49
50
51     file IS_file: TEXT
52     open READ_MODE is "F:\samp80ip.txt";
53
54
55     variable BUF_IN, BUF_OUT : LINE;
56     variable VAR_S, VAR_OUT : INTEGER :=0; --these VAR's can be 16bit vector type, to be tried
57
58     variable m, y: INTEGER := 1;
59     variable delay : bit := '1';
60
61
62     variable IS:INVECT;
63     variable COU1: CCL;
64     variable COU2: CCL_DS;
65
66 begin
67
68     if (clk'event and clk = '1') then
69
70
```

ERROR: You must specify a value for the "Target Simulator" property for this process before running.

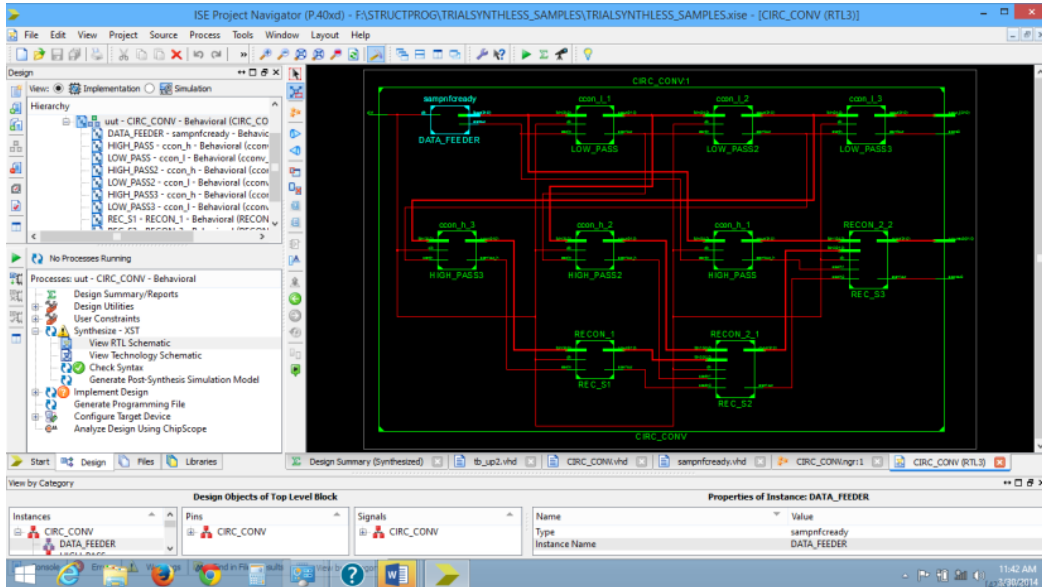
## VHDL Simulation Screen Shot



## VHDL output txt file of reconstruction Plotted in MATLAB: Screen Shot



## VHDL SYNTHESIS Screen Shot



Pic: Synthesized design: Denoising architecture of db3, VHDL Code synthesized in Xilinx ISE Design compiler (above), resources used from vertex-7 target FPGA(below fig) .

## Resources used from vertex-7 target FPGA

CIRC_CONV Project Status (03/30/2014 - 11:00:53)			
Project File:	TRIALSYNTHLESS_SAMPLES.xise	Parser Errors:	No Errors
Module Name:	CIRC_CONV	Implementation State:	Synthesized
Target Device:	xc7vx330t-1ffg1157	Errors:	No Errors
Product Version:	ISE 14.3	Warnings:	2288 Warnings (2205 new)
Design Goal:	Balanced	Routing Results:	
Design Strategy:	Min Default (unlocked)	Timing Constraints:	
Environment:	System Settings	Final Timing Score:	

Device Utilization Summary (estimated values)			
Logic Utilization	Used	Available	Utilization
Number of Slice Registers	32564	408000	7%
Number of Slice LUTs	107830	204000	52%
Number of fully used LUT-FF pairs	28841	111553	25%
Number of bonded IOBs	67	600	11%
Number of BUFG/BUFGCTRLs	1	32	3%

Detailed Reports				
Report Name	Status	Generated	Errors	Warnings
Synthesis Report	Current	Sun Mar 30 11:00:18 2014	0	2288 Warnings (2205 new)

Console Output:

```

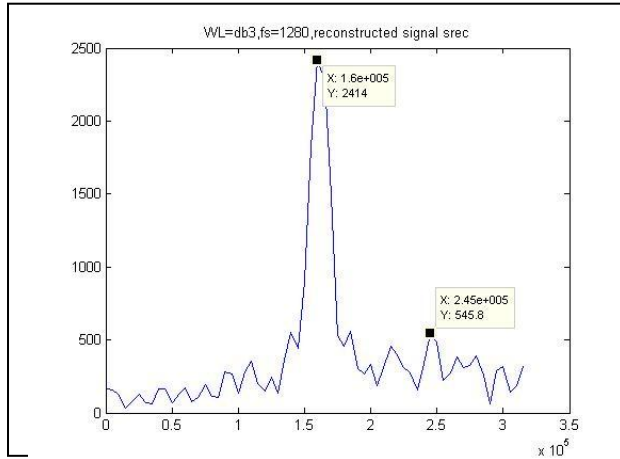
Minimum input arrival time before clock: No path found
Maximum output required time after clock: 3.792ns
Maximum combinational path delay: No path found

-----
Process "Synthesize - XST" completed successfully
    
```

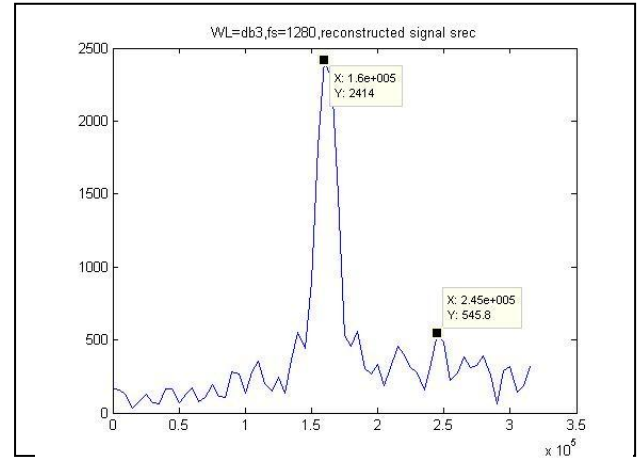


## 6.4.2 Comparison of MATLAB & VHDL outputs

MATLAB RESULT



VHDL RESULT



Results of FMCW DWT-db3 based model of MATLAB & VHDL o/p are verified and found satisfactory.

## 6.5 Applications :

The DWT with db3 wavelet based Denoising under low Signal to Noise Ratio (SNR) conditions is very much suitable for FMCW Radar Altimeter used in Anti-Radiation Missiles, Smart Bombs, Fighter aircrafts, Helicopters and other RF carrier based defense applications and Cellular communications etc..

## 6.6 Conclusion:

In view of signals with low SNR and low complexity Hardware implementation, among the various wavelets under study i.e dmey, coif, sym, & debouches such as db1, db2, db3, db4, db6, the best wavelet found is db3 (less no. of filter coefficients for better results) giving excellent performance by virtue of various results. By DWT based Denoising using db3 improved SNR appx. 5db more and Correlation, Regression, R<sup>2</sup> statistics gave very good results compared to filter methods. With less number of sample data also results are found satisfactory, which is very important for low complexity hardware based application apart from denoising non-

stationary signals for applications like FMCW Radar Altimeter, where in the method of denoising is proven to give very good results. Following is conclusion point wise.

- a. Out of dmey, coif1, sym2, & debouches db1, db2, db3, db4, db6 wavelets, db3 delivers very good results for the denoising of target frequencies while cancelling out unwanted harmonic of the ramp frequency and in view of low complexity vs. better performance designers.
- b. It outperforms over the HPF approach even under low SNR RF inputs, results in improved SNR and other parameters.
- c. With less number of sample data also results are found satisfactory, which is very important for low complexity hardware based application apart from denoising non-stationary signals for applications like FMCW Radar Altimeter, where in the method of denoising is proven to give very good results.
- d. The suggested technique is applicable in all FMCW radars which use periodic ramp signals

**Overall Conclusion : The simulations results found satisfactory, VHDL Synthesis has been done. so one can use this method.**

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