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Deviation from tri-bimaximal mixing as a result of modification of Yukawa coupling structure of constrained sequential dominance

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Abstract. Tri-bimaximal mixing (TBM) in the neutrino sector has been obtained in constrained sequential dominance where the Yukawa couplings of righ-handed neutrinos have some particular structure. Current neutrino oscillation data suggests that neutrino mixing should deviate from TBM mixing pattern. To explain it, we propose a phenomenological model by adding some small complex parameters to the Yukawa couplings of CSD. Using this we have shown that neutrino mixing angles can deviate from their TBM values. We justify the modified form of Yukawa couplings of our work by constructing a model based on flavor symmetry.

1. Introduction

Experimental observations suggest that neutrinos have very small mass and they mix among them [1]. The smallness of neutrino masses can be explained theoretically in a type I seesaw mechanism through the mediation of heavy right-handed neutrinos. To reduce the number of parameters in a seesaw model, models based on sequential dominance with two right-handed neutrinos and one texture zero in the neutrino Yukawa matrix have been proposed [2]. These models are named as CSD(n). Here n indicates a positive integer.

To explain the mixing angles in the model of CSD(n), two right-handed neutrinos are proposed and these fields have a particular Yukawa matrix structure with three lepton doublets. For the case of $n = 1$, the neutrino mixing angles are predicted to have TBM pattern [3]. The case $n = 1$ is originally called constrained sequential dominance (CSD) which is ruled out by Daya Bay and RENO when θ_{13} was found to be non-zero.

In this work, in order to get deviations from TBM values, we first modify the Yukawa coupling structure of CSD. Then we compute masses and mixing angles of neutrino masses by following some approximate diagonalization procedure which gives deviation from TBM pattern. At the end we construct a flavor model which justify the Yukawa coupling structure of our phenomenological model.

2. Sequential Dominance and CSD

We extend the standard model with three additional right-handed neutrinos and after electroweak symmetry breaking, charged lepton and neutrinos acquire mixing mass matrices. Now, working in a basis where the masses of charged leptons and right-handed neutrinos are diagonal, the mass matrix for right-handed neutrinos and the mixing mass matrix between left- and right-handed neutrinos can be written, respectively, as

$$M_R = \begin{pmatrix} M_{\rm atm} & 0 & 0 \\ 0 & M_{\rm sol} & 0 \\ 0 & 0 & M_{\rm dec} \end{pmatrix}, \quad m_D = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}. \tag{1}$$

Here a, b, c etc. can be viewed as Yukawa couplings multiplied by vacuum expectation value(VEV) of Higgs field. Now the seesaw formula for active neutrinos can be written as

$$m_\nu = m_D M_R^{-1} m_D^T. \tag{2}$$

The conditions for sequential dominance is given by [4]

$$M_{\rm atm} \ll M_{\rm sol} \ll M_{\rm dec}, \quad \frac{|e^2|, |f^2|, |ef|}{M_{\rm atm}} \gg \frac{xy}{M_{\rm sol}} \gg \frac{x'y'}{M_{\rm dec}}. \tag{3}$$

Here, $x, y \in a, b, c$ and $x', y' \in a', b', c'$. In the limit of sequential dominance, third column of m_D and the mass $M_{\rm dec}$ can be decoupled from the theory. A prediction of sequential dominance is that neutrinos have normal mass hierarchy where lightest neutrino mass is zero. In the CSD model, after the limit of sequential dominance, the Dirac and Majorana mass matrices can be taken as [2]

$$m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}. \tag{4}$$

Now plugging this into the seesaw formula of Eq. (2), we find

$$U_{\rm TBM}^T m_\nu U_{\rm TBM} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3a^2}{M_{\rm sol}} & 0 \\ 0 & 0 & \frac{2e^2}{M_{\rm atm}} \end{pmatrix}, \quad U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{5}$$

As $U_{\rm TBM}$ is ruled out, we need to modify the Dirac mass matrix of Eq. (4), which is explained in the next section.

3. Our phenomenological model

To get deviation from TBM pattern, we propose a modified form for the Dirac mass matrix of Eq. (4), which is given below.

$$m'_D = m_D + \Delta m_D, \quad m_D = \begin{pmatrix} 0 & a \\ e & a \\ e & -a \end{pmatrix}, \quad \Delta m_D = \begin{pmatrix} e\epsilon_1 & a\epsilon_4 \\ e\epsilon_2 & a\epsilon_5 \\ e\epsilon_3 & a\epsilon_6 \end{pmatrix}. \tag{6}$$

Here ϵ_i, where $i = 1 \cdots, 6$, are complex parameters. Now, with the new Dirac mass matrix m'_D, seesaw mass formula is

$$m_\nu^s = m'_D M_R^{-1} (m'_D)^T. \tag{7}$$

Since, we are in a basis where charged leptons are diagonalized, light neutrino mass matrix can be diagonalized by Pontecorvo-maki-Nakagawa-Sakata (PMNS) matrix. The PDG convention of this PMNS matrix in terms of three angles and one Dirac CP violating phase is [5]

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}. \tag{8}$$

Here $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, where $i,j = 1,2,3$. In order to simplify our calculation, we parameterize s_{12} and s_{23} as

$$s_{12} = \frac{1}{\sqrt{2}}(1+r), \quad s_{23} = \frac{1}{\sqrt{2}}(1+s). \tag{9}$$

The variables r, s, s_{13} are small, which can be noticed from the neutrino oscillation data. Now the diagonalization formula for Eq. (7) can be written as

$$m_\nu^d \equiv U_{\rm PMNS}^T m_\nu^s U_{\rm PMNS} = {\rm diag}(m_1, m_2, m_3) \tag{10}$$

In the above equation, we can expand m_ν^s and $U_{\rm PMNS}$ in power series of ϵ_i, r, s, s_{13}. After that m_ν^d need not be in diagonal form. We equate off-diagonal terms of m_ν^d to zero, which give relations for ϵ_i in terms of r, s, s_{13}. From the diagonal terms of m_ν^d, we get neutrino masses in terms of model parameters. However, in doing so, we need to take care of the hierarchy in neutrino masses which is explained below.

In our model, in the limit where ϵ_i, r, s, s_{13} tend to zero, Eq. (10) agrees with the results of Eq. (5). These results are considered as leading order expressions. We can see that the lightest mass m_1 is always zero in our calculations. As our calculation prefers normal hierarchy for neutrino masses, we can fit m_2 and m_3 to solar($\sqrt{\Delta m^2_{\rm sol}}$) and atmospheric($\sqrt{\Delta m^2_{\rm atm}}$) neutrino mass scales. So, in the diagonalization procedure, one needs to apply the following order of estimation

$$\frac{a^2}{M_{\rm sol}} \sim \sqrt{\Delta m^2_{\rm sol}}, \quad \frac{e^2}{M_{\rm atm}} \sim \sqrt{\Delta m^2_{\rm atm}}. \tag{11}$$

It is observed from the neutrino oscillation data that $\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} \sim s_{13} \sim 0.15$, which is a small quantity whose square is negligible in comparison to unity. This order of approximation is applied in our diagonalization process. Now divide Eq. (10) by $\sqrt{\Delta m^2_{\rm atm}}$ and expand the left-hand side of it in terms of small parameters ϵ_i, r, s, s_{13} and $\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}}$ up to first order. The results following from this analysis are summarized below.

$$m_1 = 0, \quad m_2 = \frac{3a^2}{M_{\rm sol}}, \quad m_3 = \frac{2e^2}{M_{\rm atm}} + \frac{2e^2(\epsilon_2+\epsilon_3)}{M_{\rm atm}}, \quad \epsilon_1 = \sqrt{2}e^{i\delta_{\rm CP}}s_{13}, \quad \epsilon_2 - \epsilon_3 = 2s. \tag{12}$$

Above equations show the deviation from TBM pattern, where non-zero and small s_{13} and s can be obtained by appropriately taking the $\epsilon_{1,2,3}$. We cannot determine r in the first-order calculations, as a result of which, we have done the second order calculation. We expand Eq. (10) up to second order in all the above mentioned small variables and after applying the above described approximations, the results we have obtained are outlined below.

$$\begin{aligned}
&m_1 = 0, \quad m_2 = \frac{3a^2}{M_{\rm sol}} + \frac{2a^2}{M_{\rm sol}}(\epsilon_4+\epsilon_5-\epsilon_6),\\
&m_3 = \frac{2e^2}{M_{\rm atm}} + \frac{4e^2}{M_{\rm atm}}(\epsilon_3+s) + \frac{2e^2}{M_{\rm atm}}(s_{13}^2+\epsilon_3^2+2\epsilon_3 s+2s^2), \quad 2\epsilon_4-\epsilon_5+\epsilon_6 = 3r,\\
&s(3s-2\sqrt{2}e^{i\delta_{\rm CP}}s_{13}) + 2\epsilon_3(s-\sqrt{2}e^{i\delta_{\rm CP}}s_{13}) = 0, \quad \sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}}e^{i\phi}[\sqrt{2}(\epsilon_5+\epsilon_6+2s)+2e^{-i\delta_{\rm CP}}s_{13}]\\
&-[2\epsilon_3(\sqrt{2}s+e^{i\delta_{\rm CP}}s_{13}) + s(3\sqrt{2}s+2e^{i\delta_{\rm CP}}s_{13})] = 0.
\end{aligned} \tag{13}$$

4. A model for Dirac mass matrix

In this section, to formulate the Dirac mass matrix of Eq. (6), we build one model based on $SU(3) \times Z_3 \times Z_3'$ by adding four $SU(3)$ scalar triplets ϕ_a, ϕ_s, ϕ_a' and ϕ_s'. We also add one singlet field ξ to explain the smallness of ϵ_i and two singlet scalar fields χ_a, χ_s to generate the Majorana masses. The particle content and charge assignments are shown in Table 1. With these charge

	ϕ_a	ϕ_s	ϕ_a'	ϕ_s'	ξ	χ_a	χ_s	ν_R^{atm}	ν_R^{sol}	L	H
$SU(3)$	3	3	3	3	1	1	1	1	1	3	1
Z_3	ω	ω^2	ω	ω^2	1	ω^2	ω	ω^2	ω	1	1
Z_3'	ω^2	ω^2	ω	ω	ω	ω	ω	ω	ω	1	1

Table 1. Charge assignments of the relevant fields under the flavor symmetry $SU(3) \times Z_3 \times Z_3'$ are given. Here, $\omega = e^{2\pi i/3}$. For other details, see the text.

assignments, the leading terms in the Lagrangian can be written as

$$\begin{aligned} \mathcal{L} &= \frac{\phi_a}{M_P}\bar{L}\nu_R^{atm}H + \frac{\phi_s}{M_P}\bar{L}\nu_R^{sol}H + \frac{\xi}{M_P}\frac{\phi_a'}{M_P}\bar{L}\nu_R^{atm}H + \frac{\xi}{M_P}\frac{\phi_s'}{M_P}\bar{L}\nu_R^{sol}H \\ &+ \frac{\chi_a}{2}\overline{(\nu_R^{atm})^c}\nu_R^{atm} + \frac{\chi_s}{2}\overline{(\nu_R^{sol})^c}\nu_R^{sol} + h.c. \end{aligned} \tag{14}$$

Here $M_P \sim 2 \times 10^{18}$ GeV is the reduced Planck scale, which is the cut-off scale of our model. The VEVs of ϕ_a, ϕ_s generate the leading order term in m_D' by choosing a particular pattern for these VEVs [2]. The VEVs of ϕ_a', ϕ_s', ξ generate sub-leading contributions to m_D', where we do not assume any pattern for these VEVs. We can take $\langle\xi\rangle/M_P \sim 0.1$, which can explain the smallness of ϵ_i in our model.

The VEV structure of all the scalars and their magnitudes are justified by doing an analysis on the full scalar potential of our model [6]. Also, with the analytic results of Sec. 3, numerically we have shown that our model is consistent with the current neutrino oscillation data [6].

5. Conclusions

In this work, we have attempted to explain the neutrino mixing in order to be consistent with the current neutrino oscillation data. To explain the TBM pattern in neutrino sector, CSD model has been proposed. Here, we have considered a phenomenological model, where we have modified the neutrino Yukawa couplings of CSD model, by introducing small ϵ_i parameters which are complex. Thereafter, we have followed an approximation procedure in order to diagonalize the seesaw formula for light neutrinos in our model. We have computed expressions, up to second order level, to neutrino masses and mixing angles in terms of small ϵ_i parameters. Using these expressions we have demonstrated that neutrino mixing angles can deviate away from their TBM values by appropriately choosing the ϵ_i values. Finally, we have constructed a model in order to justify the neutrino Yukawa coupling structure of our phenomenological model.

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