Conjugacy Relation via Group Action on the set of Fuzzy Implications

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Let Φ denote the set of all increasing bijections on $[0,1]$ and $\mathbb{I}$ the set of fuzzy implications. In [1], the authors proposed a new way of generating fuzzy implications from fuzzy implications using order automorphisms $\varphi \in \Phi$ on the unit interval via the following equation

$$I_\varphi(x,y) = \varphi^{-1}(I(\varphi(x),\varphi(y))), \qquad x,y \in [0,1]\ , \tag{1}$$

where $I \in \mathbb{I}$. Here I_φ is called the **φ-conjugate** of I.

Moreover, in the same work, they proposed also a conjugacy relation on $\mathbb{I}$, as follows:

$$I \sim J \text{ if and only if } J = I_\varphi \text{ for some } \varphi \in \Phi. \tag{2}$$

Clearly, $\sim$ is an equivalence relation on $\mathbb{I}$ and partitions $\mathbb{I}$ into conjugacy classes. Let us denote by $[I]$ the set of all φ-conjugates of I where φ varies over whole of Φ.

Recently, in [2], the authors proposed a novel method of generating fuzzy implications from fuzzy implications as follows:

Given $I, J \in \mathbb{I}$ we define

$$(I \circledast J)(x,y) = I(x, J(x,y)),\ x,y \in [0,1]\ . \tag{3}$$

Further, it was shown that the above binary operation imposes a monoid structure $(\mathbb{I},\circledast)$ on the set of all fuzzy implications.

In this present work, we show that the conjugacy relation defined via Eq. (2) is nothing but a group action of $\mathbb{S}$ on the set $\mathbb{I}$, where $\mathbb{S}$ is the set of all invertible elements of the monoid $(\mathbb{I},\circledast)$. In other words, we show that the conjugacy classes $[I]$ obtained from Eq. (2) are exactly the pieces in the partition obtained by the group action of $\mathbb{S}$ on the set $\mathbb{I}$.

References

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