Suitability of FRIs based on Generalised Operators

Mandal Sayantan[1] and Jayaram Balasubramaniam[1]

[1] *Department of Mathematics,*
Indian Institute of Technology Hyderabad,
Hyderabad-502205, India
E-mail: ma10p002@iith.ac.in, jbala@iith.ac.in

It is well known that a t-norm T and its residual implication I_T, normally denoted as the residual pair (T, I_T), play an important role in fuzzy inference systems, especially in Fuzzy Relational Inference (FRI) mechanisms. For instance, many desirable properties like the interpolativity, continuity, robustness and monotonicity of an FRI largely depend on the properties possesed by the residual pair (T, I_T).

It is also well known that a more general class of a binary operations $C : [0,1]^2 \longrightarrow [0,1]$ than t-norms give rise to fuzzy implication I_C through the following residual operation:

$$I_C(x,y) = \sup\{t \in [0,1] | C(x,t) \leq y\}, \qquad x,y \in [0,1] .$$

For the exact conditions on C for I_C to be a fuzzy implication refer to [1]. Let us denote this class of conjunctions by $\mathscr{C}$.

One can investigate the class $\mathscr{C}$ along the following two approaches: On the one hand, one can discuss the additional conditions on C so that I_C satisfies some desirable properties of fuzzy implications like neutrality, ordering and exchange principle, etc. This has already been done quite extensively, see, for instance, [2], [4]. On the other hand, one can also investigate the additional conditions required on C that would enable a (C, I_C) pair to be admitted in an FRI, see, for instance, [3]. In other words, if we call an FRI mechanism where a t-norm T is replaced by C and I_T replaced by I_C as a *generalised FRI mechanism*, then one can investigate the additional conditions required on (C, I_C) for a generalised FRI to possess the above desirable properties. In this work, we follow the second line of investigation.

To this end, firstly, we discuss the solvability of the fuzzy relational equations (FREs) of the following form: For X, Y, Z being some universes of discourse and P, Q, R being some fuzzy relations on $X \times Y, Y \times Z, X \times Z$ respectively,

$$P \circ_C Q = R \text{ and } P \lhd_C Q = R$$

where $\circ_C = \sup -C$ and $\lhd_C = \inf -I_C$ and determine the largest subset of $\mathscr{C}$ that enjoys solvability of the above mentioned FREs.

Following this, we study the generalisations of two of the most important FRIs obtained from the above two types of FREs, viz., *generalised* Compositional Rule of Inference(CRI) and *generalised* Bandler-Kohout Subproduct(BKS) mechanisms and present some necessary and sufficient conditions on C that ensures some of the desirable properties like interpolativity and continuity in the considered setting.

We illustrate the above results with some specific family of conjunctions from $\mathscr{C}$ which are not t-norms.

References

[1] Demirli K., De Baets B.: *Basic Properties of Implicators in a Residual Framework.* Tatra M. Math. Publ. **16** (1999) 31–46.

[2] Durante F., Klement E. P., Mesiar R., and Sempi C.: *Conjunctors and their Residual Implicators: Characterizations and Construction Methods.* Mediterranean Journal of Mathematics. **4** (2007) 343–356.

[3] Hájek P., Mesiar R.: *On copulas, quasicopulas and fuzzy logic.* Soft Computing. **12** (2008) 1239-1243.

[4] Króll A.: *Dependencies between fuzzy conjunctions and implications.* EUSFLAT-LFA 2011. (2011).