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Flavoured CP-asymmetry at the effective neutrino mass floor

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Abstract

Both neutrinoless double beta decay and leptogenesis require neutrinos to be Majorana fermions. A relation between these two phenomena can be derived once the mechanism of neutrino mass generation is specified. Using the current data from the neutrino oscillations, we constrain the Majorana phases of the neutrino mixing matrix by minimising the effective neutrino mass in neutrinoless double beta decay. Given these Majorana phases at the effective neutrino mass floor, we show that it is possible to obtain a large enough CP asymmetry ($\geq 10^{-8}$) required for adequate leptogenesis, without additional phases at high scale. Such scenario pushes the lower bound on M_1 (the mass of the lightest of the heavy neutrinos in the Type-I see-saw mechanism) to a higher value compared to the usual Davidson-Ibarra bound. In particular, we find that $M_1 \geq 10^{10}\,(10^9)$ GeV for the case of Normal (Inverted) hierarchy. We extend our analysis to the case when one of the heavy neutrinos decouples (two right handed neutrino models). In this case we find $M_1 \geq 10^{10}\,(10^{11})$ GeV for the case of Normal (Inverted) hierarchy.

1. Introduction

At present the Standard Model (SM) of particle physics, which is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, is considered to be the best candidate to explain elementary particles

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Table 1
Global fit 3σ ranges of neutrino oscillation parameters [10].

Parameters	Normal Hierarchy (NH)	Inverted Hierarchy (IH)
$\frac{\Delta m^2_{21}}{10^{-5}\text{ eV}^2}$	$6.79 - 8.01$	$6.79 - 8.01$
$\frac{\lvert\Delta m^2_{31}\rvert}{10^{-3}\text{ eV}^2}$	$2.427 - 2.625$	$2.412 - 2.611$
$\sin^2\theta_{12}$	$0.275 - 0.350$	$0.275 - 0.350$
$\sin^2\theta_{23}$	$0.418 - 0.627$	$0.423 - 0.629$
$\sin^2\theta_{13}$	$0.02045 - 0.02439$	$0.02068 - 0.02463$
$\delta(^\circ)$	$125 - 392$	$196 - 360$

and their interactions in nature. However, it doesn't address certain issues like sub-eV masses of three generations of active neutrinos. Moreover, it does not explain the observed baryon asymmetry of the Universe, measured to be $n_B/n_\gamma = (6.09 \pm 0.06) \times 10^{-10}$ [1,2]. Therefore, we need to go beyond the SM of particle physics to address these issues.

The current neutrino oscillation experiments [3–5] confirmed non-zero, but tiny neutrino masses and also mixing between different flavours. The mixing matrix, relating the flavour eigenstates to mass eigenstates, is called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [6–8]. It is parameterized as [9]

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_{ph}\,, \tag{1}$$

where $U_{ph} = \text{diag}(1, e^{i\alpha_1}, e^{i\alpha_2})$ and the symbols c_{ij} and s_{ij} stand for $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively. Of the three phases in U_{PMNS}, α_1, α_2 are called the Majorana phases and δ is called the Dirac phase.

In terms of the mixing matrix U_{PMNS} the neutrino mass matrix can be given as

$$m_\nu = U^\dagger m_{\text{diag}} U^* \tag{2}$$

where $m_{\text{diag}} = \text{diag}(m_1, m_2, m_3)$. Thus the neutrino mass matrix m_ν consists of nine parameters: three masses, three mixing angles and three phases. At present the oscillation experiments measure two mass square differences: namely solar (Δm^2_{sol}) and atmospheric (Δm^2_{atm}), three mixing angles θ_{23}, θ_{12} and θ_{13} to a good degree of precision. Data indicate that $|\Delta m^2_{\text{atm}}| \gg \Delta m^2_{\text{sol}}$. Without loss of generality, we can define $\Delta m^2_{\text{sol}} = \Delta m^2_{21} = m_2^2 - m_1^2$ and $\Delta m^2_{\text{atm}} = \Delta m^2_{31} = m_3^2 - m_1^2 \simeq m_3^2 - m_2^2 = \Delta m^2_{32}$. Matter effects in solar neutrino oscillations require $\Delta m^2_{21} > 0$, but, so far, the sign of Δm^2_{31} is not determined. The case of $\Delta m^2_{31} > 0$ is called Normal hierarchy (NH) and that of $\Delta m^2_{31} < 0$ is called Inverted hierarchy (IH). For NH the smallest neutrino mass (m_{min}) is m_1, whereas it is m_3 for IH. The 3σ ranges of the oscillation parameters are given in Table 1. At present the value of Dirac phase δ is quite ambiguous. T2K experiment [11] prefers a value $\delta \approx -\pi/2$ whereas NOvA experiment [12] prefers $\delta \approx 0$. The best fit values for global fits is $\delta \approx -3\pi/4$ for NH and $\delta \approx -\pi/2$ for IH [10].

The oscillation experiments do not give us any hint about the nature of neutrino being either Dirac or Majorana. However, the neutrinoless double beta decay ($0\nu\beta\beta$) experiments [13] can explore the Majorana nature of neutrinos. At present, the best lower limit on half-life of the $0\nu\beta\beta$ using ^{76}Ge is $T^{0\nu}_{1/2} > 8.0 \times 10^{25}$ yrs at 90% C.L. from GERDA [14]. For ^{136}Xe isotope, the derived lower limits on half-life from KamLAND-Zen experiment are $T^{0\nu}_{1/2} > 1.6 \times 10^{26}$ yrs [15]. The proposed sensitivity of the planned nEXO experiment is $T^{0\nu}_{1/2} \approx 6.6 \times 10^{27}$ yrs [16]. The above mentioned lower limits on $T^{0\nu}_{1/2}$ lead to an upper limit on effective neutrino mass $|m_{ee}|$ about 10^{-2} eV. In future these experiments will increase their sensitivities down to the floor of $|m_{ee}|$ *i.e.* $|m_{ee}|_{\rm min}$, obtained by minimising $|m_{ee}|$ with respect to the oscillation parameters.

Since Majorana neutrinos violate lepton number by two units they can also lead to leptogenesis. Therefore, within a given neutrino mass model, we can expect a correlation between the two lepton number violating processes, namely $0\nu\beta\beta$-decay and leptogenesis. The phenomenon of $0\nu\beta\beta$-decay depends on the effective neutrino mass: $|m_{ee}|$ which involves only the low energy neutrino parameters, while the leptogenesis involves low energy oscillation parameters as well as unknown high energy parameters. The relation between $0\nu\beta\beta$-decay and leptogenesis has been well studied in the literature for $|m_{ee}| > |m_{ee}|_{\rm min}$. See for instance [17–28]. In this work, we consider the following issues: Suppose the Majorana phases of $U_{\rm PMNS}$ take the values which lead to the worst case scenario for $0\nu\beta\beta$-decay, *i.e.* $|m_{ee}|$ takes the value $|m_{ee}|_{\rm min}$. In such a situation, is it possible to get sufficient CP asymmetry for adequate leptogenesis, **without** additional phases from the high energy scale? If the answer is affirmative, then how does the bound on M_1 (the mass of the lightest of the heavy neutrinos in Type-I see-saw mechanism) in such a scenario, compare with the usual Davidson-Ibarra bound [29]? We also consider the question of whether this bound depends on the hierarchy of the light neutrino masses and also on the number of heavy neutrinos.

Here we first minimise $|m_{ee}|$ as a function of $m_{\rm min}$ and find the ranges of values of Majorana phases α_1 and α_2 which yield $|m_{ee}|_{\rm min}$. With these phases as inputs, we compute the CP-asymmetries ϵ_1^l $(l = e, \mu, \tau)$ responsible for leptogenesis [17–19,30–45] within a frame work of type-I seesaw mechanism [46–54]. The latter requires only the addition of three right handed neutrinos N_i $(i = 1, 2, 3)$ to the SM. Since these particles are electrically neutral and have no charges under the SM gauge group, they can have bare Majorana masses M_i $(i = 1, 2, 3)$. As a result the CP-violating out-of-equilibrium decay of the lightest of these heavy neutrinos (N_1) in the early Universe could generate a net lepton asymmetry, which is then converted to observed baryon asymmetry of the Universe by electroweak sphalerons [31,55]. We obtain a lower bound on M_1 (mass of N_1) at the effective neutrino mass floor, both for NH and for IH in the scenario with three right handed neutrinos as well as in the scenario of two right handed neutrinos, by imposing the constraint $\epsilon_1^l \geq 10^{-8}$, which can give rise adequate leptogenesis. We see that the lower bound on M_1 is pushed to a higher value compared to the usual Davidson-Ibarra bound [29] due to restrictive choice of Majorana phases at the effective neutrino mass floor.

The paper is organised as follows. In the section 2, we minimise the effective neutrino mass parameter $|m_{ee}|$ as a function of $m_{\rm min}$ and obtain the allowed ranges of Majorana phases for various different values of $m_{\rm min}$. We then obtain the flavour dependent CP-asymmetry parameters in section 3 corresponding to the set of low energy parameters which minimise $|m_{ee}|$. In section 4, we repeat this calculation for the case where there are only two heavy right handed neutrinos (which corresponds to setting $m_{\rm min} = 0$). We present our conclusion in the last section 5.

2. Effective Majorana mass and its minimisation

The most promising way to find the Majorana nature of neutrinos is through $0\nu\beta\beta$ decay experiments. The $0\nu\beta\beta$ decay rate is proportional to the effective Majorana mass $|m_{ee}|$, which is given by,

$$|m_{ee}| = |m_1 \cos^2\theta_{12}\cos^2\theta_{13} + m_2\cos^2\theta_{13}\sin^2\theta_{12}e^{-2i\alpha_1} + m_3\sin^2\theta_{13}e^{-2i(\alpha_2-\delta)}|. \quad (3)$$

As we see from Eq. (3), the value of effective Majorana mass depends on light neutrino masses m_1, m_2, m_3, mixing angles θ_{12}, θ_{13} and three CP-phases δ, α_1 and α_2. Given the minimum mass ($m_{\min}$), the other two neutrino masses can be defined in terms of the Δm^2_{21} and the Δm^2_{31}. For NH, m_1 is $m_{\min}$ and $m_2 = \sqrt{m^2_{\min} + \Delta m^2_{21}}$, $m_3 = \sqrt{m^2_{\min} + \Delta m^2_{31}}$. For IH, m_3 is $m_{\min}$ and Δm^2_{31} is negative. The expressions for the other two masses are $m_1 = \sqrt{m^2_{\min} + |\Delta m^2_{31}|}$, $m_2 = \sqrt{m^2_{\min} + |\Delta m^2_{31}| + \Delta m^2_{21}}$. At present, both the mixing angles, θ_{12} and θ_{13}, are strongly constrained by the neutrino oscillation data, while the three phases, which are responsible for CP-violation [56–59], are essentially unconstrained.

First, we consider the dependence of the minimum of $|m_{ee}|$ on $m_{\min}$ qualitatively based on Eq. (3). Here we take the ranges of the three phases to be $(-\pi, +\pi)$. We need to consider the cases of NH and IH separately. For NH, we consider four different mass sub-ranges for $m_{\min} = m_1$.

i). $\underline{m_1,\ (10^{-2} - 1)\text{ eV}}$: In this case $m_1 \simeq m_2 \simeq m_3$. The first term in Eq. (3) is positive and has the largest magnitude. To obtain the minimum of $|m_{ee}|$ the second and third term have to be negative. This leads to the conditions $\alpha_1 = \pm\pi/2$ and $\alpha_2 = \delta \pm \pi/2$.

ii). $\underline{m_1,\ (10^{-3} - 10^{-2})\text{ eV}}$: Here $m_2 \gtrsim 0.01$ eV and $m_3 \simeq 0.05$ eV. Given $\sin^2\theta_{12} \approx 0.35$ and $\sin^2\theta_{13} \approx 0.02$, the $|m_{ee}|$ is dominated by the second term and the third term is negligibly small. Hence the minimum of $|m_{ee}|$ occurs for $\alpha_1 \approx \pm\pi/2$, and depends very weakly on α_2 and δ. In this case, the magnitudes of m_1 and m_2 are comparable and hence the complete cancellations between the first two terms of Eq. (3) is possible. In such a situation, it is possible for $|m_{ee}| \to 0$.

iii). $\underline{m_1,\ (10^{-4} - 10^{-3})\text{ eV}}$: Here again the second term of Eq. (3) has the largest magnitude with the first and third having similar magnitudes. Requiring both these to have sign opposite to that of the second term imposes the conditions $\alpha_1 = \pm\pi/2$ and $\alpha_2 \approx \delta \pm \pi$. The minimisation procedure leads to a lower bound on $|m_{ee}| \simeq 10^{-3}$ eV.

iv). $\underline{m_1,\ (10^{-6} - 10^{-4})\text{ eV}}$: This case is similar to the third case except that the first term is negligibly small. The minimisation condition is equivalent to the requirement that the terms two and three must have opposite sign. This occurs when $\alpha_2 = \alpha_1 + \delta - (2n+1)\pi/2$ for appropriate integer values of n. Here also we obtain a lower bound on $|m_{ee}| \simeq 10^{-3}$ eV.

Turning to the case of IH, we consider the following two sub-ranges for $m_{\min} = m_3$.

i). $\underline{m_3,\ (10^{-6} - 10^{-2})\text{ eV}}$: This case similar to first case of NH. Hence the conditions on α_1 and α_2 are same.

ii). $\underline{m_3,\ (10^{-2} - 1)\text{ eV}}$: The smallness of m_3 and $\sin^2\theta_{13}$ make the third term in Eq. (3) completely negligible and the minimization of $|m_{ee}|$ completely independent of α_2 and δ. Requiring a cancellation between the first two terms, we get $\alpha_1 = \pm\pi/2$.

To verify the qualitative deductions made above, we calculated the minimum of $|m_{ee}|$ as a function of $m_{\min}$ through simulations. For illustration, we kept the Dirac phase fixed at $\delta = -\pi/2$. As shown above, the minimization of $|m_{ee}|$ fixes $(\alpha_2 - \delta)$. Choosing a different value of δ merely changes α_2, on which there is no experimental constraint at the moment. We varied $m_{\min}$ within its sub-range, the neutrino oscillation parameters within their 3σ ranges and the two

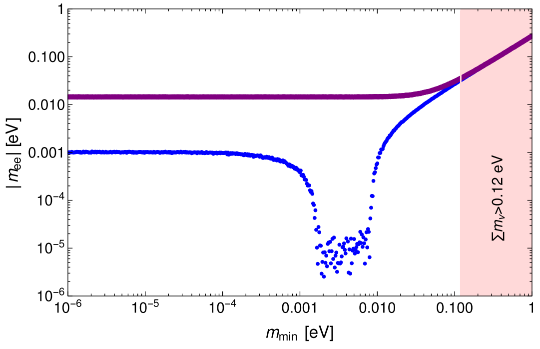


Fig. 1. Minimum of $|m_{ee}|$ as a function of $m_{\rm min}$ in case of normal hierarchy (Blue) and inverted hierarchy (Purple). The Pink shaded region defined by $m_{\rm min} > 0.12$ eV is ruled out by PLANCK data [2]. (For interpretation of the colours in the figure(s), the reader is referred to the web version of this article.)

Majorana phases α_1 and α_2 in $(-\pi, +\pi)$. We randomly chose a set of values of the neutrino parameters within their respective ranges and calculated $|m_{ee}|$. We repeated this procedure 10^7 times and picked the minimum value of $|m_{ee}|$ and the corresponding values of α_1 and α_2. The variation of $|m_{ee}|$ with respect to $m_{\rm min}$ as shown in Fig. 1, where $|m_{ee}| = 10^{-3}\,(10^{-2})$ eV for NH (IH) as $m_{\rm min} \to 0$. We note that $|m_{ee}| \to 0$ for the case of NH if $m_{\rm min}$ is in the sub-range $(10^{-3} - 10^{-2})$ eV [60–62].

The four panels of Fig. 2 show the values of α_1 and α_2 which minimise $|m_{ee}|$ for the four sub-ranges of $m_{\rm min}$ in the case of NH. The two panels of Fig. 3 shows similar results in the case of IH. The relation between α_1 and α_2, in all the six panels match those expected from qualitative discussion. The values of Majorana phases, α_1 and α_2, obtained by minimising $|m_{ee}|$ are plotted in Fig. 4 as a function of $m_{\rm min}$. We note that the values of α_1 and α_2 in Fig. 4 are consistent with those in Fig. 2 (NH) and Fig. 3 (IH). Had we chosen a value of δ other than $-\pi/2$, the value of α_1 is unaffected and the value of α_2 would be shifted by $(\delta + \pi/2)$.

3. Flavoured CP-asymmetry with three right handed neutrinos

In the previous section we derived the constraints on α_1 and α_2 as a function of $m_{\rm min}$ by minimizing $|m_{ee}|$. We utilize these values of the phases in the present section to compute the CP-asymmetry parameter ϵ_1^l of leptogenesis. In Type-I seesaw mechanism, the SM is extended by the inclusion of three right handed neutrinos, which have no gauge charges. These neutrinos can have bare Majorana masses. They can also couple to left handed lepton doublet and the Higgs doublet through yukawa couplings. These couplings give rise to a Dirac mass matrix of the neutrinos on spontaneous symmetry breaking. In this extended model, the leptonic mass terms are

$$\mathcal{L}_{\rm mass} = -\left(\frac{1}{2}\overline{(N_{iR})^c}(M_R)_{ij}N_{jR} + \frac{v}{\sqrt{2}}\overline{\ell_{Li}}(Y_e)_{ij}\ell_{Rj} + \frac{v}{\sqrt{2}}\overline{\ell_{Li}}(Y_\nu)_{ij}N_{jR} + h.c.\right), \quad (4)$$

where v is the vacuum expectation value of the Higgs. In Eq. (4) i, j run from 1 to 3, ℓ_{Li} represents the $SU(2)_L$ doublets, ℓ_{Ri} and N_{jR} are right handed charged lepton and neutrino fields respectively. The seesaw mechanism leads to a light neutrino mass matrix, $m_\nu = -m_D^T M_R^{-1} m_D$, where $m_D = Y_\nu v/\sqrt{2}$ is the Dirac mass matrix and M_R is the mass matrix of right handed neutrinos. Without loss of generality we consider M_R to be diagonal and in this basis m_D contains the rest of the physical parameters that appear in m_ν.

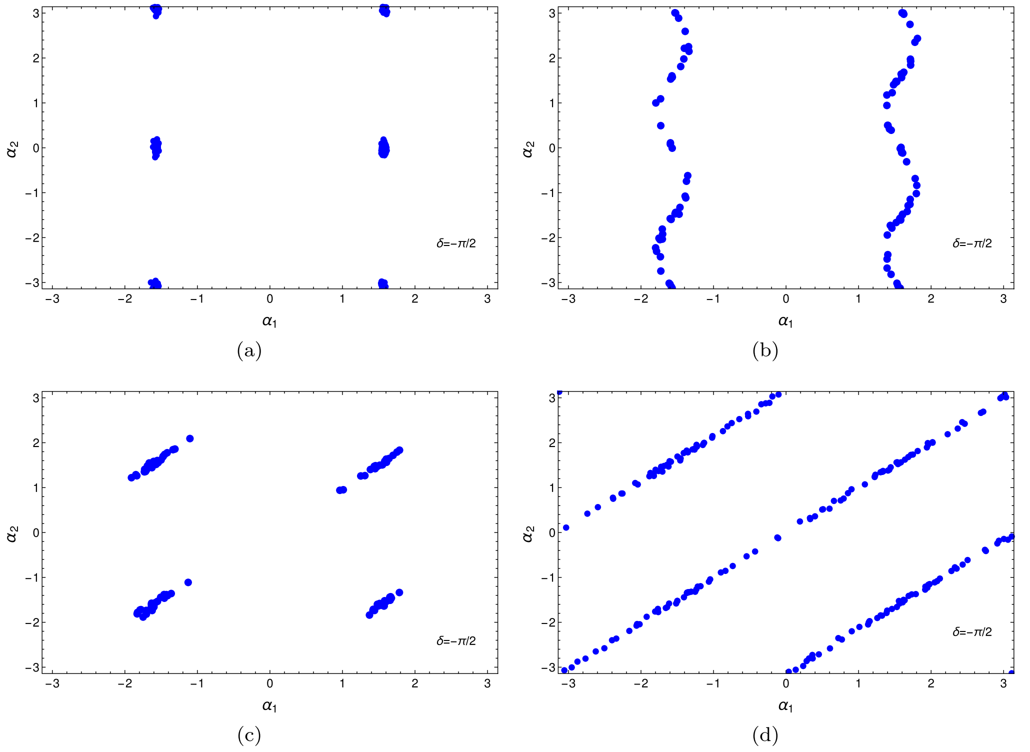


Fig. 2. The allowed values of α_1 and α_2, when $|m_{ee}|$ attains its minimum in the ranges: a). $m_{\rm min} = (10^{-2} - 1)$ eV, b). $m_{\rm min} = (10^{-3} - 10^{-2})$ eV, c). $m_{\rm min} = (10^{-4} - 10^{-3})$ eV, d). $m_{\rm min} = (10^{-6} - 10^{-4})$ eV. The hierarchy is assumed to be NH. Dirac phase δ is fixed at $-\pi/2$.

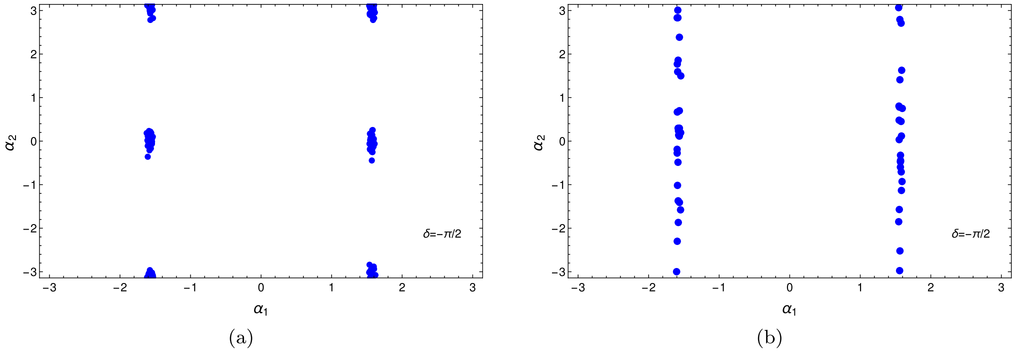
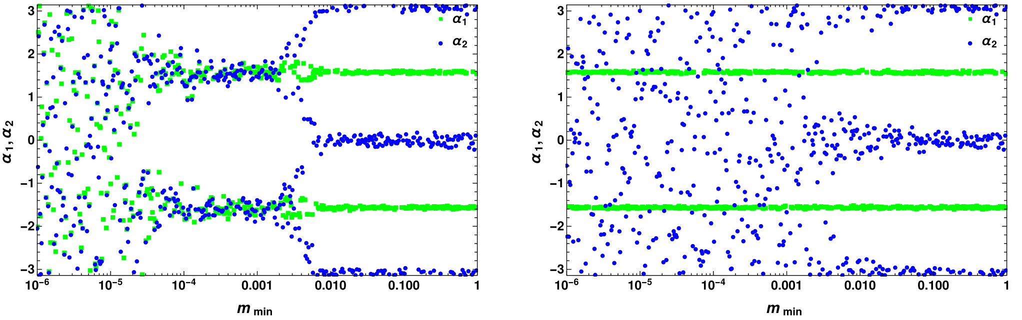


Fig. 3. The allowed values of α_1 and α_2, when $|m_{ee}|$ attains its minimum in the ranges: a). $m_{\rm min} = (10^{-2} - 1)$ eV. b). $m_{\rm min} = (10^{-6} - 10^{-2})$ eV. The hierarchy is assumed to be IH. Dirac phase δ is fixed at $-\pi/2$.

The Majorana mass of the heavy neutrinos N_i $(i = 1, 2, 3)$ can give rise to lepton number violation. Therefore, the CP-violating out-of-equilibrium decay of N_i to ℓH and $\bar{\ell} H^\dagger$ in the early Universe can give rise to a net lepton asymmetry. This lepton asymmetry is then converted to an observed baryon asymmetry via electroweak sphalerons. We assume that the masses of the



(a) (b)

Fig. 4. Values of Majorana phases obtained by minimising $|m_{ee}|$: α_1 (Green) and α_2 (Blue) versus $m_{\rm min}$ for a). Normal hierarchy, b). Inverted hierarchy. Dirac phase δ is fixed at $-\pi/2$.

right handed neutrinos have the pattern $M_1 \ll M_2 \ll M_3$, so that the lepton asymmetry arises purely due to the decay of the lightest right handed neutrino N_1.

The baryon asymmetry in a comoving volume, generated by the decay of N_1, can be given as

$$Y_B = C\epsilon_1^l \kappa \times \frac{n_{N_1}^{eq}(T \to \infty)}{s}\,, \tag{5}$$

where $C \sim \mathcal{O}(1)$ is the sphaleron conversion factor and $\kappa = \Gamma/H(T)$, where Γ is total decay rate of lightest right handed neutrino and $H(T) = (2\pi^2/45)g_* T^3$ is the Hubble expansion parameter. The factor $(n_{N_1}^{eq}/s)(T \to \infty) = 135\zeta(3)/(4\pi^4 g_*)$ with $g_* \approx 100$ being the relativistic degrees of freedom above electroweak phase transition. In a weak wash out regime where $\kappa \approx 1$, one gets $Y_B \sim \epsilon_1^l/g_* \approx 10^{-10}$. This implies that $\epsilon_1^l \sim 10^{-8}$. On the other hand, in a strong wash out regime $\kappa < 1$ and can be obtained by solving the required Boltzmann equations [38]. For a typical value $\kappa \sim 0.01$, one gets $Y_B \sim \kappa\epsilon_1^l/g_* \approx 10^{-10}$. This implies that $\epsilon_1^l \sim 10^{-6}$ which is two orders of magnitude larger than the value of ϵ_1^l in case of weak wash out regime. In the following we will be interested to obtain a lower bound on M_1 at the effective neutrino mass floor (where the Majorana phases α_1 and α_2 are obtained by minimising $|m_{ee}|$) irrespective of weak or strong wash out regime. Therefore, it is sufficient for our purpose to consider the CP-violating parameter $\epsilon_1^l > 10^{-8}$ which can give rise adequate leptogenesis. We note that the use of Boltzmann equations only yields a precise value of κ. However, it does not improve the lower bound on M_1. See for instance [38]. Hence the lower bound obtained on M_1 in case of weak wash out regime by demanding adequate baryogenesis should be valid in case of strong wash out regime too. In order to obtain the lower bound on M_1 in what follows we use Casas-Ibarra parameterization [63] for the Yukawa matrix connecting light and heavy neutrinos.

For a given flavour l, the neutrino yukawa coupling matrix can be written in Casas-Ibarra parameterization [63] as,

$$(Y_\nu)_{i\,l} = \frac{1}{v}\sqrt{M_i}\,R_{ij}\sqrt{m_j}\,U_{l\,j}^*\,. \tag{6}$$

In Eq. (6) m_j, M_i, $(j, i = 1, 2, 3)$ are mass eigenvalues of the light and heavy Majorana neutrinos respectively and U is the PMNS matrix. The matrix R in Eq. (6) is a complex orthogonal matrix. It is parameterized in terms of three complex angles z_i $(i = 1, 2, 3)$ as

$$R=\begin{pmatrix} \cos z_2 \cos z_3 & \cos z_2 \sin z_3 & \sin z_2 \\ -\cos z_3 \sin z_1 \sin z_2 - \cos z_1 \sin z_3 & \cos z_1 \cos z_3 - \sin z_1 \sin z_2 \sin z_3 & \cos z_2 \sin z_1 \\ -\cos z_1 \cos z_3 \sin z_2 + \sin z_1 \sin z_3 & -\cos z_3 \sin z_1 - \cos z_1 \sin z_2 \sin z_3 & \cos z_1 \cos z_2 \end{pmatrix}. \tag{7}$$

The CP-asymmetry generated in a particular flavour l ($l = e, \mu, \tau$), is given by

$$\epsilon_1^l = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{jk} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2}, \tag{8}$$

where m_j and m_k are appropriate light neutrino mass eigen value.

In terms of Casas-Ibarra parameterization, Eq. (8) can be written as [64]

$$\begin{aligned}
\epsilon_1^l = &-\frac{3M_1}{16\pi v^2} \frac{1}{m_1|R_{11}|^2 + m_2|R_{12}|^2 + m_3|R_{13}|^2} \\
&\times \{m_1^2 \,\mathrm{Im}[U_{l1}^* U_{l1} R_{11} R_{11}] + m_1^{1/2} m_2^{3/2} \,\mathrm{Im}[U_{l1}^* U_{l2} R_{11} R_{12}] \\
&+ m_1^{1/2} m_3^{3/2} \,\mathrm{Im}[U_{l1}^* U_{l3} R_{11} R_{13}] + m_2^{1/2} m_1^{3/2} \,\mathrm{Im}[U_{l2}^* U_{l1} R_{12} R_{11}] \\
&+ m_2^2 \,\mathrm{Im}[U_{l2}^* U_{l2} R_{12} R_{12}] + m_2^{1/2} m_3^{3/2} \,\mathrm{Im}[U_{l2}^* U_{l3} R_{12} R_{13}] \\
&+ m_3^{1/2} m_1^{3/2} \,\mathrm{Im}[U_{l3}^* U_{l1} R_{13} R_{11}] + m_3^{1/2} m_2^{3/2} \,\mathrm{Im}[U_{l3}^* U_{l2} R_{13} R_{12}] \\
&+ m_3^2 \,\mathrm{Im}[U_{l3}^* U_{l3} R_{13} R_{13}]\}.
\end{aligned} \tag{9}$$

We consider the following question: Is it possible to get large enough ϵ_1^l, with **no** phases in the matrix R and all the CP violation coming purely through the phases in the PMNS matrix, so that adequate leptogenesis occurs? A related question we consider is: If the phases in the PMNS matrix take values which minimize $|m_{ee}|$, then what effect do such phases have on ϵ_1^l and on the lower bound on M_1? To explore these questions, we set the phases in R to be zero and assume z_i to be real. Hence R becomes a real orthogonal matrix. Only the decays of the lightest right handed neutrino creates CP-asymmetry. Therefore only the elements of the first row of R enter in the expression ϵ_1^l. In our parameterization of R, these elements depend only on $\sin z_2$ and $\sin z_3$, *i.e.*, ϵ_1^l is independent of $\sin z_1$. For simplicity, we assume $\sin z_2 = \sin z_3 = \sin z$. It must be noted that the total CP-asymmetry, $\epsilon_1 = \epsilon_1^e + \epsilon_1^\mu + \epsilon_1^\tau = 0$ when the matrix R is real. Under the assumptions we made, the expression in Eq. (9) simplifies to

$$\begin{aligned}
\epsilon_1^l = &-\frac{3M_1}{16\pi v^2} \frac{1}{m_1|R_{11}|^2 + m_2|R_{12}|^2 + m_3|R_{13}|^2} \\
&\times \{\sqrt{m_1 m_2}\,(m_2 - m_1)\,\mathrm{Im}[U_{l1}^* U_{l2}]\, R_{11} R_{12} \\
&+ \sqrt{m_1 m_3}\,(m_3 - m_1)\,\mathrm{Im}[U_{l1}^* U_{l3}]\, R_{11} R_{13} \\
&+ \sqrt{m_2 m_3}\,(m_3 - m_2)\,\mathrm{Im}[U_{l2}^* U_{l3}]\, R_{12} R_{13}\}.
\end{aligned} \tag{10}$$

This expression contains various low energy parameters, the mass of the lightest right handed neutrino, the Higgs vacuum expectation value and an extra free parameter $\sin z$ which comes from high energy scale. Here we study the dependence of CP-asymmetry on the parameters $m_{\min}$ and $\sin z$ using the latest oscillation data at the floor of $|m_{ee}|$.

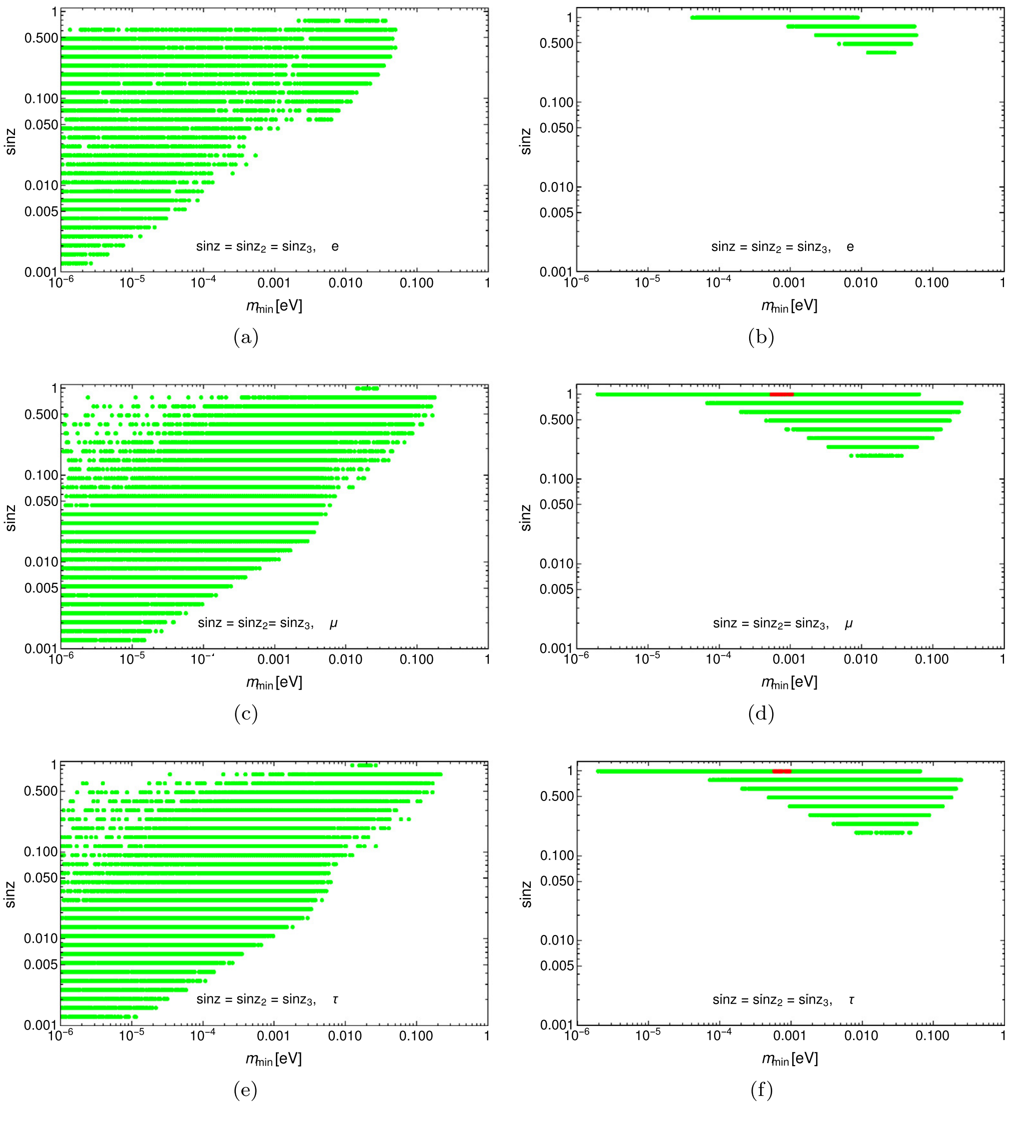


Fig. 5. Allowed points in the plane of $\sin z$ versus m_{min} when $|m_{ee}|$ is minimised, which give rise to $\epsilon_1 > 10^{-8}\ \epsilon^l$. The left column shows the figures for $l = e$(Fig.a), μ(Fig.c), τ(Fig.e), where we assumed the hierarchy to be NH and the right column shows the figures for $l = e$(Fig.b), μ(Fig.d), τ(Fig.f), where we assumed the hierarchy to be IH. The red points correspond to $M_1 = 10^9$ GeV and the green points correspond to $M_1 = 10^{10}$ GeV. In generating these figures, we set $\sin z_2$=$\sin z_3$=$\sin z$ and fixed $\delta = -\pi/2$.

Below, we present the results of our numerical study. As in section 2, we use the best-fit values for the mass-squared differences and the mixing angles. For the purpose of illustration, the value of δ is fixed to be $-\pi/2$ and the values of α_1 and α_2 are chosen to be those which minimize $|m_{ee}|$. We consider only those values of m_{min} and $\sin z$ for which $\epsilon_1 \gtrsim 10^{-8}$. We take into account the Davidson-Ibarra bound [29] by considering the values of $M_1 \geq 10^8$ GeV. Our results are shown in Fig. 5.

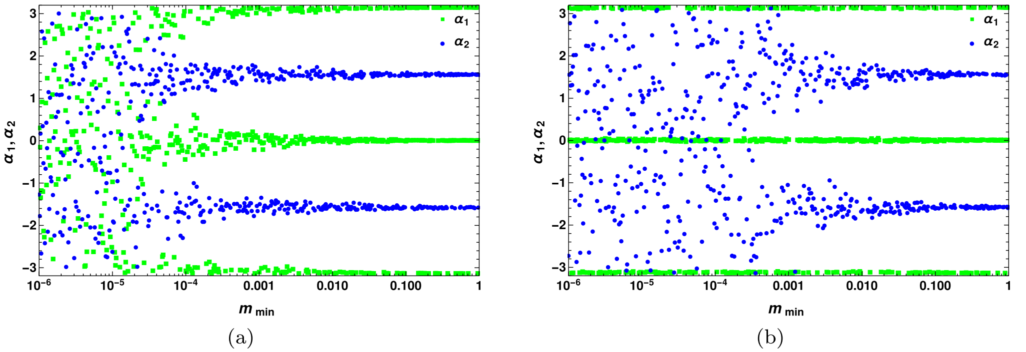


Fig. 6. Values of Majorana phases obtained by maximising $|m_{ee}|$: α_1 (Green) and α_2 (Blue) versus $m_{\min}$: a). Normal hierarchy, b). Inverted hierarchy. Dirac phase δ is fixed at $-\pi/2$.

We considered three values of $M_1 = 10^8, 10^9, 10^{10}$ GeV. When $M_1 = 10^8$ GeV, all values of ϵ_1^l $(l = e, \mu.\tau)$ are less than 10^{-8} independent of $m_{\min}$ and $\sin z$, both for NH and IH. For $M_1 = 10^9$ GeV, the asymmetries ϵ_1^μ, ϵ_1^τ become larger than 10^{-8} only for IH and that too for a single point $\sin z = 1$ and $m_{\min} \simeq 10^{-3}$ eV. For $M_1 = 10^{10}$ GeV, a large set of values of $\sin z$ and $m_{\min}$ are allowed. For NH, all three asymmetries have values in the range 10^{-8} to 10^{-7} for the $m_{\min}$ range $(10^{-6}, 0.1)$ eV and $\sin z$ range $(10^{-3}, 1)$. In the case of IH, $\epsilon_1^l > 10^{-8}$ is possible only if $\sin z > 0.2$ and $m_{\min}$ in the range $(10^{-4}, 0.2)$ eV. Hence adequate leptogenesis requires the lightest heavy neutrino mass M_1 to be $\gtrsim 10^{10}$ GeV when the oscillation parameters are at the floor of $|m_{ee}|$. From Eq. (10) we see that $\epsilon_1^l \to 0$ if the light neutrino masses are almost degenerate. Hence there are no allowed points for $m_{\min} \geq 0.2$ eV for both NH and IH.

Note that the lower bound M_1 obtained here is two orders of magnitude larger than Davidson-Ibarra bound [29]. We believe this occurs due to the following two reasons:

- The restricted choice of Majorana phases α_1 and α_2 obtained from the minimization of $|m_{ee}|$.
- The choice of R is to be real.

One way to lower this bound is to consider R to be complex [29]. However here we consider alternative values of α_1 and α_2 which does not minimise $|m_{ee}|$. As an example we maximise $|m_{ee}|$ and found the values of α_1 and α_2. These values of α_1 and α_2 are plotted in Fig. 6 as function of $m_{\min}$. The CP asymmetry parameter ϵ_1^l is computed using these values of α_1 and α_2 while all the other parameters are varied in the ranges mentioned previously. In Fig. 7 we plot the allowed values of $\sin z$ versus $m_{\min}$ which satisfy the constraint $|\epsilon_1^l| > 10^{-8}$. From this figure, we find the allowed value of $M_1 = 10^9$ GeV for both NH and IH. This lower value is applicable only if $\sin z < 0.25$ and $m_{\min} < 10^{-3}$ eV in the case of NH. On the other hand for IH, the lower value of $M_1 = 10^9$ GeV requires that $\sin z = 1$ and $m_{\min} \simeq 10^{-3}$ eV, which is similar to the situation when $|m_{ee}|$ is minimized.

In presenting our numerical results, we have fixed the value of $\delta = -\pi/2$. From the expressions of $|m_{ee}|$ and ϵ_1^e, it is easy to see that they depend on α_1 and $(\alpha_2 - \delta)$. For a different input value of δ, we can obtain the same values for $|m_{ee}|$ and ϵ_1^e by changing the value of α_2 appropriately. However, the other two CP asymmetries, ϵ_1^μ and ϵ_1^τ, have a more complicated dependence of δ, α_1 and α_2. Hence, it is worth studying how the limits of ϵ_1^l and on M_1 change if a different

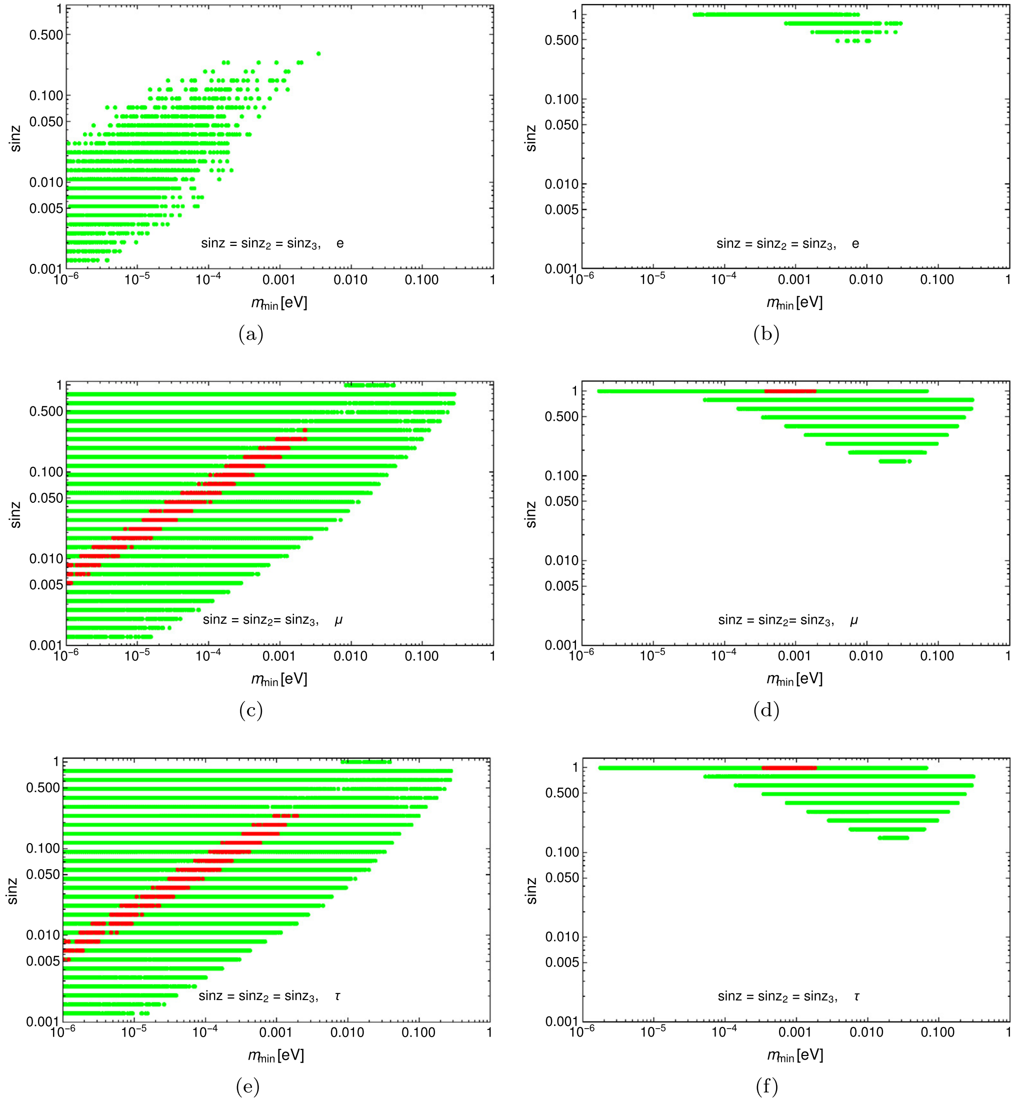


Fig. 7. Allowed points in the plane of $\sin z$ versus $m_{\min}$ when $|m_{ee}|$ is maximised, which give rise to $\epsilon_1 > 10^{-8}\ \epsilon^l$. The left column shows the figures for $l = e$(Fig.a), μ(Fig.c), τ(Fig.e), where we assumed the hierarchy to be NH and the right column shows the figures for $l = e$(Fig.b), μ(Fig.d), τ(Fig.f), where we assumed the hierarchy to be IH. The red points correspond to $M_1 = 10^9$ GeV and the green points correspond to $M_1 = 10^{10}$ GeV. In generating these figures, we set $\sin z_2$=$\sin z_3$=$\sin z$ and fixed $\delta = -\pi/2$.

value of δ is used as an input. In computing the limits on M_1 for different values of δ, we have fixed the values of α_1 and α_2 such that $|m_{ee}|$ is minimized.

We present our results for the input values of $\delta = -0.1\pi$ in Fig. 8, where $\sin z$ is plotted against $m_{\min}$. As in the case of the earlier figures, the red points are the allowed solutions for $M_1 = 10^9$ GeV and the green points are the allowed solutions for $M_1 = 10^{10}$ GeV. We note that the plots in Fig. 8 are quite similar to those in Fig. 5. Very similar features are also seen in Fig. 9 where the input value of $\delta = -0.9\pi$. Thus, we see that the CP asymmetries related

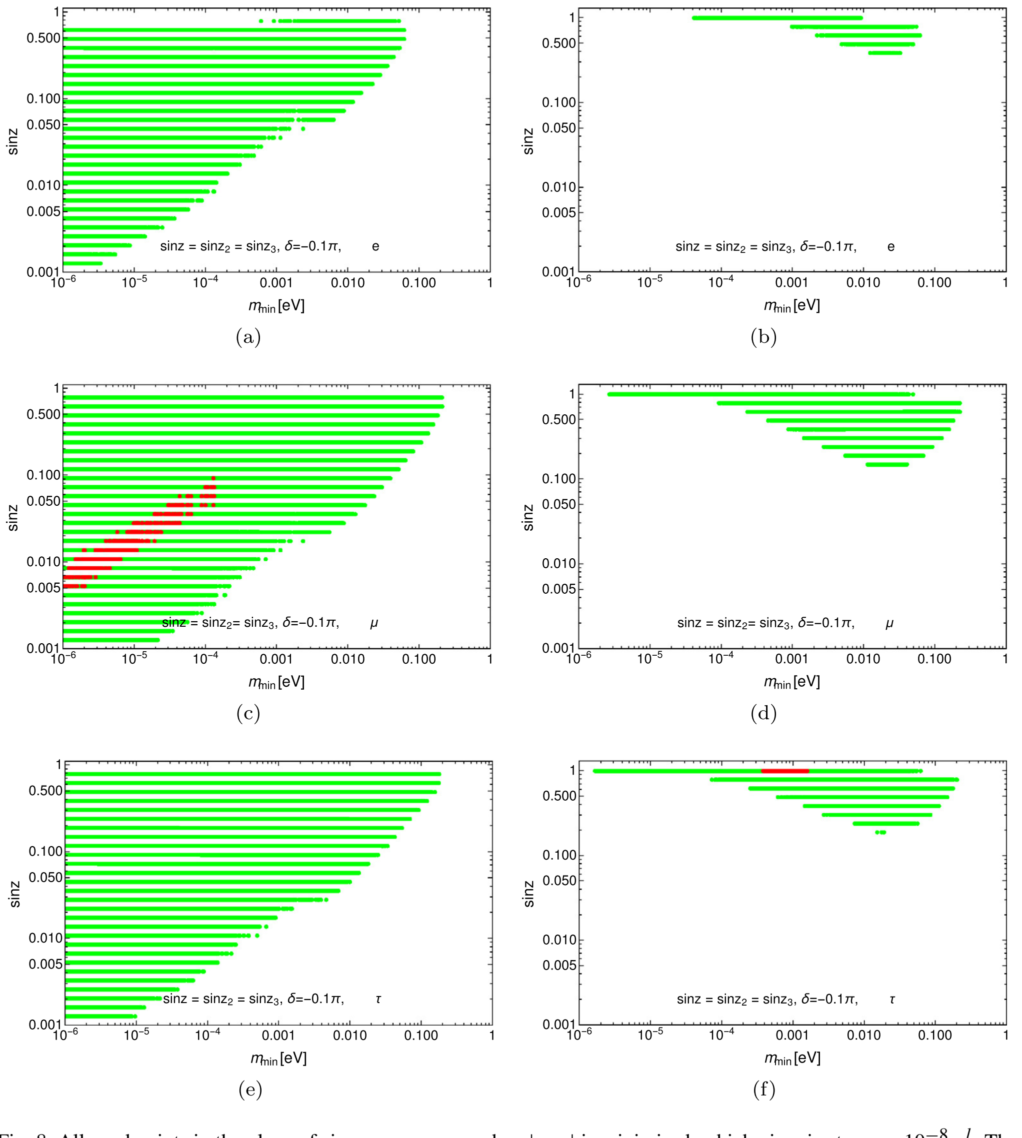


Fig. 8. Allowed points in the plane of $\sin z$ versus $m_{\rm min}$ when $|m_{ee}|$ is minimised, which give rise to $\epsilon_1 > 10^{-8}\ \epsilon^l$. The left column shows the figures for $l = e$(Fig.*a*), μ(Fig.*c*), τ(Fig.*e*), where we assumed the hierarchy to be NH and the right column shows the figures for $l = e$(Fig.*b*), μ(Fig.*d*), τ(Fig.*f*), where we assumed the hierarchy to be IH. The red points correspond to $M_1 = 10^9$ GeV and the green points correspond to $M_1 = 10^{10}$ GeV. In generating these figures, we set $\sin z_2$=$\sin z_3$=$\sin z$ and fixed $\delta = -0.1\pi$.

to leptogenesis are not sensitively dependent on the value of δ. By adjusting the values of the Majorana phases α_1 and α_2, it is possible to get adequate CP asymmetry for $M_1 \gtrsim 10^9$ GeV, for any value of δ.

Here, we also briefly discuss the effect of the approximation $\sin z_2 = \sin z_3 = \sin z$ that was made in generating the numerical results. The CP asymmetries depend on the three elements of the first row of the R matrix, R_{11}, R_{12} and R_{13}, each of which can vary in the range (0,1),

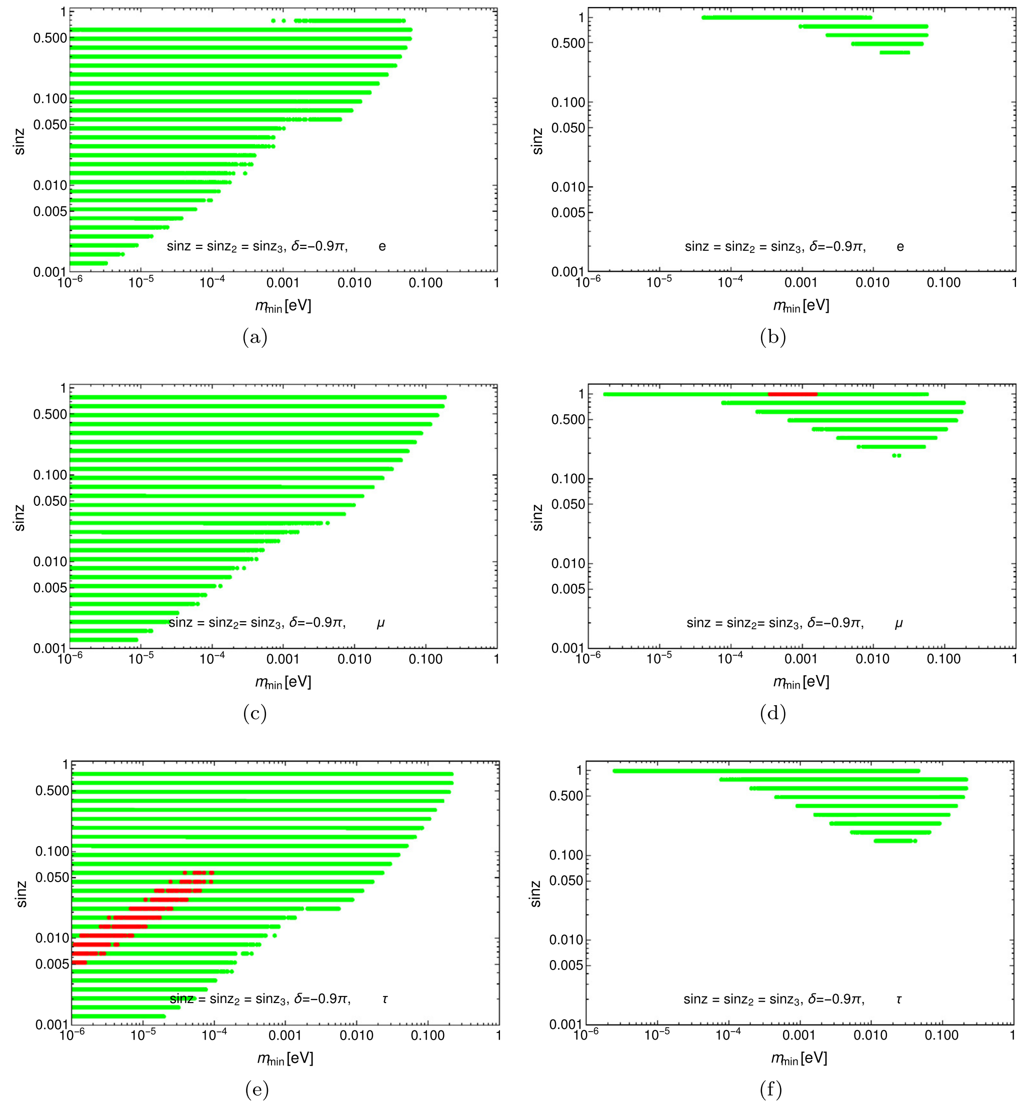


Fig. 9. Allowed points in the plane of $\sin z$ versus $m_{\rm min}$ when $|m_{ee}|$ is minimised, which give rise to $\epsilon_1 > 10^{-8}\ \epsilon^l$. The left column shows the figures for $l = e$(Fig.*a*), μ(Fig.*c*), τ(Fig.*e*), where we assumed the hierarchy to be NH and the right column shows the figures for $l = e$(Fig.*b*), μ(Fig.*d*), τ(Fig.*f*), where we assumed the hierarchy to be IH. The red points correspond to $M_1 = 10^9$ GeV and the green points correspond to $M_1 = 10^{10}$ GeV. In generating these figures, we set $\sin z_2$=$\sin z_3$=$\sin z$ and fixed $\delta = -0.9\pi$.

subject to the orthogonality constraint. With the above approximation, we have $R_{11} = \cos^2 z$, $R_{12} = \cos z \sin z$ and $R_{13} = \sin z$. When $\sin z \lesssim 1$, we see that all the three $R_{1j} \lesssim 1$. In the limit $\sin z \ll 1$, we have $R_{11} \simeq 1$ and $R_{12}, R_{13} \ll 1$. On the other hand, when $\sin z \simeq 1$, we have $R_{11}, R_{12} \ll 1$ and $R_{13} \simeq 1$. Thus, despite making the simplifying assumption of setting two angles to be the same, we are able to explore most of the allowed values of R_{11}, R_{12} and R_{13}. Hence, our results are not dependent on this simplifying approximation.

4. Flavoured CP-asymmetry with two right handed neutrinos

It is possible to generate two independent mass square differences with only two non zero light neutrino masses, *i.e.,* we can set $m_{\min} \equiv 0$. This scenario can be achieved in the limit the heaviest right handed neutrino decouples. We implement this decoupling by setting the third row of the yukawa coupling matrix in Eq. (6) to be zero. Since one of the light neutrino masses is zero, one of the Majorana phases becomes unphysical and can be set equal to zero. Hence, the CP-asymmetries depend only on two phases δ and α_1. We consider the cases of NH and IH separately.

4.1. NH ($m_1 = 0$)

In this case, the effective Majorana mass is

$$|m_{ee}| = m_2 \cos^2\theta_{13} \sin^2\theta_{12} e^{-2i\alpha_1} + m_3 \sin^2\theta_{13} e^{2i\delta}. \tag{11}$$

As before, we need a cancellation between two terms to minimize $|m_{ee}|$, which leads to the condition $\alpha_1 + \delta = (2n+1)\pi/2$. Because of the wide difference in the values of $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ the cancellation is never complete and the minimum of $|m_{ee}|$ in this case is of order 10^{-3} eV, as illustrated in Fig. 1.

The condition that the third row of the matrix Y_ν should consist of zeros leads to the following form of R,

$$R = \begin{pmatrix} 0 & \cos z & \sin z \\ 0 & -\sin z & \cos z \\ 1 & 0 & 0 \end{pmatrix}, \tag{12}$$

where we assume z to be a real angle, as we did in three right handed neutrino case. The expression for the CP-asymmetry is

$$\epsilon_1^l = -\frac{3M_1}{16\pi v^2} \frac{1}{m_2|R_{12}|^2 + m_3|R_{13}|^2} \sqrt{m_2 m_3}\,(m_3 - m_2)\; [\mathrm{Im}(U_{l2}^* U_{l3})]\, R_{12} R_{13}. \tag{13}$$

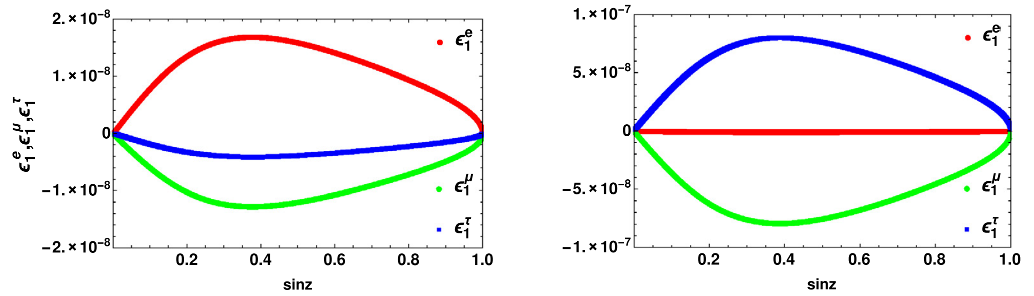
In Fig. 10 (left panel) we have plotted ϵ_1^l as a function of $\sin z$. We inputted $M_1 = 10^{10}$ GeV, $\delta = -\pi/2$ and $\alpha_1 \simeq -\pi$, with the Majorana phase being obtained by minimizing $|m_{ee}|$. For this value of M_1, $|\epsilon_1^l| \sim 10^{-8}$ is possible. Lower values M_1 do not give rise to adequate leptogenesis. We also checked if smaller values of M_1 will be allowed if the Majorana phase is fixed by maximizing $|m_{ee}|$. In Fig. 10 (right panel), we find that a value of $M_1 = 10^{10}$ GeV is required to obtain $\epsilon_1^l \simeq 10^{-8}$ for the values of the phases, $\delta = -\pi/2$ and $\alpha_1 \simeq -\pi/2$. Since $\epsilon_1^l \propto M_1$ for $M_1 < 10^{10}$ GeV we cannot get $\epsilon_1^l > 10^{-8}$.

4.2. IH ($m_3 = 0$)

In this case, the effective Majorana mass is

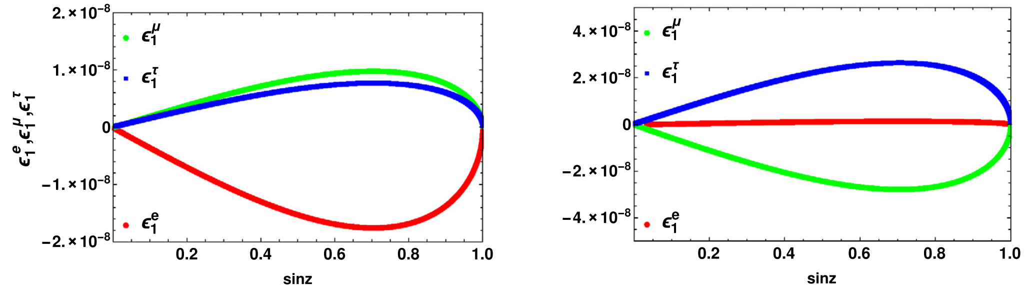
$$|m_{ee}| = m_1 \cos^2\theta_{12} \cos^2\theta_{13} + m_2 \cos^2\theta_{13} \sin^2\theta_{12}\; e^{-2i\alpha_1}. \tag{14}$$

As before, we need a cancellation between two terms to minimize $|m_{ee}|$, which leads to the condition $\alpha_1 = (2n+1)\pi/2$. An exact cancellation is not possible because $m_2 \gtrsim m_1$ and $\cos^2\theta_{12} \simeq 2\sin^2\theta_{12}$. We are led to a lower limit on $|m_{ee}|$ of the order 10^{-2} eV, as illustrated in Fig. 1.



(a) (b)

Fig. 10. The CP-asymmetry corresponding to different flavours of leptons ϵ_1^e, ϵ_1^μ, ϵ_1^τ. The inputs used are $M_1 = 10^{10}$ GeV, $\delta = -\pi/2$ and the hierarchy is NH. (a) The value of the Majorana phase is $\alpha_1 \simeq -\pi$, which minimizes $|m_{ee}|$. (b). The value of the Majorana phase is $\alpha_1 \simeq -\pi/2$, which maximizes $|m_{ee}|$.



(a) (b)

Fig. 11. The CP-asymmetry corresponding to different flavours of leptons ϵ_1^e, ϵ_1^μ, ϵ_1^τ. The inputs used are $\delta = -\pi/2$ and the hierarchy is IH. (a) The value of the Majorana phase is $\alpha_1 \simeq -\pi$, which minimizes $|m_{ee}|$ and $M_1 = 10^{10}$ GeV. (b). The value of the Majorana phase is $\alpha_1 \simeq -\pi/2$, which maximizes $|m_{ee}|$ and $M_1 = 10^{12}$ GeV.

Once again we impose the condition that the third row of the matrix Y_ν should consist of zeros. This leads to the following form of R,

$$R = \begin{pmatrix} \cos z & \sin z & 0 \\ -\sin z & \cos z & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

where we again assume z to be a real angle. The expression for the CP-asymmetry is

$$\epsilon_1^l = -\frac{3M_1}{16\pi v^2} \frac{1}{m_1|R_{11}|^2 + m_2|R_{12}|^2} \sqrt{m_1 m_2}\,(m_2 - m_1)\ [\mathrm{Im}(U_{l1}^* U_{l2})]\, R_{11} R_{12}. \tag{16}$$

As in the case of NH, we input $M_1 = 10^{10}$ GeV and $\delta = -\pi/2$. The value of α_1 is taken to be $\simeq \pi/2$, which is the smallest value that minimizes $|m_{ee}|$. In Fig. 11(left panel), we have plotted ϵ_1^l as a function of $\sin z$. Here again, a value of $M_1 = 10^{10}$ GeV is needed to obtain $|\epsilon_1^l| \sim 10^{-8}$. We also checked the parameter space for $\epsilon_1^l \geq 10^{-8}$ by maximizing $|m_{ee}|$. In Fig. 11(right panel), we need $M_1 = 10^{12}$ GeV to obtain $\epsilon_1^l \simeq 10^{-8}$, for $\delta = -\pi/2$ and $\alpha_1 \simeq -\pi$. Further smaller values of M_1 cannot give rise to adequate leptogenesis. Thus, in the case of $m_{\text{min}} \to 0$, the lower limit on M_1, needed to generate adequate leptogenesis, rises to 10^{10} GeV.

5. Conclusions

In this work, we studied the correlation between the low energy lepton number violating process $0\nu\beta\beta$-decay and the high energy lepton number violating flavour dependent leptogenesis, using oscillation parameters at the effective neutrino mass floor, which is obtained by minimizing $|m_{ee}|$ with respect to $m_{\rm min}$. This is done with the motivation of exploring the question: What effect do Majorana phases have on the CP asymmetries of the leptogenesis, if they take the values that lead to the worst case scenario for $0\nu\beta\beta$-decay? Our calculation is done in a type-I see-saw framework where three heavy right handed neutrinos are added to the SM. We expressed the Yukawa matrix connecting light and heavy neutrinos through Casas-Ibarra parameterization, which involves the light and heavy neutrino masses, the low energy neutrino oscillation parameters involved in PMNS matrix and an unknown orthogonal matrix R, usually accommodating the high energy phases. In general, the orthogonal matrix R is complex and can give rise to additional CP violation. We considered the consequences of setting the high energy phases to be zero and assuming this R matrix to be real. To keep the algebra simple, we parameterized R in terms of a single angle z. We then explored the possibility of obtaining adequate leptogenesis purely from the phases of PMNS matrix when they take the values which can minimise $|m_{ee}|$. Due to the restrictive choice of Majorana phases, we show that the lower bound on M_1 is pushed to a higher side in comparison to the usual Davidson-Ibarra bound. In particular, a right handed neutrino mass of 10^{10} GeV is needed to obtain adequate leptogenesis in case of NH. For IH, $M_1 = 10^9$ GeV is possible provided $m_3 \simeq 10^{-3}$ eV and $\sin z = 1$. However, if we choose other Majorana phases in the PMNS matrix which not necessarily minimise $|m_{ee}|$, then adequate leptogenesis purely through the PMNS matrix is possible for $M_1 = 10^9$ GeV for both NH and IH. We verified it by choosing a set of Majorana phases which are obtained by maximizing $|m_{ee}|$.

We also considered the case where the lightest neutrino mass is zero which corresponds to the case where one of the heavy right handed neutrino decouples. Here again, we explore obtaining the necessary CP asymmetry purely from the phases in the PMNS matrix, by choosing the high scale mixing matrix R to be real. When the Majorana phases are fixed by the minimization of $|m_{ee}|$, the lower bound on M_1 is found to be 10^{10} (10^{11}) GeV for NH (IH). On the other hand, if the Majorana phases are fixed by the maximization of $|m_{ee}|$, the lower limit on M_1 is found to be 10^{10} (10^{12}) GeV for NH (IH). Thus our results show that the lower bound on M_1 in two right handed neutrino models is larger than the case of three right handed neutrinos. The most likely reason for the increased lower limit on M_1 is the presence of only one Majorana phase in the case of two right handed neutrinos, as opposed to two Majorana phases in the case of three right handed neutrinos.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

In this appendix, we discuss the form of the complex rotation matrix R in the Casas-Ibarra parametrization, in the limit one of the light neutrino masses becomes zero and one of the heavy neutrino states decouples. The yukawa matrix $(Y_\nu)_{il}$ in Eq. (6), in general, is a 3×3 matrix. If the heavy neutrino state M_3 decouples, the elements of the 3th row of this matrix vanish. From Eq. (6), we obtain

$$(Y_\nu)_{3l} = \frac{\sqrt{M_3}}{v}\left[R_{31}\sqrt{m_1}(U^\dagger)_{1l} + R_{32}\sqrt{m_2}(U^\dagger)_{2l} + R_{33}\sqrt{m_3}(U^\dagger)_{3l}\right]. \tag{17}$$

Suppose the light neutrino mass m_1 is set to zero, as we should for vanishing $m_{\rm min}$ in the case of NH. Then condition that the LHS of the above equation should vanish gives rise to the constraints $R_{32} = 0 = R_{33}$. Orthogonality of R implies that $R_{31} = 1$ and $R_{11} = 0 = R_{21}$. The four remaining elements of R, R_{12}, R_{13}, R_{22} and R_{23}, form a 2×2 complex orthogonal matrix, defined by one complex angle z. In the case of vanishing $m_{\rm min}$ for IH, we need to set $m_3 = 0$. It is easy to see from Eq. (17) that the decoupling of M_3 leads to the condition $R_{33} = 1$ which makes the third row and the third column of R trivial. The upper 2×2 block of R is a complex orthogonal matrix, once again parametrized by a single complex angle z.

The above argument can be extended to a general case. Suppose we want the heavy eigenstate with mass M_i to decouple and we also want the light mass m_j to be set to zero. The requirement that the ith row of $(Y_\nu)_{il}$ should vanish leads to the condition $R_{ij} = 1$ which means that the ith row and jth column of R are trivial. The remaining four elements of R then form a 2×2 complex orthogonal matrix parametrized by a single complex angle z.

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