Journal of Number Theory 222 (2021) 426–428

Contents lists available at ScienceDirect

Journal of Number Theory

www.elsevier.com/locate/jnt

Corrigendum

# Corrigendum to "On a generalization of a conjecture of Grosswald" [J. Number Theory 216 (2020) 216–241]

Pradipto Banerjee *, Ranjan Bera

*Indian Institute of Technology – Hyderabad, Department of Mathematics, Kandi, Sangareddy, Medak, Telangana 502285, India*

A R T I C L E I N F O

*Article history:*
Received 15 November 2020
Received in revised form 17 November 2020
Accepted 18 November 2020
Available online 23 December 2020
Communicated by S.J. Miller

*MSC:*
11R32
1C08
33C45

*Keywords:*
Bessel Polynomials
Generalized Laguerre polynomials
Irreducibility
Galois group

A B S T R A C T

We extend the result of Lemma 4, [1] to the case that $e = 0$ and $\ell = 1$ which was missing in [1] but used in the proof of Theorem 1, [1].



DOI of original article: https://doi.org/10.1016/j.jnt.2020.02.013.

* Corresponding author.
*E-mail addresses:* pradipto.banerjee7@gmail.com (P. Banerjee), ma16resch11002@iith.ac.in (R. Bera).

https://doi.org/10.1016/j.jnt.2020.11.002

## 1. The correction

On page 15, [1], in *Case (ii)*, it is claimed in the fifth display that for $e \in \{0, 1\}$, the following holds.

$$\frac{1}{p-1} + \max\left\{\frac{1}{p-1}, \frac{1}{p-2\ell}, \frac{2\log(2n+\beta)}{p^s \log p}\right\} > \frac{1}{k}.$$

This was obtained by arguing that $\mu_e(g) \geq 1/k$ and that

$$\mu_e(g) < \frac{1}{p-1} + \max\left\{\frac{1}{p-1}, \frac{1}{p-2\ell}, \frac{2\log(2n+\beta)}{p^s \log p}\right\}. \quad (1)$$

For the latter estimate, we had referred to Lemma 4, [1]. The bound obtained in Lemma 4, [1], is valid only for $e \geq \ell$. While, according to (b), Corollary 4, if $e = 0$, then $\ell = 1$, i.e., $e < \ell$. Thus, in order for our arguments to work in case (ii), we must justify the validity of (1) in Lemma 4, [1], in the case that $e = 0$ and $\ell = 1$. In the present note, we achieve this.

We follow the notations of [1]. In the case under consideration, $e = 0$ and $\ell = 1$. By (b), Corollary 4 [1], this is the case if $\beta \neq -2$. We let $p$ be a prime factor of $n - \ell = n - 1$ satisfying $p > 2\ell + |\beta| = 2 + |\beta|$, as required in Lemma 4, [1]. We are to establish that

$$\mu_0(g) < \frac{1}{p-1} + \max\left\{\frac{1}{p-1}, \frac{1}{p-2}, \frac{2\log(2n+\beta)}{p^s \log p}\right\}. \quad (2)$$

We recall from [1] that

$$\mu_e(g) = \mu_{e,p}(g) = \max\left\{\frac{\nu(b_0) - \nu(b_j)}{j} : e < j \leq n\right\}$$

where $g(x) = \sum_{j=0}^{n} b_j x^j$ and $\nu(b_j)$ is the highest power of $p$ that divides $b_j$. From Lemma 4, [1], we already have that

$$\mu_1(g) < \frac{1}{p-1} + \max\left\{\frac{1}{p-1}, \frac{1}{p-2\ell}, \frac{2\log(2n+\beta)}{p^s \log p}\right\}.$$

Thus, in order to establish (2), it would suffice to show that

$$\nu(b_0) - \nu(b_1) \leq 0.$$

Next, we recall from [1] that

$$b_j = \binom{n}{j} \frac{(2n+\beta-j)!}{(n+\beta)!}.$$

Thus,

$$b_0/b_1 = \frac{(2n+\beta)!}{(n+\beta)!}\frac{(n+\beta)!}{n(2n+\beta-1)!} = \frac{2n+\beta}{n}.$$

Therefore, $\nu(b_0)-\nu(b_1) > 0$ implies that $p|(2n+\beta)$. Also, as per our hypothesis, $p$ divides $n-1$. Thus, $p$ divides $2n+\beta-2n+2 = \beta+2$. Since $p > 2+|\beta|$, it must be that $\beta+2 = 0$. But this is a contradiction since $|\beta| \neq -2$ in this case. Our assertion now follows.

## References

[1] P. Banerjee, R. Bera, On a generalization of a conjecture of Grosswald, J. Number Theory 216 (2020) 216–241.